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PRACTICAL ASTRONOMY,

NAVIGATION, NAUTICAL ASTRONOMY,

AND

METEOROLOGY.

PRACTICAL ASTRONOMY,

NAVIGATION, NAUTICAL ASTRONOMY,

AND

METEOROLOGY;

BEING

A Guide to the Scenery of the Heavens, the Planetary and Stellar Movements,

THE PRACTICE OF NAVIGATION,

AND

PHENOMENA OF THE ATMOSPHERE.

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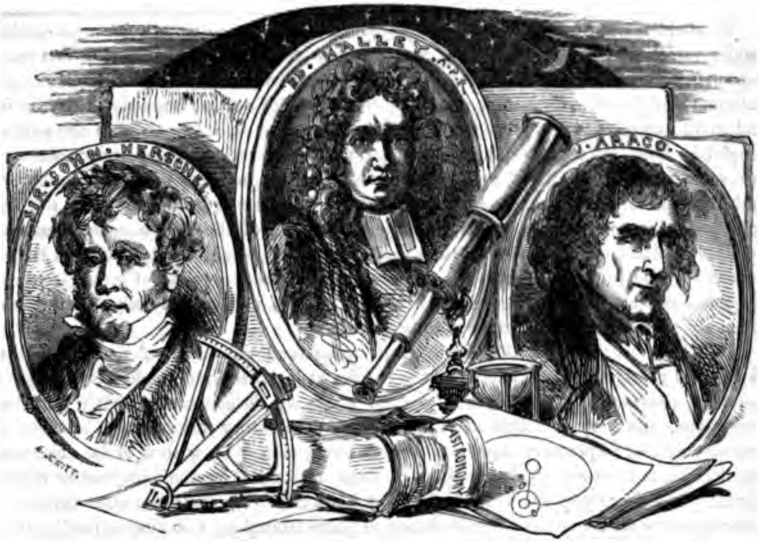

NAVIGATION AND ASTRONOMY.

Introduction.—The principles of Navigation rest almost entirely upon that part of Plane Trigonometry which is limited to the doctrine of plane triangles. A person unacquainted with the mathematical theory upon which the practical rules followed by the navigator are based, would naturally imagine, as the track of a ship is a path marked out on the surface of a *sphere*, that to calculate, from the necessary data, the length of this track, the aid of *Spherical* Trigonometry would be required. But, in general, such is not the case; and, for this reason:—spherical trigonometry is wholly concerned with the arcs of great circles of the sphere, and with the angles formed by such arcs; whereas the course of a ship at sea, unless it sail on the equator, or on a meridian, is never a great circle;—it is either a small circle, a parallel to the equator, or else a line cutting the successive meridians over which it sails, obliquely, and at the same invariable angle, so long as its *course* remains unchanged. A ship, continuing on this unchanged course, would trace out, on the surface of the sphere, a winding or spiral curve, called in navigation a *rhumb line*, and which is widely different in figure from a circle. If a vessel were to start from any point between the equator and either pole, and on a course inclined ever so little towards that pole, it would wind round the globe in this spiral path, approaching nearer and nearer to the pole, but actually arriving at it only after it had circulated round it an infinite number of times.

It is the length of a portion of such a spiral line that it is one of the objects of navigation to calculate; and it is pretty obvious that rules and formulæ, supplied by *spherical* trigonometry, can give no aid in such a calculation; since the latter science has nothing to do with the spiral curves which vessels at sea trace out.

It would seem, however, that these spirals on a *sphere*, are equally external to the proper objects of *plane* trigonometry, which recognizes only *straight* lines, drawn on a flat surface. But it must be observed that navigators are not interested in any investigations respecting the form or shape of the spiral path of a ship; but only in its *length*, and in the angle it makes with the meridians crossed by it. *Lengths*, and the angles formed by them, are of course the proper objects of consideration in plane trigonometry; and some notion may, therefore, be formed as to how it happens that a ship's course, and distance sailed, are matters for investigation by plane trigonometry, and not by spherical trigonometry. In the latter subject, the *form* of the lines concerned cannot possibly be disregarded, any more than their lengths; it would not be spherical trigonometry unless the lines of which it treats were all of them portions of great circles of the same sphere. Plane trigonometry regards lengths and angles only,—and these alone are all that navigators require to be calculated.

It is obvious, therefore, that in order to understand the *science* of navigation, the theory and practice of plane trigonometry—as far, at least, as the triangle is concerned—must be previously acquired; but modern books on Trigonometry, with but few exceptions, are very deficient in this needful preparatory instruction; so that it becomes almost imperative upon a writer on navigation, who wishes to be clear and intelligible, to conduct his reader through some amount of preliminary matter before formally entering upon the special object of his work. We shall therefore introduce the subject by an article on the calculation of plane triangles; referring to the Treatise on Trigonometry, in the volume on the MATHEMATICAL SCIENCES, for the necessary theoretical principles; except in a few cases where it may be thought expedient to use rules of operation not expressly provided for in that work: in such cases the theoretical investigation will precede the practical rule.

But we here apprise the learner that it is not our design to print extensive collections of TABLES. Even the most inexperienced of our readers must be aware that this does not come within the scope of the CIRCLE OF THE SCIENCES. Tables, moreover, require so much care in the composition, and such cautious and repeated revision in order to secure accuracy, that the first edition of a set of tables, of any extent, is seldom regarded with implicit confidence. We shall, therefore, content ourselves, and probably, at the same time, the better satisfy the demands of our readers, by referring to tables which have already stood the test of long experience. There is no doubt, however, that, to many students, tables, exclusively for the purpose of facilitating the calculations of the practical seaman, would be regarded almost as an incumbrance; most persons who read this treatise, will very likely do so for the sole purpose of becoming acquainted with the general principles of a most valuable part of practical mathematics; for the sole purpose of learning in what way a few elementary theories, in pure mathematics, are made available for objects of such commercial and national importance, as the conducting a ship across the ocean from one port to another; the daily registry of its position on the globe; the determination of its direct distance from any preceding position, however irregular its actual course between the two positions may have been; of its distance from the equator at any time, and from the meridian of Greenwich, &c., &c. These things are well worth inquiring into even by the non-professional student of science; as they give a practical value, of the highest kind, to what might otherwise seem to be but barren speculations.

Nothing can more forcibly illustrate the importance of the theoretical principles developed in the mathematical volume above referred to than the application of those principles to Navigation and Nautical Astronomy; for, aided by the Compass and the Log, and by a simple optical instrument—the construction of which those principles have suggested—they instruct the mariner to read aright the intelligence of the stars—those faithful finger-posts of heaven, which direct him unerringly on his course across a vast and dreary expanse, where no human skill can erect a signal to guide, or a beacon to warn; and where even the busy traffic of ages has left no beaten pathway behind it.

The object of the following treatise is to explain in simple language, but in a strictly scientific manner, how all this is brought about; and while it will contain everything that can be reasonably desired by the mathematical student, it will also furnish, to the practical seaman, the reasons on which his rules of calculation are founded, and the principles on which his tables are constructed; and will thus, we hope, form a useful accompaniment to whatever book of rules and tables he may choose to consult in the ordinary exercise of his profession. It is in the highest degree desirable that persons engaged in a calling so responsible should be enabled to judge for themselves of the soundness and accuracy of the directions which guide them, and not deliver themselves blindly up to authoritative rules: the amount of mathematical knowledge necessary to convert their art into a science is really very small; and the satisfaction arising from knowing a thing as an ascertained truth, instead of receiving it upon trust, is alone sufficient to compensate for the time expended on the study of it. Besides, mere printed directions can never suffice for unforeseen emergencies; and in no occupation are these more likely to arise than amidst the hazardous duties of the sailor's life. In such circumstances a little science may often be of more avail than volumes of prescribed directions; and even in the ordinary calculations which form part of the every-day routine on shipboard, time and figures are often both thrown away by the unscientific mariner, under the impression that he is increasing the accuracy of his work, when he is, in reality, only encumbering it with errors. A very meritorious writer, Lieut. Raper, justly remarks, that "very indistinct and erroneous notions prevail among practical persons on the subject of accuracy of computation, and much time is, in consequence, often lost in computing to a degree of precision wholly inconsistent with that of the elements themselves. The mere habit of working invariably to a useless precision, while it can never advance the computor's knowledge of the subject, has the unfavourable tendency of deceiving those who are not aware of the true nature of such questions into the persuasion that a result is always as correct as the computor chooses to make it; and thus leads them to place the same confidence in all observations, provided only they are *worked* to the same degree of accuracy."* These are very judicious observations; and Lieut. Raper is, as far as we know, the only person who has drawn attention, in print, to this customary waste of calculation.

The tables which accompany books on navigation are, in general, computed to an extent of decimals—usually six or seven places—much beyond what the ordinary calculations of navigation require; and the unscientific seaman uses them *all*, when half the number would, in most cases, be amply sufficient; his fault is analogous to that of the ill-taught schoolboy, who, having to multiply together two numbers of five decimals—the final decimal of each number being confessedly inaccurate—is at the trouble of computing ten decimals in his product, and fancies, if there be no error in his operation, that they are all correct, whereas five of them at least are wrong, and, therefore, worse than useless.

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* "The Practice of Navigation and Nautical Astronomy." By H. Raper, R.N., Secretary to the Royal Astronomical Society.

We shall now proceed to those preliminary matters which furnish the suitable introduction to the different sailings—plane sailing, parallel sailing, middle latitude sailing, and Mercator's sailing.

On Logarithms.—Logarithms are a peculiar kind of numbers, invented by Lord Napier, for the purpose of shortening the calculations by common figures, whenever these calculations require the operations of multiplication or division, the raising of powers, or the extracting of roots.

Logarithms are of great importance in the computations of Navigation and Nautical Astronomy; and it was mainly for the purpose of reducing the labour of such computations that Napier was led to construct a *table of logarithms.*

The peculiar character, and the practical use, of these important numbers may be briefly explained as follows, the principles of Algebra, as given in our mathematical volume, being previously understood.

It is shown in the treatise just referred to (see Algebra, page 193) that, a being any *base*, and x and y any exponents,

$$a^x \times a^y = a^{x+y}, \text{ and } a^x \div a^y = a^{x-y},$$

and, moreover, that whatever exponents m and n may be, $(a^m)^n = a^{mn}$.

Now let us suppose that any number N could be written in the form a^x, a being some chosen number fixed upon as a *base*, and x the suitable exponent to justify the equation $a^x = \text{N}$; thus, the base a might be the number 2, or 3, or 4, or any other positive number different from unity, and it is plain that, whether we can find it or not, *some* value exists for x which would satisfy the equation just written, whatever be the positive number N. There is no doubt, for instance, that a numerical value for x exists such that $2^x = 12$: the value of x must evidently lie between 3 and 4. Again, the value of x, which satisfies the condition $2^x = 6$, must evidently lie between 2 and 3. Now imagine two numbers thus represented, one of them N by a^x, and the other N′ by a^y; then, from the principles stated above, we should have, by multiplication and division,

$$a^x \times a^y, \text{ that is, } a^{x+y} = \text{NN}'; \text{ and } a^x - a^y, \text{ that is, } a^{x-y} = \text{N} \div \text{N}',$$

where we see that the *multiplication* of the two numbers N,N′ is replaced by the simple *addition* of the two exponents x and y, in the proposed notation for those numbers, and that the *division* of N by N′ is replaced by the simple *subtraction* of the exponent y from the exponent x. The proper exponent z, to be placed over the base a, so that a^z may replace the number NN′, is therefore $z = x + y$; and the proper exponent, for the number $\frac{\text{N}}{\text{N}'}$, is $z = x - y$. It appears from this, that if a table were constructed in which the numbers 1, 2, 3, &c., up to the highest number likely to occur in calculation, were inserted, and against each number n were placed the proper exponent x—that is, such a value of x that the equation $a^x = n$ should, in each case, be satisfied—the operations of multiplication and division might, by aid of the table, be converted into the simpler operations of addition and subtraction.

Such a table was actually constructed by Napier; it was afterwards improved by Briggs, who found that 10 was the most convenient number to choose for the base a; and the tables constructed to this base, and in which are inserted against each number n the value of x proper to satisfy the equation $10^x = n$, are the modern *tables of logarithms*:—the exponent x, adapted to any proposed number n, being called the *logarithm* of n; so that a table of logarithms is nothing more than a table of exponents—the base-number, to which each exponent is conceived to be attached, being 10.

For example, suppose we want the logarithm of 67:—in other words, that we require the value of the exponent x, fitted to satisfy the condition $10^x = 67$, we turn to the tables, and find the value to be $x = 1{\cdot}826075$.

In like manner, if we want the logarithm of 58, or to solve the equation $10^x = 58$, we find, from the table, that $x = 1{\cdot}763428$. We therefore say that $\log 67 = 1{\cdot}826075$, and $\log 58 = 1{\cdot}763428$; which is only another way of expressing that

$$67 = 10^{1\cdot826075}, \text{ and } 58 = 10^{1\cdot763428}.$$

The product of these numbers is $67 \times 58 = 10^{1\cdot826075 + 1\cdot763428}$; that is, $10^{3\cdot589503}$. The quotient of the first by the second is $67 \div 58 = 10^{1\cdot826075 - 1\cdot763428}$; that is $10^{\cdot062647}$. In other words, $\log (67 \times 58) = 3{\cdot}589503$, and $\log (67 \div 58) = {\cdot}062647$. Turning now to the table, we find, against the logarithm 3·589503, the number 3886, and against the logarithm ·062647, the number 1·155 . . . : we conclude, therefore, without actually performing the operation of multiplication or division, that

$$67 \times 58 = 3886, \text{ and } 67 \div 58 = 1{\cdot}155 \ldots.$$

And in a similar way may the product or quotient of any two numbers, within the limits of the table, be found: for the *product*, we *add* the logs of the factors; for the *quotient*, we *subtract* log of divisor from log of dividend. The result, in each case, is a log, against which, in the table, is the product or quotient sought; and in this way addition and subtraction is made to replace the more lengthy operations of multiplication and division.

It must be noticed, however, that the *integral* part of the logarithm or exponent is not inserted in the table—only the *decimal* part: the insertion of the integral part, or *index*, as it is called, is unnecessary, because this index is at once known from inspection of the number to which the logarithm belongs: thus, since

$$10^0 = 1,\ 10^1 = 10,\ 10^2 = 100,\ 10^3 = 1000,\ 10^4 = 10000, \text{ \&c.},$$

we know that the log of a number between 1 and 10 must lie between 0 and 1; we know, therefore, that the index of a positive number, anywhere between these limits, is 0. We see, also, that the log of a number between 10 and 100 must lie between 1 and 2; the index of such a number must therefore be 1. In like manner, since the log of a number between 100 and 1000 must lie between 2 and 3, we know that the index of such a number must be 2; and so on.

Hence the index of the log of a number consisting of but one integer—however many decimals may follow that integer—is 0; the index of the log of a number consisting of two integers, however many decimals may follow, is 1; of a number consisting of three integers, the index of the log is 2; of a number consisting of four integers, the index is 3; and, generally, of a number consisting of n integers, the index of the log is $n - 1$.

We have, therefore, only to count the figures in the integral part of a number, disregarding the decimals, and then to write down for the index of the log of that number the figure denoting what was counted diminished by 1: to this index the proper decimal part of the log, called the *mantissa*, which is furnished by the table, is to be united, when the complete log will be exhibited. Thus, for the numbers 43, 58, 32·47, 67·813, &c., the index, in each case, is 1; for each of the numbers 246, 835, 647·29, 158·72, &c., the index is 2; and so on.

If, however, the number proposed have no integral figures—that is, if it consist wholly of decimals—then the index of the log will be *negative*. For instance, suppose the number is ·3247, we may write it thus $\frac{3·247}{10}$; and, from the principles already explained, the log of the quotient thus indicated is log 3·247 — log 10; that is, log 3·247 — 1. Consequently, the index of the log of ·3247 is the index of log 3·247 diminished by 1: but as there is only one integer in the number 3·247, the index of its log is 0; therefore the index of log ·3247 is — 1. Suppose the number is ·03247; this may be written $\frac{·3247}{10}$: hence log ·03247 = log ·3247 — log 10; that is, = log ·3247 — 1: but the index of log ·3247, as just shown, is — 1; hence the index of log ·03247 is — 2. Similarly, the index of log ·003247 is — 3, that of log ·0003247 is — 4, and so on. Consequently, whatever be the number whose log is required, we find the index of that log thus:—

Count how many places the *leading figure* of the number is from the unit's place, and put down what is counted for index: the figure thus put down will be *plus* if the counting is towards the right, and *minus* if towards the left; or the rule is expressed otherwise, thus:—"Place your pen between the first and second FIGURE (not CIPHER), and count *one* for each figure or cipher, until you come to the decimal point; the number this gives will be the index. If you count to the right the index is positive, if to the left it is negative." (See MATHEMATICAL SCIENCES, page 285).

The decimal part of the log of a number is always found against that number in the table, which is to be referred to so soon as the index is written down. Because of this ready way of ascertaining what the index of the log of a number is, it is not necessary to insert more than the mantissa, or decimal part of the log, in the table: but it is to be observed that this decimal part is always *plus*. For instance, having first written down the proper index, we find, on referring to the table, that

$$\log ·3247 = -1 + ·511482,\ \log ·03247 = -2 + ·511482,\ \log ·003247 = -3 + ·511482, \&c.$$

But a more compact and convenient way of writing these logs is this, namely:

$$\log. ·3247 = \bar{1}·511482, \log ·03247 = \bar{2}·511482, \log ·003247 = \bar{3}·511482, \&c.$$

And in this form the logarithms of numbers less than unity—that is, of numbers of which all the figures are decimals—are always used in practice.

The logarithmic operation for finding a product from its factors suggests that for finding any power of a known root, or any root of a known power: thus, since n^p merely denotes the product of p factors, each equal to the number n, we have log $n^p = p \log n$; and since if $n^{\frac{1}{p}} = r$, we must have $n = r^p$, it follows that $\log n = p \log r$, and consequently that $\log r = \frac{\log n}{p}$. We thus derive the following practical rules for performing the more troublesome operations of arithmetic in a short and easy manner by help of a table of logarithms.

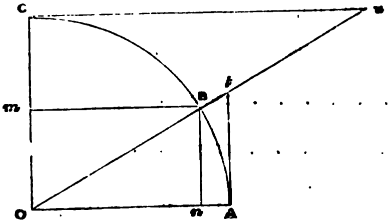
1. *Multiplication.*—From the table take the log of each factor; add these logs together: the *sum* will be the log of the *product* of the factors; and against this log in the tables will be found the product sought.

2. *Division.*—Subtract the log of the divisor from the log of the dividend: the remainder is the log of the *quotient.*

3. *Powers, and Roots.*—Multiply the log of the number to which the exponent is attached, by that exponent, whether it be integral or fractional: the result will be the log of the power or root.

As to the means which algebraists have devised for constructing such a table as that here referred to, the student may consult the section on Logarithms and Series in the volume on THE MATHEMATICAL SCIENCES: it has been thought sufficient, in this introductory article, to show the general principles on which tables of logs are based, without entering into details as to their actual formation.

On the Sines, Cosines, Tangents, &c., of Angles.—In the volume on the Mathematical Sciences (pp. 294-5) the trigonometrical terms, sine, cosine, tangent, &c., are defined in two different ways. For the purposes of navigation, the first of these ways will be found to be the more convenient, as well as the more intelligible to the learner. Taking the diagram at page 294, we may explain the lines there figured as follows:—



Sine.—The sine of an angle AOB, or of the arc AB, which measures that angle, is the straight line B*n* drawn from the extremity B of the arc, at right angles to, and terminating in, the radius OA, drawn to the commencement of the arc.

Cosine.—And the cosine of the same angle is the portion O*n* of the radius between the centre O and the foot *n* of the sine.

Tangent.—The tangent of the same angle is the line A*t*, touching the measuring arc AB at its commencement A, and terminating in the prolonged radius drawn through the extremity B.

Secant.—And this prolonged radius, that is the line O*t*, is the secant of the angle AOB.

As A is regarded as the origin or commencement of the arc which measures the angle AOB, so C is regarded as the commencement of the *complement* of that arc; that is, of the arc which, united to the former, makes up a quadrant or 90°. The sine, cosine, &c., of *this* arc (or of the angle which it measures) are the cosine, sine, &c., of the former, as is obvious; and the tangent and secant of the latter are called the cotangent and cosecant of the former; thus:—

Cotangent.—The cotangent of the angle AOB, or of the arc AB which measures it, is the line C*u*, drawn to touch the complement of that arc at C, and terminating in the secant O*t*, or in that secant prolonged.

Cosecant.—And O*u*, drawn from the centre through B, up to the cotangent, is the cosecant of the angle AOB, or of the arc AB.

The lines here defined refer to circular arcs and the angles which they measure. The sides OA, OB of the angle at O, are each equal to the radius of the circle here considered: if these sides or radii had been of any other length no change would have taken place in the angle at O; neither would any change have taken place in the *number of degrees* contained in the measuring arc; but the actual length of that arc would have varied, being longer than AB in a circle of longer radius, and shorter than AB in a circle of shorter radius. On this account the sine, cosine, &c., of an *arc* are not lines of fixed lengths for a fixed number of degrees: a degree of one circle is double the length of a degree of another of only half the radius of the former. Now, in trigonometry, angles alone, independently of all connection with their measuring arcs, are almost exclusively the subjects of consideration; and it is therefore necessary that an invariable angle should have an invariable sine and cosine, &c. It is agreed, therefore, that the trigonometrical sine, cosine, &c., should be estimated according to the scale $OA = 1$: in other words, that the abstract number 1 being the numerical representative of the radius OA, the abstract numbers, which, conformably to this scale, are the numerical expressions for B*n*, O*n*, &c., should be called the *trigonometrical* sine, cosine, &c., of the angle AOB. The lines in the preceding diagram, considered *as* lines, are the *geometrical* sine, cosine, tangent, &c., of the arc AB, or of the angle AOB *to radius* OA; but viewed as merely linear representations of abstract numbers, OA denoting the abstract unit, they are the trigonometrical sine, cosine, tangent, &c., of the angle AOB.

It is thus that the terms sine, cosine, tangent, &c., as employed in trigonometry, are regarded as *ratios;* for from what has just been said,

$$OA : Bn :: 1 : \frac{Bn}{OA}, \text{ the trigonometrical sine of O.}$$

$$OA : On :: 1 : \frac{On}{OA}, \text{ . . . . cosine . .}$$

$$OA : At :: 1 : \frac{At}{OA}, \text{ . . . . tangent . .}$$

&c. &c.

And generally any geometrical sine, cosine, &c., divided by the radius of the arc with which such sine, cosine, &c., is connected, is the trigonometrical sine, cosine, &c., of the angle measured by that arc: but in trigonometry the terms sine of an angle, cosine of an angle, &c.—without any prefix—are sufficiently explicit, because when geometrical lines are meant the fact is always stated.

The learner will find it an assistance to keep the diagram at page 7 before his mind in investigating the relations among the trigonometrical quantities now considered, regarding the lines defined above as merely the linear representatives of the abstract numbers, sine, cosine, &c., the radius being the representative of the abstract number 1; for he will have the aid of geometry to assist him in establishing the fundamental principles of trigonometry. Thus from Euclid, Prop. 47, Book I., we know that

$$Bn^2 + On^2 = OB^2,\ OA^2 + At^2 = Ot^2,\ OC^2 + Cu^2 = Ou^2,$$

so that we have from the first of these, $\sin^2 O + \cos^2 O = 1$ . . . (1)

from the second, $1 + \tan^2 O = \sec^2 O$ . . . (2)

and from the third, $1 + \cot^2 O = \operatorname{cosec}^2 O$ . . . (3)

Also from the property that the sides about the equal angles of equiangular triangles are proportional (Euc. Prop. 4, B. VI.), we find, upon comparing together the equiangular right-angled triangles BO*n*, *t*OA, as also the equiangular right-angled triangles BO*m*, *u*OC, that

$$\frac{\sin O}{\cos O} = \frac{\tan O}{1}, \text{ that is, } \tan O = \frac{\sin O}{\cos O} \quad . . . (4)$$

$$\frac{\cos O}{\sin O} = \frac{\cot O}{1}, \text{ . . . . } \cot O = \frac{\cos O}{\sin O} \quad . . . (5)$$

$$\frac{\sin O}{1} = \frac{1}{\operatorname{cosec} O}, \text{ . . . } \sin O = \frac{1}{\operatorname{cosec} O} \quad . . . (6)$$

$$\frac{\cos O}{1} = \frac{1}{\sec O}, \text{ . . . . } \cos O = \frac{1}{\sec O} \quad . . . (7)$$

These fundamental relations will suffice for our present purpose. We learn—

From (1), that $\sin O = \sqrt{(1 - \cos^2 O)}$; and $\cos O = \sqrt{(1 - \sin^2 O)}$.
From (2), that $\sec O = \sqrt{(1 + \tan^2 O)}$; and $\tan O = \sqrt{(\sec^2 O - 1)}$.
From (3), that $\text{cosec}\, O = \sqrt{(1 + \cot^2 O)}$; and $\cot O = \sqrt{(\text{cosec}^2 O - 1)}$.
From (4) (5), that $\tan O = \frac{1}{\cot O}$; and $\cot O = \frac{1}{\tan O}$.
And from (6) (7), that $\text{cosec}\, O = \frac{1}{\sin O}$; and $\sec O = \frac{1}{\cos O}$.

For the more general theory of the trigonometrical ratios, the student is referred to the treatise on TRIGONOMETRY.

On the Tables of Sines, Cosines, Tangents, &c.—The numerical values of the trigonometrical lines considered above are computed according to the scale radius = 1, and are arranged in tables called tables of *natural sines*, *cosines*, &c. As tangents, cotangents, secants, and cosecants, may be deduced with but little trouble from sines and cosines, as shown above, and as, moreover, the *natural* sines, cosines, &c., are seldom employed in navigation, the tables of these, which accompany books on that subject, usually give only the sines and cosines. But the tables which furnish the logarithms of these, being indispensable in most of the calculations performed by the mariner, are always given in a complete form, and contain the logarithmic sines, cosines, tangents, cotangents, and cosecants of all angles from 0° up to 90°. In the former tables the numerical values of the sine and cosine of every angle between 0° and 90° is less than 1, because the radius itself is only 1: the logarithms of all these sines and cosines would, therefore, have negative indices (page 5), which is an inconvenience in practice. To remedy this, the logarithmic tables are computed, not to the scale rad. = 1, but to the scale rad. = 10^{10}, which is so large as to preclude the possibility of any logarithm in the tables requiring a negative index: in these tables the index, or integral part of the log, is always inserted before the decimal part, while in the logs of numbers, as already stated, the index is omitted.

Now, in investigating the rules and formulæ of trigonometry, the argument always proceeds on the supposition that the numerical value of the radius is 1; and consequently, in the practical application of those rules and formulæ, it must be borne in remembrance, when logs are used, that the radius is taken 10^{10} times as great,—that is, instead of the log of a natural sine, cosine, &c., we employ in reality the log of 10^{10} times that natural sine, cosine, &c. For instance, if we call any natural sine, cosine, &c., p, the logarithmic tables will give us, not $\log p$, but $\log 10^{10}p$,—that is, $\log 10^{10} + \log p$; or since $\log 10^{10}$ is $10 \log 10 = 10 \times 1$, the tables will give us $10 + \log p$, so that the log of every natural sine, cosine, &c., is increased by 10. In order, therefore, that our practical results may agree with our theoretical formulæ, these superfluous 10's—introduced merely to avoid negative logs—must be suppressed at the close of our operation. If in our work as many of these logarithmic trigonometrical quantities are used subtractively as are used additively, then, of course, as the superfluous 10's destroy one another, no correction of the final result becomes necessary; but if more of these logs are additive than subtractive, as many 10's as mark the excess of additive over subtractive logs must be suppressed in the result; and if the excess is in favour of the subtractive logs, so many 10's must be introduced.

The suppression or the introduction of a 10, in any amount, is so easy a matter that the trouble attending it is not worth consideration, in comparison with the advantages gained by the avoidance of logs with negative indices.

What has now been said being understood, let us suppose that, by aid of the algebraic series and formulæ in our volume on Mathematical Science, the two sets of trigonometrical tables have been constructed,—namely, tables of natural sines, cosines, &c., and tables of logarithmic sines, cosines, &c.; and let us endeavour to see a little into the use and advantages of what is thus supplied, in the calculations of plane triangles.

And first as to the table of natural sines, cosines, &c. By referring to the diagram at page 7, we see that this table supplies, already computed for us, the numerical lengths of the sides of two similar right-angled triangles O Bn, OtA, whatever be the angle O, from O = 0°, up to O = 90°. We may be quite sure, therefore, whatever be the magnitude or shape of any right-angled triangle met with in practice, that *two* triangles, *equiangular to it*, will always be found in the table; the sides of the one, OBn, being computed to the scale OB, the hypotenuse, = 1, and the sides of the other to the same scale, the base OA, = 1. The table thus furnishes us with the perpendicular and base of every possible right-angled triangle whose hypotenuse is 1, and also with the perpendicular and hypotenuse of every possible right-angled triangle whose base is 1.

Now the sides about the equal angles of equiangular triangles are proportional (Euc. Prop. 4, Book VI.), so that if *one* side only of a triangle be known, and *all* the sides of another, equiangular to it, be given, we can compute the unknown sides of the former by simple proportion. It follows, therefore, that if one side only of any right-angled triangle, with which we may be engaged in practice, be measured, we shall be able to calculate each of the other sides, in this manner, provided only we know where to look for the triangle, equiangular to the proposed triangle, in the table.

As already observed, there are always *two* such equiangular triangles in every complete table of natural sines, cosines, &c., so that we have a choice as to which shall be compared with the triangle to be calculated; but we must, of course,—take whichever we may,—be careful to compare base with base, perpendicular with perpendicular, and hypotenuse with hypotenuse. The sides of the two triangles OBn, OtA, computed in the table, do not, however, go by these names: the base of the triangle OBn is called *cosine* of the angle O, the perpendicular is called *sine* of the angle O, and the hypotenuse is called *radius*, or 1. Also the base of the similar triangle OtA is *radius* or 1, the perpendicular is *tangent* of the angle O, and the hypotenuse is *secant* of the angle O: the angle O, expressed in degrees and minutes, is thus a sufficient guide to direct us to that part of the table where the equiangular triangle we are in search of is to be found.

It is always best to fix upon that one of the two tabular triangles in which the unit-length, or radius, corresponds to the given side of the triangle proposed for calculation. Thus, if the given side be the hypotenuse, we should compare our triangle with the tabular triangle OBn, because in this the hypotenuse is 1; we should then have to compare the perpendicular of the proposed triangle with Bn, that is with sin O, and the base with On, that is with cos O. For example, if the angle equivalent to O, and the hypotenuse were given to find the perpendicular and the base, then, referring to the necessary particulars, on the page of the table headed by the degrees and minutes in the given angle, we should be supplied with the first three terms of the proportion.

$$1 : \sin O :: \text{given hyp} : \text{required perp},$$

$$\text{Consequently the required perp} = \frac{\text{hyp} \times \sin O}{1} = \text{hyp} \times \sin O.$$

$$\text{Also, } 1 : \cos O :: \text{given hyp} : \text{required base},$$

$$\text{Consequently the required base} = \frac{\text{hyp} \times \cos O}{1} = \text{hyp} \times \cos O.$$

As the first term of each proportion is 1, all division is thus avoided; and the operation reduced merely to the multiplication together of two factors.

On the other hand, if the base were given to find the perpendicular and hypotenuse, the other tabular triangle, O*t*A, would be the better to compare with the proposed one, because the radius or unit-line here is OA, the *base*, corresponding to the side given, and the proportions would be

$$1 : \tan O :: \text{given base} : \text{required perp},$$

$$\text{and } 1 : \sec O :: \text{given base} : \text{required hyp};$$

$$\therefore \text{required perp} = \text{base} \times \tan O, \text{ and required hyp} = \text{base} \times \sec O.$$

As to the tables of logarithmic sines, cosines, &c., little need be said in addition to what has already been stated; they furnish the logarithms of the tabular numbers in the table of natural sines, cosines, &c., with the addition of 10 to each logarithm, as before explained. By using the logarithmic tables, we convert the multiplication of the two factors adverted to above, into the addition of the corresponding logarithms. Sufficient practical details of these operations will be given in the next article.

It may be noticed here, however, that in what is said above, as to the assistance afforded by trigonometrical tables, our observations have had exclusive reference to *right-angled* triangles; we shall shortly see that they are equally available for oblique-angled triangles; but it will strike the learner as a remarkable fact that, notwithstanding the complicated character of the curve, traced on the surface of the globe by the course of a ship, all the calculations in reference to the different sailings, involve only *right-angled triangles*—the simplest kind of triangles with which trigonometry has to deal; we shall therefore give a distinct article on the calculation of the sides and angles of right-angled triangles.

To Calculate the Sides and Angles of Right-angled Triangles.—The sides and angles of any triangle make up what are called the six *parts* of the triangle; and if any three of these six parts, provided they are not the three angles, be given, the remaining three can be found by calculation. In a *right-angled* triangle one of the six parts is always known—namely, the right angle, so that a side and one of the acute angles, or two sides, being measured, we can always calculate the remaining parts.

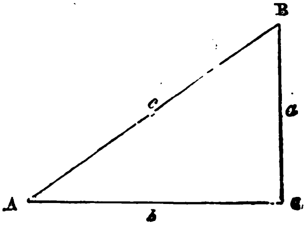
The reason that the three angles alone will not suffice for the determination of the other three parts—the three sides—is obvious from the simplest principles of geometry; for all *equiangular* triangles are alike, as respects the equality of the angles, so that, from the angles alone, we could not conclude to which one, out of an infinite variety of similar triangles, the proposed angles are considered to belong, for they belong in reality equally to all.

In a right-angled triangle the given parts must therefore be either:—

1. A side and one of the acute angles, or
2. Two of the sides, to find the remaining parts.

I. *When the Hypotenuse and one of the Acute Angles are given.*

Let the hypotenuse AB, and the angle A be given; then turning to the diagram at page 7 we should compare the given side AB, with the radius OB, as directed above; so that OB*n* would be the tabular triangle equiangular to the triangle ABC. And, therefore, if *c* denote the numerical length of AB, since the numerical length of OB is 1, AB = OB × *c*; consequently, *a* and *b* being the numerical lengths of BC, and AC, we have, since the like sides are proportional,



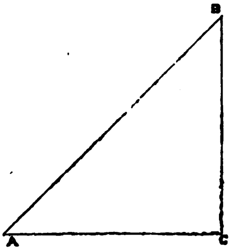
$$BC = Bn \times c, \text{ and } AC = On \times c,$$

that is,

$$a = c \sin A, \text{ and } b = c \cos A.$$

Hence the following rule:—

Rule.—Multiply the given hypotenuse by the sine of the angle at the base; the product is the perpendicular. Multiply the given hypotenuse by the cosine of the angle at the base: the product is the base.



Or work by the following formula,

perp = hyp × sin ang at base; base = hyp × cos ang at base,

which by logarithms will be,

log perp = log hyp + log sin ang at base − 10;
log base = log hyp + log cos ang at base − 10.

Example 1.—In the right-angled triangle ABC are given AB = 480 feet, and the angle A = 53° 5′; required the perpendicular BC, and the base AC.

1. To find the perpendicular BC.

Without logarithms.	With logarithms.
BC = AB sin A.	Log BC = log AB + log sin A — 10.
	— 10
Sin A, 53° 5′, = ·7995	Log AB = log 480 = 2·6812
AB = 480	Log sin A, 53° 5′, = 9·9028
639600	
31980	Log BC = log 383·8 = 2·5840
∴ BC = 383·7600	

Hence BC is 383·8 feet.

2. To find the base AC.

AC = AB cos A.	Log AC = log AB + log cos A — 10.
Cos A, 53° 5′, = ·6006	— 10
AB = 480	Log AB = log 480 = 2·6812
	Log cos A, 53° 5′, = 9·7786
480480	
24024	Log AC = log 288·3 = 2·4598
∴ AC = 288·2880	

Hence AC is 288·3 feet.

In solving the preceding example we have taken only *four* places of decimals from the tables; this is a number quite sufficient for all the ordinary purposes of navigation, and it will be remembered that the present introductory article is only preparatory to the discussion of that subject.

If, instead of the angle A at the base, the angle B at the vertex had been given, the operation would have been much the same as that above; for, since $A = 90° - B$, $\sin A = \cos B$; and $\cos A = \sin B$, because the sine and cosine of an angle are respectively the same as the cosine and sine of the *complement* of that angle; so that where we have used above sin A, we should have used its equal cos B; and where we have used cos A we should have substituted its equal sin B. This must be observed in solving the last three of the following examples.

2. In the triangle ABC (last figure) are given
   $AB = 291$, angle $A = 47° 55'$, to find AC, BC.
   Ans. $AC = 195$, $BC = 216$.
3. Given $AB = 480$, angle $A = 53° 8'$, to find AC, BC.
   Ans. $AC = 288$, $BC = 384$.
4. Given $AB = 521$, angle $B = 36° 6'$, to find AC, BC.
   Ans. $AC = 307$, $BC = 421$.
5. Given $AB = 645$, angle $B = 50° 50'$, to find AC, BC.
   Ans. $AC = 500$, $BC = 407{\cdot}4$.
6. Given $AB = 98$, angle $B = 33° 12'$, to find AC, BC.
   Ans. $AC = 53{\cdot}66$, $BC = 82{\cdot}01$.

II.—*When the base or perpendicular, and one of the acute angles are given.*

Let the base AC, and the angle A be given: then, referring to the diagram at page 7, we are to compare the given side AC with the radius, or unit-line, OA; so that OtA is the tabular triangle now to be referred to. As AC is b times OA, therefore, because the proposed and tabular triangles are equiangular, BC must be b times At, and AB b times Ot; that is,

$$a = b \tan A, \text{ and } c = b \sec A.$$

Hence the following rule:—

Rule.—Multiply the given base by the tangent of the angle at the base: the product will be the perpendicular. Multiply the given base by the secant of the angle at the base: the product will be the hypotenuse.

In a right-angled triangle either of the two perpendicular sides may be regarded as the base, and the other as the perpendicular; so that two distinct rules would be unnecessary. And as in the former case, when the vertical angle B is given instead of the base angle A, the former may take the place of the latter in the operation, provided only that we write cotan for tan, and cosec for sec.

But, as already observed at page 9, the tables of *natural* sines and cosines, which accompany books on navigation, do not, in general, furnish the natural tangents and secants; so that, if this table be employed in the work, we must make use of the relations established at page 8, namely,

$$\tan A = \frac{\sin A}{\cos A}, \sec A = \frac{1}{\cos A}; \text{ cotan } B = \frac{\cos B}{\sin B}, \text{ cosec } B = \frac{1}{\sin B}.$$

The working formulæ for the present case are as follows:

perp = base × tan ang at base; hyp = base × sec ang at base.

Or, by logarithms,

log perp = log base + log tan ang at base − 10;

log hyp = log base + log sec ang at base − 10.

The first of these formulæ, if the table referred to be limited to natural sines and cosines only, must be replaced by

$$\text{perp.} = \frac{\text{base} \times \text{sin ang at base}}{\text{cos ang at base}}; \ \text{hyp.} = \frac{\text{base}}{\text{cos ang at base}},$$

agreeably to the relations given above.

EXAMPLES.

1. In the right-angled triangle ABC, at page 12, are given the base AC = 327, and the angle A = 54° 17′: required the perpendicular BC, and the hypotenuse AB.

1. *To find the perpendicular* BC.

Without logarithms.

$$BC = \frac{AC \sin A}{\cos A}$$

$$\begin{array}{rr} \sin A,\ 54^\circ\ 17',\ = & \cdot 8119 \\ AC = & 327 \\ \hline & 56833 \\ & 16238 \\ & 24357 \\ \hline \cos 54^\circ 17' = \cdot 5838) & 265\cdot 4913(454\cdot 8 = BC \end{array}$$

(See the remarks at p.18).

With logarithms.

log BC = log AC + log tan A − 10.

$$\begin{array}{lr} & -10 \\ AC = 327 \ \ldots\ldots & 2\cdot 5145 \\ \tan A,\ 54^\circ\ 17' \ \ldots\ldots & 10\cdot 1433 \\ \hline \log BC,\ 454\cdot 8 \ \ldots\ldots & 2\cdot 6578 \\ \hline \end{array}$$

Hence BC = 454·8

2. *To find the hypotenuse* AB.

$$AB = \frac{AC}{\cos A}$$

$$\begin{array}{rl} \cdot 58,3,8) & 327\ (560 = AB \\ & 2919 \\ \hline & 351 \\ & 350 \\ \hline & 1 \end{array}$$

log AB = log AC + log sec A − 10.

$$\begin{array}{lr} & -10 \\ AC = 327 \ \ldots\ldots & 2\cdot 5145 \\ \sec A,\ 54^\circ\ 17' \ \ldots\ldots & 10\cdot 2338 \\ \hline AB,\ 560 \ \ldots\ldots & 2\cdot 7483 \\ \hline \end{array}$$

The logarithmic operation might have been performed as readily by using the formula for AB, on the left, which gives

log AB = log AC − log cos A + 10.

The work by this formula will be as follows:

$$\begin{array}{lr} & 10 \\ AC = 327 \ \ldots\ldots & 2\cdot 5145 \\ \cos A,\ 54^\circ\ 17' \ \ldots\ldots & -9\cdot 7662 \\ \hline AB = 560 \ \ldots\ldots & 2\cdot 7483 \\ \hline \end{array}$$

2. *In the triangle* ABC, *page* 12, *are given* AC = 195, *and the angle* A = 470° 55′, *to find BC and AB.* **Ans.** BC = 216, AB = 291.

3. Given AC $= 288$, and the angle A $= 53^\circ\ 8'$, to find BC and AB.
Ans. BC $= 384$, AB $= 480$.

4. Given AC $= 421$, and the angle A $= 36^\circ\ 6'$, to find BC and AB.
Ans. BC $= 307$, AB $= 521$.

5. Given AC $= 625$, and the angle B $= 41^\circ\ 15'$, to find BC and AB.
Ans. BC $= 713$, AB $= 948$.

6. Given the base of a right-angled triangle $346\frac{1}{4}$, and the opposite angle $54^\circ\ 36'$: required the perpendicular and hypotenuse.
Ans. Perp. 246·2, Hyp. 425·1.

III. *When the hypotenuse and one of the other sides are given.*

Let the hypotenuse AB, and the perpendicular BC be given; then comparing AB with the radius or unit line OB in the diagram at page 7, the tabular triangle OB*n* will be that to which the proposed triangle is to be compared: and since AB is *c* times OB, therefore BC is *c* times B*n*; that is to say,

$$a = \text{BC} = c \sin \text{A}, \text{ therefore } \sin \text{A} = \frac{a}{c}.$$

If the base were given instead of the perpendicular, then we should have

$$b = \text{AC} = c \cos \text{A}, \text{ therefore } \cos \text{A} = \frac{b}{c}.$$

The rule is therefore as follows:—

Rule.—Divide the perpendicular by the hypotenuse: the quotient will be the sine of the angle at the base. Divide the base by the hypotenuse; the quotient will be the cosine of the angle at the base.

An angle being thus determined, the remaining side of the triangle may be found by either of the preceding rules. Or, without first finding an angle, the third side of any right-angled triangle may be determined from the other two, by the 47th of Book I. of Euclid; for since $c^2 = a^2 + b^2$; $\therefore a^2 = c^2 - b^2$, and $b^2 = c^2 - a^2$. The working formulæ for finding the angles, when the hypotenuse and a side are given, are

$$\sin \text{ang at base} = \frac{\text{perp}}{\text{hyp}}; \quad \cos \text{ang at base} = \frac{\text{base}}{\text{hyp}}.$$

Or, by logarithms,

$$\log \sin \text{ang at base} = \log \text{perp} - \log \text{hyp} + 10;$$
$$\log \cos \text{ang at base} = \log \text{base} - \log \text{hyp} + 10.$$

Examples.

1. In the right-angled triangle ABC, page 12, are given the hypotenuse AB $= 480$, and the perpendicular BC $= 384$, to find the angles A, B, and the base AC

1. *To find the angles* A, B.

Without logarithms.

$$\sin \text{A} = \frac{\text{BC}}{\text{AB}}.$$

480)384(·8 = sin 53° 8'
384

With logarithms.

$$\log \sin \text{A} = \log \text{BC} - \log \text{AB} + 10$$

	10
BC, 384	2·5843
AB, 480	−2·6812
sin A, 53° 8' . . .	9·9031

Consequently $A = 53^\circ\ 8'$, $\therefore B = 90^\circ - 53^\circ\ 8' = 36^\circ\ 52'$.

2. *To find the base* AC.

$AC = AB \cos A$

$\cos A$, 53° 8′, $= \cdot 6$

$\therefore 480 \times \cdot 6 = 288 = AC$

AC is thus determined with very little trouble.

$\log AC = \log AB + \log \cos A - 10$

$- 10$

AB, 480 . . . . 2·6812

cos A, 53° 8′ . . . . 9·7781

AC, 288 . . . . 2·4593

To find the base without the aid of the angle, we have

$$AC^2 = AB^2 - BC^2 = (AB + BC)(AB - BC) = 864 \times 96 = 82944$$

$\dot{8}29\dot{4}\dot{4}$ (288 ; or by logs,

4

48)429

384

568)4544

4544

log 864 = 2·9365

log 96 = 1·9823

2)4·9188

log 288 = 2·4594

Consequently AC = 288. The sum of the two logs on the right is divided by 2, because the half of the log of a number is the log of the square root of that number (see page 6).

2. Given the hypotenuse AB = 237, and the perpendicular BC = 197, to find the angles and the base. Ans. A = 56° 13′, B = 33° 47′, Base = 132.

3. Given the hypotenuse AB = 645, and the perpendicular BC = 407·4, to find the angles and the base. Ans. A = 39° 10′, B = 50° 50′, Base = 500.

3. Given the hypotenuse AB = 400, and the base AC = 236, to find the angles and the perpendicular. Ans. A = 53° 51′, B = 36° 9′, Perp = 323.

4. Given the hypotenuse AB = 54·68, and the base AC = 35·5, to find the angles and the perpendicular. Ans. A = 49° 31′, B = 40° 29′, Perp = 41·6.

5. Given the hypotenuse of a right-angled triangle = 779·8, and the perpendicular = 725, to find the vertical angle and the base.

Ans. 21° 36′, and 287·1.

6. Given the hypotenuse = 780·6, and the base = 607·6, to find the angle at the base, and the perpendicular. Ans. 38° 53′, and 490.

IV. *When the base and perpendicular are given.*

The base AC = b, and the perpendicular BC = a being given, we are to compare the triangle with the tabular triangle OAt, of which the base OA = 1 : we thus have BC = b times At, that is

$$a = b \tan A \;\therefore\; \tan A = \frac{a}{b}.$$

Consequently the rule for finding the angle A at the base is as follows :—

Rule.—Divide the perpendicular by the base : the quotient will be the tangent of the angle at the base. Or, divide the base by the perpendicular : the quotient will be *the tangent of the* angle at the vertex. The hypotenuse may be found without first finding the angles by Euclid 47th of Book I.: thus $c^2 = a^2 + b^2$. The formulæ for finding either angle is therefore

$$\tan \text{ ang at base} = \frac{\text{perp}}{\text{base}}; \quad \tan \text{ ang at vertex} = \frac{\text{base}}{\text{perp}}$$

Or, by logarithms,

$$\log \tan \text{ ang at base} = \log \text{ perp} - \log \text{ base} + 10;$$
$$\log \tan \text{ ang at vertex} = \log \text{ base} - \log \text{ perp} + 10.$$

EXAMPLES.

1. In the right-angled triangle ABC, page 12, are given the base AC = 288, and the perpendicular BC = 384, to find the angles and the hypotenuse.

1. *To find the angles* A, B.

Without logarithms.	With logarithms.
$\tan A = \frac{BC}{AC}$	log tan A = log BC — log AC + 10.
288)384(1·3333 . . . = tan A, 53° 8′.	10
288	BC, 384 . . . . . 2·5843
96	AC, 288 . . . . −2·4594
864	tan A, 53° 8′ . . . . 10·1249
96	

∴ A = 53° 8′, and B = 90° − A = 36° 52′.

2. *To find the hypotenuse* AB.

$AB = \sqrt{(AC^2 + BC^2)}$	AB = AC sec A.
$288^2 = 82944$	∴ log AB = log AC + log sec A — 10.
$384^2 = 147456$	−10
230400(480 = AB	AC, 288 . . . . . 2·4594
16	sec A, 53° 8′ . . . . 10·2219
88) 704	AB, 480 . . . . . 2·6813
704	
00	

In the foregoing example a table of natural tangents and secants is necessary to enable us to find the required parts without logarithms, and in the shortest manner; tan A being determined as above, and then AB from the formula AC sec A.

In all works on Navigation, as also in most books on Trigonometry, the calculation of the several cases of right-angled triangles is performed exclusively by logarithms: the learner will perceive, from the illustrations furnished above, that, in general, the work is more expeditious without logarithms than with them; and that it would be of advantage to the practical navigator if tables more comprehensive than those generally given—that is, tables including the natural tangents and secants—were bound up with books on navigation. As the preceding examples show, a single reference to such a table suffices for the discovery of the unknown part; whereas *three* references to the logarithmic tables are necessary. In the latter mode of working, fewer figures may appear on the paper, but the time occupied in two searchings in the table, out of three, is saved; and, in operations of this kind, referring to tables, and transcribing the figures, constitute the *principal part of* the trouble.

The greatest amount of arithmetical work involved in any of the preceding solutions occurs in the example at page 14; but the multiplication and division operations there indicated, according to the common beaten track, may be pruned of many superfluous figures by using the *contracted* methods: thus, to multiply ·8119 by 327, in the shortest way, without sacrificing accuracy in the result, write the figures of the multiplier in reverse order thus, 723; and multiply as in the margin, rejecting from the multiplicand a figure at every step, after the first, but attending to the *carryings* from the rejected figures. And similarly for contracted division, as exemplified at the page referred to, and again here in the margin. It will be observed, however, that the whole of the division part of this work would have been saved if tan 54° 17′ had been supplied by the tables: the entire work would then have stood as here annexed.

$$
\begin{array}{r}
\sin 54^\circ\,17' = \cdotp 8119 \\
723 \\
\hline
24357 \\
1624 \\
568 \\
\hline
\cos 54^\circ\,17' = \cdotp 5838)265\cdotp 49(454\cdotp 8 \\
23352 \\
\hline
3197 \\
2919 \\
\hline
278 \\
233 \\
\hline
45 \\
46 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
\tan 54^\circ\,17^\circ = 1\cdotp 3908 \\
723 \\
\hline
41724 \\
2782 \\
974 \\
\hline
BC = 454\cdotp 80 \\
\hline
\end{array}
$$

A person but moderately expert in the simple operations of arithmetic could execute the work in the margin in less time than it would take to refer to the logarithmic tables, and to transcribe therefrom a single number. And then the numerical process, being placed permanently before the eye, is very easily revised if error be suspected. These obvious advantages are of sufficient importance to justify the entire abandonment of logarithms in those calculations of Navigation—and they include nearly all—in which right-angled triangles only are concerned. When oblique-angled triangles are the subjects of computation, the case is different; as the logarithmic operation has then, with few exceptions, the advantage over that with the natural numbers, as will be sufficiently seen in the next article.

2. In the right-angled triangle ABC, are given the base AC = 101·9, and the perpendicular BC = 195·4, to find the angles and the hypotenuse.
Ans. A = 62° 27′, B = 27° 33′, AB = 220·3.

3. Given the base AC = 659·8, and the perpendicular BC = 520·5, to find the other parts. Ans. A = 38° 16′, B = 51° 44′, AB = 840·4.

4. Given the base AC = 35·5, and the perpendicular BC = 41·6, to find the other parts. Ans. A = 49° 31′, B = 40° 29′, AB = 54·68.

5. Given the base AC = 32·76, and the perpendicular BC = 46·58, to find the vertical angle B. Ans. B = 35° 7′.

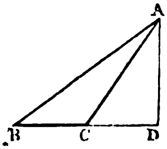
6. Given the base AC = 53·66, and the perpendicular BC = 82·01, to find the angles and the hypotenuse. Ans. A = 56° 48′, B = 33° 12′, AC = 98.

Miscellaneous Examples in the Calculation of Right-angled Triangles.

1. *The angle of elevation* ACD of the top A, of a tower, was found to be 55° 54′;

and from the station B, 100 feet from C in the same straight line DB, the angle of elevation ABD was found to be 33° 20′: what was the height of the tower?

Solution without Logarithms.



By right-angled triangles,

$$DC = AD \tan CAD,\ DB = AD \tan BAD,$$

$$\therefore DB - DC = BC = AD (\tan BAD - \tan CAD).$$

But $BAD = 90° - 33° 20' = 56° 40'$, and $CAD = 90° - 55° 54' = 34° 6'$,

$$\therefore BC = 100 = AD (\tan 56° 40' - \tan 34° 6').$$

By referring to a table of natural tangents, we find

$$\text{Tan } 56° 40' = 1{\cdot}5204$$
$$\text{Tan } 34° \ 6' = {\cdot}6771$$

$$\begin{array}{rl} \text{The difference} = 8{,}4{,}3{,}3)100 & (118{\cdot}6 = AD. \\ 8433 & \\ \hline 1567 & \\ 843 & \\ \hline 724 & \\ 675 & \\ \hline 49 & \\ 51 & \\ \hline \end{array}$$

The division here is by the contracted method, recommended above.

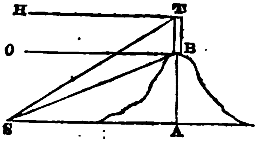
Hence the height of the tower is 118·6 feet, very nearly; it is accurately between 118·5 feet and 118·6 feet: but nearer the latter than the former.

To solve such examples as this by logarithms, as is done in the books, the computation of an oblique-angled-triangle—namely, of the triangle ABC, is necessary; and there will be five references to the logarithmic tables. In the above operation two references to a table suffice; and the subsequent arithmetical work, whatever number the dividend in the division may be, will always be trifling if the contracted method of dividing be employed.

2. From the top of a castle, 60 feet high, standing upon a hill near the sea-shore, the angle of depression HTS of a ship at anchor was observed to be 4° 52′; and the angle of depression OBS, taken from the bottom of the castle, was found to be 4° 2′. What was the distance AS of the ship?

Solution without Logarithms.

Since TH is parallel to AS, the angle AST is equal to the angle HTS (Euc. 29, I.).



For a like reason the angle ASB is equal to the angle OBS. Consequently, the angles of elevation AST, ASB are known—namely, AST = 4° 52′, and ASB = 4° 2′.

By right-angled triangles,

$$AT = SA \tan AST,\ AB = SA \tan ASB,$$

$$\therefore AT - AB = 60 = SA (\tan AST - \tan ASB).$$

By the table of natural tangents, we find

Tan AST = tan 4° 52′ = ·0851
Tan ASB = tan 4° 2′ = ·0705

The difference = ·01,4,6)60 (4100 = SA.
584
16
15
1

Consequently, the horizontal distance of the ship is 4100 feet, or 1367 yards, very nearly. By multiplying SA by tan ASB, we shall get the height of the hill—namely,

$$\cdot 0705 \times 4100 = 289 = AB.$$

3. The angle of elevation of the top of a tower was found to be 46° 30′, the place of observation from the bottom being 220 feet distant in a horizontal line: required the height of the tower. Ans. 232 feet.

4. At a horizontal distance of 45 yards from the bottom of a steeple the angle of elevation of the top was found to be 48° 12′; the height of the observer's eye was 5 feet: required the height of the steeple. Ans. 52 yards.

5. In order to find the height of a castle surrounded by a moat, the angle of elevation of its top was taken from a convenient station and found to be 46° 10′; then at a station 110 yards further off, but in the same horizontal line as the former station and the bottom of the castle, the angle of elevation was again taken and found to be 29° 56′: required the height of the castle. Ans. 141·6 yards.

6. The height of an inaccessible object being required, the angle of elevation was taken at some distance from it and found to be 51° 30′, and then a further distance of 75 feet being measured in the same horizontal line, the angle of elevation was again taken and found to be 26° 30′. What was the height of the object, and at what horizontal distance was it from the first place of observation?

Ans. height 61·97 feet; distance 49·29 feet.

7. From the top of a ship's mast, 80 feet above the water, the angle of depression of another ship's hull was taken, and found to be 20°. What was the distance between the ships? Ans. 220 feet.

8. From the top of a lighthouse, 85 feet high, reckoning from the summit of the rock on which it stands, the angle of depression of a ship at anchor was found to be 3° 38′, and the angle at the bottom of the lighthouse, or top of the rock, was found to be 2° 43′: required the horizontal distance of the ship and the height of the rock above the level of the sea. Ans. distance 5296 feet, height of rock 251 feet.

9. From the edge of a ditch, 36 feet wide, surrounding a fort, the angle of elevation of the top of the wall was found to be 62° 40′: required the height of the wall, and the length of a scaling ladder to reach from the edge of the ditch to the top.

Ans. wall 69·6 feet, ladder 78·4 feet.

10. In the year 1784, two observers on Blackheath, at the distance of exactly a mile one behind the other, observed the angle of elevation of Lunardi's balloon, at the same instant of time; these angles were 36° 52′ and 30° 58′ respectively: required the *perpendicular height of* the balloon. Ans. 3 miles.

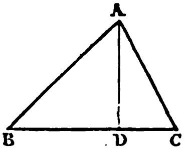
To Calculate the Sides and Angles of Oblique-angled Triangles.—In an oblique triangle the given parts must be either

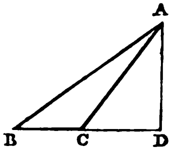
1. Two angles and a side;
2. Two sides and an angle; or,
3. The three sides, to find the remaining three parts.

I. *When the given parts are either two angles and an opposite side, or two sides and an opposite angle.*

Investigation of the Rule.

Let the triangle be ABC, and let it be either acute-angled, or obtuse-angled, as in the margin; and let the perpendicular AD be drawn, meeting the base or the base produced in D.





As in the former investigations, let the sides of the triangle, or rather their numerical measures, be represented by *a*, *b*, *c*, these letters applying to the sides opposite to the angles A, B, C, respectively. Each triangle presents now two right-angled triangles,—namely, the triangles ABD, ACD; and from each we get a distinct expression for the perpendicular AD common to both, namely,

$$\text{AD} = \text{AB} \sin \text{B}, \text{ and AD} = \text{AC} \sin \text{C};$$

so that AB sin B $=$ AC sin C, that is,

$$c \sin \text{B} = b \sin \text{C}, \therefore \frac{c}{b} = \frac{\sin \text{C}}{\sin \text{B}}$$

which shows that *any one side of a triangle is to any other as the sine of the angle opposite to the former is to the sine of the angle opposite to the latter.* The angle C, in the preceding equations, belongs to the right-angled triangle ACD, which, in the second diagram, is not the angle C of the proposed triangle ABC, but the supplement of that angle; yet, as the sine of an angle is the same as the sine of its supplement, we may regard the C, in sin C above, as the angle of the proposed triangle in each diagram.

The property enounced in italics above supplies the following rule, when of the three given parts two are opposite to one another:—

Rule.—*To find an angle.* As one of the given sides is to the other, so is the sine of the angle opposite to the former to the sine of the angle opposite to the latter.

To find a side. The sines of the given angles are to each other as the sides opposite to those angles.

If *a*, *b*, be any two sides, and A, B, their opposite angles,

$$\therefore a : b :: \sin \text{A} : \sin \text{B} = \frac{b \sin \text{A}}{a},$$

$$\text{Or, } \sin \text{A} : \sin \text{B} :: a : b = \frac{a \sin \text{B}}{\sin \text{A}},$$

$$\therefore \log \sin \text{B} = \log b + \log \sin A - \log a; \ \log b = \log a + \log \sin \text{B} - \log \sin \text{A}.$$

When an *angle* is to be determined by this rule, there is sometimes a choice of *two* angles; and the case is then called the *ambiguous case.* The ambiguity arises from the circumstance that the sought angle is to be inferred from its *sine*, and in the absence of all overruling restrictions, *two* angles have each equal claim to the *same sine:* an angle and its supplement. But if it happen that that one of the given sides which is opposite to the given angle, is *greater* than the other given side, the angle opposite to the latter must be *acute*, otherwise the triangle would have two obtuse angles, which is impossible. Also, if either of two angles are known to be obtuse, the others must, of course, each be acute. Except under these conditions, the angle sought may be either acute or obtuse; so that there are two distinct triangles determinable from the proposed given parts, these parts belonging equally to both triangles.

EXAMPLES.

1. In a triangle two of whose sides are $a = 95{\cdot}12$, and $b = 98$, and of which the angle A opposite to the former is 32° 15′, it is required to find the angle B opposite to the latter, as also the third side c.

To find the angle B.		To find the side c.	
As $a = 95{\cdot}12$	−1·9783	As sin A, 32° 15′	−9·7272
: $b = 98$	1·9912	: sin C, 114° 24′	9·9594
:: sin A, 32° 15′	9·7272	:: $a = 95{\cdot}12$	1·9783
: sin B, 33° 21′	9·7401	: $c = 162{\cdot}3$	2·2105

This example comes under the ambiguous case noticed above, for the side b, opposite to the sought angle, being greater than the side a, opposite the given angle, the angle B may be either 33° 21′ or its supplement 146° 39′, as the sine determined above belongs equally to both, and each of these angles is greater than 32° 15′. The side c is calculated here for the acute angle B = 33° 21′, for which $C = 180° - (32° 15' + 33° 21') = 114° 24'$. If it be calculated for the obtuse angle B = 146° 39, then C will be $C = 180° - (32° 15' + 146° 39') = 1° 6'$.

It will be seen that there is an inconvenience in having a *subtractive* logarithm in each of the columns; it may be replaced by an additive quantity as follows:—Instead of writing down 1·9783 from the table, write down what this number wants of 10, which is 8·0217; but instead of subtracting from 10, to get this remainder, in the ordinary way, commence with the leading figure 1 on the left, and proceed from figure to figure towards the right, subtracting each from 9, till the last, 3, is reached, which subtract from 10: this is of course the same, in effect, as subtracting the 3 from 10, in the common way, and carrying 1 from every figure in proceeding from right to left; but it is easier to write down the remainder by the former way than by the latter: thus, pointing to the 1 in the number 1·9783, we write down 8; to the 9 we write down 0; to the 7 we write down 2; to the 8 we write down 1; and, lastly, to the 3 we write down 7. The remainder thus written in place of the tabular logarithm, is called the *arithmetical complement* of that logarithm. The arithmetical complement should always be written instead of a subtractive log, and *added;* and the error of 10, thus committed, is to be corrected by suppressing 10 in the result of the addition.

The foregoing work should, therefore, be modified thus:—

To find the angle B.	
As $a = 95{\cdot}12$, Arith. Comp.	·8·0217
: $b = 98$	1·9912
: : sin A, 32° 15′	9·7272
: sin B, 33° 21′	9·7401

To find the side c.	
As sin A, 32° 15′, Arith. Comp.	·2728
: sin C, 114° 24′	9·9594
: : $a = 95{\cdot}12$	1·9783
: $c = 162{\cdot}3$	2·2105

2. Given $A = 48° 3'$, $B = 40° 14'$ and $c = 376$, to find a and b.

$$C = 180° - (48° 3' + 40° 14') = 91° 43'.$$

To find a.	
As sin C, 91° 43′, Arith. Comp.	·0002
: sin A, 48° 3′	9·8714
: : $c = 376$	2·5752
: $a = 279{\cdot}8$	2·4468

To find b.	
As sin C, 91° 43′, Arith Comp.	·0002
: sin B, 40° 14′	9·8102
: : $c = 376$	2·5752
: $b = 243$	2·3856

3. Given $a = 355$, $c = 336$, and $A = 49° 26'$, to find b and C.
Ans. $b = 465{\cdot}3$, $C = 45° 58'$

4. Given $a = 310$, $B = 62° 9'$, and $C = 41° 13'$.
$b = 281{\cdot}7$, $c = 210$, $A = 76° 38'$.

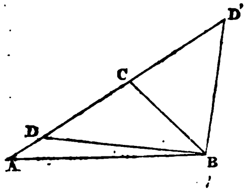
5. Given $a = 70$, $b = 104$, and $B = 44° 12'$.
$A = 27° 59'$, $C = 107° 49'$, $c = 142$.

6. Given $b = 104$, $c = 142$, and $B = 44° 12'$.
There are *two* triangles having these parts in each; the remaining parts being as here annexed.
$$\left.\right\}\begin{array}{lll} C = 72° 11' & \text{or} & 107° 49'. \\ A = 63° 37' & \text{or} & 27° 59'. \\ a = 134{\cdot}8 & \text{or} & 70°. \end{array}$$

II. *When the given parts are two sides and the included angle.*

Investigation of the Rule.

Let the two sides AC, BC, and their included angle ACB, be given, to find the remaining angles A, B, of the triangle ABC.



From the greater CA, of the two given sides cut off a part CD, equal to the less CB; and also prolong AC, till CD′ = CB: then CB = CD = CD′, and consequently, with centre C, and radius CB, a circle may be circumscribed about DBD′, so that the angle DBD′ is a right angle (Euc. 31 of III.), and therefore CDB is the complement of D′.

Now CDB = A + ABD: add CBD = CDB to each; then

$$\left.\begin{array}{l} 2\text{CDB} = \text{B} + \text{A}, \therefore \text{CDB} = \tfrac{1}{2}(\text{B} + \text{A}), \therefore \sin \text{ADB} = \sin \tfrac{1}{2}(\text{B} + \text{A}) \\ \text{And consequently since D}' \text{ is the comp. of D}, \sin \text{AD}'\text{B} = \cos \tfrac{1}{2}(\text{B} + \text{A}) \end{array}\right\} \ldots (1)$$

But CBD, the half sum of A and B, added to ABD, gives B, the greater,

$$\left.\begin{array}{l} \qquad\therefore \text{ABD} = \tfrac{1}{2}(\text{B} - \text{A}), \therefore \sin \text{ABD} = \sin \tfrac{1}{2}(\text{B} - \text{A}) \\ \text{But } \sin \text{ABD}', \text{ greater than } 90°, \text{ is the same as the sine of an angle as} \\ \quad \text{much } \textit{less than } 90° \qquad \therefore \sin \text{ABD}' = \cos \tfrac{1}{2}(\text{B} - \text{A}) \end{array}\right\} \ldots (2)$$

These relations being established, let us refer to Case I., which gives

$$\frac{AD'}{AB} = \frac{AC+CB}{AB} = \frac{\sin ABD'}{\sin AD'B} \quad . . . \ (3)$$

$$\text{and} \quad \frac{AD}{AB} = \frac{AC-CB}{AB} = \frac{\sin ABD}{\sin ADB} \quad . . . \ (4)$$

Hence, dividing (3) by (4), we have

$$\frac{AC+CB}{AC-CB} = \frac{\sin ABD' \sin ADB}{\sin ABD \sin AD'B} \quad . . . \ (5)$$

that is, by substituting for these sines their values in (1) and (2),

$$\frac{AC+CB}{AC-CB} = \frac{\cos \frac{1}{2}(B-A) \sin \frac{1}{2}(B+A)}{\sin \frac{1}{2}(B-A) \cos \frac{1}{2}(B+A)} = \frac{\tan \frac{1}{2}(B+A)}{\tan \frac{1}{2}(B-A)} \quad \text{(page 8)};$$

so that in any plane triangle, *the sum of two sides is to their difference, as the tangent of half the sum of the opposite angles is to the tangent of half their difference.* The following, therefore, is the rule:—

Rule.—As the sum of the two given sides
Is to their difference,
So is the tangent of half the sum of the opposite angles
To the tangent of half their difference.

The half difference of the unknown angles thus becomes known; and as their half sum is also known—for the whole sum is 180° minus the given angle (Euc. 32 I.)—the two angles themselves are readily discovered: the greater of the two is the half sum increased by the half difference, and the less, the half sum diminished by the half difference.

Note.—Instead of using the tangent of half the sum of the opposite angles, we may use the cotangent of half the included angle, as is obvious.

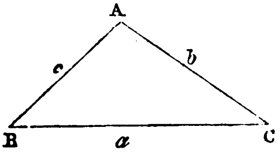
Examples.

1. In the triangle ABC are given $b = 47$, $c = 85$, and the angle $A = 52° 40'$: required the remaining parts.

$\frac{1}{2}(C + B) = 90° - 26° 20' = 63° 40'$,

and the formula for finding $\frac{1}{2}(C - B)$ is

$\therefore c + b : c - b :: \tan \frac{1}{2}(C + B) : \tan \frac{1}{2}(C - B)$



As $c + b = 132$ Arith. Comp. 7·8794
: $c - b = 38$ . . . . 1·5798
: : $\tan \frac{1}{2}(C + B)$, 63° 40′ . . . 10·3054
: $\tan \frac{1}{2}(C - B)$, 30° 11′ . . . 9·7646

$\therefore C = 93° 51'$, the sum of $\frac{1}{2}(C + B)$ and $\frac{1}{2}(C - B)$
$B = 33° 29'$, the difference.

As all the angles of the triangle are now known, the remaining side a may be found by Case I., thus:

To find the side a.

As sin B, 33° 29′ Arith. Comp. ·2583
: sin A, 52° 40′ . . . . . 9·9004
: : $b = 47$ . . . . . 1·6721

: $a = 67·74$ . . . . . 1·8308

It may be observed here that the third side of the triangle may always be determined independently of Case I., and in a manner somewhat more easily, by employing a relation furnished by the foregoing investigation, thus: the equations (3) and (4), after putting for the sines the values previously deduced, are

$$\frac{\mathrm{AC}+\mathrm{CB}}{\mathrm{AB}}=\frac{\cos\frac{1}{2}(\mathrm{B}-\mathrm{A})}{\cos\frac{1}{2}(\mathrm{B}+\mathrm{A})};\quad\frac{\mathrm{AC}-\mathrm{CB}}{\mathrm{AB}}=\frac{\sin\frac{1}{2}(\mathrm{B}-\mathrm{A})}{\sin\frac{1}{2}(\mathrm{B}+\mathrm{A})}$$

which furnish the proportions

$$\cos\tfrac{1}{2}(\mathrm{B}-\mathrm{A}) : \cos\tfrac{1}{2}(\mathrm{B}+\mathrm{A}) :: b+a : c \quad \ldots\ldots \quad (6)$$

$$\sin\tfrac{1}{2}(\mathrm{B}-\mathrm{A}) : \sin\tfrac{1}{2}(\mathrm{B}+\mathrm{A}) :: b-a : c \quad \ldots\ldots \quad (7)$$

The advantage of using either of these proportions instead of the rule in Case I. is, that all the three logarithms employed occur at the same openings of the table as the logarithms in the process for finding the angles, and they may, therefore, be readily taken out at the same time: *one* of the three—that for the sum or difference of the sides—has only to be repeated.

For the particular triangle solved above the second of these proportions is

$$\sin\tfrac{1}{2}(\mathrm{C}-\mathrm{B}) : \sin\tfrac{1}{2}(\mathrm{C}+\mathrm{B}) :: c-b : a;$$

and in order to show the facilities gained by employing it, we shall re-work the example just given by its aid.

$$\begin{array}{llrlr}
\text{As } c+b=132 & \text{Arith. Comp.} & 7{\cdot}8794 & & \text{To find the side } a. \\
:\quad c-b=\ 38 & \ldots\ldots & 1{\cdot}5798 & \ldots\ldots\ldots & 1{\cdot}5798 \\
::\ \tan\frac{1}{2}(\mathrm{C}+\mathrm{B}),\ 63^\circ\ 40' & \ldots & 10{\cdot}3054 & \ldots\ \sin\ .\ . & 9{\cdot}9524 \\
\hline
:\ \tan\frac{1}{2}(\mathrm{C}-\mathrm{B}),\ 30^\circ\ 11' & \ldots & 9{\cdot}7646 & \text{Arith. Comp. sin .} & {\cdot}2986 \\
\hline
\therefore\ \mathrm{C}=93^\circ\ 51' & & & a=67{\cdot}74\ .\ . & 1{\cdot}8308 \\
\mathrm{B}=33^\circ\ 29' & & & &
\end{array}$$

It may assist the memory in computing the side by this method, to observe that the three terms employed in the proportion for the side are merely those in the proportion for the angles *reversed*, with sine in the place of tangent.

Another use, too, may be made of the proportions (6) and (7)—namely, in the case where two angles and the interjacent side are given to find the remaining sides, as in the following example.

2. Given $\mathrm{A}=41^\circ\ 13'$. $\mathrm{B}=62^\circ\ 9'$, and $c=310$, to find a and b.

Inverting the proportions (6) and (7) we have

$$\sin\tfrac{1}{2}(\mathrm{B}+\mathrm{A}) : \sin\tfrac{1}{2}(\mathrm{B}-\mathrm{A}) :: c : b-a$$

$$\cos\tfrac{1}{2}(\mathrm{B}+\mathrm{A}) : \cos\tfrac{1}{2}(\mathrm{B}-\mathrm{A}) :: c : b+a.$$

Hence the work will stand as follows:—

$$\begin{array}{llrlr}
\text{As } \sin\frac{1}{2}(\mathrm{B}+\mathrm{A}),\ 51^\circ\ 41' & \text{Arith. Comp.} & {\cdot}1054 & \text{Arith. Comp. cos .\ .} & {\cdot}2076 \\
:\ \sin\frac{1}{2}(\mathrm{B}-\mathrm{A}),\ 10^\circ\ 28' & \ldots\ldots & 9{\cdot}2593 & \ldots\ldots\ \cos\ .\ . & 9{\cdot}9927 \\
::\quad c=310 & \ldots\ldots & 2{\cdot}4914 & \ldots\ldots\ldots & 2{\cdot}4914 \\
\hline
:\quad b-a=71{\cdot}7 & \ldots\ldots & 1{\cdot}8561 & b+a=491{\cdot}7\ .\ . & 2{\cdot}6917 \\
 & & & b-a=\ 71{\cdot}7 &
\end{array}$$

(See example 4, page 23.)

$$2\left|\begin{array}{l} 420\ ,\ 210\ =a \\ 563{\cdot}4,\ 281{\cdot}7=b \end{array}\right.$$

3. Given $a=512$, $c=907$, *and* $B=49^\circ\ 10'$, to find the remaining parts of the triangle. *Ans.* $\mathrm{A}=34^\circ\ 6'$, $\mathrm{C}=96^\circ\ 44'$, $b=691$.

4. Given $a = 95{\cdot}12$, $c = 98$, and $B = 114^\circ\ 24'$, to find the remaining parts.
Ans. $A = 32^\circ\ 15'$, $C = 33^\circ\ 21'$, $b = 162{\cdot}34$.

5. Given $a = 112$, $b = 120$, and $C = 57^\circ\ 57'$, to find the remaining parts.
Ans. $A = 57^\circ\ 28'$, $B = 64^\circ\ 35'$, $c = 112{\cdot}6$.

6. Given $b = 154{\cdot}3$, $c = 365$, and $A = 57^\circ\ 12'$, to find the remaining parts.
Ans. $B = 24^\circ\ 45'$, $C = 98^\circ\ 3'$, $a = 310$.

III. *When the given parts are the three sides.*

Investigation of the Rules.

Returning to the diagrams of Case I., page 21, we have by right-angled triangles,

$$BD = c \cos B, \text{ and } CD = b \cos C.$$

If the angle C of the triangle be acute, the cosine of it will be positive; if it be obtuse, as in the second of the diagrams referred to, it will be negative (see MATHEMATICAL SCIENCES, page 298). In the equation above, the angle C is ACD, which is acute in the second diagram; but if we replace it by the angle C of the triangle, thus making cos C subtractive, we shall have, equally for both triangles, the condition

$$a = b \cos C + c \cos B$$

And similarly,

$$b = c \cos A + a \cos C$$
$$c = a \cos B + b \cos A$$

Let these be regarded as three algebraical equations, in which the unknown quantities (usually x, y, and z) are cos A, cos B, and cos C; then, multiplying them by a, b, c respectively, we have—

$$a^2 = ab \cos C + ac \cos B$$
$$b^2 = bc \cos A + ab \cos C$$
$$c^2 = ac \cos B + bc \cos A,$$

and subtracting each of these from the sum of the other two, there results—

$$\left.\begin{aligned} b^2 + c^2 - a^2 &= 2bc \cos A \\ a^2 + c^2 - b^2 &= 2ac \cos B \\ a^2 + b^2 - c^2 &= 2ab \cos C \end{aligned}\right\} \therefore \left\{\begin{aligned} \cos A &= \frac{b^2 + c^2 - a^2}{2bc} \\ \cos B &= \frac{a^2 + c^2 - b^2}{2ac} \\ \cos C &= \frac{a^2 + b^2 - c^2}{2ab} \end{aligned}\right.$$

The expressions on the right furnish a complete solution to the present problem; but when the given sides a, b, c, consist each of several places of figures, the calculation of the fractions involves a good deal of arithmetical work. On this account, means have been contrived to change the preceding forms into others, that shall consist exclusively of *factors* and *divisors*, without any addition or subtraction operations; so that the expressions may be fitted for computation by logarithms: how this is brought about we shall explain presently. But, as logarithms may be dispensed with whenever a, b, c are conveniently small numbers, we shall first show the best form of using the above expression for cos A, in such a case.

Subtract 1 from the fraction for cos A; it then becomes,

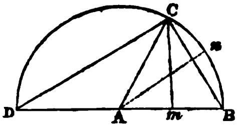
$$\frac{b^2 + c^2 - a^2}{2bc} - 1 = \frac{(b - c)^2 - a^2}{2bc} = \frac{(b - c + a)(b + c - a)}{2bc}$$

and now adding the 1 subtracted, we have—

$$\cos A = \frac{(b-c+a)(b-c-a)}{2bc} + 1 = 1 - \frac{(a+b-c)(a-b+c)}{2bc} \ldots (1)$$

which is tolerably convenient for calculation without logarithms: but a form somewhat preferable is given at page 29.

If the 1 were transposed and signs changed, the right-hand member of the last equation would be equally convenient for calculation *with* logarithms; but the left-hand member, which would then be 1 — cos A, has no corresponding logarithm in the ordinary tables;* the object is then to change 1 — cos A into some equivalent trigonometrical quantity, the log of which is to be found in the tables. A diagram will assist in showing how this is to be effected.



Suppose AB to be the trigonometrical radius—namely, AB = 1, and let BCD be the arc of a semicircle to that radius, and BAC the angle A; then A*n* bisecting the chord BC, also bisects the arc BC and the angle A, $\therefore$ BC = 2 sin ½A.

Now DCB, in a semicircle, being a right angle, we have (Euc. 8 of VI.) BD. B*m* = BC², that is, since BD = 2, and A*m* = cos A,

$$2\,(1-\cos A) = (2\sin \tfrac{1}{2}A)^2 \;\therefore\; 1-\cos A = 2\sin^2 \tfrac{1}{2}A,$$

$$\therefore\ (1),\ \sin^2 \tfrac{1}{2}A = \frac{(a-b+c)(a+b-c)}{4bc}$$

$$\therefore\ \sin \tfrac{1}{2}A = \sqrt{\frac{\frac{1}{2}(a-b+c)\frac{1}{2}(a+b-c)}{bc}}$$

a formula well adapted to computation by logarithms. If s be put to represent $\frac{1}{2}(a+b+c)$, then the two factors in the numerator will be respectively $s-b$ and $s-c$, and we shall have the expression for sin ½ A in the more compact form,

$$\sin \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}} \quad \ldots\ldots \text{(I)}$$

Again: because as proved above ½(1 — cos A) = sin² ½A, if each be subtracted from 1, we shall have,

$$\tfrac{1}{2}(1+\cos A) = 1-\sin^2 \tfrac{1}{2}A = \cos^2 \tfrac{1}{2}A,$$

$$\therefore\ 2\cos^2 \tfrac{1}{2}A = 1+\cos A = 1+\frac{b^2+c^2-a^2}{2bc} = \frac{(b+c)^2-a^2}{2bc}$$

$$= \frac{(a+b+c)(b+c-a)}{2bc}$$

$$\therefore\ \cos \tfrac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}} \quad \ldots\ldots \text{(II)}$$

and dividing (I) by (II), we have finally,

$$\tan \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \quad \ldots\ldots \text{(III)}$$

Either of the formulæ marked (I), (II), (III), will supply a rule for finding an angle from the sides being given; but it is better to work by the formula itself than to be guided by written directions. That marked (II) is a little the shortest; but when

---

* The expression 1 — cos A is called the *versed-sine* of the angle A. A table of natural versed-sines is given in Vol. II. of Dr. *Mackay's* valuable work on "The Theory and Practice of finding the Longitude."

$\frac{1}{2}$A, that is, half the sought angle, is foreseen to be very small, it will be better to use the formula (I): the reason is that the sines of angles very near 90°, or, which is the same thing, the cosines of angles very near 0°, vary from each other by such slight differences, that several of such angles have equal claim to the same sine or cosine. For instance, suppose we were led to 9·9999998 for the logarithmic sine or cosine of an angle to be found. By tables calculated to seven places of decimals, this number is the log sine of every angle, indifferently, between 89° 56′ 19″ and 89° 57′ 8″; and, consequently, the log cosine of every angle between 2′ 52″ and 3′ 41″. In such extreme cases the formula (III) is to be preferred, although for a small angle (I) may be safely employed, and (II) for a large one. In practice, however, these ill-conditioned triangles, as they are called, are always avoided, if possible; and in Navigation they are but little likely to be met with. An error of a few seconds in *this* subject is however a matter of no moment; and we advert to the peculiarity here merely to account to the learner how it happens that, although the three formulæ above are all equally correct, yet in inquiries where the minutest accuracy is necessary, one of them may, in certain rare cases, be preferable to another. The defect is not in the formulæ, but in the tables, which being calculated to only six or seven places of decimals, cannot, of course, mark distinctions of value that affect only the decimals more remote.

EXAMPLES.

1. Given the sides $a = 95·12$, $b = 162·34$, and $c = 98$, to find the angle A.

By formula (I).

$$\sin \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$a =$	95·12		
$b =$	162·34	Arith. comp.	7·7896
$c =$	98	Arith. comp.	8·0088
	2)355·46		
$s =$	177·73		
$s - b =$	15·39	. . .	1·1872
$s - c =$	79·73	. . .	1·9016
			2)18·8872
sin $\frac{1}{2}$A,	16° 7½′	. . .	9·4436
	2		
∴ A =	32° 15′		

By formula (II).

$$\cos \tfrac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$$

$a =$	95·12		
$b =$	162·34	Arith. comp.	7·7896
$c =$	98	Arith. comp.	8·0088
	2)355·46		
$s =$	177·73	. . .	2·2498
$s - a =$	82·61	. . .	1·9170
			2)19·9652
cos $\frac{1}{2}$A,	16° 7½′	. . .	9·9826
	2		
∴ A =	32° 15′		

In the first of these logarithmic operations there enters a log sine, and in the second a log cosine: each, therefore, requires a correction, as noticed at page 9; 10 should be added to the right-hand member of each of the formulæ (I), (II), when expressed logarithmically. Now *two* tens have been added to the sum of the four logs above, on account of the two complements; so that after dividing this sum by 2, for the square root, *one* ten, in excess, affects the result, which therefore supplies the 10 that ought to have been added, and thus the proper correction is provided for.

We shall now exhibit the work of the preceding example without logarithms. *The most convenient formula* for this purpose is perhaps that immediately derived from the expression for cos A at page 26, by first adding and then subtracting 1: we thus get

$$\cos A = \frac{(b^2 + 2bc + c^2) - a^2}{2bc} - 1 = \frac{(b+c)^2 - a^2}{2bc} - 1 = \frac{(a+b+c)(b+c-a)}{2bc} - 1$$

$$\begin{array}{rr}
a = 95{\cdot}12 & \\
b = 162{\cdot}34 & \\
c = 98 & \\
\hline
a + b + c = 355{\cdot}46 \;\ldots\; & 355{\cdot}46 \\
b + c - a = 165{\cdot}22 \text{ reversed} = & 22561 \\
\hline
b = 162{\cdot}34 & 35546 \\
c = 98 & 21328 \\
 & 1777 \\
129872 & 71 \\
146106 & 7 \\
\hline
15909{\cdot}32 \times 2 = & 31818{,}64)58729(1{\cdot}8457 \\
 & 31819 \\
\hline
{\cdot}8457 = \cos A,\ 32^\circ\, 15' & 26910 \\
 & 25455 \\
\hline
 & 1455 \\
 & 1273 \\
\hline
 & 182 \\
 & 159 \\
\hline
 & 23 \\
 & 22
\end{array}$$

The contracted method, so often recommended in these pages, is used throughout this operation.

It may be remarked here that in computing, as in this specimen, the first figure in the quotient will always be 1, followed by decimals; so that there will be no occasion to take any notice of decimal points, either in the dividend or divisor: it is sufficient that we know that the first figure of the quotient is always an integer (unit), and the following figures decimals. But there is no question that, in general, the operation by logarithms would be preferred. It is worthy of notice, however, that whenever a, b, c are all divisible by the same number, we may divide accordingly, and use the quotients: thus, instead of the above values, we might have taken $a = 47{\cdot}56$, $b = 81{\cdot}17$, and $c = 49$; these being the halves of those values.

2. Given $a = 174{\cdot}1$, $b = 232$, and $c = 345$, to find the angle A.
Ans. A = 27° 4′.

3. Given $a = 95{\cdot}12$, $b = 162{\cdot}34$, and $c = 98$, to find the angle B.
Ans. B = 114° 24′.

4. Given $a = 3388$, $b = 2065$, and $c = 1637$, to find the angle A.
Ans. 132° 7′.

5. Given $a = 112$, $b = 112{\cdot}6$, and $c = 120$, to find all the angles.
Ans. A = 57° 28′, B = 57° 57′, C = 64° 35′.

6. Given $a = 698$, $b = 352$, and $c = 467$, to find all the angles.
Ans. A = 116° 12′, B = 26° 54′, C = 36° 54′.

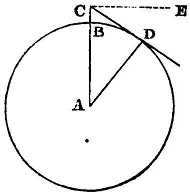
Miscellaneous Questions requiring the Solution of Plane Triangles.—

1. Being on one side of a river, and wishing to know the distance of a tree (C) on the other side, I measured a length of 500 yards along the side of the river, and set up a rod at each end (A and B): the angle at A subtended by BC was then measured, and found to be 74° 14′; and the angle at B, subtended by AC, was found to be 49° 23′: required the distance of the tree from each station.

Here the two angles at A and B are given, as also the interjacent side AB; the third angle at C is therefore known, namely, $C = 180° - (74° 14' + 49° 23') = 56° 23'$.

As sin C, 56° 23′ Arith Comp.	·0795	As sin C, 56° 23′ Arith. Comp.	·0795
: sin B, 49° 23′	9·8803	: sin A, 74° 14′	9·9833
: : AB = 500	2·6990	: : AB = 500	2·6990
: AC = 455·8	2·6588	: BC = 577·8	2·7618

2. From the top of a mountain 3 miles high, the angle of depression of the visible horizon—that is, of the circle where sky and sea appeared to meet,—was found to be 2° 13′½: it is required from this to determine the diameter of the earth.



Let A be the centre of the earth, C the summit of the mountain BC, and ECD the angle of depression of the remotest visible point of the surface of the sea, below the horizontal line CE.

The angles ACD, DCE, make up a right angle; so do the angles ACD and A, because D is a right angle (Euc. 18 of III.); therefore the angle A = the angle ECD. Now by right-angled triangles AC = AD sec A.

$$\therefore AC - AB = BC = AD (\sec A - 1); \text{ that is,}$$

$$AD (\sec 2° 13'\tfrac{1}{2} - 1) = 3, \quad \therefore AD = \frac{3}{\sec 2° 13'\tfrac{1}{2} - 1}$$

By help of a table of natural secants this expression for the semi-diameter AD is very readily calculated. To convert it into a form adapted to logarithms, we have, by putting $\frac{1}{\cos A}$ for sec A,

$$AD = \frac{BC}{\sec A - 1} = \frac{BC \cos A}{1 - \cos A}$$

But it was shown at page 27 that $1 - \cos A = 2 \sin^2 \tfrac{1}{2}A$,

$$\therefore AD = \frac{BC \cos A}{2 \sin^2 \tfrac{1}{2}A}, \quad \therefore 2\,AD = \frac{BC \cos A}{\sin^2 \tfrac{1}{2}A}$$

$$\therefore \log \text{Diameter} = \log 3 + \log \cos 2° 13'\tfrac{1}{2} - 2 \log \sin 1° 6'\tfrac{3}{4} + 10,$$

the 10 being added because there are two subtractive log sines—namely, log sin 1° 6′¾ + log sin 1° 6′¾, and only one additive trigonometrical quantity—namely, log cos 2° 13′½.

log 3	·477121
log cos 2° 13′½	9·999672
log sin 1° 6′¾ Arith. Comp.	1·711859
Repeated	1·711859
log diam, 7952½	3·900511

Hence the diameter of the earth, as deduced from these observations, is 7952½ miles.

Otherwise by a table of natural cosines.

Since, as shown above, $AD = \frac{BC \cos A}{1 - \cos A}$, we may compute by the table of natural sines and cosines as follows:—

```
                    nat cos 2° 13'½ =  ·999246
                                             3
                                       -------
   1 —·999246 = ·000754 )2·997738( 3976
                          2262          2
                          ----       ----
                           7357      7953 miles the diameter.
                           6786      ----
                           ----
                            5713
                            5278
                            ----
                             4358
                             4524
                             ----
```

3. Wanting to know the breadth of a river, I measured a base of 500 yards in a straight line close to its edge, and at each end of it I found the angles subtended by the other end, and a tree close to the opposite margin of the river to be 53° and 79° 12′: what was the breadth of the river? Ans. 529·5 yards.

4. Two ships of war intending to cannonade a fort separated from each other 500 yards, as near to the shore as possible: the angles subtended by each ship and the fort were observed from the other ship to be 38° 16′ and 37° 9′: required the distance of the fort from each place of observation. Ans. 312 yards and 320 yards.

5. The peak of Teneriffe is said to be 2½ miles above the surface of the sea: the angle formed at the top between a plumb line, which, of course, hangs perpendicular to the horizon, and the line from the eye to touch the sea at the remotest visible point, is found to be 87° 58′: required the diameter of the earth.

Ans. 7936 miles.

6. Three objects, A, B, C, whose distances apart were AC = 8 miles, BC = 7½ miles, and AB = 12 miles, were visible from a station D, in the line joining A and B; and the angle at that station, subtended by AC, was observed to be 107° 56′ 13″: required the distances of the three objects from D?

Ans. AD = 5 miles, DB = 7 miles, DC = 4·89 miles.

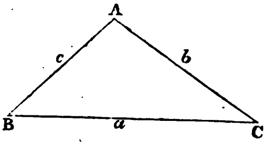
We here conclude the introductory treatise on the solution of plane triangles: it is intended solely to familiarize the learner with the business of practical calculation, and to illustrate the best and shortest methods of arranging his arithmetical operations. Many instances have been given, more especially in what concerns right-angled triangles, of the advantage of making more use of the table of natural sines and cosines than is at present customary. Valuable as logarithms unquestionably are, they do not always effect a saving of time or trouble, even in cases well fitted for their application, and we would therefore recommend that the example we have here set him be followed by *the learner: that he* would exercise his own deliberate judgment as to which kind of table will aid him in reaching his result in the readiest way, and not in all cases resort to logarithms, as is almost invariably done in works of this kind.

But whatever table he uses he should use with deliberation and care: tabular numbers should never be transcribed in a hurry, as everything depends on the accuracy with which they are written down. It should also be the habitual practice of the calculator to make all the use he can of his table when it is in his hand; and when it is not in use, he should advance his work as much as practicable before he refers to it. In order to this, as many as possible of the *arguments*, as they are called—that is, of the quantities with which the tabular numbers are to be connected—should be written down before the tables are touched: thus, in Example 1, for instance, page 28, the entire skeleton of the operation should be formed before the table of logarithms is referred to, to fill it up. Again, in Example 2, page 23, every item in both columns of the work should be written down while the tables are in hand, and they should be again referred to only after the amount of each column is found.

In the treatise on TRIGONOMETRY in the mathematical volume of the CIRCLE OF THE SCIENCES, the main object has been to develop the analytical theory of angular magnitude, and not to enter at any length into arithmetical details; the present PART, independently of the special purpose it is intended to serve in this treatise, may, therefore, be acceptable to the student as a praxis on certain theoretical principles there delivered. This collateral object has not been lost sight of in the preparation of the foregoing pages; and we have, accordingly, introduced examples in sufficient number and variety to supply the learner with all needful materials for exercise, in the ordinary calculations of Practical Trigonometry.

Before closing the introductory portion of our subject, it may be well to collect into one place the several formulæ for the solution of oblique triangles. By thus bringing them together, future reference to them will be facilitated; and the learner, by having the whole more frequently under his eye, will at length get them so impressed on his mind as eventually to dispense altogether with a formal reference to them. This amount of familiarity with his tools is a necessary qualification in a good workman.

Formulæ for the Solution of Plane Triangles.



I. $a : b :: \sin A : \sin B$

II. $a + b : a \backsim b :: \tan \frac{1}{2}(A + B) : \tan \frac{1}{2}(A \backsim B)$

$\sin \frac{1}{2}(A \backsim B) : \sin \frac{1}{2}(A + B) :: a \backsim b : c$

$\cos \frac{1}{2}(A \backsim B) : \cos \frac{1}{2}(A + B) :: a + b : c$

III. $\sin \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}, \cos \frac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$

$$\tan \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\cos A = \frac{(a+b+c)(b+c-a)}{2bc} - 1.$$

The last of these expressions is to be computed by common arithmetic, and the cosine of A to be found in the table of natural sines and cosines; all the other formulæ are adapted to logarithmic computation. Whenever the sides a, b, c, have a factor common to all, it may be cancelled, and the results used instead, in any of the formulæ III.

NAVIGATION.

General Notions of the Figure and Rotation of the Earth.—The surface of the sea is very nearly that of a perfect sphere or globe. For all the purposes of Navigation, it may be regarded as accurately so; for if the very small deviation from this figure were to be removed, no sensible difference would be made in any of the rules and operations by which the sailings of a ship are regulated, and its position determined.

That the surface of the ocean is globular may be inferred from the evidence of the senses. On whatever part of it an observer be placed, or however high above it he be raised, he sees around him an expanse in which no defect from sphericity can be discovered anywhere within the compass of his vision. He sees a receding vessel, in the distance, gradually disappearing behind the rotundity; he first loses sight of the hull, then of more and more of the upper works, till at length even the most elevated part sinks below the circle that limits his field of view. These effects being observed, at whatever spot on the surface the spectator may be, lead to the natural impression that it is a portion of a globe that is everywhere spread around him. This impression is confirmed by the unquestionable fact that, on the assumption that the surface is uniformly spherical, and under the sole guidance of rules and directions based exclusively on that assumption, ships have actually circumnavigated the earth, and that by routes so various as to leave on the mind not the slightest doubt that the inference, as to the general figure of the ocean's surface, drawn from observation of its appearance to the eye, is correct.

Again: an eclipse of the moon is caused by the passage of that body through the shadow cast by the earth, the earth being then between the sun and moon. As soon as the moon enters the shadow, the small part of her face thus obscured always presents a circular boundary; as she advances, and the obscuration increases, the boundary of the shadow enlarges, but it continues circular; and whenever the centres of the sun, earth, and moon are so nearly in the same straight line as to render the eclipse *annular*, as it is called, then the shadow projected on the moon's surface is observed to be a complete circle. These appearances being uniform, whatever part of the earth's surface is exposed to the sun's rays, we cannot resist the conclusion that the planet we inhabit does not differ in external figure from a perfect sphere, except in a very slight degree.

The earth rotates—turning once round every day; it revolves invariably about the same diameter, and completes each revolution invariably in the same time. That the rising and setting of the sun and stars are appearances really due to the diurnal rotation of the earth, and not to the motions of the celestial bodies themselves, is a truth not so obvious to the senses as the spherical figure of the earth,—at least, till very lately, no contrivance had been thought of to render this rotation visible to our eyes. But there are means now of *showing* that the earth turns round; so that we may have the same visible proof of its diurnal revolution that we have of its general figure. This will be explained when we come to treat of NAUTICAL ASTRONOMY.

That the rotation is daily performed in exactly the same invariable period of time —without the difference of a single second—is proved by innumerable observations. Age after age, the same spot on the earth, after the lapse of the same interval of time, invariably returns to the same fixed star; which could not be the case if there were the slightest irregularity in the diurnal rotation of the earth. Nor could such be the case if the diameter round which the rotation is performed were shifted.

Admitting, then, from facts such as these, that the earth is a sphere,—or that, at least, it differs from a sphere so little as for the purposes of navigation to be of no moment,—that it revolves uniformly once in twenty-four hours,—and that the diameter about which it turns is invariable, we may proceed to the following definitions:—

DEFINITIONS.

1. *Axis.*—The diameter about which the earth performs its daily revolution is called the axis of the earth: the revolution about this axis is from west to east.

2. *Poles.*—The extremities of this diameter are called the *poles* of the earth. The extremities of *any* diameter of a sphere are also called poles—the poles, namely, of that great circle of the sphere the circumference of which is at every point of it, at the same distance from each, this distance being a quadrant, or 90°: these same points are also spoken of as the poles of every *small* circle parallel to the great circle just mentioned; but from the circumference of any small circle the poles are, of course, at unequal distances.

3. *Equator.*—The *equator* is that great circle of the earth, the axis of which is perpendicular to the plane of that circle: the poles of the equator are the poles of the earth. These poles are called—one of them the NORTH POLE, and the other the SOUTH POLE. Whenever, in navigation, we speak of the poles, the poles of the *earth*—that is of the Equator—are always to be understood.

4. *Meridians.*—Every semi-circle which terminates at the two poles, and which is therefore perpendicular to the equator, is called a *meridian:* it is said to be the meridian of each place on the earth through which it passes. For the convenience of Navigation and Geography, every civilized kingdom selects one of these innumerable meridians as a meridian of reference; it is usually that which passes through the national observatory, or the principal city, and is called the *first meridian.* In this country the first meridian is that of Greenwich; in France, it is that of Paris.

The plane of every meridian (as also the plane of the equator) is conceived to be extended to the heavens, and to mark out, on the celestial sphere, the celestial meridian of the place. The apparent daily motion of the sun and stars is across the celestial meridians; the path of a ship easterly or westerly is across the terrestrial meridians; but, in general, the distinctions *terrestrial* and *celestial* are dropped: a ship cannot be on a celestial meridian, nor a star on a terrestrial meridian.

5. *Latitude.*—The latitude of a place on the surface of the earth is the distance of that place from the equator, measured on the meridian of the place. Latitude is, therefore, either north or south: a place cannot exceed 90° in latitude, this being the latitude of each pole.

6. *Parallels of Latitude.*—A small circle on the globe, parallel to the equator, is called a *parallel of latitude:* every point on the circumference of such a circle has the same latitude, as the parallel is everywhere equi-distant from the equator. The arc of a meridian, intercepted between two such parallels drawn through any two places on the globe, measures the *difference of latitude* of those places. When the places are both on the same side of the equator—that is, both north or both south,—their difference of latitude is found by subtraction; when they are on opposite sides of the equator, their difference of latitude is found by addition.

Longitude.—The longitude of any place on the earth's surface is the arc of the *equator* intercepted between the meridian of that place and the *first meridian:* it is estimated, like latitude, in degrees and parts of a degree, and is, of course, the measure of the angle at the pole included between the two meridians spoken of. When the place is to the east of the first meridian, it has east longitude; when to the west, west longitude.

As every place on the surface of the earth performs a complete revolution in twenty-four hours, 15° of longitude become the measure of one hour of time; the whole 360° of the equator, and of every parallel to it, having the measure of twenty-four hours. Longitude is thus sometimes expressed in *time;* a place 45° east of the meridian of Greenwich is, for instance, said to be three hours east of Greenwich. It is necessary to precision that the *latitude* of a place should be designated either north or south, according as it is situated in the northern or southern hemisphere; but the distinction of longitude into east and west is unnecessary and inconvenient. "It would add greatly to systematic regularity, and tend much to avoid confusion and ambiguity in computations, were this mode of expression abandoned, and longitudes reckoned invariably *westward* from their origin round the whole circle from 0 to 360°."* In the present mode of reckoning, however, longitude, like latitude, is of two denominations; so that the difference of longitude of two places is found sometimes by subtraction and sometimes by addition: by subtraction, if both places are on the same side of the first meridian; and by addition, if they lie on opposite sides: the limit of longitude, whether east or west, is of course 180°. A place of which the longitude is 180° is on the meridian opposite to that of Greenwich.

The learner must bear in mind that, in Geography and Navigation, the meridian of a place is limited by the poles of the earth: it is the semicircle, passing through the place, and terminating in the north and south poles. As already remarked, the terrestrial meridian is extended to the concavity above us, and marks out the corresponding celestial meridian. Certain stars which are observed to come to this meridian are seen to pass the opposite meridian without setting; but the distinction, noticed above, between a meridian and the opposite meridian is not preserved here: the star is generally said to come to the meridian twice—once above and once below the pole. In the case of a ship, however, sailing round the pole from one meridian to its continuation, it is always said to have arrived at the *opposite* meridian, and to have advanced 180° in longitude.

Horizon.—The horizon of any place is a plane conceived to touch the surface of the earth at that place and to be extended to the heavens—that is, to the region of the remotest of the stars. This plane is called the *sensible horizon.* A plane parallel to this, but passing through the centre of the earth, is called the *rational horizon.* These two imaginary planes, though separated from each other by an interval equal to the semi-diameter of the earth, cannot but be regarded as coincident at the distance of the stars. An eye, whether at the centre of the earth or at the point on its surface immediately over it, would see a star in precisely one and the same direction: the altitude of it, referred to the rational horizon, would be exactly the same as the altitude referred to the sensible horizon. The observer at the centre sees the star higher above *his* horizon (that is, the parallel to the sensible horizon) than the observer at the surface sees it above his, by the apparent interval between the two horizons, at the distance of the star; which interval, however, although we here call it *apparent,* is in reality too minute to appear as any interval at all.

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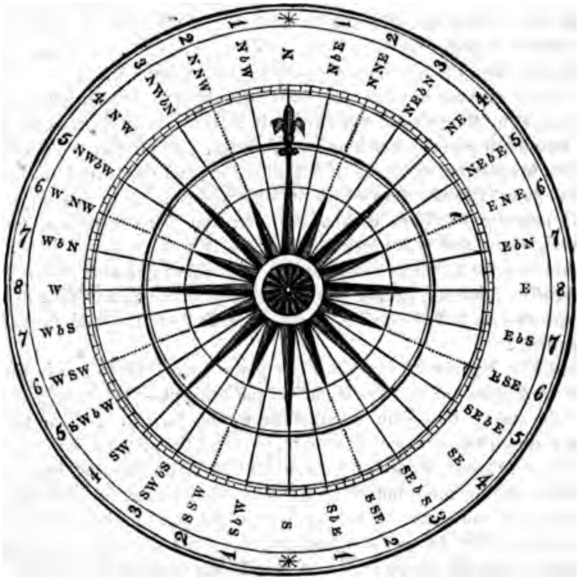
* Sir John Herschel.

An eye situated *above* the surface of the earth, as is always practically the case, has an horizon different from the sensible horizon: the latter is an extended *plane;* the former is a *conical surface*, everywhere *dipping* below the sensible horizon, and of wider visible boundary. A very moderate elevation above the surface will give a sensible increase to the observed altitude of a heavenly body, namely, the whole of the angular distance between the two horizons—the whole of what may be called the angle of the dip.

The apparent circular boundary of the sea, as seen by an eye thus elevated above its surface, is called the visible or *sea-horizon;* or more frequently by sailors, the *offing.*

The Compass.—The straight line through any place, in which the plane of the meridian of that place cuts the sensible horizon, is called the horizontal meridian, or simply the meridian line, or the north and south line; and the horizontal straight line perpendicular to this is the east and west line of the horizon. The horizon is represented in miniature by a circular card, connected with a magnetized needle, which points out the direction of the horizontal meridian, and consequently also that of the east and west line: the *points* thus marked out on the rim of the card, namely, the North, South, East and West points, are called the four Cardinal Points of the compass; the quadrantal arcs, intermediate between these, are subdivided each into *eight* equal parts called also *points*, and these again each into four equal parts called quarter-points.



The accompanying engraving is a representation of the *Mariners' Compass*, divided into its thirty-two points, with the intermediate quarter-points also marked.

This important instrument is so suspended on ship-board as always to assume a horizontal position under every change of motion in the vessel, so that the direction in *which the ship is sailing* at any time is always known by observing what *point* is in that direction; that is, in general, what point coincides with the line, through the centre of the compass-card, from stern to stem of the ship.

It is of importance to mention, however, that the magnetic needle does not point accurately north and south; the points to which it *is* directed are called the *magnetic* north and south: and the angular departure from the true north and south, at any place, is called the *variation* of the compass at that place. Its amount may be discovered, and the necessary corrections for it made, by Nautical Astronomy.

Courses.—A line on the globe, which cuts the successive meridians at the same angle, is called a *rhumb-line*; it marks the track of a ship, the constant angle referred to being called the ship's *course*. It is indicated by the compass; but if no corrections be made for variation, the course thus indicated is called the *compass-course;* after correction and allowance for what is called *deviation*, it is the *true course*. These modifications of the compass courses will be more fully noticed in the Nautical Astronomy.

Leeway.—Another correction of the course is also frequently requisite. The ship's progress is not always in the direction of her length; the wind often impels her sideways, or, as it is called, to *leeward* of the line from fore to aft. The necessary allowance for this divergence from the path indicated by the compass is the correction for *leeway;* its amount can be estimated only by practical experience.

Rate of Sailing.—The rate at which a ship sails on any course is measured by an instrument called the *log*, and a line attached to it called the *log-line*, about 120 fathoms in length. The log is a piece of wood in shape of a sector of a circle, and with its arc or rim so loaded with lead that, when thrown into the sea, it stands vertically in the water, with only its centre just above the surface.* The log-line being so attached as to keep the face of the log towards the ship, in order that it may offer the greater resistance to being dragged after it; the length of line unwound from a reel, by the advancing motion of the vessel, in *half-a-minute*, gives the distance run in that time, and thus is inferred the rate of sailing. The log-line is divided into equal parts by means of a bit of string passed through the strands and *knotted*, the number of knots showing the number of parts—each part, which is the 120th of a nautical mile, is hence called a *Knot;* so that as many knots as are run out in half-a-minute, or the 120th of an hour, so many nautical miles per hour is the ship's rate of sailing. Sailors thus say that the rate is so many knots an hour, meaning so many nautical miles an hour. A nautical mile is the 60th part of a degree of the equator or of the meridian, that is, about 6,080 feet. When the log is *hove*, about ten or twelve fathoms of line are suffered to run out before the counting commences; this is called the *stray-line*, which is an allowance for letting the log go clear of the ship, and settle in the water. As soon as the end of the stray-line, which is marked by a bit of red cloth, passes from the reel, the half-minute commences. The time is measured by a sand-glass, which runs out in thirty seconds, and which is turned when the end of the stray-line passes, and the line is stopped as soon as the sand has run out.

A Day's Work.—By a day's work is meant the ordinary daily operations at sea to determine the position of a ship, and the advances made from noon till noon; it consists in keeping a record of the compass-courses sailed in the interim, of the different rates of sailing, of the variation of the compass, the velocity and direction of currents, &c., &c., and, finally, of the latitude and longitude in. These particulars, as they become known, are recorded on the *log-board*, and the latitude and longitude, finally deduced, is called the *dead reckoning*, or the latitude and longitude *by account*. The particulars of the log-board are transferred, from day to day, to the *log-book*; and, with the addition of whatever else may give the necessary completeness to the record—the astronomical observations made to correct the dead reckoning, the state of the wind and weather, the bearings of points of land, &c., &c.,—form the *journal* of the voyage.

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* This is *the common log, still too much* used; a more accurate instrument is Massey's log.

Points of the Compass.—The following table gives the several angles which the different points of the compass make with the meridian, as also the angles for the quarter-points:—

NORTH.		POINTS.	ANGLES.	SOUTH.	
		¼	2° 49′		
		½	5° 37½′		
		¾	8° 26′		
N. *b.* E.	N. *b.* W.	1	11° 15′	S. *b.* E.	S. *b.* W.
		1¼	14° 4′		
		1½	16° 52½′		
		1¾	19° 41′		
N. N.E.	N. N.W.	2	22° 30′	S. S.E.	S. S. W.
		2¼	25° 19′		
		2½	28° 7½′		
		2¾	30° 56′		
N.E. *b.* N.	N.W. *b.* N.	3	33° 45′	S.E. *b.* S.	S.W. *b.* S.
		3¼	36° 34′		
		3½	39° 22½′		
		3¾	42° 11′		
N.E.	N.W.	4	45° 0′	S.E.	S.W.
		4¼	47° 49′		
		4½	50° 37½′		
		4¾	53° 26′		
N.E. *b.* E.	N.W. *b.* W.	5	56° 15′	S.E. *b.* E.	S.W. *b.* W.
		5¼	59° 4′		
		5½	61° 52½′		
		5¾	64° 41′		
E. N.E.	W. N.W.	6	67° 30′	E. S.E.	W. S.W.
		6¼	70° 19′		
		6½	73° 7½′		
		6¾	75° 56′		
E. *b.* N.	W. *b.* N.	7	78° 45′	E. *b.* S.	W. *b.* S.
		7¼	81° 34′		
		7½	84° 22½′		
		7¾	87° 11′		
E.	W.	8	90° 0′	E.	W.

The angles in the above table nowhere differ from the truth by more than a quarter of a minute; a degree of accuracy amply sufficient for all the purposes for which it is used. It is necessary that the learner should commit this table to memory—at least, so far as to be able to state, without reference to it, how many points any of these courses, or rhumbs, are distant from the meridian, or north and south line; as each point is 11° 15', the angle corresponding to any number of points is easily deduced. Repeating the points in order, completely round the compass, as figured at page 4, or according to this table, is called by sailors *boxing the compass.*

We shall now proceed to consider the various sailings, first, however, making a few brief remarks in reference to the data, or observed conditions upon which the computations in the following articles are, in general, to be founded.

From what has been explained in the preceding pages, the learner will perceive that the principal measurements made at sea to determine the place of a ship, independently of astronomical observations, are the measurements of the *course* sailed on, and the *rate* of sailing. From the account given of the means and instruments employed for these purposes, he must see that the results furnished by them can scarcely ever be regarded as rigorously correct. The compass-card, valuable and indispensable as it is, has no divisions upon it to distinguish angles which differ from one another by less than a degree;* such differences, therefore, in steering a ship, have to be roughly estimated by guess—the course is thus liable to error to some extent. Again, the rate of sailing, as measured by the log, is equally exposed to error from the very nature of the operation, and thus the distance run on any course cannot be determined with strict accuracy. Even if these sources of error could be obviated, yet winds, currents, swells of the sea, and the various other accidents to which a ship is exposed, and which all act as disturbing causes, would often seriously affect the correctness of the position of the vessel, as deduced from the dead reckoning.

The aim of the careful and experienced mariner is to be on the look-out for these external influences, and to allow, as best he can, for their effects; matters in reference to which there is room for the exercise of much practical tact and judgment.

These allowances being made, the courses on the log-board are modified accordingly, and, being corrected for the variation and local deviation of the compass, the actual distance made, as also the difference of latitude and longitude since the preceding noon, are computed, and the latitude and longitude, by *account*, are thus ascertained.

These last important particulars—the latitude and longitude—being derived from data so exposed to error, can be regarded, at best, as only approximately true; and therefore the properly qualified navigator loses no opportunity to correct his dead reckoning by employing the more sure and certain methods which nautical astronomy supplies. In the following articles, however, none of these can be introduced; the object of this part of our subject is to treat exclusively of what concerns the dead reckoning; and we shall, throughout, suppose that proper allowance for the variation of the compass, for the leeway, &c., have been made, and that the courses concerned are the *true* courses. The methods for ascertaining the variation of the compass must be deferred till we come to treat of Nautical Astronomy.

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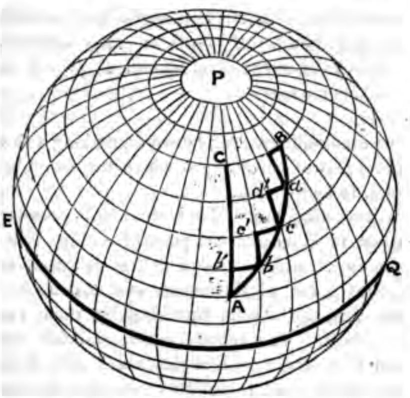
* The outer edge of the card is divided into 360 degrees; but as it is difficult to steer a ship to the nicety which these divisions imply, the marks on the extreme rim of the compass-card are seldom much attended to by *mariners.*

Plane Sailing.—*Investigation of the Theoretical Principles.*—Let the annexed diagram represent the globe of the earth, P being one of its poles, and EQ the equator. Let AB be a rhumb-line or track of a ship on a single course. Imagine this oblique path to be divided, by equidistant meridians, into portions Ab, bc, cd, &c., so small that each portion may differ insensibly from a straight line; and, as in the figure, let the parallels of latitude $b'b$, $c'c$, $d'd$, &c., be drawn. A series of triangles Abb', bcc' cdd', &c., will thus be formed on the surface of the sphere, so small that each may be practically regarded as a *plane* triangle. It is obvious that this may be conceived without any sensible violation of strict accuracy. The triangles thus described, and thus assumed to be plane triangles, are all similar; for the angles at b', c', d', &c., are all right angles, and the ship's track cuts every meridian which it crosses at the same angle. Consequently, by Euclid, Prop. 4, Book VI., we have the proportions

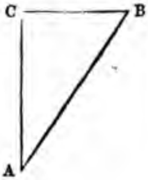


$$Ab : Ab' :: bc : bc' :: cd : cd' \text{ \&c.,}$$

and therefore, (Euclid, Prop. 5, Book V., or Prop. 10 of the Treatise on Proportion in the Circle of the Sciences)

$$Ab : Ab' :: Ab + bc + cd + \text{\&c.,} : Ab' + bc' + cd' + \text{\&c.}$$

But $Ab + bc + cd +$ &c., is the whole distance sailed on the course, and $Ab' + bc' + cd' +$ &c., is the difference of latitude AC between A, the place left, and B, the place arrived at. Consequently, if a right-angled triangle, similar to the little right-angled triangle Abb', be constructed,—that is, a right-angled triangle, in which A is the angle of the course, and if the hypotenuse AB be made to represent the distance sailed,—that is, the track AB on the globe,—then, obviously, the perpendicular AC will represent the difference of latitude, while the base CB,—the side opposite to the course,—will represent the sum of all the minute *departures* which the ship makes from the successive meridians which it crosses; for since



$$Ab : bb' :: AB : BC$$
$$Ab : bb' :: bc : cc' :: cd : dd', \text{ \&c.}$$
$$\therefore Ab : bb' :: Ab + bc + cd + \text{\&c.} : bb' + cc' + dd' + \text{\&c.}$$

and since by construction $AB = Ab + bc + cd +$ &c., therefore $BC = bb' + cc' + dd' +$ &c. The length BC is called the *departure* made by the ship in sailing from A to B; and it therefore follows that the distance sailed, the difference of latitude made, and the departure, may be represented by the sides of a right-angled *plane* triangle, the angle opposite to the departure being the angle of the course.

Hence, when any two of the four things—Distance, Difference of Latitude, Departure, and Course,—are known, the remaining two may be determined by the solution of a right-angled plane triangle; so that, as far as these particulars are concerned, the results are the same as if the ship were sailing on a plane surface, the meridians being replaced by parallel straight lines, and the perpendiculars to these taken for the parallels of latitude. It is thus that that part of Navigation which is concerned only with the four things just mentioned is called Plane Sailing.

The line BC, or the side of the right-angled triangle opposite to the course, is not the representation of any corresponding line on the globe: it is the entire sum of all the minute departures made by the ship in passing from meridian to meridian, from A up to B.

The foregoing investigation comprehends the whole mathematical theory of Plane Sailing, into which, it will be observed, the consideration of *longitude* does not enter.

The attentive reader will perceive that in replacing the spiral track of a ship's run, and the great circle arc which measures the difference of latitude made good in that run, by the hypotenuse and perpendicular of a right-angled triangle drawn upon a *plane surface*, we sacrifice not the slightest amount of accuracy. It is shown above, that if this spiral track were unbent into a straight line AB, and at one extremity, A, of this straight line a plane angle equal to that of the course were made by AC, and the perpendicular BC drawn,—it is shown that AC must accurately represent the difference of latitude and BC the departure.

It cannot be any objection to this conclusion that we have taken small triangles on the sphere to be *plane* triangles; for the reasoning fixes no limit to the degree of smallness of the sides; nor must it be understood that we have assumed the sphere, on which these triangles are assumed to be figured, to be *itself* a plane. The assumption extends only to the length of supposing the sphere to present a succession of triangular plane faces; and as each face is contracted to any degree of minuteness, the error of this supposition ultimately disappears.

As a corollary to what is proved above, we may add that—In sailing upon a single rhumb, the differences of latitude made are proportional to the distances run. And from the theory of the right-angled triangle established in the Introduction, we have all the proportions usually given on this part of the subject in books on Navigation; they are expressed as follows:—*

I. Radius : sin Course :: Distance sailed : Departure.
II. Radius : cos Course :: Distance sailed : Diff. Latitude.
III. cos Course : Radius :: Diff. Latitude : Distance sailed.
IV. Radius : tan Course :: Diff. Latitude : Departure.
V. Distance sailed : Diff. Latitude :: Radius : cos Course.
VI. Distance sailed : Departure :: Radius : sin Course.
VII. Sin Course : Radius :: Departure : Distance sailed.
VIII. Tan Course : Radius :: Departure : Diff. Latitude.
IX. Diff. Latitude : Departure :: Radius : tan Course.

If the table of natural sines, cosines, &c., be used, then Radius = 1; if the table of logarithmic sines, cosines, &c., be used—and they are employed by seamen too indiscriminately—then log Radius = 10.

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* We have thought it as well to express the rules and formulæ given in the Introduction, in the form of proportions here, as seamen are more accustomed to use them in this shape; but the reader of the Introduction will see that this Rule-of-Three arrangement of the terms employed is not necessary, and he may therefore adopt it or not, as he pleases.

In all books on Navigation the latter tables, exclusively, are referred to; but, as already stated in the INTRODUCTION, we would recommend a departure from this practice: we shall, therefore, in general, exhibit the calculations by *both* tables.

It is not considered necessary, in the examples that follow, to introduce *diagrams* of the several triangles; but the learner should always roughly sketch the suitable triangle himself, observing that, as is usual in maps, the top of the page is to be regarded as North, and the bottom as South; the right-hand side East, and the left West. In sketching his right-angled triangle, therefore, he should first draw the N. and S. line, or the horizontal meridian, and then take a portion of it for the difference of latitude, drawing, from the latitude reached, the base of the triangle, to represent the departure—to the *right* if the departure be east, and to the *left* if it be west. The hypotenuse will then represent the distance sailed, and the angle between it and the difference of latitude, the course. It will be as well to regard the vertex of this angle as at the centre of the compass-card, since it is the centre of the sensible horizon at starting on the course, and thus no mistake can be made as to which side of the meridian line the angle of the course is to lie on, or whether its opening be upward or downward.

Examples.

1. A ship from latitude 47° 30′ N., has sailed S.W. by S., a distance of 98 miles: what latitude is she in, and what departure has she made?

The course being 3 points is 33° 45′ : hence—

1. *To find the diff. lat.*

By logarithms.

As rad.	—10
: cos course, 33° 45′, . . .	9·9199
: : distance = 98	1·9912
: diff. lat. = 81·49 . . .	1 9111

Without logarithms.

diff. lat. = cos course × distance.

cos course, 33° 45′, ...	·8315
distance	98
	66520
	74835
diff. lat.	= 81·487

Hence the difference of latitude is 81·49 miles, S.

2. *To find the departure.*

By logarithms.

As rad.	—10
: sin course, 33° 45′, . . .	9·7447
:: distance = 98	1·9912
: departure = 54·44 . . .	1·7359

Without logarithms.

dep. = sin course × distance.

sin course, 33° 45′, ...	·5556
distance	98
	44448
	50004
	54·4488

Hence the departure is 54·45 miles, W.

The latitude from . . .	47° 30′ N.
The difference of lat. . .	1° 21′ S.
Latitude in	46° 9′ N.

☞ $\frac{81}{60}$ degrees = 1° 21′

Departure 54½ miles, W.

The example is here worked by *computation*; there is another way of obtaining the results—namely, by *inspection*. This latter method requires reference to a table called the Traverse Table, and which is to be found in every collection of navigation tables. The table is arranged much like a table of sines and cosines; the angles of the courses are inserted at the top of the page, when they do not exceed 45°, and at the bottom when they do, and the distances are placed down the margin. By entering the table with any given course and distance, the corresponding difference of latitude and departure can be taken out from the body of the table.

This table is a very useful one for seamen, as it computes for him the two formulæ worked by in the right-hand column of operations above; that is, entering his table with the proper course and distance, he finds, under the respective heads "Lat." and "Dep." the values of

cos course × distance, and sin course × distance

already worked out for him, usually as far as two places of decimals, which is an extent amply sufficient. A table of natural sines and cosines is thus all that is wanted to construct a traverse table. In such a table each sine and cosine is multiplied by all distances from 1 up to 120, which is the ordinary limit to which the columns of distances are carried. Distances which exceed 120 miles may be cut up into smaller distances, and the portions brought within the compass of the table.

2. A ship from latitude 47° 30′ N., sailing N.W. *b* W., finds that she has made 82 miles of departure: What is her distance run, and her latitude in?

The course being 5 points is 56° 15′, hence—

1. *To find the distance.*

By logarithms.

As sin course, 56° 15′ . . . — 9·9198
: radius 10
:: departure = 82 . . . . 1·9138
: distance = 98·62 . . . . 1·9940

Without logarithms.

dist. = dep. ÷ sin course.

sin course, 56° 15′ . . ·8,3,1,5)82 (98·6 (dep.)
7484
716
665
51
50

∴ distance = 98·6 miles.

2. *To find the diff. lat.*

By logarithms.

As tan course, 56° 15′ . . . — 10·1751
: radius 10
:: departure = 82 . . . . 1·9138
: diff. lat. = 54·79 . . . . 1·7387

Without logarithms.

diff. lat. = dep. ÷ tan course.

tan course, 56° 15′ . 1·4,9,6,6)82 (54·8
7483
717
509
118
119

∴ Diff. lat. = 54·8 miles = 55′ N.
Latitude from . . . . 47° 30′ N.
Latitude in . . . . . 48° 25′ N.

3. A ship has sailed S.E. *b* S., from lat. 37° 30′ N. to lat. 46° 8′ N., required th
distance run and the departure made.

The course being 3 points is 33° 45′; also { lat. from 47° 30′ N.
lat. in 46° 8′ N.

diff. lat. 1° 22′ N. = 82 miles.

1. *To find the distance.*

By logarithms.

As cos course, 33° 45′ .	— 9·9198
: radius . . .	10
: : diff. lat. = 82 . .	1·9138
∴ distance = 98·62 .	1·9940

Without logarithms.

dist. = diff. lat. ÷ cos course.
cos course, 33° 45′ ·8,3,1,5)82 (98·62
74835
7165
6652
∴ dist. = 98·62 miles 513
499
14
17

2. *To find the departure.*

By logarithms.

As radius . . .	— 10
: tan course, 33° 45′ .	9·8249
: : diff. lat. = 82 . .	1·9138
: departure = 54·79 .	1·7387

Without logarithms.

dep. = diff. lat. × tan course.
tan course, 33° 45′ . . ·06682
diff. lat. = 82, reversed . 28
53456
1336
departure = 54·792 mile

4. A ship from lat. 50° 13′ N., while sailing on a course between south and ea
a distance of 98 miles, makes 82 miles of departure: what course did she keep, a
what latitude did she arrive at?

1. *To find the course.*

By logarithms.

As dist. = 98 . . .	— 1·9912
: dep. = 82 . . .	1·9138
: : rad.	10
: sin course, 56° 48′ .	9·9226

Without logarithms.

sin course = dep. ÷ dist. = $\frac{82}{98} = \frac{4}{4}$
7)41
7) 5·8571
sin 56° 48′ . . ·8367

Hence the course is S. 56° 48′ E.

2. *To find the diff. lat.*

By logarithms.

As rad. . . . . . — 10
: cos course, 56° 48′ . 9·7384
: : dist. = 98 . . 1·9912

: diff. lat. = 53·66 . 1·7296

Without logarithms.

diff. lat. = dist. × cos course.
cos course, 56° 48′ . ·5476
dist. = 98, reversed . 89

49284
4381

diff. lat. = 53·665 miles = 54′

And 50° 13′ N. — 54′ N. = 49° 19′ N., the lat. in.

5. Yesterday at noon we were in lat. 38° 32′ N.; and this day at noon we were in lat. 36° 56′ N. We have run on a single course between S. and E., at 5½ knots an hour: required our course and departure.

Lat. from 38° 32′ N.
Lat. in 36° 56′ N.

Diff. lat. 1° 36′ N. = 96 miles

2)24, number of hours.
5½

120
12

132 miles, the distance.

1. *To find the course.*

By logarithms.

As dist. = 132 . . — 2·1206
: diff. lat. = 96 . . 1·9823
: : rad. 10

cos course, 43° 20′ . . 9·8617

Without logarithms.

$$\text{cos course} = \text{diff. lat.} \div \text{dist.} = \frac{96}{132} = \frac{8}{11}$$

11)8

cos course, 43° 20′ . . ·7273

∴ the course is S. 43° 20′ E. = S.E.*b*S.¾E. nearly.

2. *To find the departure.*

By logarithms.

As rad. . . . . . . — 10
: sin course, 43° 20′ . . 9·8365
: : dist. = 132 . . . 2·1206

: dep. = 90·58 . . . 1·9571

∴ departure = 90·58 miles E.

Without logarithms.

dep. = sin course × dist.
sin course, 43° 20′ . . ·6862
dist. = 132, reversed . 231

6862
2059
137

departure = 90·58 miles E.

The foregoing examples have all been solved by computation. As remarked at page 11, the same results, though with not precisely the same amount of accuracy, may be obtained by inspection of the Traverse Table. There is also a third method of proceeding, much practised by seamen, though less accurate still, by which the required conclusions may be reached: it is the method of *construction.*[*] A circle is described, and the north and south line, or the horizontal meridian, is drawn through its centre; then from a scale of chords, constructed agreeably to the radius used, which radius is of course the chord of 60° on the scale, the chord of the course is pricked off in its proper direction from the N. or S. extremity of the meridional diameter; from the same extremity a line is then drawn through the point of the circumference before marked: this is the line of distance, or hypotenusal line, and the line already drawn through the centre is the line of difference of latitude: whichever of these is given is now to be measured off from any scale of equal parts, and the right-angled triangle is then to be completed by introducing the third side, which, measured from the same scale, will give the length sought.

In this illustration, the course and one of the sides including it is supposed to be given; but if the course be unknown, and any two of the sides of the triangle given, the mode of proceeding is readily suggested from that above:—in all cases we have two parts of a right-angled triangle given to construct the triangle—a simple geometrical problem. The unmeasured parts are then to be measured, the angle of the course from the scale of chords, and the sides from the scale of equal parts already employed. We shall conclude this article with a few examples for the exercise of the learner: he will not forget that the given courses are always understood to be the true courses, that is, the compass courses corrected for variation, local deviation, and leeway. The means by which the variation of the compass is ascertained cannot be considered here, as the subject belongs to Nautical Astronomy.

Examples for Exercise in Single Courses.

1. A ship from latitude 48° 40′ N. sails N.E. by N. 296 miles: required the departure made and the latitude in.

Ans. Departure 164·4 miles E. Latitude in 52° 46′ N.

2. A ship from latitude 49° 30′ N. sails N.W. by N. 103 miles: required her departure and the latitude in.

Ans. Departure 57·2 miles W. Latitude in 50° 56′ N.

3. A ship from latitude 47° 20′ N., sails on a course between N. and E. a distance of 98 miles, and arrives at lat. 48° 42′ N.: required the course steered and the departure made. Ans. Course N. 33° 12′ E. Departure 53·7 miles E.

4. A ship sails S.E. ½ E. from latitude 15° 55′ S. till she is found by observation to be in latitude 18° 49′ S.: required her distance run, and departure.

Ans. Distance 274 miles. Departure 212 miles E.

5. A ship from latitude 34° 23′ S. sails between the south and west till she reaches latitude 36° 34′ S. and finds that she has made 75 miles of departure: required her course and distance run. Ans. Course S. 29° 47′ W. Distance 151 miles.

6. A ship from latitude 3° 16′ N. sails S.W. by W. ¼ W., till she has made 356 miles of departure: required the distance sailed and latitude in.

Ans. Distance 415 miles. Lat. in 17′ S.

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[*] There is also a fourth method—by GUNTER'S SCALE—which it is not worth while to dwell upon. Any mechanical operation with scale and compasses is necessarily affected with inaccuracy; and as reference to the Traverse Table requires less time, and furnishes truer results, it is always to be preferred next to computation.

7. A ship in latitude 3° 52′ S., is bound to a port bearing N.W. by W. ½ W., in latitude 4° 30′ N. How far does that port lie to the westward, and what is the ship's distance from it? in other words, what departure and distance must the ship make to reach it? Ans. Departure 939 miles W. Distance 1065 miles.

8. A ship, from latitude 42° 18′ N., sails S. 25° W. a distance of 150 miles: required her departure and latitude in.

Ans. Departure 68½ miles W. Lat. in 40° 2′ N.

9. If a ship sails from latitude 48° 27′ S. on a S.W. by W. course at the rate of 7 knots an hour, in how many hours will she arrive at latitude 50° S.?

Ans. In $23\frac{8}{10}$ hours.

10. A ship from latitude 55° 30′ N. sails S.W. by S. for 20 hours, and then finds by observation that she is in latitude 53° 17′ N.: required her hourly rate of sailing, and the departure she has made.

Ans. Rate 8 miles an hour. Departure 88·87 miles W.

11. A ship sails for 18 hours on a single course between the S. and W. from latitude 38° 32′ N. to latitude 36° 56′ N., at the rate of 7⅓ miles an hour: required the course, distance, and departure.

Ans. Course S. 43° 20′ W. Distance 132 miles. Departure 90·58 W.

12. A ship sails for 53¾ hours on a S.E. ¾ E. course from latitude 52° 30′ N. to latitude 47° 10′ N.: required the average rate of sailing per hour, and the departure made. Ans. Rate 10 miles an hour. Departure 432 miles E.

Compound Courses, or Traverse Sailing.—When, from contrary winds or other causes, a ship's track from one place to another is made up of several single courses, the zig-zag path it takes is called a TRAVERSE, or a COMPOUND COURSE; and the determination of the single course and distance, from the place left to the place arrived at, is called *working* or *resolving* the traverse.

To work a traverse, it is only necessary to find the difference of latitude and departure for each distinct course, as in the foregoing article, to take the aggregate of these for the whole difference of latitude and departure, and thence to find the corresponding single course and distance.

The most orderly way of proceeding is to form a little traverse table, consisting of six columns, to receive the proper entries for course, distance, diff. lat. N. and S., and dep. E. and W., as in the specimen in example 1 following. When the entries are completed, the two diff. lat. columns are added up separately, and the difference of the results taken: this difference is the whole diff. lat., which is N. or S. according as the N. or S. column gives the greater result. In like manner, the results of the two departure columns being found, their difference is the resultant departure, to be used with the whole difference of latitude, to determine the direct course and distance.

Examples.

1. A ship from latitude 51° 25′ N. has sailed the following courses, namely,

1st, S.S.E. ¼ E., 16 miles. 2nd, E.S.E., 23 miles.
3rd, S.W. *b* W. ½ W., 36 miles. 4th, W. ¾ N., 12 miles.
5th, S.E. *b* E. ½ E., 41 miles.

Required the lati*tude in, and the direct* course and distance to reach it.

TRAVERSE TABLE.

Courses.	Dist.	Diff. of latitude.		Departure.	
		N.	S.	E.	W.
S.S.E. ¼E.	16		14·5	6·8	
E.S.E.	23		8·8	21·3	
S.W. *b* W. ½W.	36		17		31·8
W. ¾N.	12	1·8			11·9
S.E. *b* E. ¼E.	41		21·1	35·2	
		1·8	61·4	63·3	43·7
			1·8	43·7	
Equivalent course, S. 18° 12′ E. Direct distance 63 miles.			59·6	19·6	

The first two columns of this table are occupied with the given courses and distances; in the other four are inserted the diff. lat. and dep. corresponding to each course and distance, taken by inspection from the Traverse Table. The results of these latter columns show that the difference of latitude made is 59·6 miles S., and the departure 19·6 miles E. And from these the latitude in, and the direct course and distance from the latitude left to the place reached, is found by computation as follows:—

Latitude left	51° 25′ N.
Diff. lat. 59·6 miles	1° 0′ S.
Latitude in	50° 25′ N.

1. *To find the direct course.*

By Logarithms

As diff. lat. = 59·6 . . .	— 1·7752
: departure = 19·6 . .	1·2923
: : radius	10
: tan course, 18° 12′ . .	9·5171

Without logarithms.

tan course = dep. ÷ diff. lat.

5,9·6)19·6(·3288 = tan 18° 12′
1788
172
1192
528
477
51
48

∴ Hence the direct course is S. 18° 12′ E.

It thus appears that if the ship had left her port on the course S. 18° 12′ E., and had kept this course unaltered for a run of 63 miles (see next page), she would have arrived at the place reached by the above traverses. This conclusion, however, is not rigorously true, though near enough for practice. See the remarks subjoined to *example 8.*

2. *To find the distance.*

By logarithms.

As sin course, 18° 12′ . .	— 9·4946
: radius	10
:: departure = 19·6 . .	1·2923
: distance = 62·75 . . .	1·7977

Without logarithms.

dist. = dep. ÷ sin course

sin 18° 12′ = ·3,1,2,3)19·6 (62·75
18738
862
625
237
219
18
16

Hence the nautical distance is 62·75 miles.

NOTE.—It will save time, and be a guard against mistake in the filling up the several columns from the Traverse Table, if, before that table is opened, a mark be put opposite to each course, and in each of the columns where the entries connected with that course are to be inserted. Thus, if N. occur in the course, mark a little cross against it in the N. column, near enough to the right-hand margin of that column to allow of room for the extract from the Traverse Table; if S. occur in the course, put a like mark in the S. column. If E. occur, mark the E. column; and if W. occur, mark the W. column. Then, when the Traverse Table is consulted, we shall have precluded the risk of writing the particulars from it in the wrong column.

2. A ship sails S.W. *b* S. 24 miles; N.N.W. 57 miles; S.E. *b* E.½E. 84 miles; and S. 35 miles: required the direct course and distance.

Ans. Course S. 43° E. Distance 57 miles.

3 A ship from latitude 50° 13′ N. has sailed the following courses, namely—

1st. W.S.W., 51 miles. 2nd. W. *b* N., 35 miles. 3rd. S. *b* E., 45 miles. 4th. S.W. *b* W., 55 miles. 5th. S.S.E., 41 miles. Required the latitude in, and the direct course and distance sailed.

Ans. Lat. in 48° 8′ N. Course S. 39° 19′ W., or S.W. *b* S. ½ W. Dist. 162 miles.

4. A ship from latitude 28° 32′ N. has run the following courses, namely—

1st. N.W. *b* N., 20 miles. 2nd. S.W., 40 miles. 3rd. N.E. *b* E. 60 miles. 4th. S.E. 55 miles. 5th. W. *b* S., 41 miles. 6th. E.N.E. 66 miles. Required the latitude in, and the direct course and distance.

Ans. The same latitude. Course due E. Distance 70·2 miles.

5. From noon to noon the following courses were run, namely—

1st. S.W. *b* S., 20 miles. 2nd. W., 16 miles. 3rd. N.W. *b* W., 28 miles. 4th. S.S.E., 32 miles. 5th. E.N.E., 14 miles. 6th. S.W., 36 miles. What difference of latitude has the ship made, and what is her direct course and distance?

Ans. Diff. lat. 50·7 miles S. Course S.W. Distance 71·7 miles.

6 A ship sails from latitude 10° 6′ S., the following courses, namely—

1st. N.N.E., 86 miles. 2nd. N., 74 miles. 3rd. E. *b* N., 53 miles. 4th. N.N.W. ½ N., 40 miles. 5th. E.N.E. ½ N., 21 miles. Required the latitude in, and direct course and distance. Ans. Lat. in 6° 34′ S. Course N. 23° 25′ E. Distance 231 miles.

7. A ship *from latitude 51° 30′ N., running* at the rate of 8 knots an hour, sails

W.S.W., 3 hours; N.W., $2\frac{1}{2}$ hours; W., 4 hours; S.W. *b* S., $2\frac{1}{4}$ hours; and N.W. $\frac{1}{2}$ W., 2 hours. Required her latitude in, and her direct course and distance.

Ans. Lat. in 51° 30′ N. Course W. Distance 90·7 miles.

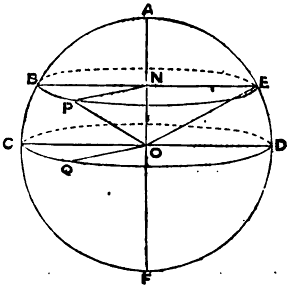
8. A ship from latitude 24° 32′ N., sails the following courses, namely—

1st. S.W. *b* W., 45 miles. 2nd. E.S.E., 50 miles. 3rd. S.W., 30 miles. 4th. S.E. *b* E., 60 miles. 5th. S.W. *b* S. $\frac{1}{4}$ W., 63 miles. Required her latitude in, her departure, and the direct course and distance.

Ans. Lat. in 22° 3′ N. Dep. 0. Course S. Distance 149·2 miles.

In this last example, the ship is said to have returned to the meridian from which she sailed, so that her course from the place arrived at to that left, is concluded to be due south. In the present case, the latitude being low, the error of this conclusion is practically of no consequence; but the balance, or aggregate of the several departures made on a traverse, is not, in strictness, the departure due to the direct course—a fact that sailors, in general, are not sufficiently sensible of. We shall advert more at length to this matter in our introductory observations to MERCATOR'S SAILING.

Parallel Sailing.—When a ship sails upon a parallel of latitude, her distance run is then the same as her departure; her difference of latitude is nothing, and her difference of longitude may be easily determined. The case is one of *parallel sailing*, the theory of which may be established in the following manner:



Taking the diagram at page 252 of the Mathematical volume of the CIRCLE OF THE SCIENCES, let C Q represent a portion of the equator corresponding to the portion B P of a parallel of latitude sailed over by a ship, the points P, Q being on the same meridian. Then O C is the radius of the equator, and N B the radius of the parallel, and the difference of longitude of B and P will be measured by the arc C Q.

Now, since similar arcs are to one another as the radii of the circles to which they belong, we have

$$\text{N B} : \text{O C} :: \text{dist. B P} : \text{diff. long. C Q}.$$

But N B is the geometrical cosine of the latitude C B, to the radius O C; consequently N B is equal to O C multiplied by the trigonometrical cosine of the angle Q O P of the latitude, that is, expressing the latitude in degrees and minutes, and not in linear measure, we have

$$\text{O C cos lat.} : \text{O C} :: \text{dist. sailed} : \text{diff. long.} \quad . \quad . \quad (1)$$

$$\therefore \text{cos lat.} : 1 :: \text{dist. sailed} : \text{diff. long.} \quad . \quad . \quad (2)$$

And it follows from this, that if the distance between any two meridians on a parallel in latitude L, be D, and the distance of the same meridians on a parallel in latitude L′ be D′, then alternating the proportion (1)

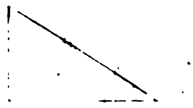
$$\cos L : \cos L' :: D . D' \quad . \quad . \quad (A)$$

(See the GEOMETRY, page 134, Prop. 2.)

The proportion (2) evidently solves the problem—Given the latitude of the parallel and the distance sailed on it, to find the difference of longitude; the solution being

$$\textit{difference of longitude} = \frac{\text{distance sailed}}{\text{cos latitude}} \quad . \quad . \quad (3)$$

So that, as in the former cases, we may connect the three things concerned in a right-angled plane triangle, the base representing the distance sailed, the hypotenuse, the difference of longitude, and the angle between the two the latitude of the parallel, because we know from the theory of the right-angled triangle that these three parts are related as in condition (3). Any problem in parallel sailing may, therefore, always be reduced to a case of right-angled triangles; and consequently may be solved, like a problem in plane sailing, by inspection of the Traverse Table. We shall only have to consider the latitude of the parallel as *course*, and the distance as *diff. lat.*; the corresponding distance in the Traverse Table will be the *diff. long.*



It must be observed, however, that the perpendicular of our right-angled triangle merely serves to connect together the three things here mentioned; it has no signification in navigation. The distance sailed on a parallel, the latitude of that parallel, and the difference of longitude between the place left and that arrived at, are related to one another as the three parts, noticed above, of a right-angled triangle, the perpendicular of which merely serves to complete the diagram in which these relations are geometrically embodied.

The proportion (2) above, is usually expressed thus:—

cos lat. : radius : : dist. sailed : diff. long.

where rad. = 1 when the table of natural sines and cosines is used; and log rad. = 10 when the logarithmic table is used. As usual, we shall exhibit the working by both tables; but it will be perceived, as in most of the computations already given, that the former table is in general to be preferred.

Examples.

1. A ship from latitude 53° 36′ N. longitude 10° 13′ E., sails due west 236 miles: required the longitude in.

To find the diff. long.

By logarithms.

As cos lat.,	53° 36′ . . .	—9·7734
:	radius . . .	10
: :	distance = 236 .	2·3729
.	diff. long. = 397·7 .	2·5995

Without logarithms.

diff. long. = dist. ÷ cos lat.

```
[cos 53° 36′ = ·5,9,3,4)236   (397·7
                       17802
                       -----
                        5798
                        5341
                        ----
                         457
                         415
                         ---
                          42
                          42
                          --
```

Hence the diff. long. is 397·7 miles = 398 miles nearly.

Reducing this to degrees, 60)398

diff. long. = 6° 38′ E. } The difference to be taken, as the lon-
long. left . . 10° 18′ W. } gitudes are E. and W.

long. in . . . 3° 40′ E.

2. A ship from latitude 32° N., sails due east till her difference of longitude is found to be 384 miles: what distance has she run?

To find the distance.

By logarithms.		Without logarithms.
As radius	.—10	Dist. = diff. long. × cos lat.
: cos lat., 32°	9·9284	cos 32° = ·8480
:: diff. long. = 384 . . .	2·5843	384 reversed, 483
: distance = 325·6 . .	2·5127	2544
		678
		34
		325·6

Hence the distance run is 325·6 miles.

3. From two ports in latitude 32° 20′ N., distant 256 miles, measured on the parallel, two ships sail directly north, till they come to the latitude 44° 30′ N.: how many miles, measured on the parallel arrived at, are they apart?

This question is to be worked by using the proportion (A).

By logarithms.		Without logarithms.
As cos lat. from, 32° 20′ Arith. Comp.	·0732	$D' = D \cos L' \div \cos L$
: cos lat. in, 44° 30′ . . .	9·8532	$\cos L' = \cos 44°\,30' = \cdot 7133$
:: first dist. = 256 . . .	2·4082	D = 256, reversed, 652
: secd. dist. = 216·1 . . .	2·3346	14266
		3567
		428
		cos L = cos 32° 20′ = ·8,4,5,0)182·61(216·1
		1690
		1361
		845
		516
		507
		9

Hence, measured on the parallel of 44° 30′ N., the ships are 216 miles apart. Their *least* distance apart is the arc of the great circle from one to the other; because an arc of a great circle of the sphere is the shortest distance between its extremities.

4. A ship from latitude 42° 54′ S. longitude 9° 16′ W., sails due west 196 miles: required her longitude in. Ans. 13° 44′ W.

5. A ship has sailed due east for 3 days on the parallel of 43° 28′; her rate of sailing has been, on the average 5 knots an hour. What difference of longitude has she made? Ans. 8° 16′ E.

6. A ship from longitude 81° 36′ W. sails due west 310 miles, and is then found by observation to be in longitude 91° 50′ W.: on what parallel has she sailed?
Ans. The parallel of 59° 41′.

7. In what parallel of latitude is the length of a degree only one-third the length of a degree at the equator? Ans. Lat. 70° 32′.

8. Two ships in latitude 47° 54′ N., but separated by 9° 35′ of longitude, both sail directly south 836 miles, and at the same rate: how many miles were they apart at starting, and how many after running the 836 miles?

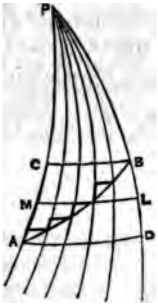
Ans. First distance 385⅓ miles; second distance 477 miles.

9. If two ships in latitude 44° 30′ N., and distant from each other 216 miles, were both to sail, at the same rate, directly south until their distance on the parallel arrived at, became 256 miles, what latitude would they be in? Ans. 32° 17′ N.

10. If a ship sail due east 126 miles, from the North Cape in latitude 71° 10′ N., and then due north, till she reaches latitude 73° 26′ N., how far must she sail west to reach the meridian from which she started? Ans. 111·3 miles.

Middle Latitude Sailing.—It has already been sufficiently seen that the principal object of plane sailing is to determine the difference of latitude made by a ship sailing upon an oblique rhumb. This sailing gives us no information respecting the change made in the ship's longitude; but if the rhumb sailed upon, instead of being oblique to the meridians crossed by it, cuts them all at right angles, as in parallel sailing, then, as just shown, the difference of longitude made may be accurately ascertained. Except in this particular case, the determination of the change of longitude made by a ship in sailing from one place to another, is a problem the strict solution of which is by no means easy. Independently of astronomical observations, there are two modes of proceeding:—one is called *Middle Latitude Sailing*, and can be regarded only as a close approximation to the truth, unless a certain correction, hereafter given, be applied to it.* The other is called *Mercator's Sailing*, and by this the problem is solved upon strict mathematical principles. Middle latitude sailing is a combination of plane sailing and parallel sailing, which are united in the following way:—

It has been seen in the theory of plane sailing, that the line called the departure is a line equal to the sum of all the elementary departures made by a ship in sailing on an oblique rhumb. Thus, if AB in the annexed diagram, be the distance sailed, the departure is made up of all the elementary portions of the parallels of latitude lying between AD and CB; it is plain, therefore, that the departure is less than AD, and greater than CB, since the meridians approach closer together the nearer the parallel is to the pole: there is, therefore, some parallel, ML, between A and C, such that the portion ML would be exactly equal to the departure; and that, in latitudes near the equator, this parallel ML cannot differ materially from the *middle* parallel. It is on the supposition that the departure is equal to the distance between the meridians left and arrived at, measured on the middle parallel, that middle latitude sailing is founded.



From a mere inspection of the diagram,—or, better still, of a common globe,—it is obvious that this supposition can differ but very little from the truth for low latitudes, and for such short distances, AB, as a day or two's run; and more especially if the angle of the course be large, so that but little difference of latitude is made, and therefore the parallels AD,

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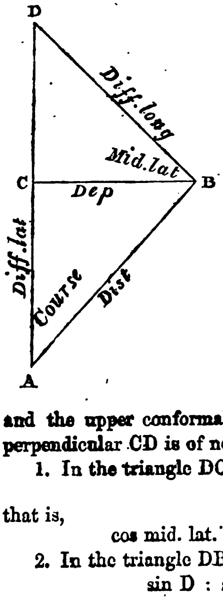
* In the year 1805, Mr. Workman published, under the sanction of the then Astronomer Royal, Dr. Maskelyne, a small and very useful table for correcting the errors of middle latitude sailing. The table is even now scarcely so well known as it ought to be; and as it removes the only objection to this mode of finding the difference of longitude, we have inserted it, a little abridged, at page 59, and would strongly rec*ommend it to the notice of* the practical navigator.

CB, pretty close together. In such favourable cases, the method of middle latitude sailing—though, if uncorrected, only an approximation to the truth—is preferable to the method of Mercator's sailing, though this is theoretically accurate, for reasons that will be hereafter shown. In high latitudes, however, this method is not to be depended on, at least for more than a single day's run, if the angle of the course be small, and the middle latitude be uncorrected; because the interval between the latitude left and that reached may be too wide to warrant the supposition that the departure is equal to the middle parallel between the meridians left and arrived at. But if the middle latitude be corrected by Workman's table, page 59, all objection will be obviated.

Investigation of the Rules for Middle Latitude Sailing.—Let AB in the preceding diagram be the track of the ship, then the difference of longitude made will be the same as if the ship had sailed from M to L, along the middle parallel ML. By the present hypothesis, the distance on this middle parallel is the departure made in running from A to B; hence, the departure being known by plane sailing, we know the length of the parallel ML: and we know, also, the latitude of that parallel. Consequently the difference of longitude may be found as in parallel sailing. Thus, if in the right-angled triangle BCD, the base represent the departure, that is ML, and the angle at the base be made equal to the latitude of ML, then the hypotenuse will represent the difference of longitude between M and L, that is, between A and B. As the base of the right-angled triangle represents the departure made, we may connect with it, as in the annexed diagram, the difference of latitude AC, and the distance AB, as in plane sailing. We shall thus have a sort of double triangle; in one triangle (the lower one here) will be represented the diff. lat. AC, the dist. AB, the angle A of the course, and the departure CB, equal in length to ML in the preceding diagram. In the other triangle will be represented by CB, the distance ML in the preceding diagram, the diff. long. BD, of M and L, and the angle CBD of the mid. lat.; the lower triangle being constructed conformably to the principles of plane sailing, and the upper conformably to the principles of parallel sailing. In the latter the perpendicular CD is of no significance.



1. In the triangle DCB we have,

$$\cos DBC : \text{radius} :: BC : DB,$$

that is,

$$\cos \text{mid. lat.} : \text{radius} :: \text{departure} : \text{diff. long.} \quad . \quad . \quad . \quad (1).$$

2. In the triangle DBA we have,

$$\sin D : \sin A :: AB : BD \qquad (\text{Introduction, p. 21})$$

that is,

$$\cos \text{mid. lat.} : \sin \text{course} :: \text{distance} : \text{diff. long.} \quad . \quad . \quad (2).$$

3. Also, in the two triangles ABC, DBC, we have,

$$AC \tan A = BC,\ BD \cos DBC = BC,$$

$$\therefore AC \tan A = BD \cos DBC, \text{ consequently} \qquad (\text{Algebra, p. 218})$$

$$AC : BD :: \cos DBC : \tan A,$$

that is,

$$\textit{diff. lat.} \ . \ \textit{diff. long.} :: \cos \text{mid. lat.} : \tan \text{course.} \quad . \quad . \quad (3).$$

The proportions marked (1) (2) (3) comprehend the entire theory of middle latitude sailing. It is scarcely necessary to mention that the middle latitude is half the difference of the extreme latitudes when both are north or both south, and half their sum when one is north and the other south.

The three proportions given above may be united in one set of equations, by which mode of expressing them, they will, perhaps, appear in a form more convenient for memory. They are as follows:—

$$\text{diff. long.} = \frac{\text{departure}}{\text{cos mid. lat.}} = \text{departure} \times \text{sec. mid. lat.}$$

$$= \frac{\text{dist.} \times \text{sin course}}{\text{cos mid. lat.}} = \text{dist. sin course sec. mid. lat.}$$

$$= \frac{\text{diff. lat.} \times \text{tan course}}{\text{cos mid. lat.}} = \text{diff. lat.} \times \text{tan course} \times \text{sec. mid. lat.}$$

But, by imprinting upon the mind the two connected triangles in the preceding diagram, any recurrence to formulæ will not be necessary on the part of any one familiar with what is taught in the INTRODUCTION to this Treatise; and this is one advantage of making the plane triangle subservient to the purposes of Navigation. However, without reference to the triangle the single equation, $\text{diff. long.} = \frac{\text{departure}}{\text{cos mid. lat.}}$ embodies all that is peculiar in middle latitude sailing; the other equations are, in fact, implied in this: instead of *departure*, the equivalent to it is substituted.

1. A ship from latitude 51° 18′ N., longitude 9° 50′ W., steers S. 33° 8′ W., till she has run 1024 miles; required the latitude and longitude in.

1. *To find the difference of latitude.*

By logarithms.		Without logarithms.	
As radius	— 10	diff. lat. = dist. × cos course	
: cos course, 33° 8′ . .	9·9229	cos 33° 8′	·8374
:: dist. = 1024 . .	3·0103	1024 reversed	4201
: diff. lat. = 857·5 . .	2·9332		8374
			167
			34
			857·5

∴ the difference of latitude is 857½ miles.

2. *To find the middle latitude.*

6,0) 85,7

14° 17′ S. = diff. latitude
51° 18′ N. = latitude left

37° 1′ N. = latitude in
2) 88° 19′ = Sum of the latitudes

44° 9½′ = middle latitude

3. *To find the difference of longitude* (*Proportion* 3).

As cos mid. lat., 44° 9½′ Arith. Comp. ·1442
: tan course 33° 8′ . . 9·8147
:: diff. lat. = 857·5 . . 2·9332
: diff. long. = 780·1 . . 2·8921

6,0)78,0
13° W. diff. long.
9° 50′ W. long. left
22° 50′ W. long. in.

Hence the place of the ship is lat. 37° 1′ N., long. 22° 50′ W. We shall now work the last proportion by using the middle latitude as corrected by the table at page 59.

Under the diff. lat. 14°, at the top of the table, and in a line with 44°, the middle latitude, we find the correction 27′; which, added to 47° 9½′, gives 44° 36½′ for the corrected middle latitude.

As cos corrected mid. lat., 44° 36½′ Arith. Comp. . ·1476
: tan course 33° 8′ . . . . . 9·8147
:: diff. lat. = 857·5 . . . . . 2·9332
: diff. long. = 786·2 . . . . . 2·8955

∴ diff. long. = 13° 6′ ∴ 9° 50′ + 13° 6′ = 22° 56′ W. long. in.
Hence the error in longitude, by taking the uncorrected middle latitude, is 6 miles.

The construction of the table here made use of, cannot be explained till we come to Mercator's Sailing. But the necessity for some such table may be readily made apparent; for since, as shown above,

diff. long. = departure × sec. mid. lat.,

it follows that if there be any error in our estimation of the departure—that is, in our considering it to be exactly equivalent to the middle latitude distance between the meridians, there will be a still greater error in the resulting difference of longitude because a *secant* is always greater than unity, when the angle to which it relates is of any value at all; and the greater the angle the greater the error. In high latitudes, therefore, where the middle latitude is considerable, the error in longitude, if left uncorrected, may be seriously wide of the truth. For example, when the difference of latitude is 20°, and the middle latitude 72°, the error in longitude would amount to nearly half a degree; that is, to nearly 30 miles. Mr. Workman's table shifts the middle latitude parallel a little higher up, as the mid. latitude parallel a little exceeds the departure in length; as will be shown presently.

2. A ship from latitude 49° 57′ N., and longitude 5° 11′ W., sails between the south and west till she arrives in latitude 38° 27′ N., and finds that she has made 440 miles of departure: required the course steered, the distance run, and the longitude in.

1. *To find the diff. lat. and mid. lat.*

Latitude left, 49° 57′ N.
Latitude in, 38° 27′ N.

Sum 88° 24′ ÷ 2 = 44° 12′ = mid. lat.
Diff. 11° 30′ = 690 miles = diff. lat.

2. *To find the course.*

By logarithms.

As diff lat = 690	— 2·8388
: departure = 440	2·6435
: : radius	10
: tan course 32° 32′	9·8047

Hence the course is 32° 32′.

Without logarithms.

tan course = dep. ÷ diff. lat. = $\frac{44}{69}$

69)44 (·6377 = tan 32° 32′
414
26
207
53
483
47

3. *To find the distance.*

By logarithms.

As sin course, 32° 32′ . . .	— 9·7306
: radius	10
: : departure = 440	2·6435
∴ distance = 818·3	2·9129

Hence the distance is 818 miles.

Without logarithms.

dist. = dep. ÷ sin course
sin 32° 32′ = 5,3,7,7,9) 440 (818·2
43023
977
538
439
430
9

4. *To find the difference of longitude* (Proportion 1).

As cos correct mid. lat. 44° 28′	— 9·8535
: radius	10
: : departure = 440 . .	2·6435
: diff. long. = 616·6 . .	2·7900

Longitude left	5° 11′ W.
Diff. long. 617 miles, = .	10° 17′ W.
Longitude in	15° 28′ W.

If the mid. lat. had not been corrected, the long. would have been about 3′ in error.

A ship from latitude 51° 18′ N., longitude 9° 50′ W., sails S. 33° 19′ W. until her departure is 564 miles: required the latitude and longitude in, and the distance sailed.

1. *To find the difference of latitude.*

By logarithms.

As tan course, 33° 19′ . .	— 9·8178
: radius	10
: : departure = 564 . .	2·7513
: diff. lat. = 858 . .	2·9335

Hence the diff. lat. is 858 miles.

Without logarithms.

diff. lat. = dep. ÷ tan course.
tan 33° 19′ = ·06,5,7)564 (858
5256
384
329
55
52

2. *To find the middle latitude.*

6,0)858

14° 18′ S. = diff. latitude
51° 18′ N. = latitude left

37° 0′ = latitude in
2)88° 18′ = sum of the latitudes

44° 9′ = middle latitude

3. *To find the distance sailed.*

By logarithms.

As sin course, 33° 19′ . . . — 9·7398
: radius 10
: : departure = 564 . . . 2·7513

: distance = 1027 . . . 3·0115

Without logarithms.

dist. = dep. ÷ sin course
sin 33° 19′ = ·5,4,9,3)564 (1027
5493
147
110
37
38

Hence the distance sailed is 1027 miles.

4. *To find the difference of longitude* (*Proportion* 1).

As cos correct mid. lat. 44° 36′ — 9·8525
: radius 10
: : departure = 564 . . . 2·7513

: diff. long. = 792·1 . . . 2·8988

6,0)79,2

13° 12′ W. diff. long.
9° 50′ W. long. left

23° 2′ W. long. in

The error in longitude here, by using the uncorrected middle latitude, would be six miles.

4. A ship in latitude 51° 18′ N., longitude 22° 6′ W., is bound to a place in the S.E. quarter, of which the nautical distance is 1024 miles, and the latitude of it 37° N.: required the direct course, as also the difference of longitude between the two places.
Ans. Course S. 33° 5′ E. Diff. long. 786½ miles.

5. A ship from St. Michael's in the Azores, lat. 37° 48′ N., long. 25° 10′ W., is bound for the Start Point, lat. 50° 13′ N., long. 3° 38′ W.: required her course and nautical distance. Ans. Course N. 51° 7′ E. Distance 1187 miles.

6. A ship sails in the N.W. quarter 248 miles, till her departure is 135 miles, and her difference of longitude 310 miles: required her course, the latitude left, and the latitude in. Ans. Course N. 32° 59′ W. Lat. left 62° 27′ N. Lat. in 65° 55′ N.

7. A ship from latitude 38° 42′½ N., long. 9° 8′½ W., sails W.S.W.½W. 168 miles: required her latitude and longitude. Ans. lat. 37° 54′ N., long. 12° 33′ W.

8. A ship sails from the Lizard, lat. 49° 57′ 44″ N., long. 5° 11′ 5″ W., for a distance of 700 miles on a W.S.W. course: required the latitude and longitude in.
Ans. Lat. in 45° 29′ 50″ N. Long. in 21° 13′ 33″ W.

TABLE OF CORRECTIONS TO BE ADDED TO THE MIDDLE LATITUDE.

Mid. Lat.	DIFFERENCE OF LATITUDE.*													
	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16
°	'	'	'	'	'	'	'	'	'	'	'	'	° '	° '
15	2	3	4	6	9	12	15	19	23	27	31	35	0 40	0 45
16	2	3	4	6	9	12	15	18	22	26	30	34	0 38	0 43
17	2	3	4	6	8	11	14	17	21	25	28	32	0 37	0 42
18	2	3	4	6	8	11	14	17	20	24	27	31	0 36	0 41
19	2	3	4	6	7	10	13	16	19	23	26	30	0 34	0 40
20	2	3	4	6	7	9	12	15	18	22	25	29	0 33	0 38
21	2	3	4	6	7	9	12	15	18	21	25	29	0 33	0 37
22	2	3	4	6	7	9	12	15	17	20	24	28	0 32	0 36
23	2	3	4	6	7	9	12	15	17	20	24	28	0 32	0 36
24	2	3	4	6	7	9	11	14	16	19	23	27	0 31	0 35
25	2	3	4	5	7	9	11	14	16	19	23	27	0 31	0 35
26	2	3	4	5	7	9	11	14	16	19	22	26	0 30	0 34
27	2	3	4	5	7	8	11	14	16	19	22	26	0 30	0 33
28	2	3	4	5	6	8	10	13	15	18	21	25	0 29	0 33
29	2	3	4	5	6	8	10	13	15	18	21	25	0 29	0 32
30	2	3	4	5	6	8	10	13	15	18	21	25	0 28	0 32
31	2	3	4	5	6	8	10	13	15	18	21	25	0 28	0 32
32	2	3	4	5	6	8	10	13	15	18	21	25	0 28	0 31
33	2	3	4	5	6	8	10	13	15	18	21	24	0 27	0 31
34	2	3	4	5	6	8	10	13	15	18	21	24	0 27	0 31
35	2	3	4	5	6	8	10	13	15	18	21	24	0 27	0 31
36	2	3	4	5	6	8	10	13	15	18	21	24	0 27	0 31
37	2	3	4	5	6	8	10	13	15	18	21	24	0 27	0 31
38	2	3	4	5	6	8	10	13	15	18	21	24	0 27	0 31
39	2	3	4	5	6	8	10	13	15	18	21	25	0 28	0 32
40	2	3	4	5	6	8	10	13	15	18	22	25	0 28	0 32
41	2	3	4	5	6	8	10	13	15	18	22	25	0 28	0 32
42	2	3	4	5	6	8	10	13	15	18	22	26	0 29	0 33
43	2	3	4	5	7	9	11	14	16	19	23	26	0 30	0 34
44	2	3	4	5	7	9	11	14	16	19	23	27	0 30	0 34
45	2	3	4	5	7	9	11	14	16	19	23	27	0 31	0 35
46	2	3	4	5	7	9	11	14	16	19	23	27	0 31	0 35
47	2	3	4	5	7	9	11	14	16	20	23	27	0 31	0 35
48	2	3	4	5	7	9	11	14	16	20	23	27	0 31	0 35
49	2	3	4	5	7	9	11	14	17	21	24	28	0 32	0 36
50	2	3	4	5	7	9	11	14	17	21	24	28	0 32	0 36
51	2	3	4	5	7	9	11	14	17	21	24	28	0 32	0 37
52	2	3	4	5	7	9	12	15	18	22	25	29	0 33	0 37
53	2	3	4	6	7	9	12	15	18	22	25	29	0 33	0 38
54	2	3	4	6	8	10	13	16	19	23	26	20	0 34	0 39
55	2	3	4	6	8	10	13	16	19	23	26	30	0 35	0 40
56	2	3	4	6	8	10	13	16	20	24	27	31	0 36	0 41
57	2	3	4	6	8	11	14	17	20	24	28	32	0 37	0 42
58	2	3	4	6	9	11	14	17	21	25	29	33	0 38	0 44
59	2	3	4	6	9	12	15	18	22	26	30	34	0 39	0 45
60	2	3	4	6	9	12	15	19	23	27	31	35	0 40	0 46
61	2	3	5	7	9	12	15	19	23	27	31	36	0 41	0 47
62	2	3	5	7	9	12	16	20	24	28	32	37	0 42	0 49
63	2	4	5	7	9	13	16	20	24	29	33	39	0 44	0 51
64	2	4	6	8	9	13	17	21	25	29	34	40	0 46	0 53
65	2	4	6	8	10	13	17	21	25	30	35	41	0 48	0 55
66	2	4	6	8	10	14	18	22	26	31	37	43	0 50	0 58
67	2	4	6	8	11	15	18	23	27	33	38	45	0 53	1 0
68	2	4	6	8	11	15	19	24	28	34	40	48	0 55	1 2
69	2	5	6	9	12	16	20	25	30	36	44	50	0 58	1 5
70	3	5	6	9	13	17	21	26	31	38	44	52	1 0	1 8
71	4	6	7	9	13	18	22	27	33	40	46	55	1 3	1 12
72	4	6	8	10	14	19	23	29	35	42	49	58	1 6	1 16

* *When the difference of latitude is 2° and under 3°, add 1'.*

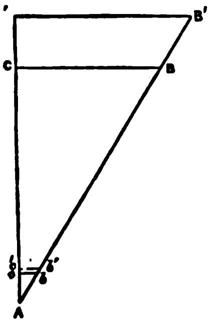
Mercator's Sailing.—The two great problems of navigation are the determining the latitude and longitude of a ship at sea; the other demands of the science are easily provided for. Of the two leading problems mentioned, the former is, in general, by far the more simple; as, by the aid of the imaginary line called the departure, we may readily discover it, either by construction or computation, by means of the common theory of the right-angled triangle, whenever the course and distance sailed are taken account of.

The departure, as used in plane sailing, is the occasion of some error in certain of the results of navigation: for instance, in the determination of the direct course and distance, as the result of a traverse. If a ship sail on several courses, and her departures east just balance her departures west, we conclude that she has returned to her meridian; but it is plain, from the principles of middle latitude sailing, that the amount of departure that would bring a ship in north latitude to any given meridian on a course inclined to the *north*, would be less than the departure necessary to bring her to the same meridian on a course inclined to the *south;* indeed, the first principles of plane sailing show that the elementary departures in the latter case exceed those in the former. And hence in a traverse, the resultant of the departures will not be the correct departure due to the resultant difference of latitude; and consequently the resultant single course will not, in strictness, be the true one.

In a day's run, the error will no doubt be in general of but little moment, and not worth taking note of practically, when we consider that the courses steered are not rigidly determinable; but it is well to apprise the learner of the mathematical shortcomings of our proceedings; and more especially to forewarn him that if *longitude* is deduced from the balance of departures, even for a single day's run, in high latitudes—latitudes, for instance, above 52° or 53°—the result may be sensibly erroneous; and in latitudes of from 60° to 70° the error may be such as to endanger the safety of the ship. Unfortunately sailors are, in general, so much guided by prejudice, and so unwilling to adopt innovations, as they think certain improvements to be, that the common practice still is to confide as implicitly in the resultant departure of a set of departures, as in the departure made in a single course. And it is probably in deference to this erroneous impression, that even the most popular books on navigation are silent on the subject. In such books the middle latitude distance of the meridians is still taken for the correct departure; though, as noticed above, the resulting error in longitude may amount to so much as 30 miles.

The place of a ship in reference to any given meridian, can be determined correctly only by correctly finding her longitude; and this is done by Mercator's sailing in a very ingenious manner, as we shall now show.

Investigation of the Theory of Mercator's Sailing.—When a ship sails upon an oblique rhumb, it has already been shown that the difference of latitude, the departure, and the distance run, are all truly represented by the sides AC, CB, AB of a plane triangle, the angle A being that of the course. The departure, CB is not the representative of any line on the sphere: it is the equivalent of all the minute departures in the diagram at page 8, united in one continuous line. Let *Abc*, in the annexed diagram, be one of the elementary triangles figured in the representation just referred to, *cb* being one of the elementary departures, and *Ac* the corresponding difference of latitude. Now *cb* being a small portion of a parallel of latitude, it will be to a similar portion of the equator, or of the meridian, as the cosine of its latitude to radius, as was proved at page 19; and this similar portion of the equator or of the meridian, measures the difference of longitude between c and b. Suppose the distance Ab prolonged to b', till the departure $c'b'$ is equal to this difference of longitude: then we shall have



$cb : c'b' :: \cos$ lat. of cb : 1 (the trig. radius).

But $cb : c'b' ::$ Ac : Ac' (Euclid Prop. 4, VI.)

$\therefore$ Ac : Ac' :: cos lat. of cb : 1 $\therefore$

$$\therefore \text{A}c' = \frac{\text{A}c}{\cos \text{lat. of } cb} = \text{A}c \times \sec \text{lat. of } cb \quad \ldots\ldots \quad \text{(I)}$$

That is, the proper difference of latitude, Ac must be increased to Ac' = Ac × sec lat. in order that the proper departure, cb, may be increased to an amount, $c'b'$, equal to the difference of longitude of c and b; in other words, the ship having made the small difference of latitude Ac, and the corresponding departure cb, must continue her course till the difference of latitude has increased to Ac × sec lat. of c, in order that her increased departure, $c'b'$, may give her difference of longitude made in sailing from A to b.

Suppose now that all the elementary distances are prolonged in this manner; it is then obvious that the sum of all the corresponding increased departures will necessarily be the whole difference of longitude made by the ship during its course from A to B. Hence, to represent the difference of longitude between A and B, we must prolong the difference of latitude AC, till the length AC′ becomes equal to the sum of all the increased elementary differences of latitude; this done, it follows that the departure C′B′, due to this increased difference of latitude, will represent the difference of longitude made in sailing from A to B.

It is self-evident that, as the departure CB, actually made by the ship, is always *less* than the difference of longitude made, there must be some more advanced departure C′B′, that would be exactly *equal* to this difference of longitude. The object of the foregoing reasoning is to discover how the increased difference of latitude, due to this departure, is to be ascertained.

Now the finding the length of AC′ implies the finding of all its elementary parts: suppose we take each of the elementary parts of AC equal to 1′, that is, that we regard Ac to be 1 nautical mile; then Ac' will be equal to 1′ × sec lat. of c. And, generally, if l be the latitude of any point in AC, the length of a minute of latitude, terminating in that point, will be increased to 1′ × sec l.

These enlarged portions of latitude are called MERIDIONAL PARTS; so that we have

$$\begin{aligned}
\text{Meridional parts of } 1' &= \sec 1' \\
2' &= \sec 1' + \sec 2' \\
3' &= \sec 1' + \sec 2' + \sec 3' \\
4' &= \sec 1' + \sec 2' + \sec 3' + \sec 4' \\
5' &= \sec 1' + \sec 2' + \sec 3' + \sec 4' + \sec 5' \\
\&\text{c.} & \qquad\qquad \&\text{c.}
\end{aligned}$$

Consequently, the meridional parts, or the proper enlargement of every portion of the meridian, measured from the equator up to any latitude, may be calculated by help of a table of natural secants: thus—

	Nat. sec.		Mer. parts.
Mer. parts of 1′ =	1·0000000		= 1·0000000
2′ =	1·0000000	+ 1·0000002	= 2·0000002
3′ =	2·0000002	+ 1·0000004	= 3·0000006
4′ =	3·0000006	+ 1·0000007	= 4·0000013
5′ =	4·0000013	+ 1·0000011	= 5·0000024
	&c.	&c.	&c.

If, therefore, a ship leave the equator, and sail upon any course till her latitude becomes 1′ and we desire to know how much she must increase her latitude in order that her corresponding increased departure may be equal to her advance in longitude, we find, from the above, that no further advance in latitude is to be made; for the difference between the departure corresponding to the 1′ of latitude already made, and the advance in longitude is, as might be expected, insensible; or to speak more rigorously, the difference is too minute to have any numerical value within the limits of seven places of decimals. If her latitude become 2′, her corresponding departure falls short of her advance in longitude by a quantity so small that she has only to increase her latitude by ·0000002 miles to render her departure exactly equal to her difference of longitude.

In like manner, when 3′ of latitude are made, a further advance in latitude to the extent of ·0000006 miles is all that must be made to render her departure the same as her difference of longitude due to the 3′ of latitude. And in this way may a table of meridional parts be calculated, minute by minute. If we enter such a table with the latitude sailed from and the latitude arrived at, and subtract the meridional parts for the lower latitude from the meridional parts for the higher, the remainder will be the *meridional difference of latitude*, or the line AC′ in the preceding diagram. If the latitude in be on the contrary side of the equator to the latitude left, then, of course, the *sum* of the meridional parts for the two latitudes will be the meridional difference of latitude.

Having thus obtained from the table the meridional difference of latitude (that is the line AC′), the difference of longitude (that is the line C′B′) is then deduced by this proportion, namely—

As radius (1) is to the tangent of the course, so is the meridional difference of latitude to the difference of longitude; or, if instead of the course, the departure (C B) be given, then the proportion will be—

As the proper difference of latitude is to the departure, so is the meridional difference of latitude to the difference of longitude.

Such are the correct principles upon which a true sea chart is constructed; but whether Gerrard Mercator, the Flemish chart-maker, was really in possession of these principles, or whether he arrived at their practical results by a series of happy mechanical experiments, is not positively known; he never divulged the methods by which he proceeded, and his secret descended with him to the grave. The first chart he published was in 1556; and it was not till the year 1590 that the true theory of its construction was expounded. This was done by Edward Wright, Fellow of Caius College, Cambridge, who did not, however, communicate his discovery to the public till the year 1590, when his "Certain Errors in Navigation Detected and Corrected," appeared; yet the key to the entire secret is furnished by the single equation marked (I) at page 29, a property so obvious, that we cannot but suppose it must have been noticed by numbers of persons engaged in these investigations, before that time; but, like the common occurrence of the falling of an apple from a tree, it was not perceived to be the germ of a theory destined to change the aspect of an important department of science, till it arrested the attention of a superior mind.

Edward Wright constructed his table of meridional parts after the process described above; that is, by actually adding secant after secant through every minute of the quadrant. But it was subsequently shown by Dr. Halley, that the meridional parts might be accurately obtained in another way—we shall explain how at the end of the present article.

It is proper to remark that a table, constructed agreeably to Mr. Wright's plan, will be more mathematically correct the smaller the elementary portions of the meridian are made; as, for instance, if the portions, instead of a minute, be only half a minute in length. Such a table was accordingly constructed by Oughtred, and further extended by Sir Jonas Moore; but the modern table of meridional parts is based upon the strictly accurate theory of Dr. Halley. Mr. Wright, however, was fully sensible that, from the intervals in his table being so great as 1′, his meridional parts, for high latitudes, erred a little in excess, and he pointed out the method of diminishing the error to any extent.

It behoves every writer on Navigation to bring the name of Edward Wright prominently forward when treating of what is called Mercator's Sailing. Mercator has, no doubt, the merit of originating the scheme of enlarging the degrees of the meridian more and more as they approach the pole, or of widening the intervals between the successive parallels of latitude; he was thus instrumental in awakening attention to a more correct way of exhibiting longitudes on a chart than was known before his time; but that he was acquainted with the true principles discovered by Wright, there is strong reason to doubt: for the degrees in his chart were not lengthened in the due proportion which those principles pointed out.

It will occur to the reader that, when the course A is a large angle, a small inaccuracy in the measure of that angle may considerably affect the length of C′B′; and, therefore, to diminish the influence of this error in the course, it is better and safer, when it differs but little from E. or W., to use the middle latitude method: there is no defect in the table of meridional parts, when the proper decimals are inserted, but in the *data* employed in connection with it.

In most collections of navigation tables, however, the decimals of the meridional parts are omitted, and the meridional parts given only to the nearest unit. It is possible, therefore, that in taking the *difference* between two numbers in the table, there may be an error of nearly a mile in the result. The equation

diff. long. = merid. diff. lat. × tan course,

shows that the corresponding error in longitude will be proportional to the error in tan course. Suppose, for instance, the course be 7 points, the tangent of which is about 5; then the error in longitude may *possibly* be 5 miles, which is of some consideration; but, as observed above, in such a large course the middle latitude method, for other reasons, should be employed. It would, however, be an improvement if the leading decimal belonging to each meridional part were actually inserted in the tables. Robertson, one of the most able and instructive writers on Navigation, *does* introduce the leading decimal;* and so, likewise, does Dr. Inman in his Nautical Tables. An extensive table of meridional parts will also be found in the quarto collection of Tables of J. De Mendoza Rios. The following short table will show the *true* meridional parts calculated agreeably to the mathematically-correct formulæ given at the end of the present article.

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* We do not understand what Robertson means when he says "But a table of meridional parts, constructed by the most accurate method, has only showed that Mr. Wright's table does nowhere

deg.	Wright's Mer. Parts.	True Mer. Parts.	deg.	Wright's Mer. Parts.	True Mer. Parts.	deg.	Wright's Mer. Parts.	True Mer. Parts.
5	300·369	300·382	35	2244·305	2244·287	65	5179·308	5178·808
10	603·048	603·070	40	2622·756	2622·690	70	5966·681	5965·918
15	910·433	910·461	45	3030·127	3029·939	75	6971·549	6970·340
20	1225·129	1225·139	50	3474·605	3474·472	80	8377·342	8375·197
25	1549·988	1549·995	55	3968·188	3967·966	85	10769·620	10764·621
30	1888·377	1888·375	60	4527·711	4527·368	89	16317·532	16299·556

We shall now re-state the proportions given in words above, and proceed to examples.

As radius : tan course : : mer. diff. lat. : diff. long.

As proper diff. lat : departure : : mer. diff. lat. : diff. long.

Examples.

1. A ship from latitude 51° 18′ N., longitude 9° 50′ W., steers S. 33° 8′ W., till she has run 1024 miles: required the latitude and longitude in.

The difference of latitude is found in example 1, page 23, to be 857½ miles, or 14° 17′ S.; and therefore the latitude in 37° 1′ N.

51° 18′ mer. parts . .	3597·5	As radius	— 10
37° 1′	2393·9	: tan course, 33° 8′ . .	9·8147
		: : mer. diff. lat. 1204 . .	3·0806
Diff. 14° 17′	1203·6	: diff. long. 785·8 . .	2·8953

Hence the diff. long. = 13° 6′ W.
long. left = 9° 50′ W.
∴ long. in = 22° 56′

2. A ship from latitude 49° 57′ N., and longitude 5° 11′ W., sails between the south and west till she arrives in latitude 38° 27′ N., and finds that she has made 440 miles of departure: required the course steered, the distance run, and the longitude in.

The course and distance are found in example 2, page 24, and the difference of longitude is found by the table of meridional parts, as follows:—

lat. left 49° 57′, mer. parts . .	3470	As diff. lat. 690 Arith. Comp. .	7·1612
lat. in 38° 27′	2503	: departure 440 . . .	2·6435
		:: mer. diff. lat. 967 . . .	2·9854
diff. lat. 11° 30′	967	: diff. long. 616·7 . . .	2·7901

Hence diff. long. = 10° 17′ W.
Long. left 5° 11′ W.
Long. in . 15° 28′.

---

exceed the true meridian parts by half a minute, and this only near the pole; for in latitudes as far as navigation is practicable, the difference is scarcely sensible."—Elements of Navigation, by John Robertson, formerly Head Master of the Royal Academy at Portsmouth, vol. 2, p. 136.

Robertson's work is a very valuable performance, and is well deserving a place in the seaman's library. It may be frequently picked up, at the book-stalls, at a very low price. For a correct table of meridional parts (the decimals omitted), the tables of Norie, Riddle, or Raper may be consulted.

3. A ship from latitude 51° 18′ N., longitude 9° 50′ W., sails S. 33° 19′ W., till her departure is 564 miles: required her longitude in.

We must first find the difference of latitude made: this, by Example 3, page 57, is 858 miles, or 14° 18′ S.: the latitude left and the latitude in will then become known, and thence the meridional difference of latitude may be found by the table of meridional parts, by aid of which the difference of longitude is determined, as below.

lat. left 51° 18′ mer. parts	3598	As diff. lat. 854 Arith. Comp.	7·0665
diff. lat. 14° 18′		: departure 564	2·7513
lat. in 37° 0′	2393	: : mer. diff. lat. 1205	3·0810
mer. parts for diff. lat.	1205	: diff. long. 792·1	2·8988

Difference of longitude	13° 21′ W.
Longitude left	9° 50′ W.
Longitude in	23° 2′ W.

Hence the longitude reached is 23° 2′ W.

4. Required the course and distance from the east point of St. Michael's to the Start Point.

To find the course we must know the meridional difference of latitude and the difference of longitude: the course being determined, the distance will be found by combining it with the proper difference of latitude, as below.

Start Point lat.	50° 13′ N.	Mer. parts	3495	Long.	3° 38′ W.
St. Michael's lat.	37° 48′ N.	Mer. parts	2453	Long.	25° 13′ W.
Diff. lat.	12° 25′	Mer. diff. lat.	1042	Diff. long.	21° 35′ W.
	60				60
Proper diff. lat.	745 miles.				1295 miles.

To find the course.

As mer. diff. lat. 1042	— 3·0178
: diff. long. 1295	3·1123
: : radius	10
: tan course, 51° 11′	10·0945

To find the distance.

dist. = diff. lat. ÷ cos course

```
cos 51° 11′ = ·6,2,6,8) 745  (1189
                        6268
                        ----
                        1182
                         627
                        ----
                         555
                         501
                        ----
                          54
```

Hence the *course is N. 51° 11′* E., and the distance 1189 miles.

It may be as well to exhibit here the work of finding the difference of longitude in Example 2, and the course in this last example, without logarithms.

For the diff. long. Ex. 2.

$$\text{Diff. long.} = \frac{\text{mer. diff. lat.} \times \text{dep.}}{\text{diff. lat.}}$$

mer. diff. lat. = 967
44
——
3868
3868
——
6,9)42548(616·6 miles
414
——
114
69
——
458
414
——
44
41
∴ diff. long. = 10° 17′ W. —

For the course Ex. 4.

$$\text{tan course} = \frac{\text{diff. long.}}{\text{mer. diff. lat.}}$$

1,0,4,2)1295(1·243 = tan 51° 11′
1042
——
253
208
——
45
42
——
3
——

Hence the course is N. 51° 11′ E.

The examples already given under the head of Middle Latitude Sailing may serve for exercises in Mercator's sailing. A comparison of the results of the two methods will also show the value of Workman's table for correcting the middle latitude. It is necessary, however, that the learner should notice that the two reasons given at page 63 both conspire to render it unadvisable to use the ordinary table of meridional parts when the course much exceeds 45°; from 50° and upwards the middle latitude method is to be preferred; and, for the first of the reasons mentioned, a small error in the course had better be in defect than in excess. We shall now explain the principles on which that table is constructed.

Construction of the Table for Correcting the Middle Latitude.—Let l represent the proper difference of latitude, l' the meridional difference of latitude, L the difference of longitude, and m the latitude in which the distance measured on the parallel between the two meridians is exactly equal to the departure. Then first by middle latitude and then by Mercator's sailing we have for the tangent of the course

$$\text{tan course} = \frac{\text{L} \times \cos m}{l} = \frac{\text{L}}{l'}, \quad \therefore \cos m = \frac{l}{l'}$$

Consequently by dividing the difference of latitude l taken in miles, by the meridional parts corresponding to that latitude, we shall get cos m, and thence m, the degrees of latitude in which the parallel between the two meridians is exactly equal to the departure. It is the difference between m and the latitude of the middle parallel that is given in the table at page 59.

By means of the corrected middle latitude, the difference of longitude, and the course, the nautical distance is found thus:—

The expressions for the departure by plane sailing, and by middle latitude sailing, are—

$$\text{departure} = \text{dist.} \times \sin \text{course} = \text{diff. long.} \times \cos m$$

$$\therefore \sin \text{course} : \cos \text{corrected mid. lat.} :: \text{diff. long.} : \text{distance}.$$

Construction of a Table of Meridional Parts by Dr. Halley's Method.—In Mr. Wright's method of constructing a table of meridional parts, every tabular number, after the first, depends upon the number previously calculated, as already shown at page 62. Dr. Halley proposed another plan by which the meridional parts corresponding to any given latitude may be computed independently of previous calculations. As the matter is of so much practical interest we shall here show how this may be effected.

To transcribe Dr. Halley's investigation (Philosophical Transactions, No. 219) would be to occupy more room than we can spare: we must therefore employ a different process; and, if the reader be unacquainted with the first principles of the Differential and Integral Calculus, he may pass it over, and omit this article altogether.

It is shown at page 61, that if a ship in latitude x vary her latitude by a small portion of the meridian Δx, and that she continue her course till her departure becomes equal to the difference of longitude due to the difference of latitude Δx, then the increased difference of latitude Δy, necessary to produce this effect, will be

$$\Delta y = \sec x \, \Delta x, \quad \therefore \frac{\Delta y}{\Delta x} = \sec x.$$

This equation is rigorously true only when Δx, and consequently Δy is diminished to the last degree of smallness; so that, employing the notation of the differential calculus, we strictly have

$$\frac{dy}{dx} = \sec x \;\therefore\; dy = \sec x \, dx$$

and the entegral of this is the equation

$$y = \log \tan (45^\circ + \tfrac{1}{2} x) \quad . \; . \; . \quad (1)$$

The logarithm here implied, is the *Napierian* logarithm. To change it into Briggs's, or a common logarithm, we must multiply it by the modulus $\frac{1}{2\cdot302585\ldots}$ (see the article on Logarithms in the CIRCLE OF THE SICENCES, page 279); we shall then get the logarithm of the *natural* tangent of $45^\circ + \frac{1}{2} x$, according to Briggs's system. But in the tables it is not the log of the *natural* tangent of $45^\circ + \frac{1}{2} x$, but this log increased by 10, that is inserted: hence, for the *common* logarithms, the equation (1) is

$$\frac{y}{2\cdot302585\ldots} = \log \tan (45^\circ + \tfrac{1}{2} x) - 10$$

$$\therefore y = 2\cdot302585 \left\{ \log \tan (45^\circ + \tfrac{1}{2} x) - 10 \right\}$$

This is the correct expression for the increased meridian length, measured in miles, in reference to a globe whose radius is 1 mile; to adapt it to the globe of the earth, we must multiply the expression by the radius of the earth, or by 3437·74679 nautical miles, for in every circle the radius is equal to 3437·74679 minutes of that circle, thus:—

$$\text{Rad. earth} \times 3{\cdot}14159^{*} \ldots = \text{length of } 180^\circ = 10{,}800 \text{ nautical miles.}$$

$$\therefore \text{Rad. earth} = \frac{10800}{3{\cdot}14159 \ldots} = 3437{\cdot}74679 \text{ nautical miles.}$$

Hence, for the number of miles in the lengthened meridian y, from the equator to the latitude x, we have the formula

$$y = 7915{\cdot}7044679 \, \{\log \tan (45^\circ + \tfrac{1}{2}x) - 10\}$$

or, since $\tan (45^\circ + \frac{1}{2}x) = \cot (45^\circ - \frac{1}{2}x)$, and that

$$\cot = \frac{\text{R}}{\tan}, \text{ and } \therefore \log \cot = 20 - \log \tan$$

the preceding formula may be otherwise written, as follows:—

$$\text{Meridional lat.} = 7915{\cdot}7044679 \, \{10 - \log \tan (45^\circ - \tfrac{1}{2} \text{ proper lat.})\} \ldots (2)$$

It thus appears, that if the log tan of half the complement of any latitude be subtracted from 10, and the remainder be multiplied by 7915·7044679, the product will be the meridional parts, in nautical miles, corresponding to that latitude; and, therefore, as observed at page 67, the meridional parts for any latitude may be computed independently of previous results.

The logarithm of the constant multiplier 7915·704 . . . is 3·8984895, so that from (2) we have

$$\log \text{merid. lat.} = 3{\cdot}8984895 + \log \{10 - \log \tan \tfrac{1}{2} \text{ comp lat.}\} \ldots (3)$$

from which formula, the *true* meridional parts for all latitudes may be calculated. A short specimen of the results has been given at page 64.†

We shall conclude this part of our subject with a few remarks upon the peculiar character of the *Rhumb line*, or, as it is sometimes called, the *Loxodromic curve.*

On the Continued Rhumb Line.—From the principles of Mercator's sailing, or from the diagram at page 61, which connects the enlarged meridian with the difference of longitude, it is clear that if a ship set out from any point on the globe, and sail on the same oblique rhumb towards the pole, it can reach it only after circulating an infinite number of times round it; for, from any point to the pole, the enlarged meridian is infinite in length, and so, therefore, is the difference of longitude due to this advance in latitude, provided longitude be measured round the globe in one uniform direction: the longitude, thus measured, being infinite, the ship must wind round the pole an infinite number of times.

But—paradoxical as the statement may appear—it is nevertheless true, that the infinite number of revolutions about the pole are performed in a finite time, and that the entire length of the spiral track of the ship is a finite line. However strange this may seem, it follows, as a necessary consequence, from the principles of *plane sailing;* for these principles correctly give,

$$\text{length of track} = \frac{\text{diff. lat.}}{\text{cos course}}$$

which is finite; and, therefore, the rate being uniform, is described in a finite time.

The matter may be explained as follows:—Whatever be the progressive rate of the ship along its undeviating course, the *times* of performing the successive revolutions about the pole continually diminish as the latitude increases; both the extent of circuit, and the time of performing it, evidently tend to zero—the limit actually attained only at the pole itself. Consequently, an infinite number of circuits must ultimately be performed to occupy a finite portion of time.

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* See GEOMETRY, page 160.

† A specimen of a chart, constructed according to these principles, is given at page 16 of the volume on Inorganic Nature, in the CIRCLE OF THE SCIENCES.

The case is somewhat analogous to those infinite descending series so frequently met with in arithmetic and algebra; they are infinite—not in their aggregate amount, but only in the number of the continually-diminishing parts into which that amount is divided, these parts becoming less and less, and ultimately vanishing altogether.

In like manner, every additional circuit the ship makes round the pole, increases the length of the previously-described track by a quantity less and less, the successive increments continually diminishing, and ultimately vanishing altogether; so that, just as in a common decreasing geometrical series, the sum of the increments—thus continually tending to, and ultimately terminating in zero—is finite; and hence the time of describing the track must be finite too.

Traverses by Mid-Latitude and Mercator's Sailing.—In order to work a traverse with a view to finding the ship's place, and the direct course and distance to it, we may proceed in either of the two ways following:—

1. Form a traverse table, in the first two columns of which insert the several courses and distances, and in the remaining columns put the corresponding differences of latitude and departures, found either by computation or by reference to the already computed traverse table, and thence determine the whole difference of latitude and departure, as also the corresponding direct course and distance, exactly as in plane sailing. (See page 41).

The traverse now being reduced to a single course, find the corresponding difference of longitude, either by mid-latitude or Mercator's sailing.

This method is less accurate than that which follows; because, as already observed (page 50), the aggregate of the several departures is not, in general, the same as the single departure due to the correct course and distance. It is better, therefore, to take the several differences of latitude and departures separately, and thus to determine the difference of longitude due to each distinct course, and thence the whole difference of longitude made. The second method is therefore this:—

2. Fill up the columns of the traverse table, as directed above; but find the aggregate of the diff. latitude columns only. By means of the successive latitudes reached at the end of each course, find the corresponding mid-latitude; and with this, and the departure, deduce the difference of longitude made at the end of each course by mid-latitude sailing, and thence the whole diff. long. made good.

But if Mercator's, instead of the mid-latitude method be employed, then the several departures need not be inserted in the traverse table at all. An example will sufficiently show the mode of proceeding by each method.

Examples.

1. A ship from latitude 38° 14′ N. longitude 25° 56′ W., has sailed the following courses and distances, namely—

1st. N.E. *b* N. ¼ E. 56 miles. 2nd. N.N.W. 38 miles.
3rd. N.W. *b* W. 46 miles. 4th. S.S.E. 30 miles.
5th. S. *b* W. 20 miles. 6th. N.E. *b* N. 60 miles.

Required the latitude and longitude in, and the direct course and distance to it.

BY THE FIRST METHOD.

Courses.	Dist.	Diff. lat.		Departure.	
		N.	S.	E.	W.
N.E. *b* N. ¼E.	56	45		33·4	
N.N.W.	38	35·1			14·5
N.W. *b* W.	46	25·6			38·2
S.S.E.	30		27·7	11·5	
S. *b*. W.	20		19·6		3·9
N.E. *b* N.	60	50		33·3	
		155·7	47·3	78·2	56·6
		47·3		56·6	
		108·4		21·6	

∴ Direct course 11° 15′, or N. *b* E.
Distance, 111 miles.

The direct course and distance, due to diff. lat. 108·4, and dep. 21·6, are found by reference to the traverse table, or **by computation, as at pages 16, 17, to be as above.**

To find the diff. long. by Mid-Latitude Sailing.

Latitude left	38° 14′ N.
Diff. lat. 108 m.	1° 48′ N.
Latitude in	40° 2′ N.
Sum of lats.	78° 16′
Half sum = mid. lat. . . .	39° 8′

Diff. long. = dep. ÷ cos mid. lat.
= 21·6 ÷ ·7757 = 28′ E.
Longitude left . . . 25° 56′ W.
Longitude in . . . 25° 28′ W.

To find the same by Mercator's Sailing.

Latitude left 38° 14′ mer. pts. .	2486
Latitude in 40° 2′	2625
Meridional diff. lat.	139

Diff. long. = mer. diff. lat. × tan course.
= 139 × ·199 = 28′ E.
Longitude left . . . 25° 56′ W.
Longitude in . . . 25° 28′ W.

BY THE SECOND METHOD (MID-LATITUDE).

Courses	Dist.	Diff. lat.		Departure.		Lats. left.	Lats. in.	Sum. lats.	Mid. lats.	Diff. long.	
		N.	S.	E.	W.					E.	W.
1st.	56	45		33·4		38° 14′	38° 59′	77° 13′	38° 36′	43′	
2nd.	38	35·1			14·5	38° 59′	39° 34′	78° 33′	39° 16′		19
3rd.	46	25·6			38·2	39° 34′	40°	79° 34′	39° 47′		50′
4th.	30		27·7	11·5		40°	39° 32′	79° 32′	39° 46′	15′	
5th.	20		19·6		3·9	39° 32′	39° 12′	78° 44′	39° 22′		5′
6th.	60	50		33·3		39° 12′	40° 2′	79° 14′	39° 37′	43′	
		155·7	47·3							101′	74′
		47·3								74′	
Diff. latitude		108·4 N.							Difference of longitude	27′ E.	

The diff. long. columns may be filled up from the traverse table, by taking the complement of each mid. lat. as a course, and, with the corresponding departure, finding the proper number of miles or minutes in the *distance* column of the table; or, which is somewhat simpler, take the middle latitude itself as a course, and seek the corresponding departure in the *diff. lat.* column of the table, against which, in the *dist.* column will be found the number of miles of diff. long. In this way the two diff. long. columns at the bottom of last page have been formed.

BY THE SECOND METHOD (MERCATOR).

Courses.	Dist.	Diff. lat.		Lats.	M. parts.	M. diff. l.	Diff. long.	
		N.	S.	38° 14′	2486		E.	W.
N.E. *b* N. ¼ E.	56	45		38° 59′	2544	58	43	
N.N.W. . .	38	35·1		39° 34′	2589	45		18·7
N.W. *b* W. .	46	25·6		40°	2623	34		50·5 *
S.S.E. . . .	30		27·7	39° 32′	2586	37	15	
S. *b* W. . .	20		19·6	39° 12′	2560	26		5·1
N.E. *b* N. .	60	50		40° 2′	2625	65	43·4	
		155·7	47·3				101·4	74·3
		47·3					74·3	

Diff. latitude 108·4 N. Difference of longitude 27·1 E.

The diff. long. columns are here also supplied from the traverse table by entering it with the given course and mer. diff. lat. taken as the proper diff lat.: the corresponding departure, furnished by the table, is the diff. long.

The longitude in, by the second method, whether the mid. latitude, or Mercator's sailing be employed, is 25° 29′ W., and the latitude and longitude left being given, the direct course and distance, and the corresponding departure, may be readily found by Mercator's sailing.

2. A ship, from latitude 52° 20′ N., and longitude 14° 38′ W., has sailed the following courses, namely—

1st. E. S.E. 43 miles. 2nd. S.W. 32 miles. 3rd. S.E. *b* S. 58 miles.
4th. S. S.W. 60 miles.

Required the latitude and longitude in.

Ans. Latitude 49° 57′ N. Longitude 13° 57′ W.

3. A ship, from latitude 52° 36′ N., and longitude 21° 45′ W., has sailed the following courses, namely—

1st. N.E. 36 miles. 2nd. N. *b* W. 14 miles. 3rd. N.E. *b* E. ½ E. 58 miles.
4th. N. *b* E. 42 miles. 5th. E. N.E. 29 miles.

Required the latitude and longitude in.

Ans. Latitude 54° 35′ N. Longitude 18° 4′ W.

4. A ship, from latitude 66° 14′ N., and longitude 3° 12′ E., has sailed as follows:—
1st. N. N.E. ½ E. 46 miles. 2nd. N.E. ¼ E. 28 miles. 3rd. N. ¾ W. 52 miles.
4th. N.E. *b* E. ¼ E. 57 miles. 5th. E. S.E. 24 miles.

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* The 40 corresponding to this in the "Lat." column is more nearly 39½, so that 50·5 is written instead of 50·7 given in the table.

Required the latitude and longitude in.

Ans. Latitude 68° 24′ N. Longitude 7° 54′ E.

5. A ship, from latitude 67° 30′ N., longitude 8° 46′ W., has sailed the following courses, namely—

1st. N.E. 64 miles. 2nd. N. N.E. 50 miles. 3rd. N.W. *b* N. 58 miles. 4th. W. N.W. 72 miles. 5th. W. 48 miles. 6th. S. S.W. 38 miles. 7th. S. *b* E. 45 miles. 8th. E. S.E. 40 miles.

Required her present latitude and longitude by Mercator's sailing.

Ans. Latitude 68° 43′ N. Longitude 11° 43′ W.

Sailing in Currents.—In what has preceded, the course of the ship is supposed to have been indicated by the compass, and that she has actually moved through the water in the direction of her length. It is plain, however, that if a current act upon the ship, she will be diverted from the course shown by the compass, unless the set of the current be itself in the direction of the ship's length, when the rate of sailing only will be interfered with.

The *set* of a current is the point of the compass *towards* which the stream runs; its rate, or velocity, is called the *drift* of the current.

The common way of ascertaining the set and drift of a current unexpectedly met with at sea, is to take a boat, if the weather permit, a small distance from the ship; and, in order to keep it from driving with the stream, to let down, to the depth of about 100 fathoms, a heavy weight attached to a rope fastened to the stem of the boat; steadiness being in this way secured, the log is hove into the current; the direction in which it is carried is observed by means of a boat-compass, and the number of knots run out in half a minute gives the hourly drift. In this way the set and velocity, or drift, of the current is estimated.

The set of the current being ascertained, we can apply it to correct the compass-course of the ship; and the drift, or velocity, of the current being known, we can apply it to correct the ship's rate of sailing as indicated by the log. The rate indicated by the log, it will be observed, is only the rate at which the ship moves faster than the log moves in the ship's direction, and, in still water, is the velocity of the ship itself, since the log remains stationary; but, as in a current, the log also moves in the direction and with the rate of the current, the absolute velocity of the ship cannot be directly determined by the log in the customary manner: yet the distance sailed, as indicated in this way by the log, and the course steered as indicated by the compass, though both modified by the action of the current, and therefore both inaccurate, are necessary helps to the determination of the true course and distance.

For the ship has, in fact, been making a sort of double course:—the wind has carried her on a certain course (shown by the compass) a certain distance, and the current has carried her a certain other course and distance in the same time; the final effect being the same as if the two courses and distances had been sailed in succession; so that the case becomes a very simple one of traverse sailing, as in the following example.

Examples.

1. A ship runs N.E. *b* N. 18 miles in three hours, in a current setting W. *b* S. two miles an hour: required the course and distance made good.

This is evidently the same as saying—A ship sails the following courses, namely—

1st. N.E. *b* N. 18 miles. 2nd. W. *b* S. 6 miles. Required the course and distance.

TRAVERSE TABLE.

Courses.	Dist.	Diff. latitude.		Departure.	
		N.	S.	E.	W.
N.E. *b* N.	18	15		10	
W. *b* S.	6		1·2		5·9
	Diff. lat. 13·8			Dep. 4·1	

And, with the difference of latitude and departure thus found, we have by plane sailing, or by the traverse table, the distance made good, 14 miles, and the course 1½ points, or N. *b* E. ½ E.

2. A ship sails N.W. 60 miles, in a current that sets S. S.W. 25 miles in the same time: required the course and distance made good.

Ans. Course N. 69° 38′ W. Distance 55½ miles.

3. A ship has sailed the following courses and distances in twenty-four hours:—

1st, S.W. 40 miles; 2nd, W.S.W. 27 miles; 3rd, S. *b* E. 47 miles; but she has been the whole time in a current setting S.E. *b* S. at the rate of 1½ miles an hour: required the ship's direct course and her distance made good.

Ans. Course S. *b* W. Distance 117 miles.

4. A ship has sailed by reckoning N.½W. 20 miles, but it is found by observation, that owing to a current she has actually sailed N.N.E. 28 miles: required the set of the current and the amount of drift upon the ship.

Ans. Set N. 64° 48′ E. Drift 14 miles.

The method described above of estimating the set and drift of a surface current, by sinking a weight, and observing the direction and velocity of the log, is often likely to lead to conclusions not strictly correct. Lieut. Walsh and Lieut. Lee, of the United States Navy, while carrying on a system of observations in connection with the wind and current charts, had their attention directed to the subject of *submarine currents*, upon which they made some interesting experiments. A block of wood was loaded to sinking, and, by means of a fishing line, was let down to the depth of one hundred and five hundred fathoms. A small float, just sufficient to keep the block from sinking farther, was then tied to the line, and the whole let go from the boat.

To use their own expressions—"It was wonderful indeed to see this *barrega* move off, against wind, and sea, and surface current, at the rate of over one knot an hour, as was generally the case, and on one occasion as much as 1¾ knot. The men in the boat could not repress exclamations of surprise, for it really appeared as if some monster of the deep had hold of the weight below, and was walking off with it."

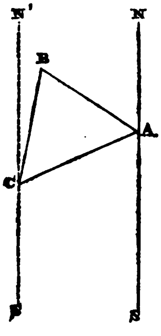
The effects of such under-currents must sometimes interfere with the results, deduced in the ordinary way, for the set and drift of a surface current, since to hidden influences, of the existence of which the observer has no suspicion, part of what he attributes to the surface current may be really due: the boat, though apparently steady, may have an imperceptible drift. Such drifts are not caused so much by the action of the under-current upon the sunken weight, as by the bellying of the line in the direction of the set. The surface current, too, if of any considerable depth, may operate in a similar way.

Admiral Sir Francis Beaufort, when in the Mediterranean, made several experiments on this subject. He says—

"The counter currents, or those which return beneath the surface of the water, are very remarkable: in some parts of the Archipelago they are at times so strong as to prevent the steering of the ship; and, in one instance, on sinking the lead, when the sea was calm and clear, with shreds of bunting of various colours attached to every yard of the line, they pointed in different directions all round the compass."

For the foregoing quotations on the under-currents of the ocean we are indebted to a recent publication of great value and ability, "The Physical Geography of the Sea," by Lieut. Maury, of the United States Navy, the author of "The Wind and Current Charts," in which the results of extensive observation and experience, in different seasons, and during a course of years, in all parts of the globe, are recorded. Speaking of these charts, the distinguished writer says—"The young mariner, instead of groping his way along till the lights of experience should come to him by the slow teachings of the dearest of all schools, would here find, at once, that he had already the experience of a thousand navigators to guide him on his voyage. He might, therefore, set out upon his first voyage with as much confidence in his knowledge as to the winds and currents he might expect to meet with as though he himself had already been that way a thousand times before."*

Oblique Sailing—Taking Departures.—From what has hitherto been delivered the learner has perceived that oblique-angled triangles may be dispensed with in all the ordinary computations which enter into the dead reckoning. But at the commencement of her voyage, before the ship is out of sight of some known point of land or other object on shore, the distance and bearing of that object is found, and thus a first course and distance obtained. This is called taking a *departure*. It is a common practice to observe the bearing by compass, and to estimate the distance by guess, and experienced navigators can in this way come pretty near the truth; but as in this treatise the object is to describe the most accurate methods of performing the various calculations of Navigation, we shall devote a short article to the more correct way of taking a departure, which is by observing two bearings of the object, and measuring by the log the distance sailed by the ship in the interim between the observations. The solution of an oblique-angled triangle will thus become necessary, as in the following example.



Examples.

1. In sailing down the Channel the Eddystone Lighthouse bore N.W., and after running W. *b* S. 8 miles it bore N.N.E.: required the ship's course and distance from the Eddystone at the last place of observation.

In the annexed diagram A represents the first position of the ship, and B the lighthouse, the bearing of which from A, that is, the angle NAB, is N.W., or 4 points. C is the second position of the ship, and the bearing of B from this position, that is, the angle N'CB, is N.N.E., or 2 points. Also the course of the ship in the interim, that is, the angle SAC, is W. *b* S., or 7 points. Hence the angle BAC is 16 points — 11 points = 5 points; also, since the angle SAC = N'CA, we have BCA = 7 points — 2 points = 5 points. Consequently the angle B is 16 points — 10 points = 6 points, so that in the triangle ABC are given AC = 8, A = 56° 15′ and B = 67° 30′, to find BC as follows:—

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* "The Physical Geography of the Sea." By M. F. Maury, LL.D., Lieut. U.S. Navy. 1855. Introduction, p. vi.

As sin B, 67° 30′ Arith. Comp. .	·0344
: sin A, 56° 15′	9·9198
: : AC = 8°	·9031
: CB = 7·2	·8573

Consequently, the distance of the Eddystone from the last place of observation is 7·2 miles; and, as the course *to* the Eddystone is N.N.E., the course *from* the Eddystone to the place of the ship must be S.S.W., the opposite point.

As the triangle BCA, in this example, happens to be isosceles, the angles at A and C being equal, a perpendicular upon AC from B will bisect AC, so that we shall have

$$CB = 4 \sec 56° \, 15' = 4 \div \cos 56° \, 15';$$

that is, CB = 4 ÷ ·556 = 7·2, as in the margin. And this is an easier way of finding the distance than that exhibited above; but when, as is usual, the triangle is scalene, the operation by logarithms, as in the specimen above, is to be preferred.

cos 56° 15′ = ·5,5,6)4 (7·2
389
11
11

2. Sailing down the Channel the Eddystone bore N.W. *b* N., and after running W.S.W. 18 miles, it bore N. *b* E.: required the course and distance from the Eddystone to both stations.

Ans. Course S.E. *b* S., distance 21·2 miles from first station. Course S. *b* W., distance 25 miles from second.

3. Coasting along shore, a headland bore N.E. *b* N.; then having run 15 miles E. *b* N., the headland bore W.N.W.: required the distance from the headland at each time of observation. Ans. First distance 8¾ miles; second 10·8 miles.

4. Two ships sail at the same time from one port; one sails E.S.E., and the other S.S.E.: required their bearing and distance when each has run 37½ miles.

Bearing from first ship N. 45° E.; distance 28·7 miles.

Great Circle Sailing—Shortening of Passages.—The great object of the navigator is to reach the port he intends to make by the shortest route. The path described by a ship, sailing on what is called a direct course from one place to another, is not the *shortest* path, unless the ship sail either on the equator or on a meridian. The oblique spiral track, or rhumb-line, of a ship, sailing in any other direction, exceeds the arc of the great circle joining the two places by an amount that becomes more and more considerable as the distance increases. The shortest distance between any two points on the globe is the arc of the great circle between them; for the curvature of this arc is less than that of any other line drawn on the surface, from point to point, and therefore approaches more nearly to a straight line.

If, therefore, a ship could sail accurately on the arc of a great circle, voyages might, in general, be considerably shortened. But a great circle cuts all the meridians it crosses in different angles, so that to keep on such a circle the course must be continually varying. The practical impossibility of changing the course every instant renders Great Circle Sailing—strictly speaking—a matter of mere theoretical speculation. Much advantage, however, has been found to arise from dividing the great circle path, from one place to another, into short portions, or stages, and to reach these in succession by the ordinary methods of sailing. A ship making these several stages, though never actually moving on the great circle, always keeps so closely in its neighbourhood as to be a considerable gainer, in point of time, in a long voyage.

Mr. Towson, of Devonport, has been at the pains of constructing tables, by which the proper successive courses necessary to cause the entire track of the ship to approximate as closely as conveniently practicable to the great circle path, may be found; they are published by the Admiralty, under the title of "Tables to facilitate the practice of Great Circle Sailing." We can do no more than allude to them here, and recommend them to the notice of the practical seaman.

But, without the aid of tables, a twelve or eighteen-inch globe would be of service in suggesting how to shape the different courses; a line stretched from one point to another, on such a globe, would show the great circle track between them. If a brass circular rim, two or three inches in diameter, made to lie close to the surface of the globe, were divided, like a compass-card, and if a tubular box inclosing a reel, with a yard of thread, were to rise from its centre, this centre might be applied to any proposed spot; and, by means of the thread, pulled out under the disc, each successive course could be ascertained: the only care being, that the north and south line of the disc—which should be marked on the only metallic diameter it need to have—be made to coincide with the meridian of the place under the centre of the instrument: we offer this suggestion as one that might, perhaps, be turned to practical account in long voyages. But, to give perfection to the problem of shortening passages, the "Sailing Directions," and "The Wind and Current Charts," by Lieutenant Maury, are of the greatest use. The winds blow and the currents run in all parts of the ocean. "These control the mariner in his course; and to know how to steer his ship on this or that voyage, so as always to make the most of them, is the perfection of navigation. When one looks seaward from the shore, and sees a ship disappear in the horizon as she gains the offing, on a voyage to India, or the antipodes perhaps, the common idea is that she is bound over a trackless waste, and the chances of another ship, sailing with the same destination the next day, or the next week, coming up and speaking with her on the 'pathless ocean,' would, to most minds, seem slender indeed. Yet the truth is, the winds and the currents are now becoming to be so well understood, that the navigator, like the backwoodsman in the wilderness, is enabled literally 'to blaze his way' across the ocean—not, indeed, upon trees, but upon the wings of the wind. The results of scientific inquiry have so taught him how to use these invisible couriers, that they, with the calm belts of the air, serve as sign-boards to indicate to him the turnings, and forks, and crossings by the way."*

We here terminate our exposition of the Principles of NAVIGATION, in so far as these are independent of astronomical observations. What has hitherto been taught the learner will remember concerns only what is called the dead reckoning, which is generally affected with errors, more or less, on account of the practical difficulty of measuring course and distance with strict precision, as well as on account of disturbing causes, either operating unseen, or, where observed, but imperfectly estimated. It is the business of Nautical Astronomy to rectify these unavoidable defects, to adjust the place of the ship, from time to time, to its true position, and thus to enable the mariner to take, as it were, a new departure, and start afresh upon each successive stage of his journey.

As an introduction to this important branch of our subject, we shall now give an article on the rotation of the earth, agreeably to our promise at page 33.

* Maury's "Physical Geography of the Sea," page 262

NAUTICAL ASTRONOMY.

Introduction.—On the Rotation of the Earth.—The common arguments in support of the doctrine that the earth has a diurnal rotation about one of its diameters give to that doctrine a degree of probability, so nearly approaching to absolute certainty, that the mind readily acquiesces in the reality of the phenomenon. Since the time of Copernicus, the evidence for the earth's rotation has been continually increasing; but still this evidence is not of that direct and positive kind that is necessary to give to it the character of demonstration. All the other hitherto-discovered planets of our system revolve on their axes, and, as might be expected as a consequence of this revolution, those of them upon which the examination can be made are seen to be flattened at their poles. It is probable, therefore, that the planet we inhabit *also* revolves on an axis; if so, it too may be expected to be flattened at its poles. Whether or not *this* is the case can be actually ascertained by experiments: these have been undertaken and repeated again and again with the greatest care, and by independent and widely different means: the results all show that the earth *is* flattened at the poles. There is thus a very high degree of probability that the earth rotates, and this is further increased by the fact that all the phenomena of the heavens are completely consistent with the hypothesis of such a rotation, that it is, moreover, the simplest hypothesis upon which the celestial appearances can be explained, and that to attempt to account for them on any other hypothesis involves the system of the universe in such intricacy and extreme complication that, judging from all the other operations of nature, we cannot bring ourselves to suppose that such complex machinery should really be the "handiwork" of an all-wise Creator when means so immeasurably simpler presented themselves of bringing about the same ends. It can scarcely be charged to the King Alphonso as a sentiment of impiety when he exclaimed, in reference to the confused astronomical systems of the ancients, "If I had been the Almighty's counsellor when he framed the universe I would have advised him better."

But, notwithstanding all this, a sensible or experimental *proof* that the earth rotates was still wanting: its general figure, as already declared (page 33), can be experimentally discovered: its superficial rotundity can be *seen*. It is very desirable that we should have the same ocular evidence of its rotation. A profound writer on Physical Astronomy has observed, that "We must, however, be content, at present, to take for granted the truth of the hypothesis of the earth's rotation. If it continues to explain simply and satisfactorily other astronomical phenomena than those already noted, the probability of its being a true hypothesis will go on increasing.

"We shall never indeed arrive at a term when we shall be able to pronounce it absolutely *proved* to be true. The nature of the subject excludes such a possibility."

This prediction of Professor Woodhouse has been falsified: we can *now* obtain sensible evidence of the rotation of the earth.

The idea of proving this interesting phenomenon to the senses occurred a few years ago to M. Faucault of Paris: it was suggested to him by accidentally observing the motion of a weight suspended by a string to a high ceiling, and which, by chance, had been set vibrating. Most of our readers must remember the sensation produced in the scientific world, in the spring of 1851, by the remarkable Pendulum Experiment of Faucault. We here propose to submit to them, without going into any minute mathematical details, what appears to us to be a conclusive and satisfactory explanation of the manner in which this experiment renders the rotation of the earth a matter of personal observation; but as we write chiefly for the young, and for those who may be supposed to be but little habituated to scientific research, we shall previously offer to their attention a few general remarks in reference to what may be called the two great postulates of astronomy: these are the rotation of the earth, and the hypothesis of gravitation.

We speak of gravitation as a physical *hypothesis*: it is not, like a proposition in Geometry, a necessary truth; nor is it an observed fact recognizable by our senses. Certain phenomena in nature are observed: they exhibit a regularity of succession, and a mutual dependence, that suggest the idea of a connecting principle and a governing law. The phenomena are *seen*, their proximate cause is *inferred*. The philosopher looks abroad upon nature, and carefully studies the facts she presents to him, the order of their recurrence, and the measure of their intensities: he retires to his closet, and endeavours to frame a law of which the appearances he has been studying shall be the outward expression—the practical manifestation. This is an *hypothesis*. An hypothesis, therefore, need not be more than coextensive with the phenomena actually observed; but it is a strong confirmation of the soundness of an hypothesis when it is found that new and unexpected phenomena are equally comprehended in it, and a stronger confirmation still when such phenomena can actually be predicted. The two fundamental hypotheses of Astronomy—the rotation of the earth, and the law of gravitation—have this character in the highest degree: every new discovery in the science has only the more firmly established the truth of both.

The learner will perceive that we could not, with any propriety, speak in this way of the truths of geometry: *they* are quite independent of the confirmations of experience, and hence the marked difference between physical and geometrical science. When Dr. Halley had predicted the return of the comet of 1682 in 1759, and Clairaut had computed, from the hypothesis of gravitation, the time when it ought to appear, its return was watched for by astronomers with the greatest interest—not from any anxiety to see the comet, but to learn how the hypothesis of gravitation would stand so severe a test; and the reappearance of the same comet in 1835 was anticipated with like interest, solely in reference to the planetary attractions—that is, to the general theory of gravitation.

"The rude supposition of the uniform revolution of the moon in a circle about the earth as a centre, led Newton at once to the true law of gravity, as extending from the earth to its companion. The uniform circular motions of the planets about the sun in times following the progression assigned by observation in Kepler's rule, confirmed the law, and extended its influence to the boundaries of our system. Every thing more refined than this,—the elliptic motions of the planets and satellites—their mutual perturbations—the slow changes of their orbits and motions, denominated secular variations—the deviation of their figures from the spherical form—the oscillatory motions of their axes, which produce nutation and precession of the equinoxes—the theory of the tides, both of the ocean and the atmosphere,—have all in succession been so many trials for life and death in which this law has been, as it were, pitted against nature: trials whose event no human foresight could predict, and where it was impossible even to conjecture what modifications it might be found to need. Even at this moment, if among the innumerable inequalities of the lunar or planetary motions any one, however small, should be discovered decidedly not explicable on the hypothesis of a force varying as the inverse square of the distance, that hypothesis must be modified till it accounts for it."*

From these statements the student will perceive that one of the two fundamental hypotheses of astronomy—the hypothesis of gravitation—is not irrefragably established like a proposition in Euclid; nor is it a truth set at rest, once and for ever, by observation and experiment. Indeed, no physical truth can be regarded as thus unalterably fixed, like a *necessary* truth of geometry. The laws of nature *may* change; the supposition of such a change would involve no such absurdity as that which would be implied in the supposition that the three angles of a triangle could ever exceed or fall short of two right angles. This truth would remain undisturbed, however the properties of matter might be modified, and even though matter were to be altogether annihilated. It is obvious that we reckon upon the continuance of the properties of matter, and the return of natural phenomena, only to the extent to which we reckon upon the permanence of the existing natural laws. And Laplace has calculated the probability that the sun will not rise to-morrow.

But assuming the unchangeableness of nature's laws, we are authorised in regarding certain of its phenomena as unalterable truths. For instance, if the planet we inhabit is clearly ascertained *now* to be a round body, we conclude that it will remain round as long as it lasts. If it is as clearly seen to rotate, we conclude, in like manner, that it will always rotate; its rotation ceases then to be an hypothesis—it becomes an observed fact, the evidence for the truth of which is not increased by the confirmations of future experience, nor by its satisfactorily accounting for whatever phenomena may be referred to such rotation. It is a matter of some importance, therefore, that the rotation of the earth is taken out of the category of hypotheses, and classed among observed physical truths, as we now proceed to show.

Faucault's Pendulum Experiment.—Let the reader conceive before him a circular table upon which, passing through its centre, the meridian line is drawn. If the earth have no rotation about an axis, this line can never change its direction; if it do rotate, the direction must continually vary, except the place of observation be at the equator: this will readily appear from the following considerations.

Let our horizontal meridian line be indefinitely extended; we shall thus have an indefinite straight line, in the plane of the terrestrial meridian, and touching the surface of the earth, the point of contact being the centre of the table;—we may, of course, regard the table-top as lying horizontally on the ground.

For any place of observation between the equator and the pole, it is obvious that, if the earth turn round its axis, this tangent line will, in one complete rotation, describe a conical surface enveloping the globe; and, as the vertex of the cone is necessarily at a finite distance, the line which generates its surface—thus always pointing to a fixed determinate point (the vertex)—must continually change its direction, which, however, it cannot do if the earth be at rest.

But if the place of observation be at the equator, what, in the case just considered, is a *conical* surface, would evidently be a *cylindrical* surface; the straight line generating it would thus always be parallel to itself, and, therefore, though the earth should really rotate, there would be no more change of direction in the meridian line than if it were at rest.

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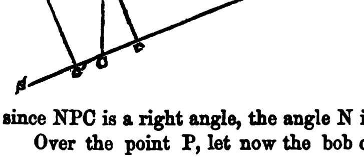
* Physical Astronomy, in the Encyclopædia Metropolitana, by Sir J. F. W. Herschel, p. 648.

If, however, the centre of the table were directly over the pole, then, taking any diameter of the table for a meridian line, the changes in its direction—if the earth rotate—would clearly be more rapid and more considerable; it would pass through a revolution of 360° for every complete rotation, and the surface described by the line would be a plane surface.

It is thus easy to see what must necessarily happen, as to the direction of the horizontal meridian, if the earth has any rotation about its axis. At the equator, the direction would remain undisturbed, and the line would generate a cylindrical surface; at a small distance from the equator, the cylinder would become a cone, and the direction of the line would regularly, but only slightly, change. At a greater distance from the equator the cone would differ more palpably from the cylinder, and the angular deviation of the line—always directed to the vertex of the cone—would be more considerable. As the place of observation approaches the pole, the cone would, as it were, widen out more and more, the deviation of the line would become greater and greater, in a given time, till at length when the pole is reached, the cone would spread into a plane, and the change of direction would be the greatest possible.

The reader will find it of much assistance, towards clearly apprehending what is to follow, if he keep constantly before his mind this idea of the enveloping cones. Every parallel of latitude is to be regarded as the circle of contact of a cone, the apex of which becomes less and less remote as the parallel approaches the pole; we shall call the cone adapted to any particular parallel the *cone of latitude* of that parallel, a designation which is sufficiently significant. The extreme cases of the cone of latitude are the cylinder and the plane—the former belongs to the equator, the latter to the pole; the angle of the former is 0, that of the latter 180°. The straight line from a point of contact to the vertex of the cone, is the horizontal meridian of that point of contact. The imaginary cone, of course, rotates with the earth, if the latter rotates, but not else; and the axis of the earth is, when prolonged, also the axis of the cone,—all this is obvious.

Let now P be the centre of the circular top of the table, PP′ half its meridional diameter, NS the axis of the earth, prolonged to meet the vertex of the cone of latitude in N. The angle N will be equal to the latitude of the point of contact P; because, if a line PC be drawn from P to the centre of the earth, the angle PCN will evidently be the complement of the latitude; consequently, since NPC is a right angle, the angle N is the latitude.



Over the point P, let now the bob of a pendulum freely hang—the longer the wire or cord by which it is suspended the better—but it must be so attached to the support above as to be equally free to vibrate in any direction. Let the pendulum be made to vibrate in the direction of the meridian line NP′; and, *if* the earth rotates, let us try to anticipate the phenomena that will be presented to us.

The point P immediately under the bob, when at rest, must rotate *slower* than the points in the meridian line more remote from the apex of the cone; so that, as the bob sweeps over these points, they must keep proceeding in *advance* of it, that is, towards the east, if P be in north latitude, as here supposed. The rate of rotation of the bob round the cone must be the same whether it oscillate or not; because the fact of its oscillating cannot interfere with any motion of revolution round the cone it may have had when hanging freely. We see, therefore, that the path of the pendulum, that is, its line of vibration, must *appear* to be gradually receding from the meridian line PP′ towards the *west*, making with PP′ an angle of deviation, which increases at every oscillation.

Actual experiment fully justifies these anticipations:—the deviation of the track of the bob is *seen* to be in complete accordance with them; and although, in the following mathematical investigation of the exact amount of deviation in a given latitude and in a given time, allowance must be made for mechanical imperfections, and for the accidental impulse to the right or left communicated to the bob by the hand in giving the initial impulse, and which will give rise to what is called an apsidal motion, yet, with only very ordinary care, the natural tendency of the pendulum sufficiently overrules these hindrances to render the deviation of its path towards the west always palpable, and always accumulative.* The rotation of the earth from west to east is the only way of accounting for this apparent deviation of the path of the bob; what we really *see* is the motion of the line PP′, receding from the track of the bob with an angular movement about the centre P, towards the east. If the pendulum can be kept swinging sufficiently long, it is plain that, as the angular deviation is always accumulative, a time will come when it shall have described a complete revolution, or 360°; or, to speak more accurately, when the radius PP′ of the table shall have revolved completely round. Let us seek to determine this time.

Time in which the Meridian Line completes a Revolution.—As the absolute direction of the oscillations remains invariable, it follows that, when the cone of latitude has turned through any angle, the original horizontal line must have turned through an equal angle; so that, when the cone has made a complete revolution, the deviation of the path of the bob, from the horizontal line, amounts to an angle equal to the plane angle given by developing the conical angle.† This, it is obvious, can never amount to so much as 360°, or a complete revolution of the table, except when the bob is suspended over one of the poles of the earth, the conic surface then becoming a *plane*, tangential to the sphere at the pole. Proceeding from this extreme limit towards the equator, the angle of the cone becomes less and less, till, on reaching the equator, it vanishes altogether, as before remarked: the cone then becomes a cylinder, and no deviation can take place.

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* In the experiment as performed at the Royal Institution of Great Britain, the impulse was not given by the hand; the bob was drawn out of the vertical by a silken thread, the end of which was fastened to a peg in the floor: a flame was then applied to the thread, and the ball set free. The author of this treatise first witnessed the experiment at the house of a scientific friend, W. H. Spiller, Esq., of Highgate; in the different repetitions of it, the suspending wire, nearly twenty feet in length, was of different metal, and the hand was employed to let the bob go; the deviation was quite palpable in eight minutes.

† The proper distinction must of course be observed by the reader between the angle of the cone and the conical angle. The angle of the cone is the plane angle of the isosceles triangle, which the section of the cone through its axis presents; the conical angle is the angle at the vertex, formed by the surface of the cone: if this surface be cut along the straight line which may be conceived to generate it, and then the surface developed or unfolded into a plane, the conical angle will become an equivalent plane angle.

It thus appears that the angle of deviation in any time, at any place, is a plane angle exactly equal to the development of the corresponding conical angle, turned through in that time by the rotation of the cone of latitude. It remains to be ascertained what part this angle of deviation is of a complete revolution, or 360°. In other words, we have to ascertain what part of 360° the developed angle at the apex of the cone of latitude amounts to.

Conceive an upright cone, unconnected with the globe, and from its apex let any two straight lines be drawn along the slant side, intercepting an arc of the base; this arc will measure a certain angle at the centre of the base. Measure from the apex along either of the two lines drawn from it, a length equal to the radius of the base, and describe with this length, as radius, a circular arc, limited by the two straight lines on the cone: this arc will evidently measure the conic angle; it will therefore be to the corresponding arc of the base, as the angle at the apex to the corresponding angle at the centre of the base; but the developed arcs are also to one another as their distances from the apex: for these distances are their radii. Hence the conic angle is to the corresponding plane angle—or angle of deviation on the horizontal plane—as the radius of the base of the cone to the length of the slant side; that is, as the sine of half the plane angle of the cone to unity. Consequently, the whole conic angle is to 360° as the sine of half the plane angle of the cone—that is, as the sine of the latitude is to unity. Hence, from what is shown above, the angle of deviation (x) of the path of the bob, in one entire revolution of the earth, is to 360° as the sine of the latitude is to unity. The *time* of this revolution is 24 hours; consequently, 24 hours divided by the sine of the latitude is the time in which the path of the pendulum makes a complete revolution in that latitude. Thus, assuming the deviation to be 360°, we have

$$360 = x \sin \text{lat.}, \therefore x = \frac{360^\circ}{\sin \text{lat.}}, \text{ in } \textit{degrees}.$$

$$\therefore x = \frac{24 \text{ hours}}{\sin \text{lat.}}, \text{ in } \textit{time}.$$

And all the more carefully conducted experiments justify this result, within those limits of difference that may reasonably be attributed to the disturbing causes adverted to above.

We have further obtained, from a few simple considerations, the following interesting proposition, namely:—

The length of the arc of the rim of the table, subtending the angle of deviation at its centre, which a pendulum oscillating over it makes during one rotation of the earth, is exactly equal to the difference between the parallel of latitude described by that centre, and the parallel described by the extremity P' of the meridional diameter of the table.

Draw Pc, P'c' perpendiculars upon the axis of the earth (last diagram) and P'D parallel to the axis NS. It has been proved above that the angle of deviation in one revolution of the earth is

$$360^\circ \sin \text{N} = 360^\circ \frac{\text{P}'c'}{\text{P}'\text{N}} = 360^\circ \frac{\text{P}'\text{D}}{\text{P}'\text{P}}$$

This angle, multiplied by the radius PP' of the table, and by 3·1416, and the product divided by 180°, is the arc of the rim of the table subtending it; that is, the measure of this arc is 2P'D × 3·1416. But twice P'D is the difference between the diameters of the two parallels described by P and P′. Hence the arc that measures the angle of deviation in one revolution of the earth is equal to the difference between the two circumferences described by P and P′. And the arc of deviation, due to any portion of a complete revolution of the earth, is equal to the difference between the two portions of parallels described by P and P′.

The same conclusion may be obtained by aid of considerations still more simple. It is plain that the difference between the circumferences of any two equidistant circles on the surface of a cone is always the same; hence, if a circle be described about the apex, with a radius equal to PP′, the circumference of it will be equal to the difference between the two circumferences described by P and P′. But this same circumference, when the cone is developed, is the arc of deviation, on the table, due to a complete revolution of the earth: hence, this arc must be equal to the difference between the two parallels described by P and P′ in a complete revolution.

It thus appears, not only that the pendulum experiment affords ocular demonstration of the rotation of the earth, but that it moreover exhibits to us the actual velocity, in linear measure, with which the point P′ proceeds in advance of P. It is the velocity with which the arc of deviation increases.

If the length of this arc, described in any interval of time, be measured, we may readily deduce the arc that would be described in a complete revolution of the earth. If the length of this arc be taken for the circumference of an entire circle, the diameter of that circle may be inferred; this diameter, applied as a chord to the circle of deviation, will subtend an arc of it, the degrees and minutes of which will be double the latitude of the place. And thus we may conceive it possible that a person, conveyed to a dungeon in some unknown part of the world, with a piece of string and a weight at hand, might form an estimate of the latitude of his position.

Application to an Unexplained Phenomenon in Falling Bodies.—The idea of the cone of latitude will subserve the purpose of accounting for a circumstance in the late M. Oersted's experiments on falling bodies, hitherto, we believe, involved in some obscurity. The following quotation is from the *Literary Gazette* of March 22, 1851:—"One of the most important observations first made by Oersted, and since then confirmed by others, was, that a body falling from a height, not only fell a little to the east of the true perpendicular—which is, no doubt, due to the earth's motion—but that it fell to the *south* of that line: the cause of this is at present unexplained. It is no doubt connected with some great phenomenon of gravitation which yet remains to be discovered."

The explanation of this phenomenon is very easy. Suppose a heavy body to be let fall from a point at a considerable height vertically over P: when it is let go, the body will have a progressive velocity towards the east *greater* than the velocity of P at the foot of the vertical; and this velocity it will preserve throughout its descent, which, from the nature of gravity, must be in a vertical plane through PC, C being the centre of the earth. Now the point P, at the foot of the vertical line, *recedes* from this plane, towards the north, during the descent of the body: it always keeps in the plane through P*c*, and perpendicular to the axis of the earth, and describes a circle whose radius is *c*P on the cone of latitude. The body, therefore, must necessarily fall towards the *south* of P, as well as towards the east. If the experiment be made in south latitude, the deviation will, of course, be north instead of south.

It is plain that in Oersted's experiment the falling body, by the rotation of the earth, and, therefore, by its own more rapid easterly motion, had advanced more towards the east, when it reached the ground, than the point P at the foot of the vertical, but it had not advanced at all towards the south; it was the foot of the vertical—the point P—that had receded towards the *north*.

These experiments prove in the most satisfactory manner that the earth really rotates on its axis: we are made sensible of this rotation in other ways also. "It is well known to engineers that when railway carriages are going north their tendency is to run off the rails on the east side; but when the train is going south, their tendency is to run off on the west side of the track; that is, always on the right hand."* In the former case, the train, at starting, is moving eastward with a velocity greater than that with which any more northerly point of the track moves; and in the latter case, it is moving more slowly towards the east than any more southerly point of the track, and hence the uniform tendency to escape the confinement of the rails towards the right.

Explanation of Terms in Nautical Astronomy.—What are called the heavenly bodies appear to an observer on the earth to occupy a surrounding spherical concavity, at the centre of which our planet is placed: the phenomena of their rising and setting are appearances which necessarily present themselves in consequence of the rotation of the earth about its axis. This apparent concavity is called the celestial sphere, and the imagination traces upon it a variety of circles analogous to those conceived to be traced on the terrestrial globe.

To assume, however, that what we call the starry heavens is really a concave sphere, whose centre coincides with that of the earth; and, therefore, that all the celestial bodies situated in it are at equal distances from that centre, would be to oppose what is well known to be truth; but the part of Astronomy with which we are at present concerned is occupied mainly with appearances, not with realities; or, we should rather say, it is chiefly occupied with the consideration of those astronomical phenomena which are independent of actual distances, and which would equally present themselves were these distances other than what they are, or all, as they appear to be, the same.

The learner will readily perceive how this assumption of a surrounding celestial sphere is perfectly consistent with correct deductions in certain departments of astronomy—all those departments, for instance, which regard only the angular distances of the stars from one another, or from the imaginary circles before alluded to. The angular distance of two objects—whether on the earth or in the heavens—is the angle formed at the eye of the observer by lines drawn to it from the objects observed. If one or both of these objects move nearer to the eye, along the line of vision, or recede further from it, it is plain that the *angular* distance of the two must remain the same;—the objects cannot in this way increase or diminish their angular separation. The observer therefore may, if he please, consider the linear distances of the two objects from his eye to be the same.

Nautical Astronomy has a good deal to do with observations of this kind—that is, with the measurement of angular distances, and but very little with linear distances. It would have *nothing* to do with linear distances if the earth were really a point, or if observations were all carried on at the centre instead of on the surface; but, as it is, the semidiameter of the earth is a linear measure of which cognizance must be taken, simply because *appearances*—in reference to the sun and moon, but not in reference to the stars—are different at the surface from what they would be at the centre. The angular distances are not exactly the same from the two points of observation.

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* Maury's "Physical Geography of the Sea," page 39.

What is here said of distances applies equally to magnitudes. The linear diameters of the sun and moon are not matters of concernment in Nautical Astronomy, only their apparent diameters, the diameters (taken in angular measure) they would appear to have to an observer at the centre of the earth.

All observations made upon these two bodies, for the purpose of determining the latitude and longitude at sea, are reduced to what they would be if the place of observation were the centre of the earth. As to the stars, it is found that observations on them, though made at the surface, would require no modification if made at the centre—the radius of the earth being a mere point in comparison to the immense distance of the stars. We shall now define the principal circles of the celestial sphere.

Axis.—The axis of the heavens is the diameter of the celestial sphere, about which the apparent diurnal rotation of the celestial sphere takes place, and which, as we have seen, is due to the real rotation of the earth. The axis of the heavens is, therefore, only the axis of the earth prolonged; and the extremities of this axis—of course the *imaginary* extremities—are the poles of the heavens.

Equinoctial.—The celestial great circle, to the plane of which the axis of the heavens is perpendicular, is called the *equinoctial*, or the celestial equator. It is traced out merely by extending the plane of the terrestrial equator to the heavens.

Meridians.—The celestial meridians are in like manner marked out by extending the planes of the terrestrial meridians; or they are semicircles terminating in the poles of the heavens, and perpendicular to the equinoctial.

Zenith, Nadir.—The zenith is that point in the heavens which is directly over the head of the spectator; or, if a straight line be drawn from the centre of the earth to any spot on its surface, and then prolonged to the heavens, the point on the celestial sphere which it would mark out is the *zenith* of that spot. The same line continued in the contrary direction would mark a point in the celestial sphere called the *nadir*. These two points, therefore, in reference to any place on the earth, are at the extremities of that diameter of the celestial sphere which is perpendicular to the plane of the horizon of that place:—that is, they are the *poles* of the horizon.

Vertical Circles.—Vertical circles of any place are those which pass through the zenith and nadir of that place; they are all perpendicular to the horizon of the place. They are hence also called circles of altitude.

The *altitude* of a celestial body is its distance above the horizon measured on the vertical circle passing through the body. The complement of the altitude is the *zenith distance*. In the case of the sun and moon, the *true* altitude is measured from the rational horizon, and is a little greater than the altitude measured from the sensible horizon. In the case of the stars, as observed at page 35, the difference in altitude is insensible, whichever horizon be referred to.

The most important of all the vertical circles of any place is the *meridian*. When a celestial object is on the meridian, its altitude is the greatest which that object can possibly have; it is called the meridian-altitude of the object.

The vertical circle which cuts the meridian at right angles, and which therefore passes through the east and west points of the horizon, is distinguished next to the meridian. It is called the *prime vertical*. When a celestial object arrives at the prime vertical, it is either due east or due west.

Azimuth.—The azimuth of a celestial object is the arc of the horizon comprehended between the meridian of the observer and the vertical circle passing through the object. The arc of the horizon here spoken of is, of course, the measure of the angle at the zenith between the meridian and the vertical through the object. Vertical circles are sometimes called azimuth circles.

Amplitude.—Amplitude is also an arc of the horizon. It is the arc comprised between the east point of the horizon and the point where the body rises, or between the west point and where it sets; the former arc is called the rising amplitude of the body, and the latter its setting amplitude. Azimuth is measured either from the north or south points of the horizon; amplitude either from the east or west. When we speak of the azimuth of a body, we refer merely to the azimuth of the vertical on which the body is, whatever its altitude on that vertical may be; when we speak of its amplitude, we refer exclusively to its position with respect to the east or west point of the horizon at rising or setting.

Declination.—The declination of a celestial object is its distance from the equinoctial, measured on the celestial meridian passing through it, and is either north or south. What is latitude as respects a point on the earth, is declination in reference to a point in the heavens. Celestial meridians are thus sometimes called circles of declination, and what are parallels of latitude on the earth become parallels of declination on the celestial sphere.

The distance of an object from the elevated pole is the polar distance of it. It is the complement of the declination when the elevated pole and the object are both on the same side of the equinoctial; but when they are on contrary sides the polar distance is the declination plus 90°. The elevation of the pole above the rational horizon of any place is always equal to the latitude of that place, for the latitude is equal to the distance of the zenith of the place from the equinoctial; the distance between the zenith and the elevated pole is, therefore, the complement of the latitude, and it is equally the complement of the elevation of the pole above the rational horizon: this elevation, therefore, is equal to the latitude of the place. Consequently the depression of the equator below the horizon, or its elevation above the horizon, in the opposite quarter, is the complement of the latitude, or, which is the same thing, the latitude is the measure of the angle which the horizon makes with the equator.

The celestial circles now defined have especial reference to the earth. The meridian and the equinoctial are merely extensions to the heavens of corresponding circles on the earth; and the vertical circles or perpendiculars to the horizon are imagined for the purpose of recording altitudes above the horizon, measured on the earth. But there are some circles peculiar to the celestial sphere; the principal of these are the *ecliptic*, or the circle of celestial longitude and the perpendiculars to it—the circles of celestial latitude.

The Ecliptic.—The ecliptic is the great circle described on the celestial sphere by the sun in its apparent annual motion about the earth: in reality, it is the path of the earth about the sun in the contrary direction; but, as already remarked, we are in this subject only concerned with the appearances. The ecliptic crosses the equinoctial at an angle subject to continuous but very small variation, determinable by observation. It is always given with the utmost attainable accuracy in the "Nautical Almanac." The obliquity at present is about 23° 27'½.

The two points where the ecliptic crosses the equinoctial are called the equinoctial points. The sun in its apparent annual course passes through these points about the 21st of March and the 23d of September; the former being the time of the vernal equinox and the latter of the autumnal equinox: these names being given because the night is then equal to the day at all places where the sun rises and sets. This is obvious, because any point in the equinoctial, by the diurnal rotation of the earth—or the apparent rotation of the heavens—is just as long below the horizon of any place as it is above it.

Celestial Longitude.—The circle on which the longitude of any heavenly body is measured is the ecliptic, not the equinoctial; and as terrestrial longitude is measured from a fixed point of the equator, the point (with *us*) where the meridian of Greenwich crosses it, so celestial longitude is measured from a fixed point in the ecliptic, namely, the vernal equinoctial point, which is called the first point of the constellation Aries.

As respects terrestrial longitude the fixed point from which the reckoning commences is only fixed for particular nations, each kingdom choosing its own: this is some inconvenience. But, as respects celestial longitude, there is perfect uniformity of reckoning among astronomers; and this reckoning—unlike that for terrestrial longitude—is carried on in one direction round the celestial sphere; so that a body may have any longitude short of 360°.

The ecliptic is conceived to be divided into twelve equal parts, called *signs*; a sign is therefore an arc of 30°. The twelve signs have the names and symbols following:—

1. ♈ Aries (The Ram).
2. ♉ Taurus (The Bull).
3. ♊ Gemini (The Twins).
4. ♋ Cancer (The Crab).
5. ♌ Leo (The Lion).
6. ♍ Virgo (The Virgin).
7. ♎ Libra (The Balance).
8. ♏ Scorpio (The Scorpion).
9. ♐ Sagittarius (The Archer).
10. ♑ Capricornus (The Goat).
11. ♒ Aquarius (The Water-bearer).
12. ♓ Pisces (The Fishes).

The first six of these signs are to the north of the equinoctial, and the others to the south: they are also called Signs of the *Zodiac*—the name given to a belt of the heavens 8° on each side of the ecliptic.

Celestial Latitude.—As the ecliptic is the circle of longitude, the perpendiculars to it, that is the great circles through the poles of the ecliptic, are the circles of latitude. The distance of a celestial object from the ecliptic, measured on one of these perpendiculars, is its latitude; it is north or south, according as the object is on the north or south side of the ecliptic.

Right Ascension.—The right ascension of a celestial object is the arc of the equinoctial intercepted between the first point of Aries and the declination circle or meridian passing through the object.

The learner will perceive that the first point of Aries is the starting-point from which both longitude and right ascension are measured; and that, what on the terrestrial globe would be longitude and latitude, on the celestial globe are right ascension and declination—the first point of Aries being substituted for the meridian of Greenwich.

Great circles, all of which pass through the poles of any of the more important great circles of the sphere, are frequently called *secondaries* to the latter. This is a very convenient term: thus, vertical circles are secondaries to the horizon; meridians, or declination circles, are secondaries to the equinoctial; and circles of celestial latitude are secondaries to the ecliptic.

On Time.—The most important portion of time, in matters connected with nautical astronomy, is the *day* and its subdivisions. There are several kinds of day referred to in astronomy; but the period occupied by a single rotation of the earth comprises, in each case, nearly the whole of the time so designated. If the heavenly bodies were all fixed, and the earth had no progressive motion, but only its present diurnal rotation on its axis, all days would be alike as to length, since the diurnal rotation is always performed in the same time; the interval between the departure from, and the return to, the meridian, of any heavenly body would then be invariably the same. But as the earth is continually shifting its place in its orbit, and that by an amount which is not uniform, the interval between two successive passages of the sun over the meridian of any place is variable. This interval is called an *apparent solar day.*

Apparent Time.—When the sun is on the meridian of any place it is *apparent noon* at that place; when it is in any other position, the angle between the meridian of the place and that on which the sun is, is called the *hour angle* from noon at that place and instant; this angle, converted into time, at the rate of 15° to an hour, is the apparent time at the place.

Mean Time.—As, on account of the inequality of the earth's motion in its orbit, the solar day is continually varying in length, a day that is the average, or mean, of these variable days is fixed upon for civil reckoning; and it is the length of such a mean day that is marked out by the twenty-four hours of a common clock or watch. This length of time is called a *mean solar day;* and any time shown by a correct clock or watch is mean solar time, or simply *mean time.* At certain periods of the year the sun will thus arrive at the meridian before the clock points to XII., and at other periods the clock will be in advance of the sun; the interval between the arrival of the index of the clock to XII., and of the sun to the meridian, is called the *equation of time.* It is given for every day in the year, at page I. of the "Nautical Almanac," for the meridian of Greenwich; that is to say, when it is apparent noon at Greenwich, on any day of the year, the almanac shows the time to be added or subtracted to obtain the corresponding mean time at that meridian.

Sidereal Time.—A sidereal day is the time occupied by one complete rotation of the earth on its axis. This interval is ascertained by observing the time elapsed between two successive passages of the same fixed star over the meridian. Such is the immense distance of the stars that the earth's change of place from day to day produces not the slightest effect upon their apparent positions, which are preserved the same as if the earth were at rest. Whatever star be observed, and whatever be the place of the earth in its orbit, it is uniformly found that the interval of two successive passages of the star over the meridian is invariable—namely, 23h. 56′ 4·09″ of mean time.*

Besides the *three* kinds of day here described, there is also the *lunar day,* which is the interval between two successive passages or transits of the moon over the meridian; the *average* length of it is 24h. 54m. But navigators have nothing to do with lunar time; what they are most concerned with are apparent time and mean time—the time that would be shown by a properly-constructed sun-dial, and the time shown by a well-regulated chronometer. The time determined by observations at sea, which is in general deduced from the sun's hour-angle with the meridian of the place, is of course apparent time. It is turned into mean time by help of the table of the equation of time at page I. of the "Nautical Almanac"—the phenomena, predicted in that important publication, for the use of seamen in finding the latitude and longitude at sea, being recorded in mean time, just like the transactions of common life.

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* The whole starry heavens have, however, a slow apparent movement, arising from a real motion of the earth distinct from its rotation on its axis. This motion causes the axis to describe a minute circle round the poles of the ecliptic in about 26,000 years; the effect is to cause the apparent approach of some stars towards the pole, and the recession of others. Thus the pole-star, as it is called, has, for many centuries, been getting nearer to the pole; it is now about 1° 34′ from it: the star will continue its approach till within about 30′, and will then recede. The physical cause of the phenomena will be noticed in treating of the Precession and Nutation, in a subsequent part of the present volume.

But there is this difference between the civil and the astronomical mode of reckoning: the civil day reckons from twelve o'clock, at *midnight*, and the whole twenty-four hours is divided into two sets of twelve, the counting recommencing at twelve o'clock, noon; but astronomers commence their day at noon, and count on through the twenty-four hours, from 0 hours up to 24 hours, when another day begins. Consequently the common or civil reckoning is always twelve hours in advance of the astronomical reckoning, both reckonings being in reference to mean time; so that to deduce the civil from the astronomical time at any instant, we have only to add twelve hours to the latter. For instance, Jan. 1, 15h. 35m., astronomical time, is Jan. 2, 3h. 35m., in the morning, civil time.

It is indispensably necessary that the learner have clear conceptions of apparent and mean time: the former is at once ascertained by observation of the true sun's hour-angle from the meridian; the latter is not pointed out by nature, but is arbitrarily chosen for practical convenience—its measure is not ascertained immediately from observation, but computed from the actual phenomena. Astronomers conceive an imaginary sun, called the *mean* sun, to move uniformly in the equinoctial, and with a motion in right ascension exactly equal to the real sun's mean or average motion in right ascension, so that the interval between two consecutive transits of the mean sun is a mean solar day—the mean, that is, of all the variable solar days of the year of the true sun. It is the motion of this imaginary sun that is measured by a chronometer: it completes every revolution in exactly twenty-four common hours; the twenty-four hours completed by the real sun—which twenty-four hours is a variable interval—is the apparent solar day. The twenty-four hours completed in one revolution of a star is a fixed and invariable period; and, as already remarked, is about 3m. 56s., mean time, less than a mean solar day. The sidereal day commences when the first point of Aries is on the meridian, and continues till its return.

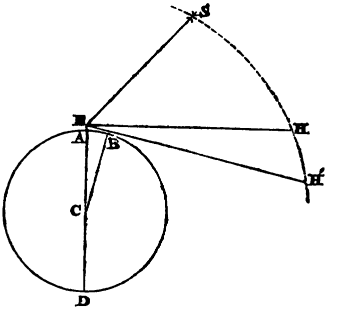
On the Corrections to be Applied to the Observed Altitudes of Celestial Objects.—The true altitude of a celestial object is the angular distance of it above the horizon of the place of observation. It is of course measured (by a quadrant or sextant) in degrees and minutes, the altitude of the zenith being 90°.

The altitude is estimated not from the visible but from the sensible horizon of the observer, and in the case of the sun or moon it is measured up to the centre of the body. The *observed* altitude is the angular distance of the visible horizon from the lower or upper limb of the body, so that a correction has to be made for the dip or depression of the horizon, and for the semidiameter of the body. If the object be a star, then there is no correction for semidiameter. These corrections being made, the result is called the *apparent altitude* of the body.

Some other corrections are necessary to obtain the *true* altitude from the apparent; these we shall speak of presently.

Dip of the Horizon.—As the eye is always elevated above the surface of the sea, the visible horizon dips below the sensible horizon, and forms an angle with it. It is the amount of this angle which must be subtracted from the observed altitude. Let E be the place of the observer's eye, and S the position of the celestial object whose altitude is to be found. The visible horizontal line is EH′, the true horizontal line EH; the altitude of S, as shown by the instrument, is the angle SEH′, instead of the angle SEH; the angle HEH′, by which the latter angle is increased, is the *dip*, which must be subtracted from the observed altitude to give the apparent altitude SEH.

The angle HEH′ is equal to the angle C, since the angle CEB is the complement of each. The height, AE, of the eye being known, as also the radius CA of the earth, EB becomes known for $EB^2 = ED \times EA$ (EUCLID, Prop. 36, Book III.), so that the amount of the angle of depression can always be found when the height of the eye above the surface is given.



Thus let r be put for the radius of the earth and h for the height of the eye above its surface; then, as just shown,

$$EB^2 = (2r + h)h = 2rh, \text{ very nearly,}$$

the quantity h^2 being omitted as insignificant in relation to $2rh$. Hence, because by right-angled triangles, $\sin C = \frac{EB}{EC}$, and since C being always very small—only a few minutes—the arc may be taken for its sine, we have

$$\text{Dip} = \frac{\sqrt{2rh}}{r+h} = \frac{\sqrt{2rh}}{r} \text{ (very nearly)} = \sqrt{\frac{2h}{r}}$$

which is the length of the arc (to radius 1) that measures the angle of the dip due to the height h. This length, for different values of h, is converted into minutes, and in this way the correction for dip is calculated for different altitudes of the eye, and the results arranged in a table.

Semidiameter.—The foregoing correction for the dip of the horizon having been applied to the altitude of the point observed, if this point be the uppermost or lowermost point of the disc of the sun or moon, a correction for the semidiameter of the body must be applied in order to obtain the apparent altitude of the centre. As the measurements in the present subject are all *angular* measurements, the correction here adverted to is the angle subtended at the eye by the semidiameter of the observed body. This angle, both for the sun and moon, is given in the "Nautical Almanac." In the case of the moon, the diameter is seen under a greater angle as she approaches towards the zenith; for at the zenith she is nearer to the observer than when she is in the horizon by a semidiameter of the earth; and such is the comparative nearness of the moon that this difference in her distance makes a sensible difference in her apparent magnitude. The semidiameter given in the "Nautical Almanac" is the horizontal semidiameter, or that under which she would be seen when in the horizon; or, which is the same thing, it is the angle subtended by the semidiameter at the centre of the earth. As this semidiameter increases with her altitude, the increase being so much as one-sixtieth part of the whole when the moon is in the zenith, for she is about sixty semidiameters of the earth off, the amount of increase for any altitude is found by multiplying one-sixtieth of the moon's linear semidiameter by the sine of the altitude, and in this way the table entitled "Augmentation of the Moon's Semidiameter," and given in some collections of Nautical Tables, is constructed: it supplies the proper correction, to be applied additively to the horizontal semidiameter, to obtain the semidiameter at the given altitude.*

On account of the great distance of the sun, the variation of his semidiameter, as he increases in altitude, is too minute to give any correction: it is practically insensible.

The corrections for dip and semidiameter being thus applied, the result is the apparent altitude of the centre. As to the stars, the only correction of the observed altitude of a star, to reduce it to the apparent altitude, is the correction for dip. It remains to be shown how the true altitude is obtained from the apparent altitude: this requires two additional corrections—one for refraction, and the other for parallax.

Refraction.—The atmosphere which surrounds the earth is of variable density, the lower parts being compressed by the weight of the upper. A ray of light, therefore, from a celestial object passes through a medium, which opposes some obstruction to its free passage, the density of the medium increasing as the ray advances from the upper to the lower regions of the atmosphere, where it meets the eye of the observer. This disturbance causes the ray to be deflected or bent, the deflection being greater as the density of the medium through which it passes increases. Instead, therefore, of reaching the eye in a direct line from the object, it begins to curve as soon as it enters the atmosphere, and the curvature increases as it reaches the earth: the direction of the object from which it proceeds—which direction is judged of by the last direction the ray takes, and in which it enters the eye—thus appears to be different from its true direction: the object always, except when in the zenith, seems *higher* than it really is. Consequently, the correction for this *refraction*, as it is called, of the rays of light, like that for dip, is always *subtractive*. At the horizon it is greatest, for the rays become more bent the more obliquely they enter a refracting medium; when they strike upon it perpendicularly they are not bent at all. These facts are proved by many optical experiments.

The refraction takes place entirely in the vertical plane; for contiguous to this plane, to the right and left, the medium being the same, there is nothing to divert it from its path either on one side or the other. Refraction, therefore, like dip, affects *altitudes* only: tables for the corresponding correction of the altitude, from the horizon to the zenith, are given in all nautical collections: these, however, are computed for the *mean* state of the atmosphere; and it must occur to the reader that, as this state is continually varying in certain latitudes, it becomes necessary to modify the numbers in the table, when the true altitude of a celestial object is required with the utmost accuracy, by taking note of the actual state of the atmosphere, as indicated by the thermometer and barometer. To the table of mean refractions a table of these corrections is generally annexed. When the *latitude* only of the ship is required, the correction of the mean refraction is of comparatively little consequence; but in determining the longitude, by lunar observations, it is deserving of attention.

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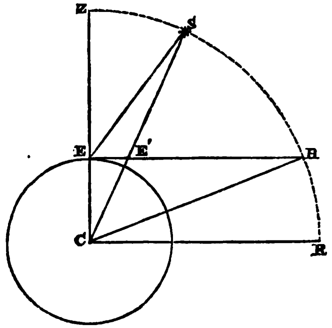
* This correction, amounting only to a few seconds, is frequently omitted in Nautical Tables, and therefore often neglected at sea.

When the corrections now explained are applied to the observed altitude of a celestial object, the result is the true altitude of it above the sensible horizon of the observer, and it now only remains to reduce this to the altitude above the *rational* horizon of the place, as if the object were observed from the centre of the earth instead of from the point on its surface immediately above the centre. In the case of a star, the altitude would be the same, whether the observation be made on the surface or at the centre—the change of position being insensible in reference to objects so remote as the stars; but, for the sun and moon, especially for the latter, the angle subtended at the body by the radius of the earth, called its *parallax in altitude*, is of sensible amount.



Parallax.—Let S represent the place of the celestial body observed from the surface of the earth at E; the observed angle SEH, when corrected for dip, semidiameter, and refraction, will be the true altitude of the centre of S above the sensible horizon, EH, and the angle SCR will be the true altitude of the centre above the rational horizon CR. It is the difference of these angles that is called the parallax in altitude. If the body be in the sensible horizon, as at H, then the difference spoken of will be the whole angle HCR: this is called the horizontal parallax. For any other position of the body the parallax is less—diminishing as the object approaches the zenith, and vanishing at that elevation. Since parallax in altitude = SE′H − SEH = ESC, the parallax is always the angle subtended by the semidiameter of the earth at the object; and since the true altitude above the rational horizon is

$$SCR = SE'H = SEH + ESC,$$

the correction for parallax in altitude must be applied *additively* to the true altitude above the sensible horizon to obtain the true altitude above the rational horizon. The sun's horizontal parallax is always about 9″; the moon's horizontal parallax varies considerably, and is given, together with her semidiameter for every noon and midnight, at page III. of the "Nautical Almanac." And from the horizontal parallax being known, the parallax in altitude is easily found thus. Referring to the triangle SEC, we have the proportion

$$SC : EC :: \sin SEC : \sin ESC = \frac{EC}{SC} \sin SEC;$$

but sin SEC = sin SEZ = cos SEH, and as EC, SC are constant, it follows that the sine of the parallax in altitude varies as the cosine of the altitude; that is,

$$1 : \text{cos alt.} :: \text{sin hor. par.} : \text{sin par. in alt.}$$

The parallax being always a very small angle, it is usual to substitute the seconds in the arc for the sine, so that we have

$$\text{par. in alt. in seconds} = \text{hor. par. in seconds} \times \text{cos altitude.}$$

And in this way the table for parallax in altitude is constructed.

We have now explained the necessary corrections for reducing the observed altitude of a celestial object to its true altitude as seen from the centre of the earth. When the object is the sun or moon, these corrections are four in number, namely, for dip, semidiameter, refraction, and parallax in altitude; when it is a star there are only two corrections, namely, those for dip and refraction. The "Nautical Almanac" furnishes the necessary particulars for the other two corrections when either the sun or moon is observed: the semidiameter of the moon, as seen from the centre of the earth, is given for intervals of twelve hours throughout the year; its value for any intermediate time is to be found by proportion, and it is the same for the horizontal parallax. In the "Explanation" which accompanies the "Nautical Almanac," every needful information is given as to how values which vary continuously may be determined for any proposed time from the recorded values at stated intervals: thus—

To find the moon's semidiameter and horizontal parallax at 6h. A.M. (that is, *before* noon) on Feb. 23, 1846, at a place 15°, or 1h., to the east of Greenwich.

The civil time at the place, expressed in mean astronomical time, is Feb. 22d., 18h., from which, subtracting 1h., because the place is to the east of Greenwich, we have Feb. 22d., 17h. for the corresponding time at Greenwich, or 5h. after midnight. Proceeding from the semidiameter given for midnight of the 22nd, we must compute the proportional part of the variation in 12 hours, due to the time elapsed, viz., 5h.; thus the semidiameter for midnight, or 12h. of the 22nd, is 16′ 31″·6, and for the 23rd, at noon, or 24h., it is 16′ 34″·7; the difference, 3″·1, is the variation in 12 hours. Therefore,

12h. : 5h. : : 3″·1 : 1″·3,

which *added* (because the quantities are increasing) to 16′ 31″·6, gives 16′ 32″·9 for the moon's semidiameter at the time proposed. Similarly, the horizontal parallax at midnight of the 22nd is 60′ 39″, and at noon of the 23rd, it is 60′ 50″·4; the difference 11″·4 is the variation in the 12 hours which include the given time: therefore,

12h. : 5h. : : 11″·4 : 4″·75 or 4″·8,

which *added* (because the quantities are increasing) to 60′ 39″, gives 60′ 43″·8 for the horizontal parallax required. And if with this horizontal parallax, and the apparent altitude of the moon, we enter the table entitled "Moon's Parallax in Altitude," we shall obtain the parallax in altitude. But in most nautical tables the two corrections for refraction in altitude and parallax in altitude are combined, and the results tabulated under the head of "Correction of the Moon's Apparent Altitude," and this is the preferable arrangement when the true altitude is to be deduced.

Besides the foregoing corrections for obtaining the true altitude of a celestial object from the observed altitude, the observed altitude itself generally requires a little correction for the known error of the instrument (quadrant or sextant) employed in taking the altitude: "Human hands or machines never formed a circle, drew a straight line, or erected a perpendicular;"* there are, in consequence, unavoidable departures from strict mathematical accuracy in all mechanical constructions. The shortcomings may be discovered and allowed for, though not remedied—just as the gain or loss of a chronometer may be discovered, though to construct one, without gain or loss, be a practical impossibility. The *index error*—as it is usually called—of the instrument, is to be allowed for before any of the astronomical corrections are introduced; it is not constant, but varies with temperature. The following examples will sufficiently show how the several corrections are applied.

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* Sir John Herschel's "Treatise on Astronomy," page 69. See also "Mathematical Sciences," page 69.

Examples of Correcting Altitudes at Sea.—1. Suppose the observed altitude of a star to be 47° 10′, the height of the eye 18 feet, and the index error of the instrument to be 3′ 12″ *subtractive*: required the true altitude.

Observed altitude	47° 10′ 0″
Index error	− 3′ 12″
	47° 6′ 48″
Dip of the horizon	− 4′ 11″
Apparent altitude	47° 2′ 37″
Refraction	− 53″
True altitude	47° 1′ 44″

The refraction here taken from the table is that for the mean state of the atmosphere. If the height of the barometer and thermometer be observed, the mean refraction may be corrected accordingly by aid of a table usually placed beside that for the mean state of the atmosphere.

2. The observed altitude of the sun's lower limb on a certain day was 16° 33′, the height of the eye was 17 feet, the index error was 3′ *additive*; the barometer stood at 29 inches, and the thermometer at 58°: required the true altitude of the sun's centre, his semidiameter, as given in the "Nautical Almanac" for the day, being 16′ 17″.

Observed alt. sun's L. L. . .	16° 33′ 0″
Index error . . .	+ 3′ 0″
	16° 36′ 0″
Dip . . .	− 4′ 4″
Apparent alt. L. L. . . .	16° 31′ 56″
Refraction . . .	− 3′ 10″
Correction for barometer . .	− 7″
„ thermometer . .	− 3″
True alt. of L. L. above sensible horizon	16° 28′ 36″
Semidiameter (Naut. Alm.) .	+ 16′ 17″
Parallax in altitude . .	+ 8″
True alt. of sun's centre . .	16° 45′ 1″

The corrections for the barometer and thermometer being, as in this example, always very small, they are not attended to at sea when the latitude is the only thing to be determined. The error in the latitude arising from omitting these small quantities is too trifling to be of any consequence.

3. The observed altitude of the moon's lower limb (index error allowed for) is

31° 18′; the horizontal parallax, from the "Nautical Almanac," 58′ 37″; semidiameter, 15′ 58″; and the height of the eye 16 feet: required the true altitude of the moon's centre.

Observed alt. moon's L. L.	31° 18′ 0″
Dip	− 3′ 50″
Semidiam. 15′ 58″ Augmentation 8″	+ 16′ 6″
App. alt. moon's centre	31° 30′ 16″
Cor. for par. and ref. (hor. par. 58′ 37″, alt. 31½°)	+ 48′ 26″
True alt. moon's centre . . .	32° 18′ 42″

4. The observed altitude of the moon's upper limb, corrected for index-error, was 41° 25′; the horizontal parallax, 55′ 40″; semidiameter, 15′ 10″; and the height of the eye, 15 feet: required the true altitude of the moon's centre.

Observed alt. moon's U. L.	41° 25′ 0″
Dip	− 3′ 42″
Semidiam. 15′ 10″ Augmentation 10″	− 15′ 20″
App. alt. moon's centre . .	41° 5′ 58″
Correction for parallax and refraction . .	− 40′ 51″
True alt. moon's centre . . .	41° 46′ 49″

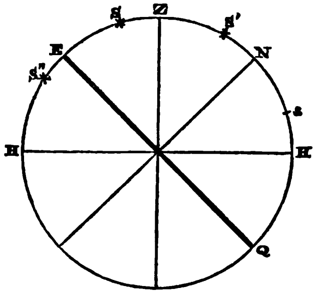
To Determine the Latitude at Sea from the Meridian Altitude of a Celestial Object whose Declination is known.—The determination of the latitude of the ship by means of the altitude, when on the meridian, of a celestial object of known declination, is the easiest, and in general the safest, method for the purpose. The observations and the subsequent calculations being but few, they may be readily accomplished, and with but little liability to error in the result. This method, therefore, is always used at sea, whenever foggy or cloudy weather does not render it impracticable.

The celestial object observed must be one of which the declination is given in the "Nautical Almanac," for the meridian of Greenwich. This declination may be reduced to the meridian of the ship—or rather to the time at Greenwich corresponding to that at the ship—by turning the longitude by account into time, and then applying the variation for declination due to that time, as explained at page 93; the hourly variation of the sun's declination is given in the "Nautical Almanac," at page I. of each month. The longitude by account is always sufficiently near the truth for the determination of this element, though greater precision is required for the moon than for the sun, as the declination of the former changes more rapidly. The declination of the object observed being thus known at the time of observation, and, from its altitude, the zenith distance of it being also known, we have the distance of the object both from the equinoctial and from the zenith of the ship: consequently, the distance of the zenith from the equinoctial—that is, the distance of the ship from the equator—becomes known either by simple addition or subtraction; and thus the latitude is found.

The object observed may be either above the pole or below it; that is, it may be on either the mid-day or the mid-night portion of the celestial meridian of the place; and, in the former case, it may be in either of the three following positions in reference to the equinoctial and the zenith, namely:—

1. The zenith and the object observed may both be on the same side of the equinoctial, and the object nearer to the equinoctial than the zenith is, as at S, in the diagram below.

2. The zenith and the object being on the same side of the equinoctial, the zenith may be nearer to the equinoctial than the object is, as at S′.



3. The zenith and the object may be on different sides of the equinoctial, as at S″.

Let NZH be the meridian, Z the zenith of the ship, N the elevated pole—the north pole suppose—and EQ the equinoctial.

First.—Let the object observed be at S, between the zenith and the equinoctial, and both *north* of the equinoctial; then for the latitude EZ, we have EZ = ES + SZ: that is,

1). The latitude is equal to the declination, *plus* the coaltitude.

Second.—Let the object be at S′, on the contrary side of the zenith, then the latitude EZ is EZ = ES′ − S′Z; that is,

(2). The latitude is equal to the declination, *minus* the coaltitude.

Third.—Let the object be at S″, on the contrary side of the equinoctial; that is, let its declination be *south*, then the latitude is EZ = S″Z − ES″: that is,

(3). The latitude is equal to the coaltitude, *minus* the declination.

Hence, if we call the coaltitude, or zenith distance of the object, *north*, when the zenith is north of it, and *south*, when the zenith is south of it, we shall have the following rule for all the three cases—namely,

Rule.—When the object observed is above the pole, if the zenith distance and the declination have the *same* name, that is, be both north or both south, their *sum* will be the latitude. If the zenith distance and the declination have *different* names, their *difference* will be the latitude, of the same name as the greater.

It is assumed above that the north is the elevated pole; but if it be the south, then by merely writing south for north and north for south, the reasoning will remain the same: the rule will require no modification.

It is obvious that the elevation of the pole above the horizon of any place is always equal to the latitude of that place, for ZN is equally the complement of the latitude EZ, and of HN the elevation of the pole. The elevation of the equator, or the angle it makes with the horizon, is also clearly equal to the colatitude.

It should be remarked, that when celestial objects are near the horizon they are in a position less favourable for observation than when they are more elevated, because the refraction near the horizon is very variable in its effects.

Latitude from the Sun when above the Pole.—The sun is the celestial object most commonly appealed to by mariners for the determination of the latitude of the ship: it is more frequently visible in the day-time, when, except in bad weather, the sea horizon is more clearly defined, and the corrections to be applied to the observed, in order to get the true altitude, are few and simple. The corrections for a star are still fewer; but, as the horizon is usually getting obscure when the stars begin to appear, a star is, in general, less favourable for the purpose than the sun: the moon, however, is often on the meridian under favourable circumstances, but the corrections necessary are more in number, and require to be made with greater care. We shall give suitable directions for each of these objects separately, detailing the proper corrections to be made preparatively to inferring the latitude, as briefly indicated by the general rule given above.

Rule 1. From the longitude by account find the *apparent* time at Greenwich: this is called the Greenwich *date*.

2. From page I. of the Nautical Almanac get the noon-declination at Greenwich; and, by means of the hourly difference in declination there given, and the previously found Greenwich date, find the correction due to that date; the declination at the time of observation will thus be discovered.

3. Apply to the observed altitude the proper corrections for dip and semi-diameter; the apparent altitude of the centre will then be obtained; and the corrections for refraction and parallax will reduce the apparent to the true altitude.

4. Mark the zenith distance N. or S. according as it is N. or S. of the sun: then, if the declination and zenith distance have the same marks, their sum will be the latitude; if they have different marks, their *difference* will be the latitude.

Note.—The first step in the foregoing rule requires the conversion of degrees, minutes, &c., of longitude into time, which is readily done from the known relations—

$$15^\circ = 1\text{h},\ 15' = 1\text{m},\ 15'' = 1\text{s};$$

for from these it is evident that if we multiply the degrees of longitude by 2, and then divide by 30, the quotient will be hours, and so many *thirtieths* of an hour; so that twice these thirtieths will be the additional minutes of time.

In like manner, if we multiply the minutes of longitude by 2, and divide by 30, the quotient will be the minutes of time, and so many thirtieths; that is, twice that number of seconds of time.

Suppose, for instance, the longitude is 93° 37′ 41″, we may easily convert it into time, as in the margin, thus: multiplying the degrees, minutes, and seconds by 2, we have 186° 74′ 82″.

Dividing each, separately, by 3, cutting off the unit figure of each for the 0 suppressed in the 30, we have for the first quotient 6 hours and 6 thirtieths—that is, 6h 12m; for the second quotient, 2 minutes and 14 thirtieths—that is, 2m 28s; and for the third quotient, 2·73s—decimals of a second being always used instead of *thirds*: hence the time corresponding to 93° 37′ 41″ of longitude is 6h 14m 30·73s.

3)18,6°	7,4′	8,2″
6	12	
	2	28
		2·73
6h	14m	30·73s.

Examples.

1. On April 27, 1853, in north latitude, and in longitude 87° 42′ W. the observed meridian altitude of the sun's lower limb was 48° 42′ 30″ (zenith N), the index

correction was + 1′ 42″, and the height of the eye 18 feet: required the latitude.

1. *For the apparent time at Greenwich.*

Long. by account . . . 87° 42′ W.
2
3)17,4 8,4
5 48
2 48
5h. 50m. 48s.

The variation for this time to be added, as the declination is increasing.

2. *For the sun's declination.*

Dec. at app. noon (Naut. Alm.)
13° 43′ 53″ N. var. in 1h. 47″·7 increasing
+ 4′ 38″ 6
13° 48′ 31″ N. in 6h. . . 286·2
in 10m. . . 7·9
in 5h. 50m. 278·3 seconds
or 4′ 38″

Observed altitude of sun's L.L.	48° 42′ 30″
Index cor. + 1′ 42″ Dip . . — 4′ 11″ Semi-diam. + 15′ 54″	+ 13′ 25″
App. alt. of sun's centre	48° 55′ 55″
Refraction and parallax	— 44″
True alt. of centre	48° 55′ 11″ 90°
True zenith distance	41° 4′ 49″ N.
Sun's reduced declination	13° 48′ 31″ N.
Latitude	54° 53′ 20″ N.

As noticed in the Introduction, page 32, it is always advisable to make all the use we can of Tables when they are once in hand. The first table referred to in the foregoing operation is that given at page I. of the Nautical Almanac for the sun's declination at apparent noon at Greenwich, with the hourly variation; the semi-diameter should be taken out at the same time, and inserted in its proper place, but seamen in general use invariably 16′ for the sun's semi-diameter. A blank form of the several particulars in such operations is of considerable assistance, as the work is thereby facilitated, and the risk of mistake diminished. The "correction in altitude"—that is, the allowance for refraction and parallax—should be taken from the Nautical Tables at the same time as that for dip in working examples; though at sea the dip generally differs but little throughout a voyage.*

It may be remarked, that in computing the declination of the sun for the time of observation, seconds of time may be disregarded, and even a few minutes is of no practical consequence: indeed, the longitude by account is only an approximation to the correct longitude.

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* In some cases, however, when the voyage is long, and the complement of men considerable, the dip may sensibly increase. Captain Parry, after wintering in "Winter Harbour," in 1820, estimated the consumption of stores and provisions to have amounted, in the "Hecla," to about seventy tons: the dip must therefore have increased.

But, as a little consideration will be sufficient to show, the error in the estimated longitude must be very great indeed to occasion any error of practical importance in the declination of the sun at the instant of observation. If only the altitude be observed with proper care, the computer may trust to the accuracy of the resulting latitude without any nice determination of his longitude: 1° of longitude is only 4° of time.

2. On August 14, 1846, in north latitude, and in longitude 51° W., the meridian altitude of the sun's upper limb was 47° 26′ (zenith N.), the index correction − 1′ 47″, and the height of the eye 20 feet: required the latitude.

1. *For the apparent time at Greenwich.*

Long. by account . . . . . 51° W.
2
3)10,2
App. time at Greenwich . . . 3h. 24m.

The variation for this time must be subtracted, as the declination is decreasing.

2. *For the sun's declination.*

Dec. at app. noon (Naut. Alm.)

14° 25′ 28″ N.	var. in 1h.	46″·51 dec.
− 2′ 38″		3
14° 22′ 50″ N.	in 3h.	139·53
	in 12m.	9·30
	in 12m.	9·30
	2′ 38″ =	158·13 sec.

Observed altitude of sun's U. L.	47° 26′ 0″
Index cor. — 1′ 47″ Dip. . . — 4′ 24″ Semi-diam. — 15′ 49″	— 22′ 0″
App. Alt. of centre	47° 4′ 0″
Refraction and parallax	— 47″
True alt. of centre	47° 3′ 13″
	90°
True zenith distance	42° 54′ 47″ N.
Sun's reduced declination	14° 22′ 50″ N.
Latitude	57° 17′ 37″ N.

3. On November 8, 1846, in longitude 62° E., the meridian altitude of the sun's lower limb was 57° 12′ 30″ (zenith S.), the index correction was + 1′ 36″, and the height of the eye 30 feet: required the latitude.

1. *For the apparent time at Greenwich.*

Long. by account . . 62° E.
2
3)12,4
App. time at Greenwich 4h. 8m. before noon.

The variation for this time must be subtracted, as the declination is greater at noon than before noon.

2. *For the sun's declination.*

Dec. at app. noon (Naut. Alm.)

16° 33′ 34″ S.	var. in 1h. . .	43″·36 inc.
— 3′ 0″		4
16° 30′ 34″ S.	in 4h. . .	173·44
	in 8m. . .	6·19
	3′ =	179·63 sec.

Observed altitude of sun's L.L.	57° 12′ 30″
Index cor. + 1′ 36″ Dip. — 5′ 24″ Semi-diam. + 16′ 10″	+ 12′ 22″
App. alt. of centre	57° 24′ 52″
Refraction and parallax	— 32″
True alt. of centre	57° 24′ 20″ 90°
True zenith distance	32° 35′ 40″ S.
Sun's reduced declination	16° 30′ 34″ S.
Latitude	49° 6′ 14″ S.

It may be remarked here, that in reducing the declination to the time at Greenwich when the observation is made, the hourly variation should, in strictness, be taken equal to 44·06, which is the average variation during the *preceding* 24 hours, or from the noon of Nov. 7 to the noon of Nov. 8, because the 4h. 8m., for which the correction is made, is a portion of *this* 24 hours; but the difference in the correction would, amount only to about 2½ seconds. Such small quantities are not worth attending to in the present problem, as the altitude of a celestial object, taken at sea, cannot be measured to within a few seconds of the truth; and if the latitude can be deduced to the nearest minute, it is all that can be reasonably expected.

To save the trouble of the preliminary reductions here adverted to, a table is given in most nautical collections, by entering which with the sun's noon-declination and the longitude by account, the correction for declination is found with accuracy sufficient for the purposes of navigation. The table referred to is, in Norie's Tables, the twenty-first; we shall use it in the following example.

4. On November 21, 1841, in longitude 165° E., the meridian altitude of the sun's lower limb was observed to be 47° 38′ (zenith N.), the index correction was — 1′ 15″, and the height of the eye 17 feet: required the latitude.

For the declination at the Greenwich time of observation.

Sun's declination at noon, Nov. 21 (Naut. Alm.)	19° 57′ 55″ S.
Correction for longitude 165° E.	— 5′ 52″
Sun's reduced declination	19° 52′ 3″ S.

Observed altitude of sun's L.L.	47° 38′ 0″
Index cor. — 1′ 15″ Dip. — 4′ 4″ Semi-diam. + 16′ 13″	+ 10′ 54″
App. alt. of centre	47° 48′ 54″
Refraction and parallax	— 46″
True alt. of centre	47° 48′ 8″ 90°
True zenith distance	42° 11′ 52″ N.
Sun's reduced declination	19° 52′ 3″ S.
Latitude	22° 19′ 49″ N.

Latitude from a Star above the Pole.—A fixed star changes its declination so slowly that its variation, even in a month, is scarcely sensible; no correction, therefore, for longitude will be necessary; the declination, as given in the Nautical Almanac on the day of observation, may be taken as that at the time of observation. As a fixed star has no parallax in altitude, the only corrections will be those for dip and refraction; the rule for deducing the latitude is therefore as follows:—

RULE 1. Apply to the observed altitude the corrections for dip and refraction; the result will be the true altitude, and this subtracted from 90° will give the true zenith distance.

2. Mark the zenith distance N. or S. according as it is N. or S. of the star; then if the declination and zenith distance have the same marks, their sum will be the latitude; if they have different marks, their difference will be the latitude.

Examples.

1. January 22, 1846, the meridian altitude of Arcturus was observed at sea to be 43° 27′ (zenith north), the index correction was + 2′ 10″, and the height of the eye 20 feet; required the latitude.

Observed altitude of star	43° 27′ 0″
Index cor. + 2′ 10″ Dip . . − 4′ 24″	− 2′ 14″
App. alt. of star	43° 24′ 46″
Refraction	− 1′ 0″
True alt. of star.	43° 23′ 46″ 90°
True zenith distance	46° 36′ 14″ N.
Star's declination, Jan. 22, 1846	19° 58′ 59″ N.
Latitude	66° 35′ 13″ N.

2. On February 12, 1853, the meridian altitude of α Hydræ was observed to be 47° 24′ 30″ (zenith north), the index correction was − 2′ 10″, and the height of the eye 17 feet. Required the latitude.

Observed altitude of star	47° 24′ 30″
Index cor. − 2′ 10″ Dip . . − 4′ 4″	− 6′ 14″
App. alt. of star	47° 18′ 16″
Refraction	− 52″
True altitude	47° 17′ 24″ 90°
True zenith distance	42° 42′ 36″ N.
Star's declination, Feb. 12, 1853	8° 1′ 29″ S.
Latitude	34° 41′ 7″ N.

3. On July 16, 1845, the meridian altitude of Fomalhaut was found to be 73° 36′½ (zenith north), the index correction — 30″, and the height of the eye 24 feet. Required the latitude.

Observed altitude of the star	73° 36′ 30″
Index cor. — 30″ Dip . — 4′ 49″	— 5′ 19″
App. alt. of star	73° 31′ 11″
Refraction	— 16″
True altitude	73° 30′ 55″
	90°
True zenith distance	16° 29′ 5″ N.
Star's declination, July 16, 1845	30° 26′ 20″ S.
Latitude	13° 57′ 15″ S.

NOTE.—In some nautical tables the corrections for dip and refraction are united under the head of "Correction of star's observed altitude."

Latitude from the Moon above the Pole.—The moon's declination is given in the Nautical Almanac for every hour of the day, and the time of her meridian passage from day to day. These elements are computed for *mean time*, the reckoning being from mean noon at Greenwich. Hence, to find the moon's declination corresponding to the time of taking her meridian altitude at sea, we must know the time at Greenwich at the instant of observation. This is ascertained as follows:—

As the motion of the moon in her orbit is eastward, her transit over the meridian of any place is delayed from day to day. In consequence of this retardation she will pass the meridian of a place to the west of Greenwich later in the day at that place than she passes the meridian of Greenwich, and her transit over a meridian to the east of Greenwich will take place earlier in the day. How much later, or how much earlier, will be ascertained by converting the longitude of the meridian into time, and applying the corresponding proportional part of the daily variation as furnished by the Nautical Almanac. By help of the short table in the next page, and the daily change in the time of transit, the correction to be added to the Greenwich time of transit, to obtain the time of transit over a meridian west of Greenwich, or to be subtracted to obtain the time of transit over a meridian east of Greenwich, may be at once found.

The daily variation, as given by the Nautical Almanac, is to be sought for in the top row of figures, and the longitude of the place in the marginal column on the left; the proper correction of the Greenwich mean time to reduce it to the mean time of transit at the place is then to be taken out from the body of the table. It is sufficient that the daily variation be taken to the nearest minute.

It must be observed, however, that in the case of the moon, there must not be the same indifference as to the accuracy of the longitude by account as is allowable in the case of the sun; the following examples will show that on account of the moon's more rapid change in declination, a comparatively short interval of time makes a sensible difference in this element.

Table for finding the time of the moon's transit over a given meridian when the time of the transit at Greenwich is known, and the daily variation of the time.

° Longitude	DAILY VARIATION IN MINUTES OF TIME OF MOON'S TRANSIT.													
	40m.	42m.	44m.	46m.	48m.	50m.	52m.	54m.	56m.	58m.	60m.	62m.	64m.	66m.
10	1	1	1	1	1	1	1	1	1	2	2	2	2	2
20	2	2	2	2	3	3	3	3	3	3	3	3	3	4
30	3	3	4	4	4	4	4	4	4	5	5	5	5	5
40	4	4	5	5	5	5	6	6	6	6	6	7	7	7
50	5	6	6	6	6	7	7	7	7	8	8	8	9	9
60	6	7	7	7	8	8	8	9	9	9	10	10	10	11
70	7	8	8	9	9	9	10	10	10	11	11	12	12	12
80	9	9	9	10	10	11	11	12	12	12	13	13	14	14
90	10	10	11	11	12	12	13	13	13	14	14	15	15	16
100	11	11	12	12	13	13	14	14	15	15	16	17	17	18
110	12	12	13	14	14	15	15	16	16	17	18	18	19	19
120	13	14	14	15	15	16	17	17	18	19	19	20	20	21
130	14	15	15	16	17	17	18	19	19	20	21	21	22	23
140	15	16	17	17	18	19	20	20	21	22	22	23	24	25
150	16	17	18	19	19	20	21	22	22	23	24	25	26	26
160	17	18	19	20	21	21	22	23	24	25	26	26	27	28
170	18	19	20	21	22	23	24	25	25	26	27	28	29	30
180	19	20	21	22	23	24	25	26	27	28	29	30	31	32

By the aid of this table the latitude from a meridian altitude of the moon when above the pole may be found as follows :—

RULE 1.—From the Nautical Almanac take out the time of the moon's passage over the meridian of Greenwich on the given day, as also the daily variation.

2.—From the longitude by account, and the foregoing table, reduce this to the time, at the place, of the moon's passage over the meridian of the ship; the time of observation, at the place where that observation is made, will thus be found.

3.—From the ship's time and longitude find the corresponding time at Greenwich.

4.—Find now the moon's declination at that time from the Nautical Almanac, computing the variation for the odd minutes by means of the difference in declination for 10m.

5.—From page III. of the month, take out the moon's semidiameter, and increase it by the "Augmentation" given in the Nautical Tables. The correction for index-error, dip, and semidiameter, will reduce the observed altitude of the limb to the apparent altitude of the centre.

6.—To the apparent altitude of the centre add the correction in altitude, that is, the parallax in altitude minus the refraction, and the true altitude of the centre will be obtained. Subtract this from 90°, and then, by adding or subtracting the moon's declination at the time, as in the case of the sun, the latitude of the place will be ascertained.

Examples.

1. On May 27, 1846, in longitude 49° W., the meridian altitude of the moon's lower limb was found to be 47° 18′ 30″ (zenith S.) the index correction was + 1′ 40″, and the height of the eye 20 feet: required the latitude of the ship.

1. *For the mean time at Greenwich when the observation was made.*

Moon's transit at Greenwich, May 27 . .	1h. 56m.	Daily variation, 48·7m.
Cor. for long. 49° W. and 48m. variation .	+ 6m.	
Time at ship when alt. was taken . .	2h. 2m.	
Long. 49° W. in time	3h. 16m.	
Time at Greenwich when alt. was taken .	5h. 18m.	

2. *For the moon's declination at that time.*

Declination May 27, at 5h. . . .	18° 56′ 16″ N.	Var. in 10m., 26″·55 (dec.)
Decrease in 18m. = 2″·655 × 18 =	— 48″	
Declination at 5h. 18′	18° 55′ 28″ N.	

3. *For the true altitude of the Moon's centre, and thence the latitude.*—At page III. of May, in the Nautical Almanac, the moon's semi-diameter at noon on the 27th is 15′ 1″; and her horizontal parallax at the same time is 55′ 16″. Hence we get the latitude thus:—

Observed alt. of moon's L. L.		47° 18′ 30″
Index cor. . . .	+ 1′ 40″	+ 12′ 32″
Dip	— 4′ 24″	
Semi-diameter + 11″ for augmen.	+ 15′ 16″	
App. alt. of moon's centre		47° 31′ 2″
Parallax and refraction		+ 36′ 10″
True alt. of moon's centre		48° 7′ 12″ 90°
True zenith distance		41° 52′ 48″ S.
Moon's declination		18° 55′ 28″ N.
Latitude		22° 57′ 20″ S.

It is plain that whatever be the celestial object observed, the error in the latitude will be the sum of the errors in the zenith distance and declination when both are of the same name, and the difference when they are of contrary names.

The latitude at sea is seldom computed to seconds, as the exact longitude and time at the ship, as inferred from the dead reckoning, generally deviates from the truth. It is customary, therefore, to aim at deducing the latitude only to the nearest minute: thus, in the present example the latitude would be concluded to be 22° 57′ S.

2. On Dec. 7th, 1840, in longitude 16° W. at 10h. 43m. apparent time, the meridian altitude of the moon's lower limb was 83° 7′ (zenith S.), the index correction was — 1′ 55″, and the height of the eye 16 feet; required the latitude to the nearest minute. Here the time at the ship being given, we shall not require to take out of

Nautical Almanac the time of the moon's meridian passage at Greenwich: the nwich date, or mean time at Greenwich when the altitude was taken, is found :—

1. *For the mean time at Greenwich when the observation was made.*

Apparent time at ship, Dec. 7	10h. 43m.
Long. 16° W. in time	1h. 4m.
App. time at Greenwich	11h. 47m.
Equation of time (Naut. Alm. p. I.) . .	— 8m.
Mean time at Greenwich	11h. 39m.

2. *For the Moon's declination at that time.*

Declination Dec. 7, at 11h. . . .	24° 45′ N. (inc.)	Var. in 10′, 80″
Increase in 39m. = 8·× 39 = . .	+ 6′	
Declination at 11h. 39m.	24° 51′ N.	

3. *For the true altitude of the Moon's centre, and thence the latitude.*

Observed alt. of moon's L. L.		83° 7′
Index cor. .	— 2′	
Dip . . .	— 4′	+ 11′
Semi-diam. .	+ 17′	
App. alt. moon's centre		83° 18′
Parallax and refraction		+ 7′
True alt. of moon's centre		83° 25′
		90°
True zenith distance		6° 35′ S.
Moon's declination		24° 51′ N.
Latitude		18° 15′ N.

Examples for Exercise.

OTE.—In the following examples the latitude is to be determined to the nearest te.

. On the 2nd of May, 1833, the meridian altitude of the sun's lower limb was 0′ (zenith N.), the index correction — 2′ the height of the eye 20 feet, and the tude by account 32° E.

lso sun's dec. May 2, 15° 23′ 21″ N. (inc.) Hourly difference 45″.

emi-diameter 15′ 53″ (Nautical Almanac).

equired the latitude to the nearest minute. Ans. Latitude 58° 53′ N.

On Jan. 9, 1840, in longitude 116° W., the meridian altitude of the sun's upper was found to be 69° 14′ (zenith N.), index error 0, and height of the eye 27 feet. ired the latitude to the nearest minute.

un's dec. Jan. 9, 22° 12′ S. (dec.). Correction for long. 116° W., — 3′. Semi- . 16′. Ans. Latitude 1° 2′ S.

3. On May 15th, 1828, the meridian altitude of the star Spica was observed to be 33° 17′ (zenith N.), index correction + 1′ 10″; the star's declination was 10° 16′ S., required the latitude to the nearest minute, the dip being — 5.

Ans. Latitude 46° 32′ N.

4. The meridian altitude of the star Rigel was observed to be 85° 4′ (zenith N.), the index correction was + 2′, and the star's declination 8° 22′ 45″ S.; the height of the eye was 20 feet: required the latitude to the nearest minute.

Ans. Latitude 3° 24′ S.

5. On Feb. 19th, 1823, in longitude 40° W., the meridian altitude of the moon's lower limb was observed to be 55° 8′ (zenith N.), the index correction was — 2′, and the height of the eye 16 feet.

The moon's passage over the merid. of Greenwich, Feb. 19th, was 6h. 56m.
" " " " " 20th, was 7h. 59m.

Declination, Feb. 19th, at noon	26° 38′ 17″ N.
" " midnight	26° 54′ 39′ N.
Semi-diameter	16′ 12″
Horizontal parallax	59′ 32″

Required the latitude to the nearest minute. Ans. Latitude 61° 1′ N.

6. On Nov. 12, 1853, at 2h. 20m. mean time, in longitude 60° 42′ W., the meridian altitude of the moon's lower limb was observed to be 30° 30′ 40″ (zenith N.), the index correction was + 10′ 42″, and the height of the eye 16 feet.

Moon's declination, Nov. 12, at 6h. . . .	2° 44′ 20″ N.
" " " 7h. . . .	2° 57′ 38″ N.
Semi-diameter	15′ 12″
Horizontal parallax	55′ 13″

Required the latitude to the nearest minute. Ans. Latitude 61° 11′ N.

To determine the latitude from the meridian altitude of a celestial object when below the pole.—In the diagram at page 96, let *s* be the position of a celestial object when on the meridian of the place whose zenith is Z, and below the elevated pole N; the altitude of it will be H*s*, and *s* N will be the complement of its declination Q*s*. Consequently, the altitude added to the co-declination will be the latitude H N of the place whose zenith is Z (page 86). The sun is on the meridian of any place, below the pole, 12 hours after the apparent noon at that place, consequently 12 hours, increased or diminished by the longitude in time, according as the place is W. or E. of Greenwich, will be the apparent time at Greenwich when the observation was made; and the declination corresponding to this time may be found as in the foregoing examples.

For a star, the declination will be the same as that given for the day in the Nautical Almanac; since, as before remarked, the change in the declination of a fixed star is insensible till after the lapse of several days.

For the moon, the time of transit over the mid-day portion of the meridian of the place may be found as at page 103, and this time increased by 12 hours and by half the daily difference of time, will be the time of her returning to the meridian below the pole; and the proper reduction being made for longitude, as in the case of the sun, the time at Greenwich, and thence the corresponding declination may be found. The rule for computing the latitude is therefore as follows:—

Rule.—1. Find the declination of the object at the time of observation.

2. To the observed altitude apply the proper corrections for deducing the true altitude.

3. To the true altitude add the co-declination; the sum will be the latitude, of the same name as the declination.

Latitude from the Sun when below the Pole.—As the rule just given applies equally to the sun, moon, or star, special directions for each case will be unnecessary; we shall, therefore, give a practical illustration of the mode of working for each object separately, and then add an example or two for exercise. The following is an example when the object is the sun:—

Example.—On June 18, 1853, in north latitude, and in longitude 96° W., the meridian altitude of the sun's lower limb, at apparent midnight, was observed to be 8° 36′, the index correction was — 2′, and the height of the eye 20 feet: required the latitude.

For the declination at the Greenwich time of observation.

Apparent time at ship, June 18,	12h. 0m.
Longitude 96° W. in time	6h. 24m.
Apparent time at Greenwich	18h. 24m.
Sun's declination at noon, June 18	23° 25′ 36″ N. (inc.)
Correction for 18h. 24m.	49″
Declination at time of observation	23° 26′ 25″ N.
	90°
Co-declination	66° 33′ 35″
Observed alt. of sun's L.L.	8° 36′ 0″
Index cor. — 2′ Dip — 4′ 24″ Semi-diam. 15′ 46″	9′ 22″
App. alt. of centre	8° 45′ 22″
Refraction and parallax	— 5′ 49″
True alt. of centre	8° 39′ 33″
Sun's co-declination	66° 33′ 35″
Latitude	75° 13′ 8″ N.

Latitude from a Star when below the Pole.—In northern latitudes, the star called the pole star is very convenient for the purpose of finding the latitude of the ship, and is, therefore, frequently observed by mariners for this object. The following is an example:

Example.—On March 1st, 1823, the observed altitude of the pole star when on the meridian below the pole, was 30° 7′, the index correction was + 2′, and the height of the eye 18 feet: required the latitude.

Observed altitude of pole star		30° 7′ 0″
Index cor. + 2′ Dip . . − 4′ 11″		− 2′ 11″
		30° 4′ 49″
Refraction		− 1′ 38″
True altitude of star		30° 3′ 11″
Co-declination Mar. 1, 1823		1° 38′ 2″
Latitude		31° 41′ 13″ N.

Latitude from the Moon when below the Pole.—The preliminary corrections for the moon, as already seen, are more numerous than those for the sun or for a star; a specimen of them is given in example 1 page 104. These corrections are of course the same whether the moon be observed on the meridian above the pole or below it. In the following example the moon's declination is found as at the page just referred to, so that the reductions need not be repeated here.

Example.—On the 27th of May, 1846, in longitude 49° W., the meridian altitude of the moon's lower limb, when below the pole, was found to be 7° 12′, the index correction was — 1′ 40″, and the height of the eye 20 feet: required the latitude.

The moon's declination at the time when the observation was made was found at page 104 to be 18° 55′ 28″ N.: hence the operation for finding the true altitude of the moon's centre, and thence the latitude of the place is as follows:

Observed alt. of moon's L.L.		7° 12′ 0″
Index cor. .	— 1′ 40″	
Dip . . .	— 4′ 24″	+ 9′ 12″
Semi-diam. augmented	+ 15′ 16″	
App. alt. of centre		7° 21′ 12″
Parallax and refraction		+ 47′ 31″
True alt. of moon's centre		8° 8′ 43″
Co-declination		71° 4′ 32″
Latitude		79° 13′ 15″ N.

Examples for Exercise.

1. On June 28, 1841, in longitude 126° W., the altitude of the sun's lower limb at midnight was 6° 28′, the index correction was + 2′ 15″, and the height of the eye 20 feet: required the latitude to the nearest minute, the sun's declination at noon, Greenwich time, on the 28th being 23° 17′ 59″ N. decreasing, and his semi-diameter 15′ 45″. Ans. Latitude 73° 19′ N.

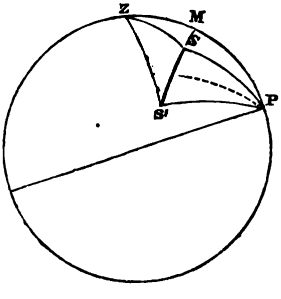
2. The observed altitude of the sun's lower limb, when on the meridian below the pole was 7° 5′; its declination at the time of observation was 23° 8′ 17″ N., its semi-diameter 15′ 45″, and the height of the eye 20 feet: required the latitude, the index error of the instrument being 0. Ans. Latitude 74° 1′ N.

3. June 1st, 1823, the observed altitude of the star Capella, when on the meridian

below the pole was 10° 1' 30", its declination was 45° 48' 27" N., and the height of the eye 17 feet: required the latitude. Ans. Latitude 54° 4' N.

The foregoing examples exhibit the different means of determining the latitude at sea by taking the altitude of a celestial object when upon the meridian of the ship. Whenever practicable, the sun is always the object observed; and its mid-day altitude is that which is to be preferred, because at mid-day it attains the greatest elevation, and the refraction is less liable to variation from the mean state of the atmosphere. The altitude below the elevated pole can be taken only in high latitudes, where the sun is above the horizon during the whole twenty-four hours for a part of the summer, and therefore the horizon clearly visible: the obscurity of the horizon often precludes the possibility of accurately measuring the altitude of a star; and on account of its rapid change in declination, the moon is less suited for the purpose of deducing the latitude, when there is much uncertainty as to the longitude of the ship, or the time at Greenwich, when the observation is made. In preparing for a meridian altitude, the observer holds himself in readiness, before the object attains its greatest elevation, and continues to observe it till it ceases to rise and appears for a moment stationary; at this instant its altitude is noted, and it is regarded as upon the meridian. Strictly speaking, however—at least as respects the sun and moon—the centre of the object is not necessarily exactly on the meridian when its altitude is the greatest or least; for the change in declination may more than counterbalance its change in altitude during the few minutes which precede its meridian transit, especially in the case of the moon; but these differences are too trifling to lead to any error of practical importance. It may, however, happen that the object becomes hidden by a cloud at the time of transit, so that the meridian observation cannot be made: it is of importance, therefore, to take note of the altitudes before the transit, as an altitude *near* the meridian may be made available for the determination of the latitude, as we are now about to show: we shall first, however, explain how to find the latitude generally when the object observed is off the meridian, provided the hour angle, which its declination-circle makes with the meridian of the ship, is known.

To find the Latitude from the Declination, Altitude, and Hour Angle.—Let Z be the zenith and PMZ the celestial meridian of the ship, P the elevated pole, and S the place of the heavenly body off the meridian; then PS is the co-declination, ZS the co-altitude, and PZ the co-latitude. In the spherical triangle PZS, there is supposed to be known the three parts, PS, ZS, and the hour angle P, which, for the sun, is the time from noon. Hence, by Spherical Trigonometry, PZ the co-latitude may be found (MATHEMATICAL SCIENCES, page 411). But the following method, by right-angled triangles only, will be more easily recollected.



Draw SM, perpendicular to the meridian: we shall then have two right-angled triangles PMS and ZMS, and applying Napier's rules to these (MATHEMATICAL SCIENCES, page 409) we have—

From the triangle PMS, taking P for middle part, and PS, PM, for adjacent parts,

$$\cos P = \cot PS \tan PM \therefore \tan PM = \cos P \tan PS \quad . \quad . \ (1)$$

Also, from the same triangle, taking the hypotenuse PS for middle part, and PM, SM, for opposite parts,

$$\cos PS = \cos PM \cos SM \quad . \quad . \quad (2)$$

And from the triangle ZMS, taking in like manner the hypotenuse ZS for middle part, and ZM, SM for opposite parts,

$$\cos ZS = \cos ZM \cos SM \quad . \quad . \quad (3)$$

Therefore dividing (2) by (3), in order to get rid of SM, we have

$$\frac{\cos PS}{\cos ZS} = \frac{\cos PM}{\cos ZM} \therefore \cos ZM = \cos PM \cos ZS \sec PS \quad . \quad . \quad (4)$$

As ZM is thus expressed in terms of the given quantities, and as PM is also in like manner known from (1), the difference (or the sum, if M fall between P and Z) PZ, that is, the co-latitude becomes known.

The formulæ (1) and (4) to be combined are

$$\left.\begin{aligned} \tan PM &= \cos \text{hour angle} \times \text{cotan dec.} \\ \cos ZM &= \cos PM \sin \text{alt.} \times \text{cosec dec.} \end{aligned}\right\} (A)$$

To know the hour-angle P, it is necessary to know the time at the ship. The chronometer shows the mean time at Greenwich; and hence, by help of the longitude by account, we may find the mean time of observation nearly; this, reduced to apparent time, by applying the correction for the equation of time taken from the Nautical Almanac, will make known the time from apparent noon at the ship. The longitude by account is, however, most likely affected with error, and it is therefore desirable that the altitude off the meridian be taken at such a time as that a small error in the hour angle may have the least influence on the determination of tan PM. Now, whenever we have to employ the cosine of a small angle, and have reason to suspect that the angle itself has not been accurately determined, the error in the cosine will be smaller as the angle itself is smaller; for the cosines of arcs near the beginning of the quadrant differ very little from one another within the limits of several seconds, and the difference becomes less as the arc approaches to zero (see INTRODUCTION, page 28).

It follows, therefore, that when the time at the ship is only approximately known, the altitude should be taken when the object is as near to the meridian as it is likely to be before being obscured by clouds.

The formulæ just established is, we see, generally applicable, however distant from the meridian the observed object may be, provided we know the apparent time at the ship; and we see also, that they may be employed when the time *nearly* is known, provided the object be pretty close to the meridian. But for this latter case there is a special method somewhat more convenient, which may be investigated as follows:—

Referring to the preceding diagram we have, by the fundamental theorem of Spherical Trigonometry, which expresses the relations among the three sides and one of the angles of a spherical triangle,

$$\cos ZS = \cos PZ \cos PS - \sin PZ \sin PS \cos P$$

$$\therefore \cos P = \frac{\cos PS - \cos PZ \cos PS}{\sin PZ \sin PS}$$

Let z be the zenith distance that S would have when upon the meridian, and z' the difference between this meridian zenith distance and that ZS actually observed, that is, let $ZS = z + z'$; then the equation above is

$$\cos P = \frac{\cos(z + z') - \cos PZ \cos PS}{\sin PZ \sin PS}$$

Subtracting each side from 1, we have

$$1 - \cos P = 2\sin^2 \tfrac{1}{2} P = \frac{\sin PZ \sin PS + \cos PZ \cos PS - \cos (z + z')}{\sin PZ \sin PS}$$

$$= \frac{\cos (PZ \backsim PS) - \cos (z + z')}{\sin PZ \sin PS}.$$

Now PZ $\backsim$ PS, that is, the difference between the co-latitude and the co-declination is equal to Z, the meridian zenith distance, because the co-latitude PZ is always equal either to PS + z, or PS − z, or z − PS, the latter being the case when S is below the pole (see the diagram at page 96). Consequently

$$2\sin^2 \tfrac{1}{2}P = \frac{\cos z - \cos (z + z')}{\sin PZ \sin PS} = \frac{\cos z - \cos z \cos z' + \sin z \sin z'}{\sin PZ \sin PS}.$$

If the difference z′, between the meridian zenith distance and that actually observed, be so small that cos z′ may be regarded as equal to 1, then we shall have

$$2\sin^2 \tfrac{1}{2}P = \frac{\sin z \sin z'}{\sin PZ \sin PS}$$

$$\therefore \sin z' = 2 \sin PZ \sin PS \operatorname{cosec} z \sin^2 \tfrac{1}{2}P.$$

The number of seconds in the arc z′ is very nearly equal to the number of times sin z′ contains sin 1″ : consequently,

$$\text{No. of seconds in } z' = \frac{2}{\sin 1''} \sin PZ \sin PS \operatorname{cosec} z \sin^2 \tfrac{1}{2}P$$

$$= \frac{2}{\sin 1''} \cos \text{lat.} \cos \text{dec.} \operatorname{cosec} z \sin^2 \tfrac{1}{2} \text{ hour angle.}$$

To apply this formula, we must of course know z, approximately: this is deduced from the latitude by account. By the aid of this approximate latitude and the small hour-angle P, we may therefore discover what correction z′ must be applied to the zenith distance actually observed to reduce it to the meridian zenith distance, from which the corrected latitude is easily obtained, as in the examples already given.

A result still more perfect may in general be arrived at, by proceeding anew with this corrected latitude, writing it in the formula in place of the latitude by account, and thus getting a more accurate correction for the reduction of the observed to the true meridian zenith distance. The additional work for this purpose will be but trifling.

The formula just deduced, furnishes the following rule for deriving the latitude by aid of the latitude by account, and from the observed altitude when near the meridian of a celestial object whose declination is known. The number 5·615455 is the logarithm of $\frac{2}{\sin 1''}$.

Latitude from an Altitude near the Meridian, the Declination, the Hour Angle, and the Latitude by Account.—Rule 1. Take the declination of the object for the Greenwich time by account, and add it to the latitude by account when they are of different names; otherwise, take the difference of the two: the result is the meridian zenith distance by account.

2. If the object be the sun, the apparent time from noon is the hour-angle: for any other object, add the sun's right ascension, to the apparent time since preceding noon. The difference between this sum and the object's right ascension is the hour-angle.

3. Add together the following logarithms :—

The constant logarithm 5·615455,
log cosine of the latitude by account,
log cosine of the declination,
log cosecant of the mer. zenith dist., as deduced from the two latter,
twice log sine of half the hour-angle.

The sum, rejecting the tens from the index, is the logarithm of a number of seconds, which subtracted from the true zenith distance, deduced from the observation, gives the meridian zenith distance. If this and the declination are of the same name, their sum, otherwise their difference is the latitude, of the same name as the greater.

Examples.

1. In latitude 48° 12′ N. by account, when the sun's declination was 16° 10′ S., at 0h. 16m. p.m., apparent time, the sun's true zenith distance was 64° 40′ N.: required the latitude.

Constant log			5·615455
Latitude by acct. . .	48° 12′ N.	cos . . .	9·823821
Declination . . .	16° 10′ S.	cos . . .	9·982477
∴ Mer. zen. dist. acct. .	64° 22′	cosec . . .	10·044995
Half hour-angle in deg.	2° 0′	2 sin . . .	17·085638
			log 2·552386

	60)357″
Reduction	— 5′ 57″
Zen. dist. from observation .	64° 40′ 0″ N.
True mer. zen. dist . . .	64° 34′ 3″ N.
Declination	16° 10′ 0″ S.
Latitude	48° 24′ 3″ N.

This example is from Mr. Riddle's Treatise on Navigation and Nautical Astronomy. We shall now solve it anew by putting the latitude here deduced in place of the latitude by account.

Constant log			5·615455
Latitude . . .	48° 24′ 3″ N.	cos . . .	9·822117
Declination . . .	16° 10′ 0″ S.	cos . . .	9·982477
Mer. zen. dist. . .	64° 34′ 3″	cosec . .	10·044268
Half hour-angle . .	2° 0′ 0″	2 sin . .	17·085638
Reduction . .	355″	log . .	2·549955

The difference between this and the former reduction is only 2″, so that the corrected latitude is 48° 24′ 5″ N.

It may here be remarked, that as fractions of a second are disregarded in the "Reduction," the logarithms used in finding it need be taken out of the table only to the nearest minute of each of the angular quantities.

If the formulæ marked (A) at page 110 be applied to the preceding example, the work will be as follows:—

Referring to the diagram at page 109, in connection with the formulæ (A) we have

1. tan PM = cos hour ang. cotan dec.

cos	4° 0′	. . . . 9·998941
cotan	16° 10′	. . . . 10·537758
tan PM	73° 47′ 45″	. . . . 10·536699
ZM	64° 36′ 20″	

2. cos ZM = cos PM cosec dec. sin alt.

cos	73° 47′ 45″	. . . 9·445699
cosec	16° 10′	. . . 10·555280
sin	25° 20′	. . . 9·631326
cos ZM	64° 36′ 20″	. . . 9·632305

138° 24′ 5″ = 90° + latitude, since lat. and dec. have different names.
90°

48° 24′ 5″ = latitude N., the same as before.

In this example the latitude is north, and the declination south, so that PM, in the general investigation of the formulæ, is in this particular case P′M, and therefore P′M + ZM — 90° is the distance of Z above the equinoctial, that is, it is the latitude of the ship.

In finding the latitude by the above formulæ (A), it is of course necessary to ascertain on which side of Z the foot M of the perpendicular from S falls, that is, whether ZM is to be added to or subtracted from PM; but whether the correct co-latitude is PM — ZM or PM + ZM, can be matter of doubt only when ZM is so small as to make it of little consequence which be taken. But as the method by the rule is free from all ambiguity, it is to be preferred when the object is near the meridian.

2. In latitude 50° 50′ N. by account, when the sun's declination was 11° 41′ 58″ N. at 12m. 3s. from apparent noon, the sun's true altitude was 50° 52′ 29″: required the latitude. Ans. Latitude 50° 47′ 49″ N.

3. At 3h. 5m. 36s. apparent time, the sun's true altitude was 35° 4′ 7″, and his declination 10° 54′ 26″ N.: required the latitude from the formulæ (A).

Ans. Latitude 50° 48′ 23″ N.

4. At 18m. 45s. from apparent noon in latitude 8° S. by account, the sun's true altitude was 74° 16′, and his declination 23° 27′ S.: required the latitude to the nearest minute. Ans. Latitude 8° 23′ S.

5. In longitude 0° 45′ W., at 11h. 2m. 32s. apparent time, the observed altitude of the pole star was 51° 22′, the index correction being — 2′. For apparent noon at Greenwich, on the day of observation, the Nautical Almanac gave the following particulars:—

Sun's right ascension, 6h. 51m. 11⅓s.
Star's right ascension, 1h. 1m. 41s.
Star's declination 88° 26′ 56″.

Required the latitude to the nearest minute by the formulæ (A).

Ans. Latitude 51° 47′ N.

Artificial Horizon.—The problem just discussed will be found very useful at sea whenever the mariner, on account of cloudy weather coming on near noon, is disappointed of a meridian altitude. As the object may be obscured though the horizon may be clear and well defined, so on the other hand the celestial body may be visible and in a position well suited for observation, and yet a haze may obscure the horizon. In such a case an artificial horizon is employed: this is a shallow trough of quicksilver, protected from agitation from the air by a glass covering or roof.

The observer, placing himself at a convenient distance from this horizon, so that both the celestial object and its reflected image may be distinctly seen, measures with his sextant the angular distance between the two; and as the real object is as much above the horizontal plane as the image is below it, he thus gets *double* the altitude, and has no correction to make for *dip*: the angle read off from the instrument, being corrected for the index-error and divided by 2, gives the apparent altitude of the point observed.

To get a *meridian* altitude of the sun is one of the principal items in a "day's work" at sea, for correcting the latitude by the dead reckoning; if the weather interfere with this operation, then an observation off the meridian is sought for, and the latitude inferred by help of the apparent time at the ship, as explained in the preceding article. The reader will bear in remembrance that a ship carries the mean time at Greenwich with her: the ship's chronometer, when the known daily gain or loss is applied to it, supplies this important information. The *apparent* time is deduced therefrom by means of the equation of time given in the Nautical Almanac, and the apparent time at the ship is ascertained by turning the longitude into time; and thus the hour-angle of the object observed from the meridian is found. The longitude by account may be somewhat in error; but the trifling inaccuracy in the resulting time at the ship will have no important influence on the latitude deduced from it.

But valuable as the chronometer is, yet, like all human contrivances, it is subject to accidents and exposed to derangements from circumstances beyond our control. It is of great importance, therefore, to be able to find the position of a ship at sea independently of its aid: this we have seen, as far as *latitude* is concerned, may be done by means of meridian altitudes. It may also be done by aid of *two* altitudes of the same celestial object taken off the meridian. This is technically called the problem of *double altitudes*: we proceed now to investigate its principles.

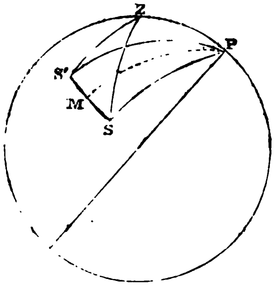
Latitude from Two Altitudes of the Sun, and the Time between the Observations.—Scarcely any problem in nautical astronomy has received more attention from scientific men than the problem of *double altitudes*; and, as the calculations involved in it are much longer than those for finding the latitude from a single altitude, various tables to facilitate the operations have been constructed. The determination of the latitude, by help of such tables, is what is called the *indirect* method of solution; and, like all such methods, it is not so strictly correct as the direct method by trigonometry. Delambre carefully examined all the rules he was acquainted with for the solution of the present problem, and he came to the conclusion that the rigorous process by spherical trigonometry was to be preferred, as well for brevity as for accuracy of result. The investigation of the direct method is as follows:—

Let P be the elevated pole, Z the zenith of the ship, and therefore Z P its co-latitude. Let S, S′ be the two places of the sun when the altitudes are taken; then drawing the great circle arcs, as in the annexed diagram, or in that at page 109, we shall have the following quantities given, namely,—

$$\left.\begin{array}{ll}\text{The co-declinations} & PS,\ PS' \\ \text{The co-altitudes . .} & ZS,\ ZS' \\ \text{The hour-angle . .} & SPS'\end{array}\right\}\ \text{To find } ZP.$$

There are three spherical triangles to consider, namely—

1. The triangle PSS′, in which are given the two sides PS, PS′, and their included angle, to find the third side SS′, and one of the remaining angles, as, for instance, the angle PSS′.



2. The triangle ZSS′, in which are given the three sides, to find the angle S′SZ; from which, and the previously-found angle PSS′, the angle ZSP becomes known, so that we have,

3. The triangle ZSP, in which are given two sides and their included angle, to find the third side ZP.

Before proceeding to the solution of these triangles, the observed altitudes must of course be reduced to the true altitudes, as in the former examples; and since the ship most probably sails on during the interval between the two observations, an additional correction becomes necessary, in order to reduce the first altitude to what it would have been if taken at the place of the second observation. This correction for the ship's run will become known, provided we know the number of minutes the ship has sailed directly towards or directly from the sun in the time between the two observations; and they may be ascertained as follows:—

Take the angle included between the ship's course and the sun's bearing at the first observation; and considering this angle as a course, and the distance sailed as the corresponding distance, find by the traverse table, or by calculation, as in plane sailing, the difference of latitude; this difference of latitude, expressed in minutes, will be the number of minutes by which the ship has approached to or receded from the sun, so that we shall know how many minutes must be added to or subtracted from the first altitude to reduce it to what it would have been if taken by another observer at the place of the second observation.

If the angle between the ship's track and the bearing of the sun be less than 90°, the ship will obviously be approaching towards the sun, in which case the correction of the altitude, determined as above, must be added; but if the angle exceed 90°, it must be subtracted: if it be exactly 90° no correction of the altitude will be necesary for the ship's change of place.

But a correction of the elapsed time may be requisite for the change of longitude; this change, converted into time, must be added to the elapsed time between the two observations, if the ship have sailed eastward, and subtracted if she have sailed westward.

These are the corrections necessary to fit the three triangles above for trigonometrical calculation. And to simplify the work of finding SS′, without any material sacrifice of accuracy, we may regard the declinations of the sun at the times of observation, as both equal to the declination it has at the middle time between them; the shorter this time is, the less will the supposition affect the precision of the result.

Regarding the above mentioned corrections to have been made, we shall now give an example of the trigonometrical operation.

Examples.

The two corrected zenith distances are,

$$ZS = 73^\circ\ 54'\ 13'', \text{ and } ZS' = 47^\circ\ 42'\ 51'',$$

the corresponding co-declinations of the sun are

PS = 81° 42′ N., and PS′ = 81° 45′ N.,

and the interval of time between the observations is three hours: required the latitude.

For the more easy determination of SS′, let it be regarded as the base of an isosceles spherical triangle, of which each of the equal sides is ½(PS + PS′) = 81° 43′ 30″, the vertical angle at P being 3h. or 45°; then if the perpendicular PM be drawn, the triangle PMS will be right-angled, and we shall have given

$$PS = 81^\circ\ 43'\ 30'' \text{ and } P = \frac{45^\circ}{2} = 22^\circ\ 30'$$

to find SM = ½ SS′ as follows.—

1. *In the triangle* PMS *to find* SM.

sin PS,	81° 43′ 30″	. . . . .	9·9954547
sin P,	22° 30′ 0″	. . . . .	9·5828397
sin SM,	22° 15′ 11″·3	. . . . .	9·5782944
	2		
∴ SS′ =	44° 30′ 22″·6		

2. *In the triangle* PSS′ *to find the angle* PSS′.

sin SS′,	44° 30′ 22″·6	Arith. Comp.	·1542898
sin PS′,	81° 45′ 0″	,, ,,	9·9954822
sin SPS′	45° 0′ 0″	,, ,,	9·8494850
sin PSS′	86° 38′ 58″	,, ,,	9·9992570

3. *In the triangle* ZSS′ *to find the angle* ZSS′.

ZS′,	47° 45′ 51″		
sin ZS′,	73° 54′ 13″	Arith. Comp.	·0173686
sin SS′,	44° 30′ 22″·6	Arith. Comp.	·1542898
	2)166° 10′ 26″·6		
½ sum =	83° 5′ 13″·3		
sin (½ sum — ZS),	9° 11′ 0″·3	. . . .	9·2030206
sin (½ sum — SS′),	38° 34′ 50″·7	. . . .	9·7949179
			2)19·1695969
sin (½ ZSS′,	22° 36′ 26″·4	. . . .	9·5847985
∴ ZSS′ =	45° 12′ 52″·8		
PSS′ =	86° 38′ 58″		
∴ PSZ′ =	41° 26′ 5″·2		

4. *In the triangle* ZSP *to find* ZP. (See the investigation below).

tan PS, 81° 42′ 0″ . .	10·8359917
cos PSZ, 41° 26′ 5″·2 . .	9·8748930
cot ω, 11° 0′ 41″·2 . .	10·7108847

ZS=73° 54′ 13″

ω+ZS=84° 54′ 54″·2

cos PS	9·1594354
sin ω Ar. Comp.	·7189551
sin (ω + ZS)	9·9982874
sin ZP, 48° 49′ 59″·7 . . .	9·8766779

Hence the latitude is 48° 50′

As ZP is here found by a method not always given in works on Spherical Trigonometry, we shall subjoin the investigation of it. Representing, as usual, the three sides of a spherical triangle by a, b, c, and the angle included by the first two by C, we have, by the fundamental formula of Spherical Trigonometry (MATHEMATICAL SCIENCES, p. 408),

$$\cos c = \cos a \cos b + \sin a \sin b \cos C.$$

Or, since $\sin a = \cos a \dfrac{\sin a}{\cos a} = \cos a \tan a$,

$$\cos c = \cos a (\cos b + \tan a \sin b \cos C).$$

Put cot ω for $\tan a \cos C$, that is, let

$$\tan a \cos C = \cot \omega = \frac{\cos \omega}{\sin \omega} \quad . \; . \; . \; . \; . \quad (1)$$

Then we shall have

$$\cos c = \cos a \, \frac{\sin \omega \cos b + \sin b \cos \omega}{\sin \omega}$$

$$\text{that is, } \cos c = \frac{\cos a \sin (\omega + b)}{\sin \omega} \quad . \; . \; . \; . \; . \quad (2)$$

The expressions (1) and (2) are those calculated above. In the preceding solution more attention is paid to minute quantities than is at all necessary in actual practice at sea, where fractions of a second are of course disregarded. Yet in lengthy operations seconds themselves ought not to be entirely neglected, except in those confessedly approximative methods which the indirect processes, by peculiar *tables*, generally are. The only departure from strict mathematical rigour in the foregoing work is in the first part of it, where the arc SS′ is supposed to be equal to an arc subtending the same angle at P, the sides of this angle being the mean between the two slightly differing co-declinations PS, PS′. It is plain, from the small amount of this difference, that the fictitious arc cannot vary in length from the real arc SS′ by a quantity deserving of any consideration, when the time between the observations is not unreasonably great.

By any one familiar with the practical solution of spherical triangles, the method here illustrated will be preferred to the indirect methods adverted to above, for the operation, though rather long, lays no burden upon the memory, and is moreover free from the inaccuracies—small inaccuracies no doubt—which indirect methods are always affected with. A distinguished practical navigator, Captain Kater, after the example of Delambre, gives the preference to the direct process: the preceding exercise is taken from the former writer; but the latter part of the solution is conducted differently.

2. On the 7th of February, 1846, in latitude by account 35° N. and longitude

47° W., at 8h. 9m. 4s. A.M. mean time, the altitude of the sun's lower limb was 36° 10′, and his bearing S. ¼ E.: after running N.E. 27 miles, the altitude of the lower limb at 11h. 30m. 18s. was observed to be 41° 20′; the height of the eye was 20 feet: required the latitude of the ship when the second observation was made.

	declination		
From the Naut. Alm.	Feb. 7 at noon	15° 19′ 32″ S.	Semi-diam. 16° 14′.
	„ 8 . . .	15° 0′ 40″ S.	

1. *For the true altitudes of sun's centre.*

First alt. sun's L.L. . . .		36° 10′ 0″
Dip . . — 4′ 24′ Semi-di. 16′ 14′	. .	+ 11′ 50″
App. alt.		36° 21′ 50″
Refraction and par. . . .		— 1′ 10″
True altitude		36° 20′ 40″

Second alt. sun's L.L. . .	41° 20′ 0″
Dip and semi-diam. . . .	+ 11′ 50″
Apparent alt.	41° 31′ 50″
Refraction and par. . . .	— 58″
True altitude	41° 30′ 52″

The angle between the sun's bearing S. ¼ E. and the course N.E. is 11¾ points, so that the ship has sailed within 4¼ points of the direction opposite to the sun, a distance of 27 miles. With this distance and 4¼ points as a course, the Traverse Table gives 18′ for the corresponding difference of latitude, which is the number of minutes the ship has receded from the sun during the interval of the observations: hence these 18′ must be *subtracted* from the first true altitude to reduce it to what it would have been if taken by another observer at the place of the second observation: consequently the true altitudes at this second place, taken at the stated times, are

36° 2′ 40″ and 41° 30′ 52″

∴ ZS = 53° 57′ 20″ and ZS′ = 48° 29′ 8″.

2. *For the co-declinations* PS, PS′.

Ship's time Feb. 6 .	20h. 9m. 4s.	
Long. in time . .	3h. 8m. 0s. W.	
Time at Greenwich .	23h. 17m. 4s.	
Dec. Feb. 6	15° 38′ 9″	var. — 46″·5
	17′ 49″	23
Declination	15° 20′ 20″ S.	1395
	90°	930
PS =	105° 20′ 20″	6)106·95
		17·82

Ship's time Feb. 6 . .	23h. 30m. 18s.	
Long. in time	3h. 8m. 0s.	
Gr. time Feb. 7 . . .	2h. 38m. 18s.	
Dec. Feb. 7	15° 19′ 32″	var. — 47″·2
	1′ 58″	2½
Declination	15° 17′ 34″ S.	944
	90°	236
PS′ =	105° 17′ 34″	6)11·80
		1·97

The decimal part of the minutes, in the correction for declination, is reduced to seconds by multiplying by 60.

3. *For the angle* SPS′.

Time of first observation .	8h. 9m. 4s.	3h.	= 45°
„ second . .	11h. 30m. 18s.	21m.*	= 5° 15′
Interval of time . .	3h. 21m. 14s.	14s.	= 3′ 30″

∴ the angle SPS′, in degrees, = 50° 18′ 30″

ce in the isosceles triangle PMS we have for each of the equal sides 3 + PS′) = 105° 18′ 57″, and for half the vertical angle 25° 9′ 15″.

1. *In the triangle* PMS *to find* SM.

sin PS,	105° 20′ 20″	9·9842479
sin P,	25° 9′ 15″	9·6284454
sin SM,	24° 11′ 58″	9·6126933
	2	
SS′ =	48° 23′ 56″	

2. *In the triangle* PSS′ *to find the angle* PSS′.

sin SS′,	48° 23′ 56″	Arith.	Comp.	·1262231
sin PS′,	105° 17′ 34″	„	„	9·9843431
sin SPS′,	50° 18′ 30″	„	„	9·8862044
sin PSS′,	83° 1′ 17″	„	„	9·9967706

3. *In the triangle* ZSS′, *find the angle* ZSS′.

ZS′,	48° 29′ 8″			
sin ZS,	53° 57′ 20″	Arith. Comp.		·0922566
sin SS′,	48° 23′ 56″	Arith. Comp.		·1262230
	2)150° 50′ 24″			
½ sum =	75° 25′ 12″			
sin (½ sum — ZS),	21° 27′ 52″	„	„	9·5633906
sin (½ sum — SS′),	27° 1′ 16″	„	„	9·6573607
				2)19·4392309
sin ½ ZSS′	31° 37′ 27″	„	„	9·7196154

∴ ZSS′ = 63° 14′ 54″
PSS′ = 83° 1′ 17″

∴ PSZ = 19° 46′ 23″

Divide by 4, and reckon every unit of remainder as 15 *minutes*, if minutes be the dividend; or *nds*, if seconds be the dividend.

4. *In the triangle* ZSP, *to find* ZP.

tan PS 105° 20′ 20″	10·5617760		cos PS	9·4226245
cos PSZ 19° 46′ 23″	9·9736081		sin ω, Ar. Comp.	·5530929
			sin (ω, + ZS)	9·7864647
cot ω 16° 15′ 2″	10·5353841			
			sin ZP, 35° 20′	9·7621821
ZS = 53° 57′ 20″				
ω + ZS = 37° 42′ 18″			Hence the latitude is 35° 20′ N.	

NOTE. It *may* happen, in low latitudes, that the arc SS′, if prolonged, would cut the meridian PZ between P and Z, in which case the angle PSZ will not be the difference between the angles PSS′, ZSS′; and the sum of these angles must be taken instead. In cases of doubt, therefore, the last portion of the work should be modified on this second supposition, and that one of the two resulting latitudes taken which best agrees with the latitude by account. (See the diagram, page 109).

It may be also noticed that the error of a few minutes in the estimated mean time at the ship will be of no consequence in deducing the declination; but as an error in the elapsed time, and therefore, in the corresponding polar angle, is to be avoided, the elapsed time should be taken from the chronometer, or a good watch.

3. The two corrected altitudes of the sun are 42° 14′ and 16° 5′ 47″; the corresponding declinations are 8° 16′ 30″ N., and 8° 15′ N., and the time between the observations is three hours: required the latitude of the place.

Ans. Latitude 48° 54′ 27″ N.

4. The two corrected zenith distances are 54° 39′ and 19° 59′; the corresponding declinations are 5° 31′ 6″ S. and 5° 28′ 54″ S., and the interval of time 2h. 20m.: required the N. latitude. Ans. Latitude 1° 29′ 28″ N.

5. In latitude 29° 10′ S. by account, and longitude 124° W., the sun being obscured at noon, its altitude was taken at about 20m. past noon, the chronometer at the time showing 9h. 49m. 20s.: at 10h. 44m. 45s., by the same chronometer, the altitude was again taken. In the first observation the altitude of the upper limb was found to be 45° 33′; in the second the altitude of the lower limb was 42° 8′ 30″, at which second observation the sun bore N. ¼ E. The ship's course between the observations was N.W. ¾W., and her run 6 miles; the allowance for dip was — 4′ 30″, and the Nautical Almanac gave the following particulars for noon of the day at Greenwich:—

Sun's declination from Naut. Alm. 16° 34′ 4″ N., hourly var. 42″·8.
Semi-diameter from ditto, 15′ 52″.

Required the latitude of the ship when the greater altitude was taken.

Ans. Latitude 28° 0′ S.

NOTE.—When, as in this last example, the latitude at the first observation is to be found, and the sun's bearing is taken at the second observation, the point *opposite* to that of the ship's course from the first position, is to be used in reducing the second altitude to what it would have been if taken at the first position of the ship.

If the sun's true bearing, or *azimuth*, could be taken with precision at either place of observation, there would be no necessity for a second altitude, because in the spherical triangle ZPS, formed by the co-latitude ZP, the co-declination PS, and the co-altitude ZS, the angle Z would then be given; and this, together with ZS and PS, is sufficient for the determination of ZP.

It is more convenient to observe the bearing when the sun is low than when it is high, and the result can be the better depended upon; therefore, in the problem of double altitudes, the bearing of the sun is always taken when the less of the two altitudes is taken.

The method of finding the latitude here explained may of course be applied to a star as well as to the sun, the interval between the observations being expressed in sidereal instead of in solar time; but as in general the horizon increases in obscurity as the star becomes more clearly visible, *two* observations of a star, with a sufficient interval of time between them, can seldom be accurately made. The following, therefore, is a more suitable problem for the purpose in view.

Latitude from the Altitudes of two Fixed Stars observed at the same time.—There are two advantages connected with this method of deducing the latitude: the first is, that as no allowance is to be made for change of place in the ship all error arising from inaccuracies in the sun's bearing, and the course and distance steered, is avoided; the second is, that the risk of losing another observation, from unfavourable weather, is not incurred when both observations are made at the same time.

When the altitudes of the two stars are to be taken by one person, the mode of proceeding is this:—The altitude of one star is taken, and the time by the watch noted; the altitude of the other star is then taken, and the time noted; after a short interval, the altitude of the second star is again taken, and the time noted. Thus the change of altitude of the second star, in a known interval of time, will be found; and therefore the correction for the time when the first star was observed may be found by proportion.

As to the principles of solution, they are the same as in the former problem: the co-declinations or polar distances PS, PS′ of the two stars are given; the angle P between these is also given, this angle in time being the difference of the right ascensions of the two stars. We may hence compute the third side SS′ of the spherical triangle. As the co-declinations may differ considerably, this third side must not be calculated on the supposition that the mean of the co-declinations may be taken for each of the other two sides. It must be found, from the distinct co-declinations themselves. The remaining part of the operation is the same as in the former problem.

Examples.

1. In north latitude 52° 30′ by account, the corrected zenith distances of Capella and Sirius were as follows:—

Capella, ZS = 29° 14′ 24″. Sirius, ZS′ = 72° 5′ 48″.
Pol. dist., PS = 44° 11′ 39″. Pol. dist., PS′ = 106° 28′ 40″.

Difference of the two right ascensions, 1h. 33m. 45s. = 23° 26′ 11″ = P: required the latitude of the ship.

1. *In the triangle* SPS′ *to find* SS′.

tan PS, 44° 11′ 39″ . . .	9·9877822	cos PS	9·8555080
cos P, 23° 26′ 11″ . . .	9·9626072	sin ω Ar. Comp.	·1271225
		sin (ω + PS′)	9·6300961
cot ω, 48° 15′ 56″ . . .	9·9503894		
106° 28′ 40″		cos SS′, 65° 47′ 55″	9·6127266
ω + PS′ = 154° 44′ 36″			

2. *In the triangle* PSS′ *to find the angle* PSS′.

sin SS′,	65° 47′ 55″	Arith. Comp.	·0399527
sin PS′,	106° 28′ 40″	„	9·9817868
sin SPS′,	23° 26′ 11″	„	9·5995891
sin PSS′,	24° 43′ 3″	„	9·6213286

3. *In the triangle* ZSS′ *to find the angle* ZSS′.

ZS′ =	72° 5′ 48″			
sin ZS =	29° 14′ 24″	Arith. Comp.		·3111631
sin SS′ =	65° 47′ 54″*	Arith. Comp.		·0399536
	2)167° 8′ 6″			
½ sum =	83° 34′ 3″			
sin (½ sum — ZS) =	54° 19′ 39″	„	„	9·9097504
sin (½ sum — SS′) =	17° 46′ 9″	„	„	9·4845601
				2)19·7454272
sin ½ ZSS′ =	48° 14′ 29″	„	„	9·8727136

∴ ZSS′ = 96° 28′ 58″ } to be added
PSS′ = 24° 43′ 3″ }

121° 12′ 1″ supplement = 58° 47′ 59″ = PSZ.

In the triangle ZSP *to find* ZP.

tan PS,	44° 11′ 39″ . . .	9·9877822	cos PS	9·8555080
cos PSZ,	58° 47′ 59″ . . .	9·7143559	sin ω Ar. Comp.	·0490919
			sin (ω + ZS)	9·9995830
cot ω,	63° 16′ 3″ . . .	9·7021381		
ZS =	29° 14′ 24″ . . .		sin ZP, 53° 19′ 23″	9·9041829
(ω + ZS) =	92° 30′ 27″			

Hence the latitude is 53° 19′ 23″ N.

In connexion with the foregoing method of solving the problem of double altitudes, there are some points of theoretical interest to which the learner should attend. In the second step of the work the object is to find the angle PSS′; the operation conducts us to the *sine* of this angle, and as the angle answering to a given sine may be either acute or obtuse, we have no right, independently of controlling conditions, to assume it to be the one any more than the other.

In the example just solved, we have arbitrarily taken it to be acute, simply for convenience, without examining into any overruling circumstances. Now, in the present problem we may always take a similar liberty; for although the proper angle may really be the supplement of that we put down, the subsequent step will not be affected by this circumstance: the angle PSZ, which it is the object of that step to find, will still be the sum or difference of ZSS′, PSS′ or the supplement of the sum.

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* A second is deducted from this arc, in order that there may be an even number of seconds in the sum of the three, so as that, in taking the half, fractions of a second may be avoided; the omission of a second can have no sensible influence on the result.

But it is well to show how all doubt respecting the species of the angle PSS′ may be avoided.

The fundamental formula of spherical trigonometry, applied to the triangle PSS′, gives

$$\cos \mathrm{PSS}' = \frac{\cos \mathrm{PS}' - \cos \mathrm{PS} \cos \mathrm{SS}'}{\sin \mathrm{PS} \sin \mathrm{SS}'}$$

and as it is matter of indifference which of the two stars, or which of the two places of the sun we mark S or S′, we may always consider, in this formula, that cos PS′ is *numerically greater* than cos PS, in which case the numerator of the above fraction, and consequently the fraction itself (since its denominator is positive), will have the same sign as cos PS′; hence *taking* PS′ *for that of the two co-declinations whose sine is the less, the opposite angle* PSS, *will always be of the same species as* PS′,—that is, they will either be both acute or both obtuse.

In the example just solved, cos PS′ is negative, and cos PS, cos SS′ are both positive; consequently the fraction is negative, and therefore the angle PSS′ is obtuse—namely, 155° 16′ 57″, instead of 24° 43′ 3″, as we have taken it above, and we see that by *subtracting* 96° 28′ 58″ we get the same result—namely, 58° 47′ 59″.

Whatever method of solving the problem of double altitudes be employed, there is always, in particular instances, a degree of uncertainty as to whether the sum or difference of a particular pair of arcs or angles is to be taken, and which uncertainty can be removed only by reference to the latitude by account.

The method proposed by Mr. Ivory, and which has been put in a very commodious form by Mr. Riddle, in his Navigation and Nautical Astronomy, is perhaps the shortest of the correct processes, when the object observed is the sun; but the investigation of it is very long and complicated, and the several steps of the work are far less easy of recollection than those above: it has the advantage, however, of deciding the ambiguity here mentioned in the case of the sun, with but very little trouble.

But in the case of a pair of stars, the uncertainty is not so easily removed; an amount of calculation, equivalent to a repetition of the work involved in the last step above, has to be gone through.

Now it occurs to us that the best and most satisfactory way of coming at the proper value of the angle PSZ, is to directly compute this angle (in imitation of step 2 above) from the three sides of the triangle PSZ—namely, the co-declination PS, the co-altitude ZS, and the co-latitude by account PZ. This extra work may be regarded as, at least an approximate, verification of the entire process up to the end of step 3; and, in executing it, seconds in the several arcs need not be attended to: it will be sufficient to take each side to the nearest minute.

The resulting value of the angle PSZ would be a safe guide to the value of it which step 3 ought to give; and the concluding step might then be worked without any misgiving.

It may be further remarked here, in reference to a double altitude of the *sun*, that when the true co-latitude PZ is obtained, we may combine it with the co-declination PS, and the co-altitude ZS, to determine the hour-angle ZPS, the apparent time from noon when the altitude furthest from the meridian was taken: the correction for the equation of time being applied, the result will be the mean time at the ship when S was observed. The chronometer, proper allowance being made for its daily loss or gain, will show the mean time at Greenwich; and the difference between the two will be the longitude in time.

But this important subject will be more fully discussed in the next following PART of the present treatise; we shall only add here, that an altitude near the meridian is not eligible for determining *time*, because in that position altitudes vary so slowly that a small error in altitude may occasion a comparatively large error in time.

Example 2.—In latitude 58° N. by account, the true altitude of Capella was 69° 23′, and at the same time the true altitude of Sirius was 16° 19′; the following particulars were also given by the Nautical Almanac.—

Right Ascension of Capella 5h. 2m. 18s. Declination 45° 47′ N.
" " Sirius 6h. 36m. 33s. " 16° 27′ S.
Required the latitude. Ans. Latitude 57° 8′ N.

The Deviation of the Compass.—The mariner's compass is subject to two kinds of disturbance, which operate against its true north and south direction. One is a local disturbance attributable to the attractive influence of the iron in the ship, which, when unequally distributed in reference to the magnetic meridian, or the line along which the compass would otherwise settle, causes a deviation from that line. The effect of this local attraction is called the *deviation* of the compass. The amount of the angular disturbance or aberration is ascertained by experiment, before the ship proceeds on her voyage; and a table is then formed, which gives the proper correction for different positions of the ship's head. But Mr. Barlow, of the Royal Military Academy, Woolwich, has devised an apparatus for counteracting the effect of local attraction: a small iron plate is so placed near the compass, vertically and behind it, as that its influence upon it may neutralize the action of the iron in the ship; the proper situation for the plate being ascertained by experiment when the ship is in harbour. For an account of this contrivance, and the mode of applying it, the reader is referred to Barlow's "Essay on Magnetic Attractions." It may be observed, however, that much care is requisite in fixing the plate; and that if the distribution of the iron in the ship be afterwards changed, the plate must be readjusted; but this inconvenience must attach to every method of estimating the deviation from the results of observations previously made, under particular conditions as to the amount and position of the disturbing materials. Local deviation has sometimes been found to be so great that, in a ship loaded with iron, the course by compass has been known to differ from the proper course by so much as 30°.

The Variation of the Compass.—The vertical plane passing through the direction of the horizontal needle at any place marks out the *magnetic meridian* of that place, just as the vertical plane through the north and south points of the horizon marks out the geographical meridian of the place. The angle traced on the horizontal plane by the vertical planes just mentioned, measures the *variation of the compass* at the place. There are but very few places on the surface of the earth where the needle points directly to the north and south points of the horizon, or where the variation is zero. The line in which such places lie is called the line of no variation, and is a curve of very complicated form. It was not till probably some centuries after the discovery of the compass that the needle was suspected ever to lie out of the plane of the geographical meridian: it seems first to have been observed by Columbus, *in 1492.*

The variation of the compass is not only different at different places; but what is more remarkable, it is not constant even at the same place. At London the variation was *eastward* till about the middle of the seventeenth century; in 1659 the needle pointed due north and south, the variation being zero, so that at that time the magnetic coincided with the geographical meridian. After this the variation became *westerly*, and it continued to increase till the year 1818, when it appears to have attained its greatest limit of westerly variation—namely, 24° 30′. Since then the variation has been slowly diminishing, and it is now about 23° *west.*

In order to ascertain the variation of the compass at any place, it is necessary that we find by computation, or by some means independent of the compass, the *true bearing* of a celestial object, then observe the magnetic or compass bearing; the difference of the two will be the variation of the compass; including, however, the local deviation, unless this have been neutralized, as explained above.

There are two forms of the problem: the object whose bearing is observed may be either in the horizon or above it. In the former case it is the magnetic *amplitude* that is observed, in the latter case it is the *azimuth*: in both cases the latitude of the place is supposed to be known.

To find the Variation from an observed Amplitude.—It has already been seen that the angle which the equinoctial makes with the horizon of any place is the co-latitude of that place (page 86). This angle is therefore given; and as the object observed is in the horizon, and its declination or distance from the equinoctial known, we have given the perpendicular (the declination) and the opposite angle (the colatitude) of a right-angled spherical triangle to find the hypotenuse (the true amplitude); and the difference between this and the observed amplitude is the variation, including the local deviation when not previously counteracted. Refraction causes objects to appear in the horizon when, on an average, they are 33′ below it; consequently, the compass-amplitude should be taken when the sun's centre, or the star selected, is about 33′ *plus* the dip, above the sea-horizon; or, allowing 16′ for the sun's semidiameter, the altitude of the sun's lower limb should be about 17′ + dip.

Examples.

1. In January, 1830, in latitude 27° 36′ N., the rising amplitude of Aldebaran was observed, by compass, to be E. 23° 30′ N.: required the variation of the compass.

By the Nautical Almanac the declination of Aldebaran was 16° 9′ 37″ N.; and since by right-angled triangles, we have

$$\sin \text{declination} = \sin \text{amplitude} \times \cos \text{latitude}.$$

$$\text{therefore, } \sin \text{amplitude} = \frac{\sin \text{declination}}{\cos \text{latitude}}$$

so that the logarithmic computation is as follows:

		10
sin declination	16° 9′ 37″ . . .	9·44455
cos latitude	27° 36′ 0″ . . .	— 9·94753
sin amplitude	18° 18′ 16″ . . .	9·49702
Magnetic amplitude	23° 30′ 0″	
	5° 11′ 44″	

As the star is thus farther by 5° 11′ 44″ from the magnetic east towards the north than from the true east, the magnetic east has therefore receded thus much towards the south, and consequently the magnetic north towards the east—hence the variation of the compass is 5° 11′ 44″ E. But the following directions will serve for all cases.

When the object is rising, the true amplitude is always estimated from the east, and when it is setting, from the west, and towards the north or south according as the declination is north or south.

If the true amplitude, and that by compass, be both north or both south, their difference will be the variation, but if one be north and the other south, their sum will be the variation, easterly when the true amplitude is to the right, and westerly when it is to the left of the observed amplitude.

2. On Feb. 15, 1841, in latitude 43° 36′ N., and longitude 20° W., the setting amplitude of the sun's centre was observed to be W. 6° 45′ N., at 6h. 52m. p.m., apparent time: required the variation of the compass.

Time at ship . .	6h. 52m.
Long. in time . .	1h. 20m. W.
Time at Greenwich .	8h. 12m.

Sun's dec. at noon Naut. Alm.	12° 37′ 18″ S.
Correction for 8h. 12m.	— 6′ 49″
Sun's dec. at time of obs.	12° 30′ 29″ S.

		10
sin declination	12° 30′ S.	9·33534
cos latitude	43° 36′ . . .	—9 85984
sin amplitude W.	17° 23′ S.	9·47550
Magnetic amplitude W.	6° 45′ N.	
Variation .	24° 8′ West, the true amplitude being to the left of the magnetic.	

The true amplitude is always of the same name—north or south, as the declination, as is obvious.

3. In latitude 21° 14′ N., when the sun's declination reduced to the time at ship, was 19° 18′ 6″ S., its magnetic amplitude at rising was E. 35° 20′ S.: required the variation of the compass. Ans. Variation 14° 34′ West.

4. In latitude 26° 32′ N. and longitude 79° W., the sun's centre was observed to set W. 4° 17′ S., the time at the ship was about 6h. p.m.: the sun's declination at noon Greenwich time, was 30′ and the hourly increase of declination 1′: required the variation of the compass. Ans. Variation 3° 31′ E.

5. In latitude 48° 20′ N. the star Rigel was observed to set 9° 50′ to the northward of the west point of the compass: the star's declination was 8° 25′ S.: required the variation of the compass. Ans. Variation 22° 33′ W.

Variation of the Compass from an Azimuth.—The computation of the azimuth of a celestial object requires the solution of an oblique-angled spherical triangle, the three sides being given to find an angle. The three sides are the co-latitude, the co-declination, and the zenith distance or co-altitude of the object: the azimuth is measured by the angle at the zenith, or that of which the sides are the co-latitude and the co-altitude; in other words, it is the angle at the zenith between the meridian and the vertical circle through the object, agreeably to the definitions (page 85). In north latitude, the true azimuth is reckoned from the south point of the horizon; and in the south latitude, from the north point, towards the east, when the altitude is increasing, and towards the west when the altitude is decreasing.

The observed or magnetic azimuth being rekoned from the same point as the true azimuth, if both are east or both west, their difference will be the variation; but if one is east and the other west, their sum will be the variation. The variation is east or west, according as the true azimuth is to the right or to the left of the observed azimuth.

Examples.

1. On June 9, 1853, at about 5h. 50m. a.m., in latitude 50° 47′ N. and longitude 99° 45′ W., the bearing of the sun by compass was S. 92° 36′ E., when the altitude of his lower limb was 18° 35′ 20″, the index correction was + 3′ 10″, and the height of eye 19 feet: required the variation of the compass.

1. *For the Co-declination and the Co-altitude.*

Time at ship, June 8 . .	17h. 50m.	Obs. alt.	18° 35′ 20″
Long. 99° 45′ W. in time .	6h. 39m.	Index cor	+ 3′ 10″
Time at Greenwich, June 9	0h. 29m.		18° 38′ 30″
Sun's dec. June 9 at noon, Greenwich time . . .	22° 57′ 36″	Dip — 4′ 17″ Semi. 15′ 46″ } . . .	+ 11′ 29″
Hourly var. + 11″·8 ∴ correction for 29m. . . .	+ 6″	App. alt. . . . . Ref. and par.	18° 49′ 59″ — 2′ 41″
Dec. at time of observation	22° 57′ 42″ 90°	True alt.	18° 47′ 18″ 90°
Co-declination	67° 2′ 18″	Co-alt.	71° 12′ 42″

2. *For the true Azimuth.*

Co-declination . . .	67° 2′ 18″		
Sin co-altitude . . .	71° 12′ 42″	Arith. Comp.	·0238109
Sin co-latitude . . .	39° 13′ 0″	Arith Comp.	·1991079
	2)177° 28′ 0″		
Sin .	88° 44′ 0″	,, ,,	9·9998939
Sin (88° 44′ — co-dec) .	21° 41′ 42″	,, ,,	9·5678091
Cos . .	51° 52′ 4″ 2	,, ,,	9·7406218
True azimuth S. . .	103° 44′ 8″ E.		
Observed azimuth S. .	92° 36′ 0″ E.		
Variation	11° 8′ 8″ West, the true azimuth being to the left of the observed.		

2. In latitude 48° 50′ N. the true altitude of the sun's centre was 22° 2′ the

declination at the time of observation was 10° 12′ S., and the magnetic bearing S. 161° 32′ E. Required the variation of the compass. Ans. Variation 22° 40′ 40″ W.

3. November 19, 1835, in latitude 50° 22′ N., longitude 24° 30′ W., at about 9 o'clock in the morning, the altitude of the sun's lower limb was 8° 10′, and the bearing by compass S. 21° 18′ E.; also the height of the eye was 20 feet Required the variation to the nearest minute.

Sun's dec. at noon, Nov. 9, G. T. 19° 23′ S. Semi diameter 16′.

Variation of dec. in 1h. + 40″, so that 2′ must be subtracted for three hours before noon. Ans. Variation 24° 12′ West.

NOTE.—In the foregoing examples, if the local deviation of the compass remain uncorrected, what is named *variation* will be compounded of variation proper and deviation. Each result must then be regarded as the difference between the uncorrected compass-bearing and the true bearing.

It is proper to remark also, in concluding this division of our subject, that a single altitude or a single azimuth taken at sea is not considered so trustworthy as the mean of several; consequently, when all attainable accuracy is desired, the observations are repeated at short intervals of time (a minute or so), and the mean result of the set is taken for *the* observation employed in the calculation, the mean of the times being the corresponding time of that observation.

Attention to the local deviation of the compass is a matter of great practical importance; and Barlow's correcting plate is a valuable check to its influence. About a quarter of a century ago the Thetis, with a million of dollars and other treasure on board, sailed under the most favourable prospects from Rio Janeiro; the next day, in consequence of unfavourable wind, they tacked ship, in full confidence of being clear of land; the fatal mistake was first discovered by the jib-boom striking a high perpendicular cliff; all the three masts were at once sent over the side, and vessel and cargo were lost. In a paper in the Phil. Trans. for 1831, Mr. Barlow shows that the local deviation of the compass was exactly such as to be likely to occasion this great mistake in the ship's reckoning. The distance the vessel had run was about 80 miles; and assuming the local attraction to have been equal to that of the Gloucester—a similar ship—she would have passed five miles nearer to the scene of her destruction—Cape Frio—than she would have done had the compass been undisturbed by the attraction of the vessel, or had this disturbance been counteracted by any neutralizing apparatus.

For further particulars on the preceding portion of Nautical Astronomy, the student may consult the treatises of Norie, Riddle, Raper, and Inman; as also the recent publication of Mr. Jeans, of the Royal Naval College, Portsmouth.* The comprehensive work of Robertson too, already recommended at page 64, though of rather old date, would be a valuable addition to the mariner's library. On the variation and local deviation of the compass, Professor Barlow's work on "Magnetic Attractions" should be read; as likewise Commander Bain's "Essay on the Variation of the Compass," and the interesting treatise on Magnetism in the "Library of Useful Knowledge." Some judicious remarks on these matters will also be found in Lieut. Raper's treatise before referred to.

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* Dr. Inman's Nautical Tables will be found of much service to the practical navigator, and Mr. Jeans's "Navigation and Nautical Astronomy" is a suitable accompaniment to them. In *Professor* Inman's tables the meridional parts are carried to two places of decimals, which give *them an advantage* over most other collections.

ON FINDING THE LONGITUDE AT SEA.

Introduction.—To determine the longitude of a ship at sea, is justly regarded as the greatest achievement of Nautical Astronomy; it is often considered, too, as the most important achievement; but since both the latitude and the longitude are equally necessary to enable the mariner to ascertain the position of his ship on the ocean, one of these determinations has, in fact, no claim to superior importance over the other as respects its practical value to the navigator.

A higher degree of consequence has been attached to the problem of finding the longitude, solely because of the greater difficulties with which the solution of it is beset, and of the larger demand made upon the resources of science—both mechanical and astronomical—for the means of overcoming them.

We have seen in the preceding PART, that there are several ways of determining latitude: of these the simplest is that wherein the data are the altitude and declination of a celestial object when on the meridian. So there are several methods of determining longitude, but in each of these we have two distinct problems to solve instead of one: the problems are, 1st, to find the time at the ship; and, 2nd, to find the time at Greenwich The difference of the times is the longitude of the ship in time. The first of these problems is comparatively easy, the second has exercised the ingenuity and tasked the exertions of the greatest minds; and two very different forms of solution have resulted from these efforts.

As the great object to be accomplished is to discover, at any instant at the ship, what time it is at Greenwich, or at the meridian from which longitude is reckoned, it is natural that the problem of finding the longitude should have more especially engaged the attention of chronometer-makers; and, accordingly, when, in 1714, the first parliamentary reward was offered for the solution of this problem, within certain limits, a most laudable emulation was called forth among the more scientific of this class of artists. The pecuniary stimulus offered by Government was this, namely, £10,000 for a method which should determine the longitude to within 60 miles of the truth, £15,000 if the method should give the longitude within 40 miles, and £20,000 if within 30 miles. The most successful competitor for these rewards was John Harrison, who, with indefatigable industry, applied himself to the construction of a chronometer that should keep time with sufficient accuracy to accomplish the last of these objects. In 1736 his first chronometer was subjected to trial in a voyage to Lisbon; and in 1739 a second, still more perfect, was produced; but in 1758 he completed a third, which he affirmed to be so accurate as to entitle him to the highest reward offered by the Commissioners of Longitude. In compliance with Mr. Harrison's request, the Admiralty directed the watch to be tried on a voyage to Jamaica, in the ship Deptford, which sailed from Portsmouth, November 18, 1761, and arrived at Port Royal, Jamaica, January 19, 1762. The time at Portsmouth—for the watch had been set to Portsmouth time—was found, on the 26th of the same month, to be 5h. 2m. 47s., as shown by the chronometer, at the instant of noon at Port Royal; and the exact difference of longitude, in time, between the same two places, as found by careful astronomical observations, was *5h. 2m. 51s.*; differing from the determination of the chronometer by only 4s., which, in the parallel of Jamaica, is less than one nautical mile.

On January 28, 1762, the chronometer was sent back in the Merlin, which arrived at Portsmouth on March 26. On the passage, the ship encountered a violent storm, and the chronometer had to be removed to a place where it was likely to suffer from exposure. The time of mean noon, as shown by the chronometer, April 2, was 11h. 58m. 65s. Hence, from Nov. 6, 1761, to April 2, 1762, during which period the chronometer had passed through a variety of climates, and been subjected to violent agitations of the sea, its error was no more than 1m. 53s·5, or 28⅜ minutes of longitude in time, which, in the parallel of Portsmouth, does not amount to 18 nautical miles.

On this occasion Mr. Harrison received £5000, and another trial voyage was proposed, namely, from Portsmouth to Barbadoes, which was made in 1764, and which proved so satisfactory that an additional £5000 was ordered to be paid to him, and the remainder of the highest reward promised as soon as he had sufficiently explained the principles of his time-keeper to enable another artist to construct one as good. Mr. Harrison accordingly explained fully the whole of his mechanism to a properly qualified committee; and Mr. Kendal, a member of that committee, was directed to construct a chronometer on Harrison's principles. The instrument he produced was committed to the care of Mr. Wales, on his voyage round the world with Captain Cook, in the years 1772, 1773, and 1774; and it so fully justified Harrison's expectations in this severe trial, that the House of Commons ordered the remaining £10,000 to be paid to him.

Subsequently to this, chronometers were constructed by various persons, upon independent principles and of equal merit; those by Mudge, Arnold, and Earnshaw were considered to deserve special distinction, and accordingly £3000 were paid by Government to each of these makers; but no parliamentary reward has since been given for chronometers.

At the present day, chronometers fully equal to any of those here adverted to are readily to be procured, and from the best makers, even superior time-keepers may at all times be obtained. The difference of longitude between Greenwich and New York, as recently determined by the chronometers of Mr. Dent, agreed with the results of astronomical observations to within three-quarters of a mile!

Such, then, is the degree of perfection to which this department of mechanical art has been brought; so that the determination of the time at Greenwich, at any instant, in any part of the globe, would be an easy matter, provided only that a chronometer could be ensured against all external interference with its action. But at sea a piece of machinery so delicate is peculiarly exposed to deranging influences—changes of climate—the jerks and vibrations of the ship—local attraction, &c., &c., all have their influence, and however these ordinary hindrances to the correct performance of the watch may be provided against, yet an unavoidable accident may at any time render the machine useless. The safety of navigation requires, therefore, that—valuable as the chronometer is—other means of finding the longitude, independent of its aid, should be devised; and these are furnished by what are called the LUNAR OBSERVATIONS.

By aid of tables of the moon's motion—first supplied to the British Admiralty by Professor Mayer of Gottingen (1755-62), and subsequently improved by Mason, Bradley, and others—the angular distances of the moon from the sun, and from the principal stars that lie near her path, are predicted for every three hours of time, and for *several* years in advance. These "Lunar Distances," are given in the Nautical

Almanac. The motion of the moon in her orbit is so rapid—about 13° in 24 hours—that her advance to, or recession from a star in her path, is a measurable quantity even in the lapse of a few seconds of time: by means of the three-hourly distances computed in the Nautical Almanac for Greenwich time, her distance from any one of the selected stars, at any intermediate instant of Greenwich time, can be easily found by simple proportion; and, conversely, any intermediate distance being known, the corresponding Greenwich time can, in a similar manner, be found.

Like as in all the other angular measurements of Nautical Astronomy, the observer is considered to be situated at the centre of the earth: a lunar distance, therefore, being taken at sea, and reduced to this point, the mariner has only to turn to his "Almanac," and from the distances there given, to compute, by proportion, the time at Greenwich when the distance of the same objects was what he has found it to be. The Greenwich time, at the instant of observation is thus discovered, and thence,—his own time being known—the longitude of the ship is ascertained.

The heavens thus supply the navigator with an unerring chronometer—a time-piece that needs no winding up—that never gets out of repair, nor ever requires re-adjustment—that can neither be deranged by accident, nor be deteriorated by neglect:—Placed beyond the reach of sublunary vicissitudes, heat and cold, storm and tempest affect it not: these indeed may temporarily cloud and obscure its face, but they can neither disturb its mechanism nor alter its rate; and we know that it will run down only when our concerns with TIME are at an end.

From these introductory remarks, the student will readily anticipate the business that is now before us. Whether we consult a chronometer, or take a lunar distance, for the purpose of finding the time at Greenwich, the time at the ship is an indispensable element in the determination of the longitude; the problem first to be solved, therefore, is *to find the local time.*

On Finding the Time at Sea.—At first sight it would appear that the time at the ship, most easily determined, is the time of *noon*, or when the sun comes to the meridian; but, for a minute or two before and after the sun's transit, the change in altitude is so small, except in very low latitudes, that it is not possible at sea to detect the instant of the meridian passage, and the same may of course be said in reference to any other celestial object.*

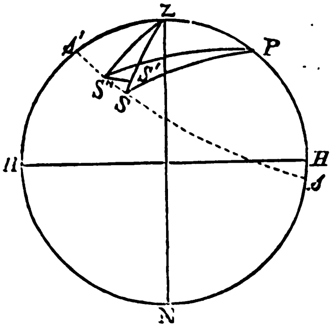
In determining the *latitude* by a meridian observation, the error of a minute or so in the time of apparent noon is of no consequence; for, as just remarked, the altitude of the object—which is all that the latitude is concerned with—is pretty nearly constant for a few minutes in the neighbourhood of the meridian; but, as respects the *longitude*, the error of a single minute in the time would occasion an error of fifteen minutes of longitude, It is obvious, therefore, that when time is to be deduced from altitude, with a view to the determination of the longitude, the position of the celestial object to be observed should be so chosen—when circumstances are such as to permit of a choice of position—as that a small error in the altitude may have the least effect possible on the hour-angle, or time from noon. It is of importance, therefore, to consider the following preliminary problem:

---

* It may be remarked, moreover, that the altitude of the sun, the moon, or a planet, is not necessarily always the greatest when on the mid-day meridian of the place of observation; nor necessarily the least, when on the midnight meridian: the body's change of declination may be such as to prevent this.

To determine upon what vertical a Celestial Object must be, in order that a small error in the Altitude may have the least effect on the Time.

Let S be the place of the celestial object observed, but by a small error in taking the altitude, let it be referred to S′. Draw S′S″ parallel to the horizon, and meeting the parallel of declination s s′, in the point S″.



Then, when the object is at S″, it will really have the co-altitude ZS″ equal to its supposed co-altitude ZS′, when it was actually not at S′, but at S; so that in the determination of the hour-angle P from the co-latitude PZ, the co-declination PS″, and the erroneous co-altitude ZS″, the small angle SPS″ will measure the amount of error in the time.

As the triangle SS′S″ is of course exceedingly small, since the error of observation is not purposely made, it may be regarded as a rectilinear triangle, right-angled at S′; therefore SS′ = SS″ sin S″, and SS″ = sin PS ∠ SPS″, for SS″, the distance passed over by the object, may be compared to the distance sailed on a parallel of latitude by a ship, making the difference of longitude SPS″, and we know that in this case (page 50)

cos lat. : 1 :: dist. : diff. long. ∴ dist. = cos lat. × diff. long., ∴ SS′ = sin PS∠SPS″

$$\text{Hence SS}' = \sin S''\sin PS\angle SPS'' \quad \therefore SPS'' = \frac{SS'}{\sin S''\sin PS} \quad . . . (1)$$

Now the angle S″, that is the angle S′S″S, is equal to the angle ZSP, because S′ being a right angle, S′SS″ is the complement of each; and therefore, from the relation between the sides and angles of a spherical triangle, we have

$$\sin S'' : \sin SZP :: \sin PZ : \sin PS$$

$$\therefore \sin S''\sin PS = \sin PZ\sin SZP \quad . . . (2)$$

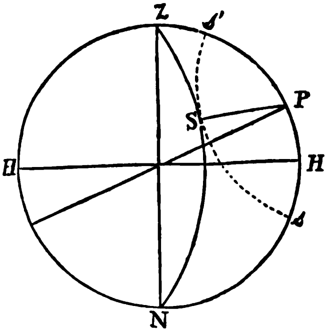
Substituting the second member of (2) in (1) we therefore have

$$SPS'' = \frac{SS'}{\sin PZ\sin SZP} = \frac{\text{error in altitude}}{\text{cos lat. sin azimuth}} \quad . . . (3)$$

This expression for the error in time will obviously be the least possible when the sine of the azimuth is the greatest possible; hence, if the co-declination be sufficiently great, or the co-latitude sufficiently small for the object to cross the prime vertical ZN, above the horizon, in its progress towards the meridian or towards the horizon, the most favourable time for an observation of the altitude will be when the prime vertical is reached—that is, when the object is due east or due west—the azimuth being then 90°. If other circumstances—weather, proximity to the horizon, &c.—be unfavourable, then the nearer the position of the object to the prime vertical the better.

But if the co-latitude PZ exceed the co-declination Ps′, then the object cannot arrive at the prime vertical; the azimuth PZN will then be the greatest, and have the greatest sine, when the object is at S, the point in which the azimuth circle ZSN touches the parallel of declination ss′. It is plain that in this position the apparent motion of the object is nearly perpendicular to the horizon (next diagram).

The triangle PSZ is right angled at S, for PS is the shortest arc from P to ZN, and *is, therefore, at right angles* to ZN; in this triangle the two sides PS, PZ, the co-declination and the co-latitude, are supposed to be given: we may, therefore, find the angle Z, or the bearing the object ought to have, so that the error in altitude may affect the time as little as possible; or we may find the angle P, the time from apparent noon, when the observation should be made. If it be not practicable to take the altitude when the object is in the most favourable position, then a position as near to it as possible should be chosen; and it must be remembered that, throughout the interval between the best position and the meridian, the nearer the object is to the meridian, the more unfavourably is it situated for the purpose of computing the time from an altitude; the formula (3) sufficiently shows this.



This formula (3) will of course serve for determining the error in the time consequent upon any assumed error in the altitude, at any observed azimuth; but as the angle SPS' is expressed in minutes of the equinoctial, as appears from (1) above, it is necessary to divide by 15, to bring the measure into minutes of time: so that

$$\text{Error of app. time in minutes} = \frac{1}{15}\cdot\frac{\text{error of altitude}}{\text{cos lat. sin}\cdot\text{azimuth}}$$

For example. Suppose the latitude to be 51° 31′ N., the azimuth of the object S 48° 10′ E., and that the error of altitude is estimated at 10′: required the error in the apparent time.

The above formula put into logarithms is,

log. error of time = log. error of alt. — log. cos lat. — log. sin azimuth — log. 15 + 20

(see page 9)

10	1·0000
cos 51° 31′ Arith. Comp.	·2060
sin 48° 10′ Arith. Comp.	·1278
15 Arith. Comp.	8·8239
1″438	·1577

Hence the error in time is
1′·438 = 1′ 26″

It will, of course, be observed that if the latitude and azimuth remain the same, as also the error in altitude, the error in time will remain the same, however the altitude may vary.

Time at Ship deduced from a Single Altitude.—If the object observed be the sun, the hour angle which its circle of declination, at the instant of observation, makes with the meridian is the apparent time from that meridian. By applying the equation of time, given in the Nautical Almanac, and reduced by means of the longitude by account, to the estimated instant of observation, the apparent may be converted into mean time.

If the object be a star, the hour angle its circle of declination makes with the meridian is a portion of sidereal time; it is the difference between the right ascension of the star and the right ascension of the meridian. When the star is to the west of the meridian its R. A. (right ascension) must be increased by the hour angle; when it is to the east its R. A. *must be diminished* by the hour angle: the result is the R. A. of the meridian, and the difference between this and the sun's R. A. at the time of observation is the time from the same meridian, and which is apparent, or mean time, according as the sun's R. A. is taken from p. I. or p. II. of the Naut. Alm.

If the object be the moon or a planet, the apparent and mean time are obtained just as in the case of a star.

To determine the hour angle P from which the mean time at the ship is thus deduced, there are given in the spherical triangle PZS, the co-latitude PZ, the co-declination PS, and the co-altitude ZS; hence, putting s for half the sum of these three sides, we have for $\cos \frac{1}{2} P$.

$$\cos \tfrac{1}{2} P = \sqrt{\frac{\sin s \sin (s - ZS)}{\sin PZ \sin PS}} \text{ (TRIGONOMETRY, p. 405).}$$

The co-altitude ZS, used in working this formula, is usually deduced from a set of altitudes taken within a minute or two of each other, as in the following examples:

Examples.

1. *Time deduced from the Sun.*—On September 23, 1845, in latitude 50° 30′ N., and longitude by account 110° W., a set of altitudes of the sun were taken as below; the index correction was — 3′ 20″, and the height of the eye 20 feet: required the mean time at the ship.

NOTE.—Besides a chronometer or two, a ship always carries a good common watch, marking seconds, by which, what may be called the time at ship *by account*, or the estimated time, is kept. This may be regulated by the meridian altitudes, or by the double altitudes, or, as already noticed, by means of the chronometer and estimated longitude. For the purposes to which the watch is applied, an error of a few minutes is of no consequence; in the present problem the estimated time is used to get the declination and equation of time. The times recorded below are the mean times at the ship as shown by the *watch*.

Altitudes of the Sun.	*Times per Watch.*
11° 4′ 40″	4h. 43m. 42s.
11° 2′ 25″	4h. 44m. 35s.
10° 59′ 45″	4h. 45m. 24s.
10° 56′ 30″	4h. 46m. 19s.
4)44° 3′ 20″	4)19h. 0m. 0s.

Mean obs. alt.		11° 0′ 50″
Index	— 3′ 20″	
Dip	— 4′ 24″	+ 8′ 14″
Semi-diam.	15′ 58″	
App. alt.		11° 9′ 4″
Ref. and Par.		— 4′ 38″
True alt.		11° 4′ 26″
		90°
∴ ZS =		78° 55′ 34″

Time nearly	4h. 45m. 0s. p.m.
Long. 110 W.	7h. 20m.
Mean time at G.	12h. 5m.
Sun's dec. at noon Green. time	0° 6′ 56″ S. v. + $58\frac{1}{4}''$
Cor. for 12h. 5m. $(58\frac{1}{4}'' \times 12\frac{1}{12})$	11′ 47″
	0° 18′ 43″ S.
	90°
PS =	90° 18′ 43″

Also since PZ = 39° 30′, the computation of the hour-angle P, will be as follows, namely :—

ZS,	78° 55′ 34″		
Sin PZ,	39° 30′ 0″	Arith. Comp.	·1964895
Sin PS,	90° 18 43″	Arith. Comp.	·0000064
	2)208° 44′ 17″		
Sin s,	104° 22′ 9″	. . . . .	9·9861968
Sin (s — ZS),	25° 26′ 35″	. . . . .	9·6330782
			2)19·8157709
cos. ½ P,	36° 0′ 47″		9·9078854
	2		
∴ P =	72° 1′ 34″		
In time =	4h. 48m. 6s.	Apparent time at ship	
Equation of time	— 7m. 52s.		
	4h. 40m. 14s.	Mean time at ship.	

Equation of Time.

Sep. 23 7m. 42s. var. ″·858,[*]

Cor for 12h., + 10s.

7m. 52s.

In this result a quarter of a second has been disregarded, and we conclude that the watch is about 5m. *fast*.

It is of importance in discussing operations of this kind, the results of which are required to be brought out with the utmost attainable accuracy, that the student's confidence in such results be not shaken by the fact that certain of the data with which he works are confessedly erroneous, and are never more than approximations to the truth; the estimated longitude and the estimated time at the ship are, of course, more or less affected with error. He must take notice that these two elements do not enter directly into the trigonometrical process: they merely affect the preparatory reductions for declination and equation of time; quantities that vary so little, even in a large interval of time, that a considerable error in the estimation of this interval occasions but an inconsiderable error in the proper corrections. Hence, with but ordinary care in the dead reckoning, and the proper accuracy in taking the altitude, the time at sea, determined as above, may be depended upon as correct, not only to the nearest *minute*, but even to the nearest *second*, notwithstanding the acknowledged fact that certain of the data are only approximately true.

To illustrate this, let us modify the foregoing operation by applying the above correction to the watch, or by putting it back 5m.; then the mean time at Greenwich, when the observed altitude of the sun at the ship was 11° 0′ 50″, will be 12h. The correction for declination will therefore be 58″ ½ × 12 = 11′ 42″; hence, subtracting 5″ from the above value of PS, we shall have, for the more correct value, PS = 90° 18′ 38″; and the work, modified in accordance with this change in the co-declination of the sun, will stand as follows :—

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[*] This is the variation of the equation of time in one hour, as given in the *Nautical Almanac*.

ZS,	78° 55′ 34″		
sin PZ,	39° 30′ 0″	Arith. Comp.	·1964895
sin PS,	90° 18′ 38″	Arith. Comp.	·0000064
	2)208° 44′ 12″		
sin s,	104° 22′ 6″	. . . .	9·9861985
sin (s — ZS)	25° 26′ 32″	. . . .	9·6330650
			2)19·8157594
cos ½ P,	36° 0′ 51″		9·9078797
	2		
∴ P =	72° 1′ 42″		

In time = 4h. 48m. 6s.¾ } ∴ 4h. 40m. 14s.¾ is the correct mean
Equation of time — 7m. 52s. } time at the ship.

The time, according to the former result, was 4h. 40m. 14s.¼; for, as stated above, the fraction of a second was suppressed: we see, therefore, that the error of 5m. in the *estimated* time occasions an error of only half a second in the *correct* time. Suppose now that not only the estimated time, but that the estimated longitude is also affected with an error equivalent to 5m. of time—that is, an error of 1°¼ in longitude, which is a large error; and suppose that both errors conspire, making the mean time at Greenwich 12h. 10m. The correction for declination will then be 58″⅓ × 12⅙ = less than 11′ 52″; then PS will be 90° 18′ 48″, and the modified work will be as follows:—

ZS,	78° 55′ 34″		
sin PZ,	39° 30′ 0″	Arith. Comp.	·1964895
sin PS,	90° 18′·48″	Arith. Comp.	·0000065
	2)208° 44′ 22″		
sin s	104° 22′ 11″	. . . .	9·9861958
sin (s — ZS),	25° 26′ 37″	. . . .	9·6330871
			2)19·8157789
cos ½ P,	36° 0′ 45″	. . . .	9·9078894
	2.		
∴ P =	72° 1′ 30″		

In time = 4h. 48m. 6s. } ∴ 4h. 40m. 14s, is the mean time
Equation of time — 7m. 52s. } at the ship.

Hence, notwithstanding the above errors in the estimated time and longitude, the time at the ship is correctly deduced to within three quarters of a second of the truth. Whenever there is a very considerable difference between the time per watch, and the calculated time at the ship, it will be prudent to repeat the work with the corrected time, as in the second of the foregoing operations.

2. *Time deduced from a Star.*—On June 3, 1842, at 12h. 9m. p.m., nearly (mean *time) in latitude* 50° 48′ N., and longitude by account 1° 6′ 3″ W., the observed

altitude (or the mean of a set of altitudes) of α Bootis, west of the meridian, was 89° 53′ 30″, with the artificial horizon: the index correction was — 10″; required the mean time at the ship.

As the artificial horizon was used, there will be no correction for dip (page 114): the angle shown by the instrument when corrected for index error is double the altitude from the rational horizon.

Angle by Instrument . .	89° 53′ 30″	Estimated time . . .	12h. 9m.
Index correction . . .	— 10″	Long. 1° 6′ W. . . .	4m.
	2)89° 53′ 20″	Mean time at G. . .	12h. 13m.
App. alt. of star	44° 56′ 40″	Sun's R. A. at noon, or the sidereal time .	4h. 46m. 7s·1
Refraction	— 58″	Correction for 12h. 13m.	+ 2m. 0s·4
True alt.	44° 55′ 42″		
	90°	Sun's R. A. at time of obs.	4h. 48m. 7s·5
∴ ZS =	45° 4′ 18″		

Stars R.A. 14h. 8m. 30s·5, Dec. 20° 0′ 15″ N., ∴ PS = 69° 59′ 45″. Also PZ = 39° 12′.

ZS,	45° 4′ 18″		
sin PZ,	39° 12′ 0″	Arith. Comp.	·1992628
sin PS,	69° 59′ 45″	Arith. Comp.	·0270256
	2)154° 16′ 3″		
sin s,	77° 8′ 2″	. . . . .	9·9889570
sin (s — ZS),	32° 3′ 44″	. . . . .	9·7249635
			2)19·9402089
cos ½ P,	21° 1′ 1″½	. . . . .	9·9701044
	2		

∴ P = 42° 2′ 3″ or 2h. 48m. 8s·2 of sidereal time.

Stars R. A.	14h. 8m. 30s·5
R.A. of meridian . .	16h. 56m. 38s·7
R.A. of sun	4h. 48m. 7s·5
Mean time at ship . .	12h. 8m. 31s·2

It will be observed here that the hour angle P is expressed in sidereal time, and in like manner the right ascensions of the star, the meridian, and the mean sun are all expressed in sidereal time: hence, 12h. 8m. 31s·2 is the distance of the mean sun from the meridian in *sidereal time; and this time* in reference to the *mean* sun is mean time.

Examples for Exercise.

1. On January 12, 1840, in latitude 30° 55′ N. and longitude 14° W., by account the mean of a set of altitudes of the sun's lower limb was 22° 25′ 33″, the corresponding time by watch was 9h. 31′ 33″ a.m.; the index correction was + 4′ 30″, and the height of the eye 30 feet. Also the Nautical Almanac gave the following particulars, namely—

Sun's dec. Jan. 11, .	21° 54′ 30″ S.	Equa. of time Jan. 11, . .	8m. 3s.
Hourly variation, .	— 23″·42	Hourly variation, . . .	+ 0m. 1s.

Sun's semidiameter 16′ 16″

Required the mean time at the place of observation.

Ans. 9h. 41′ 24″ a.m.

2. On April 18, 1844, in latitude 50° 48′ N. and longitude by account 1° 0′ W., the mean of a set of altitudes of the sun's lower limb (with artificial horizon) was 76° 16′ 46″, the corresponding time by watch was 9h. 18m. a.m.: the index correction was — 3′ 46″; and the Nautical Almanac gave the following particulars, namely—

Sun's dec. April 17, .	10° 36′ 49″ N.	Equa. of time April 17,	0m. 31s·3
Hourly variation . .	52″·38	Hourly variation . .	0m. 0s·573

Sun's semidiameter 15′ 56″.

Required the mean time at the place of observation. Ans. 9h. 18m. 16s. a.m.

3. On April 26, 1840, in latitude 29° 47′ 45″ S. and longitude by account 31° 7′ E., at 2h. 19m. 41s. a.m. by watch, the true altitude of the star Altair was 25° 14′ 20″ to the east and north: required the mean time at the place of observation.

R.A. of Altair 19h. 43m. Decli. 8° 26′ 47″ N. Sun's R.A., or sidereal time at mean noon, 2h. 18m. 21s·1. Ans. 1h. 45m. 11s. a.m.

NOTE.—As remarked at page 134, the time may be found in a similar way when the object is the moon instead of the sun or a star; but, on account of the rapidity of the moon's motion in right ascension and declination, this body is the least eligible for the purpose of discovering the time at sea. The approximate time and the approximate longitude which, as we have seen in the case of the sun (page 138), may be employed with safety, may lead to considerable uncertainty when the object is the moon, whose declination changes sometimes more than 2′ in 10m. of time.

Time Deduced from Equal Altitudes.—There is another way of determining the time which ought to be briefly noticed. It is called the method of *equal altitudes*: the following exposition of it is principally from Lieut. Raper's "Practice of Navigation and Nautical Astronomy"—a work which well deserves the attention of the practical seaman.

Since the altitude of a body which does not change its declination varies exactly at the same rate while rising on the E. side of the meridian, as while falling on the W. side, the same altitude occurs at the same hour-angle on each side of the meridian, and the middle point of time, between the instants of two equal altitudes, is the instant at which the body passes the meridian.

In the case of the sun, the middle point of time, or the mean of the observed times of equal altitudes A.M. and P.M., is apparent noon. In the case of a star, or other body, the mean of these times corresponds to the R.A. of the star when upon the meridian, or the sidereal time, which may easily be converted into apparent time, or mean time.

Since the sun changes his declination sensibly in large intervals of time, two equal altitudes, A.M. and P.M., do not correspond to equal hour-angles; and it therefore becomes necessary to apply to the mean of the observed times a correction which is called the *Equation of Equal Altitudes*. A table of the equation of equal altitudes is to be found in all Nautical Tables. Also at sea, the change of place of the ship, in the intervals between the altitudes, will generally render another correction necessary. When the course made good is true E. or W., the ship changes her longitude only, by the portion of time which she gains or loses on the sun in the interval. This change, provided it is made good equally on both sides of the meridian introduces no correction; and the only question is, the time by watch when the interval is half expired.

But when the ship changes her latitude, the same altitude no longer corresponds to the same time from noon, and the correction adverted to becomes necessary.

This method has some advantages:—it is independent of the terrestrial refraction, provided this remains unchanged in the interval employed; and the correction when necessary requires the latitude and altitude to be but roughly known. Within the tropics, the interval may in general be very small, on account of the rapid change of the altitude, and the correction for change of latitude may, in such cases, be omitted. In high latitudes, however, the ship's change of latitude considerably alters the time from noon, at which the second altitude (equal to the first) is taken: hence, in such high latitudes the method is less useful.

The general mode of proceeding is this:—Observe the sun's altitude shortly before noon, and note the time. Note the instant of the the same altitude in the afternoon. For greater accuracy, several pairs of equal altitudes should be obtained. Take the mean of the A.M. and P,M. times by watch: this, when the ship does not change her latitude, is the time by watch of apparent noon. To render any correction for change of latitude unnecessary, it would be desirable to alter the course of the ship, so as to preclude this change, when the interval between the observations is but short, as it should be to warrant the dispensing with a correction for change of declination.

On account of the constancy of its declination, a star is the best suited for the purpose of the present problem; but a change in the state of the atmosphere, during the interval of the observations, may interfere with the precision of the result, especially in the case of low altitudes; attention to the indications of the barometer and thermometer is therefore necessary to the attainment of accuracy. But the observations, to be free from all source of error, should be made on shore; for if the interval be large, there is, at sea, some uncertainty as to the correction for change of latitude; and if the interval be small, there is some uncertainty as to the altitudes, especially in high latitudes, on account of the slowness with which altitude changes near the meridian.

On finding the Longitude by a Chronometer.—From what is shown in the preceding articles, it appears that the time at the ship can be accurately obtained by means of altitudes of the sun or fixed stars, assisted by the longitude, by dead reckoning, and the time by estimation; and therefore all that is necessary for the determination of the ship's longitude is, that we know also the time at Greenwich at the same instant. If we can only discover what o'clock it is at the same instant at two different places on the globe, however distant, we may at once infer the difference of longitude of those places. We have only to convert the difference of time into degrees, at the rate of 15° to one hour, to effect this object. To find the time at Greenwich at any instant is therefore to solve the problem of the longitude; and as noticed at page 129, it is to supply this important information that so much skill and perseverance have been bestowed on the *chronometer*.

It is not necessary to the perfection of this instrument that it should show the exact time at Greenwich, which, in fact, a chronometer never does; what is required is, that its action be uniform and regular. The difference between Greenwich mean time at any instant and the time shown by the chronometer, is the *error* of the chronometer at that instant, and its average gain or loss in 24 hours is its *mean daily rate*. There is a depôt established at the Royal Observatory, Greenwich, for the reception of such chronometers as the makers choose to send, and where their errors and rates are determined by the Astronomer Royal; and few persons would now purchase a chronometer for use at sea without the proper official certificate of these particulars. If the original error on Greenwich mean time be allowed for, and the proper correction be made for the accumulated daily rate, the Greenwich time at any instant will of course be correctly obtained, provided that nothing has disturbed the action of the chronometer since it left the observatory.

It will have been observed in the foregoing articles, that the estimated longitude and time at the ship have been employed solely for the purpose of getting the Greenwich time when the altitude was taken, with sufficient accuracy to enable us to get the co-declination or polar distance of the object observed; and we have seen that even a considerable error in the estimated quantities will not sensibly affect the precision of the result. But if the chronometer can at all be depended upon, the Greenwich time of the observation will become known at once, and, unless great derangement of the chronometer have taken place, with more accuracy than it can be found by the estimated longitude and time at the ship. The problem just disposed of may therefore be correctly solved by aid of even an indifferent chronometer; but if the time at Greenwich, after the corrections for error and rate, be not correctly furnished by the instrument, there will of course be a proportionate error in the longitude inferred from it.

We are anxious that the student should clearly understand how it happens that, from erroneous data, we obtain accurate results even in a matter of so much delicacy as the determination of the time at sea. The errors referred to can affect only two elements of the calculation—the declination and the equation of time; and from the slowness with which these vary, a few minutes, more or less, can have no sensible influence.

On finding the Rate of a Chronometer.—It is evident that if the rate of a chronometer remained invariable, the time at Greenwich could always be accurately deduced from it: but a sudden jerk or concussion, change of climate, local attraction, and even the motion of transporting it from the observatory to the ship, may cause the rate to vary. The scientific navigator, therefore, avails himself of every favourable opportunity of ascertaining the rate of his chronometer. This is best done when the ship is in harbour, and in the neighbourhood of a fixed observatory, at which the rate may be ascertained like as it was at Greenwich. For this purpose the chronometer need not be carried on shore. The time may be taken from it by a good seconds-watch, and the watch then carried to the observatory and compared with the mean-time clock, the error of the chronometer, on the mean time at the place, will thus be seen: and this comparison being repeated day after day, the daily deviation from the exact amount of the error will be discovered, and hence the mean daily rate. The ship then sails afresh with the time at the new meridian, and therefore—the longitude of that meridian being known—with the Greenwich time.

Or without resorting to a fixed observatory the rate may be ascertained, when the ship is in harbour, by finding the mean time at the place, as explained in last problem. A comparison of this with the mean time shown by the chronometer will, in like manner, show the error on mean time at the place; and the observations being repeated day after day, the rate will become known. The altitudes may in general be obtained with greater precision by taking them on shore, by means of an artificial horizon, the chronometer time being carried there by the watch.

Every attention should be paid on ship-board to circumstances likely to interfere with the rate of the chronometer, it should be kept as much as possible out of the influence of changes of temperature, and its horizontal position should remain undisturbed; to secure this permanence of position it is frequently suspended on jimbals, like the compass. It should be wound up by the action of the key alone, the chronometer itself being kept steady; if the instrument be turned as well as the key, the motion of the balance wheel will be affected, and the rate disturbed. We have hitherto spoken of *the* chronometer, as if ships in general carried only one; but in long voyages, or in voyages undertaken for scientific purposes, ships usually take several chronometers as checks to one another, and to provide against accident. Captain Fitzroy, in his surveys of the coast of South America, took so many as twenty-two chronometers. The apartment in which the chronometers are kept is called the chronometer-room, and the temperature of it is sometimes regulated by lamps, with the aid of the thermometer. This should be the permanent depository of the time-keepers, and they should be meddled with only for the purpose of winding them up, which should be at the same regular interval; and, of course, they should not be allowed to run down.

The error and rate of a chronometer being found, as explained above, the longitude is determined as in the following example from Captain Kater's article in the Encyclopædia Metropolitana.

Examples.

1. On the 2nd of June, 1823, the true altitude of the sun's centre was 30° 2′, when the chronometer showed 5h. 1m. 0s.; the latitude was 40° 5′ N.; and the sun's declination at the time of observation 22° 9′ 17″ N. The chronometer on the 20th of May preceding was 45s. slow for Greenwich time: its daily rate was 2s·1 losing. Required the longitude of the ship.

1. *Apparent Time at Greenwich from Chronometer.*

	h.	m.	s.
Time by chronometer	5	1	0
Slow April 20		+	45
Loss from April 20 to June 2		+	25·2
Mean time at Greenwich	5	2	10·2
Corresponding equation of time	+	2	31 4
App. time at Greenwich	5	4	41·6

2. *Apparent time at ship from triangle* PZS.

Here ZS = 59° 58′, PS = 67° 50′ 43″, PZ = 49° 55′, to find P.

ZS,	59° 58′ 0″	Arith. Comp. ·0333098
sin PS,	67° 50′ 43″	Arith. Comp. ·1162768
sin PZ,	49° 55′ 0″	
	2)177° 43′ 43″	
sin s,	88° 51′ 51″	9·9999146
sin (s — ZS),	28° 53′ 51″	9·6841666
		2)19·8336678
cos ½ P,	34° 20′ 17″·6	9·9168339
	2	
∴ P =	68° 40′ 35″·2	
	2	
	3)13,6 8,0 7,0·4	
	4 32	
	2 40	See page 97.
	2·3	
App. time from noon at ship	4h. 34m. 42s.·3	
App. time at Greenwich (by chron.)	5h. 4m. 41s.·6	
Longitude W. in time	0h. 29m. 59s.·3	
29m. =	7° 15′	
59s.·3 =	14′ 49″·5	See page 119.
LONGITUDE W.	7° 29′ 49″·5	

2. On the 28th of May, 1823, in latitude 0° 50′ N., the mean of several obs altitudes of the star Antares, when eastward of the meridian was 30° 42′, an corresponding time by the chronometer was 9h. 35m. 43s. The chronometer wa fast 1m. 5s. on April 20 at noon at Greenwich, and its daily gain was 5s·4. height of the eye was 16 feet. Required the longitude of the ship.

1. *Apparent time at Greenwich from Chronometer.*

	h. m. s.
Time by chronometer	9 35 43
Fast April 20	— 1 5
Gain from April 20 to May 28	— 3 27·5
Mean time at Greenwich	9 31 10·5
Corresponding equation of time	+ 3 9·2
App. time at Greenwich	9 34 19·7

2. *Apparent time at Ship from triangle* PZS.

	°	′	″
Mean of observed altitudes	30	42	0
Dip		— 3	50
Star's apparent altitudes	30	38	10
Refraction		— 1	39
Star's true altitude	30	36	31

	°	′	″		
ZS,	59	23	29		
sin PS,	116	1	37	Arith. Comp.	·0464395
sin PZ,	89	10	0	Arith. Comp.	·0000459
	2)264	35	6		
sin s,	132	17	33	. . . . .	9·8690669
sin (s — ZS),	72	54	4	. . . . .	9·9803665
					2)19·8959188
cos ½ P,	27	29	32 2	. . . . .	9·9479594
∴ P =	54	59	4 2		
	3)10,8	11,8	,8		
	3	36			
		3	56·3	See page 97.	

olar angle in sid. time .	3h	39m	56s·3
tar's R.A.	16h	18m	31s·2
..A. of meridian	12h	38m	34s·9
un's R.A. May 28, at 9h 34m 20s app. time	4h	19m	17s
ıpp. time at ship	8h	18m	17s·9
ıpp. time at Greenwich .	9h	34m	19s·7
ong. W. in time	1h	15m	1s·9

∴ Long. W. = 18° 45′ 28″

Examples for Exercise.

On September 23, 1845, in the afternoon and in latitude 50° 30′ N., the observed ıde of the sun's lower limb was 11° 0′ 50″, the index correction being — 3′ 20″, he height of the eye 20 feet. The chronometer showed 11h. 59m. 30s.; it was on Greenwich mean time 45s·5 on August 21, and its daily rate was 5s·7 losing; un's declination at the time of observation being 0° 18′ 41″ S., and the equation ne 7m. 52s·3; also the sun's semi-diameter was 15′ 58″: required the longitude.

Ans. Long. 110° 18′ 10″ W.

2. In the afternoon of October 18, 1841, in latitude 15° 46′ N. the observed altitude of the sun's lower limb was 12° 40′: the chronometer showed 11h. 12m. 42s. Greenwich time of Oct. 17: its error Aug. 12 at noon was 5m. 26s. slow, and its daily rate was 12s·5 gaining: required the longitude, the index correction being + 2′ 30″, the height of the eye 16 feet, and the following particulars being furnished by the Nautical Almanac, namely—

Sun's dec. Oct. 17, 9° 18′ 8″ S. Hourly diff. + 54″·81. Eq. of time 14m. 34s·3. Hourly diff. + 0″·483. Sun's semidiameter 16′ 5″.

Ans. Long. 83°51′ 45″ E.

3. On September 10, 1844, in latitude 48° 20′ N., an afternoon altitude of Arcturus was observed to be 31° 5′ 40″, the star west of the meridian: the chronometer showed 5h. 1m. 28s.: its rate, Aug. 25, was 4s·3 gaining, and it was 2m. 40s. slow on Greenwich mean time. The index correction was — 4′ 10″, and the height of the eye 20 feet: also the sun's R.A. at the time of observation was 11h. 19m. 18s., the star's R.A. 14h. 8m. 34s·65, and its declination 19° 59′ 44″ N. Required the longitude.

Ans. Long. 32° 8′ 20″ E.

On Finding the Longitude at Sea by Lunar Observations.—The reader has sufficiently seen that the difference of longitude between two places on the surface of the earth is virtually the same as the difference of time between those two places at the same absolute instant. Suppose, for example, that a rocket could be projected so high, and give out light of such intensity, as to be seen at the instant of explosion by two observers thirty or forty miles apart: the explosion takes place, of course, at a certain instant of absolute time: the mean-time clock at Greenwich might mark this instant—in the local reckoning of that place—as 3h. 20m.: an observer to the east of Greenwich might find the time at *his* place to be 3h. 22m.: the difference in the local reckoning would thus give a difference of 2m. in the time, although both observers saw the flash at the same absolute instant; and we should conclude that the easterly place of observation had half a degree, or 30 nautical miles of east longitude. It is the same in reference to any other phenomenon; let only the time at Greenwich be noted which marks the instant of its occurrence, and also the time at any other locality which marks the same instant; the difference of the times thus noted will be the longitude, in time, of that other locality from Greenwich.

The heavenly bodies furnish signals in abundance analogous to the rocket-signal here imagined, and astronomers have only to make their selection suitably to the circumstances of the observers: what are called *the lunar observations* are the best adapted to determine the longitude at *sea*. Certain stars, lying in or very near to the moon's path, are chosen, and the distance of the moon from each of these, as also from the sun at noon (Greenwich time), and at every interval of three hours, is predicted and recorded in the Nautical Almanac for every day in the year. An observer at sea measures one of these distances, and refers to the Nautical Almanac for the Greenwich time when the same phenomenon happened; he finds, most likely, that *his* distance is intermediate between a certain *pair* of the three-hourly distances; he knows, therefore, that the Greenwich time is intermediate between the recorded hours; and, just as in the case of any other varying quantity—given in the almanac for regular intervals of time—he calculates the intermediate time corresponding to the intermediate distance by proportion. The motion of the moon is found to be sufficiently uniform, for a period of three hours, to justify an intermediate position being inferred in this way, and it is sufficiently rapid to render her change of place sensible even in two or three seconds of time; a trifling correction for variable motion will, however, be noticed hereafter.

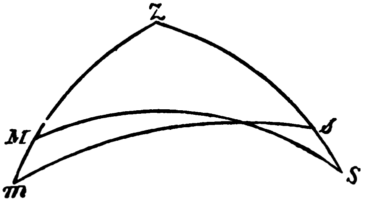
The lunar observations thus enable the mariner to discover the time at Greenwich independently of the chronometer, and thence to determine the error of his timekeeper; and, therefore, by taking the difference between this error and that last recorded, to apply the proper correction to the mean daily rate. The lunar observations *alone* will not enable the navigator to find his longitude; but they enable him to find what o'clock it is at Greenwich at the instant the lunar distance was taken, and thus to correct his chronometer.

Knowing, in this way, the time at Greenwich, we can very accurately compute the declination of the sun at the time of observation, or its right ascension, if the distance be that between the moon and a star, and thence, by means of the latitude of the place, supposed to be already known, we can find, as explained at pages 133 and 138, the time at that place, and thence the longitude.

It must be remarked that the *observed* distance between two celestial objects is not the *true* distance, any more than their observed altitudes are the true altitudes; these latter—the true altitudes—it is necessary to find, in order, from the observed, to compute the true distance; for it is the true distance, as measured from the centre of the earth, that is predicted in the Nautical Almanac; in other words, it is the observed distance cleared from the effects of parallax and refraction. Such being the case, the reader will perceive that the deduction of the Greenwich time, from an observed distance, is not wholly independent of all local information to be afforded by the ship's account: the estimated time at the ship, and the estimated longitude, are evidently useful for the purpose of enabling us to get the semi-diameter and horizontal parallax of the *moon* with greater precision, for these quantities, especially the latter, sensibly vary from noon to midnight, and they are elements which necessarily enter the calculations for reducing the moon's observed altitude to the true altitude.

We shall now investigate formulæ for computing the true distance of the moon's centre from that of the sun, or from a fixed star, by means of the apparent and true altitudes of the two objects, and their apparent distance—that is, the observed distance corrected for semi-diameters.

Investigation of Formulæ for Clearing the Lunar Distance.—In the annexed diagram, let Z represent the zenith of the place of observation, and ZM, ZS the two verticals on which the objects are observed. Let *m*, *s* be the observed places of the moon and sun, or of the moon and a fixed star, and let M, S be their true places. As the moon is depressed by parallax more than it is elevated by refraction, M will be above *m*; but the sun, on the contrary, being more elevated by refraction than depressed by parallax, S will be below *s*.



The corrections for dip, index error, and semi-diameter being applied, observation gives the apparent zenith distances Zm, Zs, and the apparent distance ms of the objects themselves, and the proper corrections for parallax and refraction being applied to the apparent altitudes, we get the true zenith distances ZM, ZS, and the object of the present investigation is to determine the true distance MS by computation.

Let d stand for the apparent distance;
D . . : true distance
a, a' . . . apparent altitudes
A, A' . . . true altitudes

By Spherical Trigonometry (MATH. SCIENCES, page 403) the triangle MZS gives for cos Z the following expression, namely—

$$\cos Z = \frac{\cos D - \sin A \sin A'}{\cos A \cos A'}$$

and the triangle m Z s gives in like manner

$$\cos Z = \frac{\cos d - \sin a \sin a'}{\cos .\ a \cos a'}$$

Hence, for the determination of D we have the equation

$$\frac{\cos D - \sin A \sin A'}{\cos A \cos A'} = \frac{\cos d - \sin a \sin a'}{\cos a \cos a'}$$

from which we have for cos D the expression

$$\cos D = \frac{\cos d - \sin a \sin a'}{\cos a \cos a'} \cos A \cos A' + \sin A \sin A'.$$

But since $\cos (a + a') = \cos a \cos a' - \sin a \sin a'$, this is the same as

$$\cos D = \frac{\cos d + \cos (a + a') - \cos a \cos a'}{\cos a \cos a'} \cos A \cos A' + \sin A \sin A' ;$$

$$= \left\{\frac{\cos d + \cos (a + a)}{\cos a \cos a'} - 1\right\} \cos A \cos A' + \sin A \sin A$$

If the -1 within the braces be suppressed, we may restore it by adding $\cos A \cos A'$ at the end, as is obvious. Hence

$$\cos D = \frac{\cos d + \cos (a + a')}{\cos a \cos a'} \cos A \cos A' - \cos (A + A')$$

The numerator of the fraction in this expression being the sum of two cosines, we may replace it by a product from the formula at page 309 MATHEMATICAL SCIENCES, namely—

$$\cos \theta + \cos \phi = 2 \cos \tfrac{1}{2} (\theta + \phi) \cos \tfrac{1}{2} (\theta \backsim \phi)$$

so that we have

$$\cos D = \frac{2 \cos \frac{1}{2} \left\{ (a + a') + d \right\} \cos \frac{1}{2} \left\{ (a + a') \backsim d \right\}}{\cos a \cos a'} \cos A \cos A' - \cos (A + A')$$

Subtract each side of this equation from 1, then since by Trigonometry (MATH. SCIENCES, p. 310),

$$1 - \cos D = 2 \sin^2 \tfrac{1}{2} D, \text{ and } 1 + \cos (A + A') = 2 \cos^2 \tfrac{1}{2} (A + A')$$

the equation after division by 2 becomes

$$\sin^2 \tfrac{1}{2} D = \cos^2 \tfrac{1}{2} (A + A') - \frac{\cos \frac{1}{2} \left\{ (a + a') + d \right\} \cos \frac{1}{2} \left\{ (a + a') \backsim d \right\} \cos A \cos A'}{\cos a \cos a'}$$

$$= \cos^2 \tfrac{1}{2} (A + A') \left\{ 1 - \frac{\cos \frac{1}{2} \left\{ (a + a') + d \right\} \cos \frac{1}{2} \left\{ (a + a') \backsim d \right\} \cos A \cos A'}{\cos a \cos a' \cos^2 \frac{1}{2} (A + A')} \right\}$$

Or, calling the second of these factors $1 - \sin^2 \theta$, that is, $\cos^2 \theta$, we have

$$\sin^2 \tfrac{1}{2} D = \cos^2 \tfrac{1}{2} (A + A') \cos^2 \theta$$

$$\therefore \sin \tfrac{1}{2} D = \cos \tfrac{1}{2} (A + A') \cos \theta$$

which is known by the name of the formula of Borda. This formula consists, we see of two parts: the first part determines the arc θ, and the second by means of θ computes D. The working model is therefore this, namely:—

$$\left.\begin{aligned} \sin\theta &= \sqrt{\frac{\cos\frac{1}{2}\{(a+a')+d\}\cos\frac{1}{2}\{(a+a')\backsim d\}\cos A\cos A'}{\cos a\cos a'\cos^2\frac{1}{2}(A+A')}} \\ \sin\tfrac{1}{2}D &= \cos\tfrac{1}{2}(A+A')\cos\theta \end{aligned}\right\} \text{(I)}$$

from which we see that the entire work for determining the true distance D may be conducted by logarithms.

It may be worth while to remark, that we were fully justified in assuming that the fraction in the foregoing expression for $\sin^2\frac{1}{2}D$ could be represented by $\sin^2\theta$; for this was only assuming that the fraction must be less than unity, or that the expression within the braces is positive, which it necessarily is, because $\sin^2\frac{1}{2}D$ is necessarily positive. A desirable feature in this method of clearing the lunar distance is, that sines and cosines are the only trigonometrical quantities employed in it; a circumstance which not only affords relief to the memory, but at the same time facilitates reference to the tables.

Rule for Clearing the Lunar Distance.—The steps for calculating D from the formula just established may be described in words as follows:—

RULE 1. Add together the apparent distance and the apparent altitudes; take the difference between half their sum and the apparent distance, and underneath the result write the true altitudes; and, leaving a gap for two lines, then write half the sum of the true altitudes: the *whole* sum may be interposed in the space thus left.

2. These directions having been complied with, it will be found that *nine* arcs have been written down, the apparent distance being the first. Then disregarding the first arc, as also the fourth arc, which is the sum of the preceding three, write opposite to each of the other *seven*, towards the right, the word *cosine*, prefixing, however, "*complement*" to the first two. Refer to the table of sines and cosines for the quantities thus indicated, putting a *plus* mark after the last cosine, and a *minus* mark in front of it.

3. Add up the first six of the numbers thus placed in column, divide the sum by 2, and subtract the cosine previously marked with the *minus*; the remainder will be sin θ.

[These three precepts conduct us, therefore, to the value of the first of the two expressions in the formula (1) above: the fourth precept following enables us to compute the second expression].

4. Take out cos θ, putting a *plus* mark after it, and add it to the cosine with the + after it; the sum will be sin ½ D; taking, therefore, the corresponding arc ½ D out of the tables, and multiplying it by 2, we have the true distance sought.

Examples.

1. Suppose the apparent distance between the centres of the sun and moon to be 83° 57′ 33″, the apparent altitude of the moon's centre 27° 34′ 5″, the apparent altitude of the sun's centre 48° 27′ 32″, the true altitude of the moon's centre 28° 20′ 48″, and the true altitude of the sun's centre 48° 26′ 49″: required the true distance.

Here d = 83° 57′ 33″, a = 27° 34′ 5″, a' = 48° 27′ 32″
A = 28° 20′ 48″, A′ = 48° 26′ 49″

and proceeding by the rule, the work will stand thus:—

d	83° 57′ 33″		
a	27° 35′ 5″	comp. cos	·0523390
a'	48° 27′ 32″	comp. cos	·1783835
	2)159° 59′ 10″		
½ sum	79° 59′ 35″	cos	9·2399686
½ sum ∽ d	3° 57′ 58″	cos	9·9989587
A	28° 20′ 48″	cos	9·9445275
A′	48° 26′ 49″	cos	9·8217187
A + A′	76° 47′ 37″		2)39·2358960
			19·6179480
½ (A + A′)	38° 23′ 48″½	− cos	9·8941654+
θ	31° 57′ 53″½	sin	9·7237826
θ		cos	9·9285870+
½ D	41° 40′ 27″	sin	9·8227524

∴ D = 83° 20′ 54″

In the foregoing operation more attention is paid to minute quantities than is necessary in actual practice. Fractions of a second are of course always disregarded at sea: seconds themselves are indeed very frequently neglected; but this negligence in problems like the present is by no means to be commended. A table of sines and cosines computed to every 10 seconds—such as the "Tables Portatives" of Callet—will enable the computer to take account of his seconds, without entailing upon him any extra work worth mentioning. Tables of logarithmic sines and cosines, computed to every ten seconds, will also be found in the Nautical Tables of Inman and Mackay.

The reader will have observed that, agreeably to what is recommended at page 32 of the INTRODUCTION, the preceding rule directs that the table employed should not be applied to till the work can be advanced no further without its aid; and that when it is once in hand all the extracts from it, previously to the determination of θ, should be made before it is laid down. As a and A are always pretty close together, cos A may be taken out of the table immediately after comp. cos a; and as a' is in like manner in close neighbourhood to A′, cos A′ may be taken out immediately after comp. cos a'.

The above is, of course, a rigorous method of clearing the lunar distance from the effects of parallax and refraction, and provided only that the data furnished by the observations be correct, is perfectly accurate. By help of an auxiliary table, like that which forms table XLII., in the second volume of Dr. Mackay's treatise on the Longitude, the operation may be shortened. The table alluded to supplies the value of the expression log $\frac{\cos A \cos A'}{\cos a \cos a'}$, which is called the "Logarithmic Difference," and unlike some of the subsidiary tables for abridging the calculations required in the direct methods of working the present problem, it may always be used with confidence; this part of the trigonometrical operation may therefore be shortened without any sacrifice of accuracy. In general, however, the indirect methods of clearing the lunar distance, though often shorter than the direct methods, are proportionally deficient in *precision. And* the author conceives that the strict accuracy of the results, and the unhesitating confidence that may consequently be always placed in the conclusions of the direct methods, by spherical trigonometry, produce a degree of satisfaction which more than compensates for the trouble of a few additional references to those common tables with which every calculator may reasonably be expected to be abundantly familiar.

It will, no doubt, occur to the reader, that instead of comp. cos, secant may be used; observing to reject the index 10 from each secant.

2. The apparent distance of the moon's centre from the star Regulus was 63° 35′ 14″, the apparent altitude of the moon's centre 24° 29′ 44″, the apparent altitude of the star 45° 9′ 12″, the true altitude of the moon's centre 25° 17′ 45″, and the true altitude of the star 45° 8′ 15″: required the true distance.

d	63° 35′ 14″		
a	24° 29′ 44″	comp. cos	·0409617
a′	45° 9′ 12″	comp. cos	·1516803
	2)133° 14′ 10″		
½ sum	66° 37′ 5″	cos	9·5986359
½ sum ∽ *d*	3° 1′ 51″	cos	9·9993921
A	25° 17′ 45″	cos	9·9562230
A′	45° 8′ 15″	cos	9·8484402
A + A′	70° 26′ 0″		2)39·5953332
			19·7976666
½ (A + A′)	35° 13′ 0″	— cos	9·9122099 +
θ	50° 11′ 23″	sin	9·8854567
θ		cos	9·8063479 +
½D	31° 32′ 17½″	sin	9·7185578

∴ D = 63° 4′ 35″

On Making the Observations.—In taking a lunar distance at sea, it is desirable that there should be *three* observers; one of these measures the distance between the limbs of the sun and moon, or between the limb of the moon and the selected star, and the other two take the altitudes of the objects at the instant the distance is obtained. A single distance, and a single corresponding pair of altitudes, is not in general considered as sufficient; but a set of distances, and a set of corresponding altitudes, are usually taken, allowing as little time as possible to intervene between each observation: the mean of the distance, and the corresponding means of the altitudes, are then employed in the calculation.

If, however, there be but one observer, it will be necessary that he have the aid of an assistant, to note by the watch intervals of time, the observer proceeding as follows. 1, Let the altitude of the *sun or star be taken*; 2, then the altitude of the moon; 3, a set of distances; 4, another altitude of the moon; 5, another altitude of the sun or star; the times of each observation being noted. Then let the means of the distances and times of observing them be taken; and reduce the altitudes to the mean time thus found by proportion. These are the directions given by Norie; but Lieut. Raper, an experienced practical navigator, recommends as follows:—When the observer is alone, he will first observe the altitude of the object farthest from the meridian, then that of the other object, and then the distance, concluding with the altitudes in the reverse order; the reason of this order is, that the outer object preserves uniformity in its change of altitude for a longer time than the other, and consequently its altitude may be reduced, by simple proportion, to an intermediate time with less error than the altitude of the other object; we may add, however, that it is desirable that the outer object should not be the moon.

When the ship has much motion, the observer fixes himself in a corner, or lies on his back on the deck, to take the distance, in order to remove, as much as possible, the sense of bodily effort and inconvenience which disturbs the eye and the attention. Three or five distances, at least, should be taken; less precision is necessary in taking the altitudes than in observing the distance, so that one altitude of each object, taken with ordinary care, will in general suffice, when the time at the ship is not to be deduced as well as the true distance.

Other Methods of Clearing the Distance.—As the problem of deducing the true lunar distance from the observed distance is one of such note and importance in Nautical Astronomy we shall present the reader with some other methods, all like that of Borda just given, conducted by the rigorous principles of spherical trigonometry, and requiring only the common logarithmic and trigonometrical tables; that is, only the logarithms of numbers, and the natural and logarithmic sines and cosines.

The first of these additional methods which we shall offer is that of Delambre, the formula for which occurs in the investigation of the method given at page 146, namely—

$$\cos \mathrm{D} = \frac{2 \cos \tfrac{1}{2} \left\{(a + a') + d\right\} \cos \tfrac{1}{2} \left\{(a + a') \backsim d\right\} \cos \mathrm{A} \cos \mathrm{A}'}{\cos a \cos a'} - \cos (\mathrm{A} + \mathrm{A}').$$

The logarithm of the first expression in this formula is calculated as in the method of Borda; then the natural number answering to that logarithm is taken out of the table, and the natural cosine of (A + A′) subtracted from it; the remainder is the natural cosine of the true distance. As this is a sort of mixed method, requiring reference to both logarithmic and natural cosines, attention must be paid to the change of radius in passing from one to the other: the expression which it is here proposed to find the common logarithm of, will supply four additive logarithms and two subtractive ones; hence two tens, or 20, must be suppressed in the result (page 9); but if the arithmetical complements of log cos a, log cos a', be *added*, then four tens, or 40, must be suppressed, and the result will then be the ordinary logarithm of the expression, and from the number answering thereto, the nat cos (A + A′) is to be subtracted, as the formula indicates.

Taking the example at page 148, in which

$$d = 83° \, 57' \, 38'', \quad a = 27° \, 34' \, 5'', \quad a' = 48° \, 27' \, 32''$$
$$\mathrm{A} = 28° \, 20' \, 48'', \quad \mathrm{A}' = 48° \, 26' \, 49''$$

The operation by the above method of Delambre will stand arranged as follows:—

d 83° 57′ 33″ . . . comp. cos ·0523390
a 27° 34′ 5″ . . . comp. cos ·1783835
a' 48° 27′ 32″ log. 2 ·3010300

2)159° 59′ 10″

½ sum 79° 59′ 35″ . . . . . . cos 9·2399686
½ sum ∽ d 3° 57′ 58″ . . . . . . cos 9·9989587
A 28° 20′ 48″ . . . . . . cos 9·9445275
A′ 28° 26′ 49″ . . . . . . cos 9·8217187

log ·3442921 = $\bar{1}$·5369260 (40 suppressed)
A + A′ 76° 47′ 37″ nat. cos ·2284595

D 83° 20′ 54″ nat. cos 1158326

ther formulæ may be investigated as follows :—

eferring to the two expressions for cos Z at page 146, we have

$$+\cos Z = \frac{\cos D + \cos A \cos A' - \sin A \sin A'}{\cos A \cos A'} = \frac{\cos D + \cos (A + A')}{\cos A \cos A'}$$

$$+\cos Z = \frac{\cos d + \cos a \cos a' - \sin a \sin a'}{\cos a \cos a'} = \frac{\cos d + \cos (a + a')}{\cos a \cos a'}$$

$$\therefore \frac{\cos D + \cos (A + A')}{\cos A \cos A'} = \frac{\cos d + \cos (a + a')}{\cos a \cos a'}$$

$$\therefore \cos D = \left\{\cos d + \cos (a + a')\right\} \frac{\cos A \cos A'}{\cos a \cos a'} - \cos (A + A') \quad \ldots\ldots (1)$$

rom the same two expressions for cos Z, we also have

$$-\cos Z = \frac{\cos A \cos A' + \sin A \sin A' - \cos D}{\cos A \cos A'} = \frac{\cos (A \backsim A') - \cos D}{\cos A \cos A'}$$

$$-\cos Z = \frac{\cos a \cos a' + \sin a \sin a' - \cos d}{\cos a \cos a'} = \frac{\cos (a \backsim a') - \cos d}{\cos a \cos a'}$$

$$\therefore \frac{\cos (A \backsim A') - \cos D}{\cos A \cos A'} = \frac{\cos (a \backsim a') - \cos d}{\cos a \cos a'}$$

$$\therefore \cos D = \left\{\cos d - \cos (a \backsim a')\right\} \frac{\cos A \cos A'}{\cos a \cos a'} + \cos (A \backsim A') \quad \ldots\ldots (2)$$

he formula marked (1) and (2) are tolerably commodious by help of the common ithmic tables; as an exemplification of their advantages, we shall solve the mple at page 149 by each. In this example the following quantities are given, ely—

$$d = 63° \, 35' \, 14'', \; a = 24° \, 29' \, 44'', \; a' = 45° \, 9' \, 12''$$
$$A = 25° \, 17' \, 45'', \; A' = 45° \, 8' \, 15''$$

Determination of D *by formula* (1).

			Logarithms.
d 63° 35′ 14″	nat. cos ·4448349 +		
a 24° 29′ 44″		comp. cos	·0409617
a' 45° 9′ 12″		comp. cos	·1516803
$a + a'$ 69° 38′ 56″	nat. cos ·3477722 +		
	natural number ·7926071		$\bar{1}$·8990579
A 25° 17′ 45″		cos	9·9562230
A′ 45° 8′ 15″		cos	9·8484402
	natural number ·7877042		$\bar{1}$·8963631
A + A′ 70° 26′ 0″	nat. cos ·3349034 —		
D 63° 4′ 35″	nat. cos ·4528008		

With the exception of the middle logarithm in the column on the right, the sum of the numbers in that column may be found at once by a table of the Logarithmic Difference as already noticed at page 148. Of all the auxiliary tables employed in Nautical Astronomy, or at least in that part of it with which we are here occupied, the table alluded to is perhaps the most useful; the element it furnishes enters into nearly all the methods of clearing the lunar distance.

As far as the author knows, the method just given is new; but rules for clearing the distance are so numerous, that it is more than probable it has appeared elsewhere. In working by this method, it is best to take the formula itself as a guide, and to attend to the algebraic sign of the factor $\{\cos d + \cos(a + a')\}$: if this should be negative, the first natural number above is still to be treated as positive; the second natural number, which is the product of the two factors in the formula, will then, however, be negative, and from this negative number the nat. cos (A + A′) is to be subtracted as above.

Determination of D *by formula* (2).

d 63° 35′ 14″	nat. cos ·4448349 +		
a 24° 29′ 44″		comp. cos	·0409617
a' 45° 9′ 12″		comp. cos	·1516803
$a \backsim a'$ 20° 39′ 28″	nat. cos ·9357042 —		
	natural number ·4908693		$\bar{1}$·6909659
A 25° 17′ 45″		cos	9·9562230
A′ 45° 8′ 15″		cos	9·8484402
	natural number ·4878329		$\bar{1}$·6882711
A $\backsim$ A′ 19° 50′ 30″	nat. cos ·9406341 +		
D 63° 4′ 35″	nat. cos ·4528012		

The first of the preceding natural numbers is, by the formula, negative, though treated as positive. In consequence of this, the second natural number—which is the product of the two factors in the formula—is also negative, so that cos D is the *difference* between the last nat. number and cos $(A \backsim A')$.

This last method of clearing the distance is the same as that proposed by Mr. Keith, in his treatise on Trigonometry, and, like the one previously given, is capable of abridgment by means of the table of "Logarithmic differences."

The reader will observe that the natural cosine of D, arrived at above, differs by four units in the seventh place of decimals from the natural cosine of D in the former method, although the resulting arcs differ only by a fraction of a second. Occasional insignificant discrepancies of this kind must be expected, in working the same example by different methods with tables. In looking at the "differences" in tables, it will be seen that in some parts the difference between two consecutive numbers is large, and in other parts small. In proportioning for an intermediate number, in the former case a comparatively large difference will have but small influence, while in the latter case a comparatively small difference may have a very sensible effect. But a discrepancy in the seventh place of decimals is of no moment even in working to seconds.

All the preceding methods of clearing the lunar distance are independent of subsidiary tables; a large collection of compendious rules for working the problem by special tables will be found in Dr. Mackay's valuable treatise on the Longitude; and a very short form of operation is also given by Mr. Woolhouse in the Appendix to White's Ephemeris, 1856.

Before leaving the present problem, it may be useful to observe that the altitudes of the objects are not required to that precision with which the distance should be taken: this is a desirable circumstance, because, from the frequent obscurity of the sea horizon, it is more difficult to get the altitudes accurately than the distance. If reference be made to any of the formulæ given in the preceding pages, it will be seen that d remaining the same, a small alteration in the values of a, a', and the same alteration in those of A, A′, cannot produce any sensible effect upon the value of D: the factor $\frac{\cos A \cos A'}{\cos a \cos a'}$, the logarithm of which is, in nautical tables, called the Logarithmic Difference, is always very nearly equal to unit, as is obvious; and this is the principal reason why a small error in the altitudes does not sensibly effect the distance. It is of importance, however, that the proper corrections be carefully applied to the observed altitudes to obtain the true altitudes, even though the former should not have been taken with precision—the *relative* values of the observed and true altitudes must still be preserved. With a view to the determination of the longitude from the lunar distance, accuracy in these corrections is of much importance; and the neglect even of those depending upon the state of the atmosphere, as indicated by the barometer and thermometer, will sometimes occasion an error of more than thirty minutes of longitude.

From what is here said, the learner will perceive that a few seconds may always be safely added to or subtracted from the observed altitudes, if such a modification be found to facilitate the calculation, the seconds may, for instance, be always made a multiple of 10. The observed altitudes may indeed, without occasioning any appreciable error in the result, be taken to the nearest minute, and even an imperfect altitude, that may err from the truth by so much as two or three minutes, may still be employed with safety, provided the time *is not to be* computed as well as the distance. In most cases an error of altitude to the extent of 10 minutes will affect the resulting distance by less than that number of seconds. It is also worthy of notice that the apparent distance itself may be so modified as to be rendered free from seconds, provided that, when D is deduced, the seconds of error, whether in defect or excess, be applied in the reverse way to D.

But except in the merely approximative methods of solution, seconds cannot be wholly dispensed with throughout the work: the corrections for deducing the observed altitudes to the true will always introduce them. By means, however, of the tables already referred to, in which the trigonometrical quantities are computed to every ten seconds of the arc or angle, the proper allowance for the odd seconds may always be made with but comparatively little extra trouble.

Examples for Exercise.

1. The apparent distance between the moon's centre and a star is 64° 36′ 40″; the apparent altitude of the moon's centre 44° 38′; the apparent altitude of the star 11° 51′; the true altitude of the moon's centre 45° 15′ 38″; and the true altitude of the star 11° 46′ 33″: required the true distance. Ans. D = 64° 46′ 14″.

2. The apparent distance between the centres of the sun and moon is 108° 14′ 34′; the apparent altitude of the moon's centre 24° 50′; the apparent altitude of the sun's centre 36° 25′; the true altitude of the moon's centre 25° 41′ 39″; and the true altitude of the sun's centre 36° 23′ 50″: required the true distance. Ans. D = 107° 32′ 1″.

3. The apparent distance between the centre of the moon and a star is 51° 28′ 30″; the apparent altitude of the moon's centre 12° 30′ 4″; the apparent altitude of the star 24° 48′ 17″; the true altitude of the moon's centre 13° 20′ 40″; and the true altitude of the star 24° 46′ 14″: required the true distance. Ans. D = 51° 9′ 48″.

4. The apparent distance between the moon's centre and a star is 31° 12′ 26″; the apparent altitude of the moon's centre 8° 26′ 13″; the apparent altitude of the star 35° 40′; the true altitude of the moon's centre 9° 20′ 45″; and the true altitude of the star 35° 38′ 49″: required the true distance. Ans. D = 30° 23′ 56″.

5. The apparent distance of the centres of the sun and moon is 90° 21′ 17″; the apparent altitude of the moon's centre 5° 17′ 9″; the apparent altitude of the sun's centre 84° 7′ 20″; the true altitude of the moon's centre 6° 9′ 14″; and the true altitude of the sun's centre 84° 7′ 15″: required the true distance. Ans. D = 89° 29′ 18″.

Note.—The method of finding the true distance between the centre of the moon and a star is the same as that for the distance between the moon's centre and a planet; in the case of a star, the true altitude is deduced from the apparent altitude by applying the correction for refraction merely; but in the case of a planet, a correction may be necessary for the parallax in altitude. But in general this correction may be neglected, as it is usually too small to be of much importance.

On Computing the Altitudes.—It sometimes happens that, though circumstances may be favourable for taking a lunar distance, yet the obscurity of the horizon may present an obstacle to the observations for altitude; in such a case the true altitudes will have to be computed, and from these the apparent altitudes may be deduced, by applying the corrections the contrary way. To compute an altitude it is necessary to know the hour-angle, or angular distance, of the object from the meridian. If the object be the sun, this angle is the apparent time from the meridian; and as the altitude, as already remarked, is not required with precision, the estimated time at the ship will answer for the purpose. If the object be the moon or a star, the sun's right ascension, at the instant, must be increased or diminished by the apparent time, according as this time, is p.m. or a.m., to get the right ascension of the meridian; the difference between this and the right ascension of the object is the hour-angle or the meridian distance of the object. The hour-angle being thus found, a formula for the altitude may be investigated as follows:—

Let P be the hour-angle, l the co-latitude, z the co-altitude, and p the polar distance or co-declination. Then the fundamental formula of spherical trigonometry gives,

$$\cos \mathrm{P} = \frac{\cos z - \cos l \cos p}{\sin l \sin p}$$

$$\therefore \cos z = \cos l \cos p + \sin l \sin p \cos \mathrm{P}; \text{ but } \cos \mathrm{P} = 1 - 2 \sin^2 \tfrac{1}{2} \mathrm{P}$$

$$\therefore \cos z = \cos l \cos p + \sin l \sin p - 2 \sin l \sin p \sin^2 \tfrac{1}{2} \mathrm{P}$$

$$\therefore \cos z = \cos (l \backsim p) - 2 \sin l \sin p \sin^2 \tfrac{1}{2} \mathrm{P} \ . \ . \ . \ (1)$$

By means of tables of natural and logarithmic sines and cosines, the co-altitude z may be obtained from this formula; but to adapt it wholly to logarithms, let 1 be added to each side of the equation, then remembering that $1 + \cos \mathrm{A} = 2 \cos^2 \tfrac{1}{2} \mathrm{A}$, we have, upon dividing by 2,

$$\begin{aligned} \cos^2 \tfrac{1}{2} z &= \cos^2 \tfrac{1}{2} (l \backsim p) - \sin l \sin p \sin^2 \tfrac{1}{2}\mathrm{P} \\ &= \cos^2 \tfrac{1}{2} (l \backsim p) \left\{1 - \sin l \sin p \sin^2 \tfrac{1}{2} \mathrm{P} \sec^2 \tfrac{1}{2} (l \backsim p)\right\} \end{aligned}$$

Put $\cos^2 \mathrm{M}$ for the expression within the braces; then for computing the altitude a, that is the complement of z, we have the following formula,—namely,

$$\left.\begin{array}{l} \sin \mathrm{M} = \sin \tfrac{1}{2} \mathrm{P} \sec \tfrac{1}{2} (l \backsim p) \sqrt{\sin l \sin p} \\ \sin \tfrac{1}{2} (a + 90^\circ) = \cos \tfrac{1}{2} (l \backsim p) \cos \mathrm{M} \end{array}\right\}$$

or, which is perhaps a little more convenient,

$$\left.\begin{array}{l} \sin \mathrm{M} = \dfrac{\sin \frac{1}{2}\mathrm{P}}{\cos \frac{1}{2}(l \backsim p)} \sqrt{\sin l \sin p} \\ \sin \tfrac{1}{2}(a + 90^\circ) = \cos \tfrac{1}{2} (l \backsim p) \cos \mathrm{M} \end{array}\right\} \ . \ . \ . \ . \ (2)$$

Suppose, for example, that by means of the estimated time at the ship, and the longitude by account, the moon's angular distance from the meridian is found to be 33° 30′, at the time of taking a lunar distance in latitude 38° 14′, the moon's co-declination reduced to the time, being 64° 13′ 13″; required the apparent altitude by computation, the obscurity of the horizon preventing an observation. Working by the formula (2) the operation is as follows, where $\mathrm{P} = 33^\circ\ 30'$, $p = 64^\circ\ 13'$, $l = 51^\circ\ 46'$:

$$\begin{array}{rlrlrl} l\ 51^\circ\ 46' & . \ . \ . & \tfrac{1}{2} \sin & 4{\cdot}947572 & & \\ p\ 64^\circ\ 13' & . \ . \ . & \tfrac{1}{2} \sin & 4{\cdot}977228 & & \\ \tfrac{1}{2}\mathrm{P}\ 16^\circ\ 45' & . \ . \ . & \sin & 9{\cdot}459688 & & \\ \tfrac{1}{2}(l \backsim p)\ \ 6^\circ\ 13\tfrac{1}{2}' & . & \text{comp.} \cos & {\cdot}002568 & . \ . \ . & \cos\ 9{\cdot}997432 \\ \hline & & \sin & 9{\cdot}387056 & . \ . \ . & \cos\ 9{\cdot}986683 \\ \hline & & \tfrac{1}{2}(a + 90^\circ) = & 74^\circ\ 36' & . \ . \ . & \sin\ 9{\cdot}984115 \\ & & & 2 & & \\ \hline & & \therefore a = & 59^\circ\ 12' & \text{the true altitude} & \\ & & \text{correction for parallax and refraction} & -\ 28' & & \\ \hline & & & 58^\circ\ 44' & \text{the apparent altitude.} & \end{array}$$

It is obvious, that in strictness, the correction for parallax and refraction should be taken out of the tables in accordance with the *apparent* altitude, not the true altitude: the correction above, therefore, belongs to an altitude somewhat too great, so that only an approximate apparent altitude is in reality deduced from the true altitude. The proper correction may, however, be found by again entering the table with this approximate apparent altitude; we shall thus get 27′ 54″ for the correction instead of 28′, so that the correct apparent altitude is 58° 44′ 6″.

Unless the time at the ship is to be deduced, precision is not necessary in computing the true altitude:—seconds may always be disregarded, and the result found to the nearest minute as above. But seconds should not be neglected in the corrections.

It is of importance that the practical navigator have a due appreciation of the value of small quantities in the computation of the true lunar distance: it is only in the apparent altitudes that precision can be dispensed with. In finding *latitude*, if the result be brought out to the nearest minute the demands of practice will be fully satisfied, as the error cannot exceed half a mile; but an error of only twenty-two seconds in the lunar distance will, on the average, occasion an error of ten minutes of longitude, which, except in high latitudes, is equivalent to seven or eight miles, and within 36° of latitude the error would range from eight to ten miles. Too much pains and caution cannot therefore be exercised in observing a lunar distance: the instrument should be of the very best description, and the observer should have a well-disciplined eye; but the corresponding altitudes may be safely taken by a less skillful person.

An interesting incident is related by the late Captain Basil Hall, an officer who was highly accomplished in the science of his profession. He once sailed from San Blas, on the west coast of Mexico; and after a voyage of eight thousand miles, occupying eighty-nine days, he arrived off Rio Janeiro, having in this interval passed through the Pacific Ocean, rounded Cape Horn, and crossed the South Atlantic, without making any land or even seeing a single sail, with the exception of an American whaler off Cape Horn. When within about a week's sail of Rio, he set seriously about determining, by lunar observations, the longitude of his ship, and then steered his course accordingly by those common principles of navigation which may be safely employed for short distances between one known position and another.

Having arrived to within what he considered from his computations to be about fifteen or twenty miles of the coast, he hove to, at four o'clock in the morning, to await the break of day, and then bore up, proceeding cautiously onward on account of a thick fog which enveloped the ship. As this cleared away the people on board had the satisfaction of seeing the great Sugar-loaf Rock, which stands on one side of the harbour's mouth, so nearly right a-head that they had not to alter their course above a point, in order to hit the entrance of the harbour! This was the first land they had seen for three months, after crossing so many seas, and being set backwards and forwards by innumerable currents and foul winds. The effect on all on board was electric, and the admiration of the sailors was unbounded.

Something in this remarkable case may have been due to a compensation of small errors; but it is only fair to conclude that the accuracy with which the ship's position was ascertained, was almost entirely attributable to the precision with which the lunar distances were taken, and the care with which the computations were executed.

To determine the Longitude from the Lunar Observations.—In the *preceding articles* we have shown at considerable length how the true distance between the moon and the sun, or a fixed star, as seen from the centre of the earth, may be determined from the observed distance taken from the surface. It was also shown, by means of the altitude of a celestial object whose declination is known, and the latitude of the place where the altitude is taken, how the time at that place may be found.

The determination of a lunar distance necessitates the determination of the altitudes of the objects whose distance is observed, so that the data for finding the true lunar distance involves likewise the data for finding the time when that distance had place. But, as already remarked, if these two objects be sought to be accomplished, the altitude employed for the purpose of ascertaining the time at the ship must be taken with a degree of precision which is not indispensable in "working the lunar:" much more care and accuracy is requisite in observations of altitude for *time* than for either latitude or lunar distance; and on this account the time at ship is in general determined independently by one or other of the methods explained at pages 133, 134, &c., either shortly before or shortly after the distance is taken. By means of the chronometer or watch the interval between the time thus found and the instant of taking the distance is known, and the proper correction for change of longitude in that interval being made, we get the time corresponding to the distance.

We have just said that the altitude—or rather the set of altitudes—for determining the time are taken *shortly* before or after the observations for the distance, whenever these observations are not themselves sufficiently accurate for the purpose. It is desirable that the interval should not be large, because the difference of longitude in that interval may be large too, and our estimation of its amount is, therefore, liable to a larger error. But as it is important to get the altitude as precise as possible, the situation of the object should be as near the prime vertical—that is, as nearly due east or due west as possible (page 132). There is thus room for the exercise of some judgment: the object selected for the determination of the time at ship should be near the prime vertical, and it should reach this position shortly before or shortly after the observation for the lunar distance. Whenever the weather is so favourable as to render all risk of losing the anticipated observation but very small, it is prudent to wait till these conditions are fulfilled, provided other circumstances are such as to allow of the delay. In general, the object best suited to the determination of the time at sea is the sun, as its declination, though varying with the time, changes so slowly as to be deducible accurately enough for the purpose from the estimated time, or time by account, as sufficiently shown at page 136; a fixed star, however, is very suitable for the purpose, whenever it is nearly due east or west, and the horizon clearly enough defined to admit of an accurate observation of its altitude.

The object of the lunar distance is to find the time at Greenwich at the instant that distance has place; and the object of the other, or extra observation, is to find the time at the same instant at the ship. The difference of the two times gives the longitude in time. The Greenwich time is obtained by comparing the true distance, deduced from the observation, with the nearest predicted distance, supplied by the Nautical Almanack, in the way that will be explained presently. It may be well, however, first to show what will be the average effect on the inferred longitude of a given error in the determination of the lunar distance.

Effect on the Longitude of an Error in the Lunar Distance.—The mean diurnal motion of the moon in her orbit is 13°·1764: at certain times it is about 2° slower, and at *other times 2° quicker;* but this is her average rate of motion; so that, on the average, 360° of longitude (or 24h. of time) correspond to 13°·1764 of the moon's progress. Hence, to find the error in longitude produced by an error of a seconds in the distance between the moon and a fixed star in her path, we have the proportion

$$13^\circ{\cdot}1764 : 360^\circ :: a : \frac{360a}{13{\cdot}1764} = 27{\cdot}322a$$

which shows that the error in the longitude is 27·322 times the error in the distance. Thus an error of 10″ in the lunar distance causes, on the average, an error of 273″·22 in the resulting longitude, or 4′ 33″·22; and an error of 1′ in the distance causes, on the average, an error of 27′ 19″·3 in the longitude.

The more rapidly the moon moves, the less is the effect upon the longitude of a given error in the lunar distance: the most rapid change in three hours is very nearly 1° 48′, or 1°·8; so that for any error a in the distance, the corresponding error in longitude, in the most favourable case, is $\frac{45a}{1{\cdot}8} = 25a$. Hence, in the most favourable case, the error in longitude corresponding to an error of 10″ in the lunar distance is 250″ = 4′ 10″; and the error in longitude caused by an error of 1′ in the lunar distance is 25′. As every inaccuracy in the distance becomes thus increased twenty-five fold in the longitude deduced from it, even in the most favourable case, the reader will at once perceive the importance of securing precision in this element. Several distances ought to be carefully observed, and the mean of them all employed; and scrupulous attention should be paid to the *corrections* of the altitudes, though, as already remarked, the altitudes themselves need not be taken with the utmost nicety.

To obtain the time at Greenwich corresponding to the true distance determined at sea, requires an operation in simple proportion. To facilitate this operation, Dr. Maskelyne—to whom, indeed, navigators are indebted for originating the Nautical Almanac—contrived the table of "Proportional Logarithms," to be found in every collection of nautical tables. As proportional logarithms have not as yet been alluded to in the present treatise, it will be necessary to say a word or two about them here.

Proportional Logarithms.—These logarithms are derived from the logarithms in common use, thus:—From the logarithm of 10800, the number of seconds in 3 hours, subtract the logarithm of a, any portion of time, in seconds, less than 3 hours: the remainder is called the proportional logarithm of a; in other words,

$$\text{Prop. log. } a = \text{common log. } \frac{10800.}{a}$$

If $a = 10800$, then Prop. log. a = com. log. $1 = 0$; so that proportional logarithms are, in fact, complements of the ordinary logarithms to the number log 10800; just as the arithmetical complements of log sines and log cosines are complements to 10.

As already remarked, the lunar distances given in the Nautical Almanac, are calculated for every three hours of interval. Suppose a distance is determined at sea, at some Greenwich time within one of these intervals; we seek in the Almanac for the nearest distance, preceding, in order of time, the given distance, and take the difference between it and the given distance: call this d, and the difference between the two three-hourly distances, intermediate to which the given distance occurs, call D: then by proportion we should have for the time x^{h}, corresponding to the given distance,

$$D : d :: 3^{\text{h}} : x^{\text{h}}$$

and therefore in common logarithms we should have

$$\log x^{h} = \log 3^{h} + \log d - \log D.$$

But since $3^{h} = 10800''$, the proportional logarithm of which is 1, we should have in proportional logarithms

$$\text{Prop. } \log x^{h} = \text{Prop. } \log d - \text{Prop. } \log D.$$

In the Nautical Almanac Prop. log D is inserted between the distances there given, at the beginning and end of every three hours, so that by subtracting this given proportional logarithm from the proportional logarithm of d taken out of the table we get a proportional logarithm, answering to which in the table is the portion of time to be added to the hour of the earliest distance: the result is the Greenwich mean time corresponding to the given distance.

For example: Suppose it were required to find the Greenwich mean time at which the true distance between the Moon and Pollux was 32° 30′ 25″, on January 14, 1846.

By inspecting the distances in the Nautical Almanac for that year, we find against January 14, and opposite the name of the proposed star, Pollux, the following row of lunar distances:

Midnight.	Pr. log.	xv$^{\text{h.}}$	Pr. log.	xviii$^{\text{h.}}$	Pr. log.	xxi$^{\text{h.}}$	Pr. log.
31° 49′ 25″	3368	33° 12′ 18″	3342	34° 35′ 41″	3319	35° 59′ 31″	3298

from which it appears that the time at Greenwich corresponding to the given lunar distance was between midnight and XV hours, the nearest distance preceding, in order of time, the given distance is therefore the distance at midnight; we therefore proceed as follows:—

Distance at midnight	31° 49′ 25″	Prop. log. of diff.	3368 —
Given distance	32° 30′ 25″		
Difference	0° 41′ 0″	Prop. log.	6425
Portion of time after midnight	1h. 29m. 2s.	Prop. log.	3057

Hence the Greenwich mean time, when the distance was as stated above, was 13h. 29m. 2s.

If the distance increased with perfect uniformity during the interval within which the given distance is found, the Greenwich time corresponding to that given distance, determined as above, would be strictly correct, but as such is not the case, a correction should be applied to the time so found, for the variation of the differences of the distances. A table for obtaining such corrections of the approximate interval of time as found above, is given in the Nautical Almanac. In the example above, the correction comes out 8s. additive, so that the correct Greenwich mean time is 13h. 29m. 10s.; the neglect of this correction would, however, occasion an error of only 2′ in the longitude; but in extreme, and therefore of course, unusual, cases, the error in longitude, from a neglect of this correction, might amount to so much as 12′ in the longitude.

Besides the use of proportional logarithms in connection with the lunar problem, they also serve to point out the star which is most favourably circumstanced for accurate observation; that star being to be preferred which has the *least* proportional logarithm opposite to it: for as already shown (page 158), the greater the velocity of the moon from or towards a star, the greater is the reliance to be placed on an observation of the distance; in other words, the less is the effect of a small error in the distance upon the longitude. It is a property of proportional logarithms to decrease as the natural numbers answering to them increase; a smaller proportional logarithm, therefore, indicates a greater velocity of the moon, or greater variation of distance in the interval, upon which the value of the observation depends.

We shall add another example or two of finding the Greenwich mean time, corresponding to a given lunar distance on a given day.

2. On August 2nd, 1836, the distance of the moon from the planet Mars was found from an observation at sea to be 56° 30′ 8″: the Nautical Almanac gave

Aug. 2 at 3h., distance	57° 43′ 59″	Pr. log of diff.	2948 —
Given distance	56° 30′ 8″		
Difference	1° 13′ 51″	Pr. log	3869
Portion of time after 3h.	2h. 25m. 36s.	Pr. log	921

Hence the mean time at Greenwich was 5h. 25m. 36s.

3. On April 7, 1831, the true distance between the sun and moon was found to be 65° 54′ 48″: the Nautical Almanac gave

April 7 at noon, Distance	67° 0′ 8″	P. L. of diff.	3051 —
Given distance	65° 54′ 48″		
Difference	1° 5′ 10″	P. L.	4412
Portion of time after noon	2h. 11m. 35s.	P. L.	1361

Hence the Greenwich mean time was 2h. 11m. 35s.

4. On November 22, 1853, the true distance of Saturn from the moon was found to be 77° 52′ 45″: the Nautical Almanac gave

Nov. 22 at 3h. Distance 77° 14′ 40″, and P. L. = 6745

Required the Greenwich mean time. Ans. 4h. 13m. 55s.

The proportional logarithms annexed to the Lunar Distances in the Nautical Almanac, and most of the tables of them inserted in books on Navigation, are limited to four places of figures; in certain parts of the table, this number of places is too few to show any difference for two, or even three consecutive *arguments*: thus the proportional logarithms of 2h. 41m., 2h. 41m. 1s., 2h. 41m. 2s., are all the same, namely, 484: this is a defect. In Dr. Inman's Nautical Tables, the proportional logarithms are given to five places of figures, so that the logarithms of consecutive quantities are not confounded. If five-figure proportional logarithms are used in finding the Greenwich mean time corresponding to a lunar distance, then the proportional logarithm of the difference between the consecutive lunar distances in the Almanac must be sought for in the table, as the Almanac gives it to only four places of figures. Thus, taking the example just given,

Given distance	77° 52′ 45″		
Nov. 22, at 3h., Distance	77° 14′ 40″	from Naut. Alm:	
„ 6h. „	78° 47′ 24″		
Difference of dist. in Alm.	1° 32′ 44″	Prop. log	28804 —
Diff. of given dist. and that at 3h.	0° 38′ 5″	Prop. log	67454
Time after 3 hours	1h. 13m. 55s.	Prop. log	38650

Hence the Greenwich mean time was 4h. 13m. 55s.

Examples of determining the Longitude from Lunar Observations.

foregoing articles contain all that is necessary for the determination of the the ship, and the time at Greenwich, at the same absolute instant; and it has ufficiently explained how these two determinations lead immediately to the ry of the longitude, as in the following examples :—

mple 1. On the 1st of September, 1846, the true distance of the star Antares e moon's centre was 39° 48′ 20″, the mean time at the ship being 8h. 14m. 12s.: d the longitude.

time at the ship at the instant the lunar distance was taken being known, e merely to find the Greenwich time corresponding to that distance. We seek, re, among the predicted lunar distances in the Nautical Almanac, for that one ng it in order of time, which is the nearest to it, and take the difference n the two. The proportional logarithm, annexed to the distance extracted e Nautical Almanac, subtracted from the proportional logarithm of this dif. will give the proportional logarithm corresponding to a portion of time to ed to the time at which the distance taken from the almanac had place. ult will be the time at Greenwich corresponding to the given time at the Thus :—

t. 1, at 6h., distance (Naut. Alm.) . . .	38° 50′ 28″	P. L.	2287 —
Given distance	39° 48′ 20″		
Difference	0° 57′ 52″	P. L.	4928
Time after 6h. . . .	1h. 37m. 59s.	P. L.	2641
∴ Time at Greenwich .	7h. 37m. 59s.		
Time at ship	8h. 14m. 12s.		
Longitude E. in time .	0h. 36m. 13s.		
Then page 119, 36m. =	9° 0′ 0″		
13s. =	3′ 15″		
∴ Longitude	9° 3′ 15″ East.		

his example the mean time at the ship is supposed to have been determined by other of the methods already explained, and the true lunar distance to have luced from the observed, as shown in the preceding pages. But it will be ive to exhibit, in a connected form, the operations necessary for ascertaining itude from the ship's account, in combination with the astronomical observa- ve shall, therefore, proceed to an illustration or two of this kind, conducting ess by the following steps :—

he first object will be to get, from the ship's account, the approximate time at ich when the observations are made.

elp of this approximate Greenwich date we shall find semi-diameter, hori- arallax, declination, &c., at the instant of observation, sufficiently near the r our ultimate purpose, because, as already seen, these elements vary so slowly that even a large error in the time can never affect their values except in a very trifling degree.

2. The next thing will be to apply the necessary corrections to the observed in order to obtain the apparent and true altitudes and the apparent distance. Sufficient data will thus be obtained for deducing both the time at ship and the time at Greenwich, and it is matter of indifference which of these requisites is determined first, when the time at the ship is deduced from either the sun or a star.

An error in the ship's account will but very slightly affect the time at ship when the observation is made, unless this time be determined from the moon (page 138); but the altitude employed must be taken with care.

An error in the ship's account will, in like manner, but very slightly affect the lunar distance, and thence the time at Greenwich; for the moon's semi-diameter, though dependent on the time, varies too slowly to cause its approximate value, deduced from the estimated Greenwich time, to differ from its true value, at the instant of taking the distance, by any appreciable amount; and as far as the *altitude* is concerned, it has been seen that this difference is of no moment.

Example 2. On February 12, 1848, at 2h. 36m. p.m., mean time by estimation, in latitude 53° 30′ S., and longitude by account 15° 30′ E., the following lunar observation was taken:—

Obs. alt. sun's L.L.		Obs. alt. moon's L.L.		Obs. dist. nearest limbs.	
	29° 17′ 26″		25° 40′ 20″		99° 27′ 30″
Index cor.	— 2′ 10″	Ind. cor.	— 1′ 10″	Ind. cor.	— 50″
	29° 15′ 16″		25° 39′ 10″		99° 26′ 40″

The height of the eye above the sea was 20 feet: required the longitude.

Ship time, Feb. 12	2h. 36m.
Longitude E. in time	1h. 3m.
Approximate Greenwich time	1h. 33m.

Referring now to the Nautical Almanac, we take out the two semi-diameters for noon of Feb. 12: the approximate Greenwich time differs too little from noon to render any correction necessary; the sun's declination and the moon's horizontal parallax are taken out at the same time: we thus have

From the Nautical Almanac.

Sun's semi-diam.	16′ 13″
Moon's at noon	15′ 58″
Hor. par. noon 58′ 36″ var. in 12h., — 13″	
Var. in 1½h.	— 1″·6
Hor. par. at est. time .	58′ 34″

Sun's dec. noon . . .	13° 52′ 18″ S.
Cor. for 1h. 33m. . .	— 1′ 17″
Dec. at estimated time .	13° 51′ 1″ S.
	90°
Polar dist. PS =	76° 8′ 59″

Hourly diff. of sun's dec. .		— 49″·87
30m.	$\frac{1}{2}$ . . .	24·94
3	$\frac{1}{10}$ . . .	2·49
		— 77·3

For the Apparent and True Altitudes.

Sun's altitude.

s. alt. L.L. . . .	29° 15′ 16″
p — 4′ 24″ mi. + 16′ 13″	+ 11′ 49″
p. alt.	29° 27′ 5″ (a)
f. and par. . . .	— 1′ 35″
ue alt.	29° 25′ 30″ (A)
	90°
Coalt. ZS =	60° 34′ 30″

Moon's altitude.

Obs. alt. L.L. . . .		25° 39′ 10″
Dip	— 4′ 24″	+ 11′ 41″
Semi. + 7″ aug.	+ 16′ 5″	
App. alt.		25° 50′ 51″ (a′)
Par. and ref. . . .		+ 50′ 44″
True alt.		26° 41′ 35″ (A′)

As it is proposed to deduce the time from the sun, the coaltitude of the moon is not wanted.

For the Time at the Ship.

'o compute the time, we have the co-altitude ZS = 60° 34′ 30″, the co-latitude = 36° 30′, and the co-declination, or polar distance, PS = 76° 8′ 59″; therefore, 135, the work is as follows :—

ZS,	60° 34′ 30″		
sin PZ,	36° 30′ 0″	Arith. Comp.	·2256124
sin PS,	76° 9′ 0″	Arith. Comp.	·0128140
	2)173° 13′ 30″		
sin s,	86° 36′ 45″	. . . .	9·9992405
sin (s — ZS),	26° 2′ 15″	. . . .	9·6424232
			2)19·8800901
cos ½ P,	29° 25′ 7″	. . . .	9·9400450
	2		
P =	58° 50′ 14″	Equa. of time Feb. 12,	14m. 33s. +
	2	,, ,, 13,	14 32
	3)11,6° 10,0′ 2,8″	∴ correction	0
	3 52		
	3 20·9		
App. time at ship	3h. 55m. 21s.	∴ Mean time 4h. 9m. 54s.	
Equation of time	14m. 33s.		
Mean time at ship	4h. 9m. 54s.		

'rom this result it appears that the ship's account must be very considerably in . It may be advisable therefore, as recommended at page 136, to repeat the fore-g operation with the corrected time here determined. Whether or not this repeti- be absolutely necessary, we shall be able to ascertain after finding the mean time reenwich, as follows :—

For the time at Greenwich.

Obs. dist.	99° 26′ 40″	d,	99° 58′ 58″		
Sun's semi.	16′ 13″	a,	29° 27′ 5″	. . comp. cos	·0600
Moon's semi.	16′ 5″	a′,	25° 50′ 51″	. . comp. cos	·0457
d =	99° 58′ 58″		2)155° 16′ 54″		

½ sum,	77° 38′ 27″	. . . cos	9·3304
½ sum ∽ d,	22° 20′ 31″	. . . cos	9·9661
A,	29° 25′ 30″	. . . cos	9·9400
A′,	26° 41′ 35″	. . . cos	9·9510
A + A′,	56° 7′ 5″		
			2)39·293
			19·646
½ (A + A′),	28° 3′ 32″½	. . — cos	9·945
θ,	30° 9′ 40″	. . . sin	9·701
θ,	. . . . . .	. cos	9·936
½ D =	49° 43′ 43″	. . . sin	9·882
∴ D =	29° 27′ 26″	the true distance	
Distance at noon,	98° 38′ 0″	Prop. log. diff.	·2725 -
	49′ 26″	Prop. log. „	·5612
Mean time after noon at Greenwich	1h. 32m. 35s.	Prop. log. „	·2887
Mean time at ship	4h. 9m. 54s.		
Longitude E. in time . . .	2h. 37m. 19s.		

2h. = 30°
37m. = 9° 15′
19s. = 4′ 45″

Longitude E. 39° 19′ 45″

It appears from this result that although the ship's longitude by the dead rec
is nearly 24° in error, yet the error in the estimated time happens to be such t
supposed time at Greenwich, namely 1h. 33m., is so nearly equal to the corre
there, that the operation for the time at the ship need not be repeated with t
rected data: we may safely conclude from what is done above that the mean t
the ship at the instant of observation is 4h. 9m. 54s. p.m., and that the longit
39° 19′ 45″ east. Such a large error as the above, as well in the estimated t
in the longitude, could arise only from a long continuance of foul weather,
preventing observations of the heavenly bodies, left the ship entirely dependen
the dead reckoning. And without taking into consideration the effect of un

currents, leeway, &c.; even the deviation of the compass alone from the magnetic action of the vessel, might in a few weeks lead a ship astray to the above extent.*

Under the circumstances here imagined, it would be better to take the Greenwich time at once from the chronometer, if this can at all be depended on, as already recommended at page 140; for the longitude and time by account are employed only as subsidiary to finding an approximation to the time at Greenwich when the observations are made. If the Greenwich time, as determined from the lunar distance, differ considerably from the chronometer time, the operation for the time at the ship should be repeated with the more correct Greenwich time thus obtained, and thence the longitude accurately deduced.

2. On June 2, 1849, at 10h. 17m. p.m., mean time by estimation, in latitude 50° 51′ N., and longitude by account 41° W., the following observations were taken:

Regulus W. of meridian

	Observed altitude		Obs. alt. moon's L.L.		Obs. dist. nearest limb
	20° 21′ 40″		31° 11′ 0″		72° 36′ 30″
Index	— 3′ 50″	Index	— 4′ 10″	Index	— 9′ 10″
	20° 17′ 50″		31° 6′ 50″		72° 27′ 20″

The height of the eye was 20 feet: required the longitude

Ship time June 2 . . .	10h. 17m.
Longitude W. in time . . .	2h. 44m.
Approximate Greenwich time	13h. 1m.

Referring now to the Nautical Almanac, we take out the moon's semidiameter and horizontal parallax for midnight of June 2, the right ascension of the mean sun at noon, and the right ascension and declination of the star.

From the Nautical Almanac.

Moon's semi-diam. midnight	14′ 49″	Sun's R.A. noon .	4h. 43m. 21s.
Horizontal parallax .	54′ 23″	Cor. for 13h. . .	+ 2m. 8s.
Star's R.A. 10h. 0m. 20s.			
Decl. . . .	12° 42′ 4″N.	Sun's R.A. at 13h. .	4h. 45m. 29s.

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* In iron ships, propelled by steam, the local attraction is subject to great and frequent changes, and the compass is so powerfully acted upon by this fluctuating influence—the effect of which is often unsuspected—that the most fearful consequences sometimes result,—the true course of the vessel, being, from this cause, widely different from that indicated. At the conclusion of the present PART, we shall give a brief abstract of Dr. Scoresby's researches and suggestions in reference to this very important subject. If science fail to apply a remedy to counteract the ever-changing influence of iron steamers on the compass, this useful instrument had almost as well be abandoned in such ships. The mere heeling of the vessel will greatly affect the local deviation, and the continual strain and vibration to which a vessel under steam is subjected, so influences its magnetic condition as to render the compass a dangerous guide. The ship Tayleur, which was wrecked in the Irish Channel in Januray 1854, was led out of her proper course by these unsuspected influences. The captain relied upon his helm compass, and 290 persons perished. The original deviation in this iron steamer was so great as 60°, and this was subjected to changes that might amount to as much as between 40 and 50 degrees. It is easy to conceive that, in the open ocean, a ship, misguided in this way, and precluded from adjusting her position by astronomical observations, through a long continuance of bad weather, might err even to the extent supposed in the above example.

For the Apparent and True Altitudes.

Star's Altitude.	
Obs. alt.	20° 17′ 50″
Dip	− 4′ 24″
App. alt. . . .	20° 13′ 26″ (*a*)
Ref.	— 2′ 37″
True alt. . . .	20° 10′ 49″ (A)
	90°
Co. alt. ZS = . .	69° 49′ 11″

Moon's Altitude.		
Obs. alt. . .		31° 6′ 50″
Dip	− 4′ 24″	+ 10′ 32″
Semi. +7 Aug.	14′ 56″	
		31° 17′ 22″(*a*′)App. alt.
Par. and Ref. .		+ 44′ 54″
		32° 2′ 16″(A′)True alt.

For the Time at the Ship.

To compute the time, we have the star's co-altitude ZS = 69° 49′ 11″, its polar distance, or co-declination PS = 77° 17′ 56″, and the co-latitude PZ = 39° 10′, to find the star's hour angle P, which, since the star is west of the meridian, if added to the star's right ascension will be the right ascension of the meridian; and since the time is p.m., this right ascension diminished by that of the mean sun will be the mean time at the ship (see page 137).

ZS,	69°	49′	11″		
sin PS,	77°	17′	56″	Arith. Comp.	·0107592
sin PZ,	39°	10′	0″	Arith. Comp.	·1995728
	2)186°	17′	7″		
sin s,	93°	8′	33″	. . .	9·9993464
sin (s — ZS),	23°	19′	22″	. . .	9·5975972
					2)19·8072756
cos ½ P,	36°	46′	24″	. . .	9·9036378
			2		
P =	73°	32′	48″		
			2		
	3)14,6	6,4	9,6		
	4	52			
		2	8		
			3		

Star's hour angle in time	4h.	54m.	11s.
Star's right ascension .	10h.	0m.	20s.
R.A. of meridian . .	14	54	31
R.A. of mean sun . .	4	45	29
Mean time at ship . .	10	9	2

For the Time at Greenwich.

In computing the true lunar distance in this example, we shall adopt the recommendation at the bottom of page 158, and so modify the observed, or rather the apparent altitudes, that the seconds in each altitude may be a multiple of 10: whatever be added or subtracted in this way must of course be in like manner applied to the true altitudes. We shall also introduce a similar modification into the apparent distance, and apply the necessary correction to the true distance: the object of these changes is to reduce the work of computing for odd seconds.

Obs. dist,	72°	27′	20″
Moon's semi.		14′	56″
d =	72°	42′	16″

d,	72°	42′	20″		
a,	20°	13′	20″	. . comp. cos	·0276310
a′,	31°	17′	20″	. . comp. cos	·0682577
2)	124°	13′	0″		
½ sum,	62°	6′	30″	. . . . cos	9·6700614
½ sum ∽ d,	10°	35′	50″	. . . . cos	9·9925289
A,	20°	10′	43″	. . . . cos	9·9724907
A′,	32°	2′	14″	. . . . cos	9·9282440
A + A′,	52°	12′	57″	. . . . .	2)39·6592137
					19·8296068
½ (A + A′),	26°	6′	28″	. . . — cos	9·9532608+
θ,	48°	47′	0″	. . . . sin	9·8763460
θ,	. . .	. . .	. . .	. . . . cos	9·8188250+
½ D =	36°	16′	34″	. . . . sin	9·7720858

∴ subtracting the 4″ added to d, D = 72° 33′ 4″ the true distance.

Distance at 12h.	72°	19′	35″	. . Prop. log. diff.	·3017 —
Difference		13′	29″	. . Prop. log.	. 1·1255
Mean time after 12h. at Greenwich .	0h.	27m.	0s.	. . Prop. log.	. ·8238
∴ Greenwich mean time	12h.	27m.	0s.		
Ship mean time	10h.	9m.	2s.		
Longitude W. in time	2h.	17m.	58s.		

2h. = 30°
17m. = 4° 15′
58s. = 14′ 30″

Longitude W. 34° 29′ 30″

In each of the foregoing examples the mean time at the ship is obtained from an altitude of the sun or a star, and as remarked at page 161, it is matter of indifference when such is the case whether the time at the ship or the time at Greenwich is determined first; but as the moon's right ascension and declination change very rapidly comparatively to the sun's in a given interval, if the estimated time at ship be much in error it is plain that the elements just referred to, computed to this estimated time, may differ so much from the truth as to very sensibly affect the accuracy of the ship's time, and thence of the longitude. We would therefore recommend when, in consequence of the other object, the sun or a star, being too near the meridian for the purpose of finding from it the time, the moon's altitude must be employed, that the time at Greenwich be found before that at the ship is computed, as in the following example:—

3. On May 22, 1844, at 11h. 15m. a.m. mean time nearly, in latitude 50° 48′ N., and longitude by account 1° W., the following lunar observation was taken when the moon was E. of the meridian:—

Obs. alt. sun's L.L.		Obs. alt. moon's L.L.		Obs. distance N.L.	
	57° 53′ 0″		22° 53′ 2″		56° 26′ 6″
Index cor.	+ 35″	Index cor.	— 20″	Index cor.	— 35″
	57° 53′ 35″		22° 52′ 42″		56° 25′ 31″

The height of the eye was 24 feet: required the longitude:—

Ship time, May 21 . . .	23h. 15m.
Longitude W. in time . .	4m.
Approximate Greenwich time .	23h. 19m.

From the Nautical Almanac.

Moon's semi-diam., 21st, midnight . .	15′ 1″·3	. . Hor. par.	55′ 7″·6
22nd, noon . .	15′ 5″·7	. . . .	55′ 23″·7
	4″·4		16″·1
Correction for 41m. before noon of 22nd .	·25	. . . .	·9
Semi-diam. at 23h. 19m. May 21,	15′ 5″·4	. . . Hor. par.	55′ 22″·8

Sun's semi-diameter at noon, May 22, 15′ 49″

For the Apparent and True Altitudes.

Sun's altitude.

Obs. alt. L.L. . . .		57° 53′ 35″
Dip . .	— 4′ 49″	+ 11′ 0″
Semi .	+ 15′ 49″	
App. alt. . . . . : .		58° 4′ 35″ (*a*)
Ref. and par. . . .		— 36″
True alt.		58° 3′ 59″ (A)

As it is proposed to deduce the time from the moon, the co-altitude of the sun is not wanted.

Moon's altitude.

Obs. alt. L.L. . . .		22° 52′ 42″
Dip .	— 4′ 49″	10′ 22″
Semi	+ 15′ 11″	
Aug.	+ 5″·5	
App. alt.		23° 3′ 4″ (*a*′)
Par. and ref. . . .		+ 48′ 38″
True alt.		23° 51′ 42″ (A′)
		90°
Co-altitude . . .		66° 8′ 18″ = ZM

For the Time at Greenwich.

s. dist.	56° 25′ 31″	d, 56° 56′ 31″		
oon's semi.	15′ 11″	a, 58° 4′ 35″	. . . comp. cos	·2767183
n's semi.	15′ 49″	a', 23° 3′ 4″	. . . comp. cos	·0361386
d =	56° 56′ 31″	2)138° 4′ 10″		

½ sum, 69° 2′ 5″	. . . . . . cos	9·5536429
½ sum ∽ d, 12° 5′ 34″	. . . . . . cos	9·9902543
A, 58° 3′ 59″	. . . . . . cos	9·7234034
A′, 23° 51′ 42″	. . . . . . cos	9·9611955
A + A′, 81° 55′ 41″		2)39·5413530
		19·7706765
½ (A + A′), 40° 57′ 50″½	. . . . − cos	9·8780168+
θ, 51° 21′ 13″	. . . . . sin	9·8926597
θ, ...	cos	9·7955409+
½ D = 28° 8′ 13″½	. . . . . sin	9·6735577

∴ D = 56° 16′ 27″ the true distance.

Distance at 21h.	55° 15′ 36″	. . Prop. log. diff.	. ·3221 —
Difference .	1° 0′ 51″	. . . .	·4710

Mean time after 21h. at Greenwich, 2h. 7m. 45s. . . . ·1489
∴ Greenwich mean time . . . . 23h. 7m. 45s.

aving thus determined the time at Greenwich when the lunar distance was taken, n compute the right ascension and declination of the moon to greater precision the time by account could be expected to give. It so happens that in this par- r example the time by account is very nearly the same as that deduced from the vations; but yet there would be a sensible difference in the resulting longitude, computation of the ship time had preceded that for the Greenwich time.

is only in reference to the moon that much accuracy in the time and longitude count is of any consequence.

For the Mean Time at the Ship.

Sun's R.A.	Moon's R.A.	Moon's declination.
t . . 3h. 56m. 53s.·8	23h. . . 7h. 55m. 24s.	23h. . . 17° 5′ 12″ N.
. . . + 3m. 46s.	7m. 45s. . + 16s.	7m. 45s. . — 1′ 2″
. 45s. 1s.·3		
. = 4h. 0m. 41s.	R A. = 7h. 55m. 40s.	Dec. = 17° 4′ 10″ N.

The data for computing the moon's hour-angle with the meridian are therefor follows, namely,

ZM = 66° 8′ 18″, PZ = 39° 12′, PM = 72° 55′ 50″

ZM, 66° 8′ 18″

PZ, 39° 12′ 0″ comp. sin ·1992628

PM, 72° 55′ 50″ comp. sin ·0195650

2)178° 16′ 8″

s, 89° 8′ 4″ . . . sin 9·9999503

s — ZM, 22° 59′ 46″ . . . sin 9·5918086

2)19·8105867

½ P, 36° 28′ 46″ . . . cos 9·9052933

∴ P = 72° 57′ 32″ the moon's hour angle

2

3)14,5° 5,5′ ,4′

4h. 50m.

1m. 50s. (see page 97)

Moon's hour-angle in time	4h. 51m. 50s. east of the meridian
Moon's R.A.	7h. 55m. 40s.
R.A. of meridian	3h. 3m. 50s.
Sun's R.A.	4h. 0m. 41s.
Sun's hour-angle with meridian	0h. 56m. 51s. before noon of the 22nd
	24h.
Mean time at ship	23h. 3m. 9s. May 21st
Mean time at Greenwich	23h. 7m. 45s.
Longitude W. in time	0h. 4m. 36s. ∴ Longitude W. 1° 9′.

Although the Greenwich time by account, differs by only a few minutes from Greenwich time as determined by the observation, yet the longitude would differed by more than 4′ from the result here arrived at, if the ship time had determined before the Greenwich time. It thus appears, that it is in all cases prudent to compute the time at Greenwich before that at the ship, when the latt to be deduced from the *moon*.

Examples for Exercise.

1. On March 25, 1847, at 3h. 30m. p.m., mean time nearly, in latitude 52° and longitude by account 33° W., the following lunar observation was taken:—

Obs. alt. sun's L.L.	Obs. alt. moon's L.L.	Obs. dist. nearest limbs
23° 10′ 20″	23° 50′ 10″	112° 56′ 30″
Index cor. — 6′ 10″	Index cor. + 5′ 0″	Index cor. — 4′ 20″

eight of the eye was 20 feet, and the following particulars were furnished by utical Almanac, namely,

Sun's dec.	Equa. of time	Moon's semi.	Hor. par.	Sun's semi.
5, 1° 40′ 56″ N.	6m. 13s·8 *addit.*	noon 25th, 14′ 59″	noon 54′ 59″	16′ 3″
6, 2° 4′ 29″ N.	5m. 55s·2	midnight 14′ 55″	mid. 54′ 44″	

Also distance at 3h., 111° 33′ 34″; at 6h., 112° 57′ 16″.

ed the longitude of the ship. Ans. 32° 59′½ W.

On January 9, 1851, at 7h. 50m. p.m., mean time nearly, in latitude 49° 40′ N., gitude by account 10° E., the following lunar observation was taken :—

of Pollux E. of merid.	Obs. alt. moon's L.L.	Obs. dist. farthest limb
37° 10′ 10″	31° 50′ 10″	108° 20′ 0″
cor. — 1′ 10″	Index cor. + 1′ 20″	Index cor. + 1′ 30″

ight of the eye was 18 feet, and the following particulars were supplied by the al Almanac, namely,

Right ascen. mean sun	Moon's semi.	Hor: par.
Jan. 9, 19h. 13m. 42s·27	noon 14′ 55″·8	54′ 47″·2
	mid. 15′ 0″·2	55′ 3″·6

ar's R.A. was 7h. 36m. 12s., and its dec. 28° 22′ 46″ S. Also, the distance at s 103° 8′ 4″, and at 9h. 101° 37′ 51″. Required the longitude of the ship.

Ans. 10° 19′ 15″ E.*

examples now given will, we think, sufficiently illustrate the practical oper- for determining the longitude at sea by a lunar observation. In the Nautical ac, the lunar distances are given for the planets Mars, Venus, Jupiter, and , as well as for the fixed stars near the moon's path. The calculations for the stance are, of course, the same for a planet as for a fixed star; and, in deducing e at ship from the planet, we must proceed exactly as in deducing the time e moon; that is to say, we must find the planet's hour-angle, and thence, by of its right ascension, taken from the Nautical Almanac, we must find the scension of the meridian: the difference between this and the right ascension of an sun will—as in last example—be the sun's hour-angle with the meridian of ce, that is to say, the time at the ship.

ither of the celestial objects observed be near the horizon, where the refraction ct to considerable irregularities, the *mean* refraction, which is that generally ed, should be modified according to the state of the atmosphere, as shown by ometer and thermometer;—a table for properly correcting the mean refraction e found in every collection of nautical tables. A neglect of this correction ne of the objects observed is not more than 8 or 9 degrees above the horizon, casion an error of 1′ in the lunar distance, and this error may be sufficient to ce one of upwards of 30′ in the longitude.

ngitude from Occultations and Eclipses of Jupiter's Satellites.— as now been delivered comprehends all the essentials for finding the latitude gitude at sea. Other celestial phenomena besides those here dwelt upon, may

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r more examples see Mr. Jeans's Navigation and Nautical Astronomy, whence the above have en.

be occasionally made available for determining the longitude, as, for instance, an eclipse, or an occultation of a fixed star or planet by the moon. Eclipses are of too infrequent occurrence to be of much service to the navigator; but the passage of the moon over the stars and planets in her path is continually occurring, and this occultation of the object by the moon would furnish a very convenient means of finding the Greenwich time, and thence the longitude, if the motion of the ship did not in general preclude the possibility of keeping the telescope steadily directed to the moon's edge.

It is plain that at the instant of the occultation, that is, at the instant of the disappearance of the star or planet by the interposition of the moon—called the *immersion*—the apparent right ascension of the moon's occulting limb must be the same as the right ascension of the occulted star. By removing the effect of parallax, the moon's true right ascension at the instant of the star's immersion may therefore be found, and the Greenwich time corresponding to this right ascension may thence be deduced.

The eclipses of Jupiter's satellites answer a similar purpose, since the entrance of a satellite into the shadow of the planet is a phenomenon which takes place at the same absolute instant, wherever on the surface of the earth the immersion be observed, and so likewise does the re-appearance of the satellite, or its emersion.

The Greenwich time, when these immersions and emersions are predicted to happen, are given in the Nautical Almanac; so that if the ship time, when any such phenomenon occurs be known, the longitude may be at once obtained.

But here again, as in the case of occultations of the stars by the moon, the frequent impracticability of keeping a telescope sufficiently steady for the accurate observation of the phenomena at sea, renders this short and otherwise convenient method of finding the longitude of but very limited application. In a calm sea, or in harbour, a telescope of sufficient magnifying power may, of course, be used without inconvenience. Of the four satellites, the *first*, or that which is at the least distance from the planet, is the best adapted for the purpose of determining the longitude, on account of its more rapid motion; it revolves round Jupiter, and is eclipsed by the shadow of the planet once in every forty-two hours: and the instants of immersion and emersion are capable in general of being much more accurately noted than the instants of contact of the earth's shadow with the moon's limb.

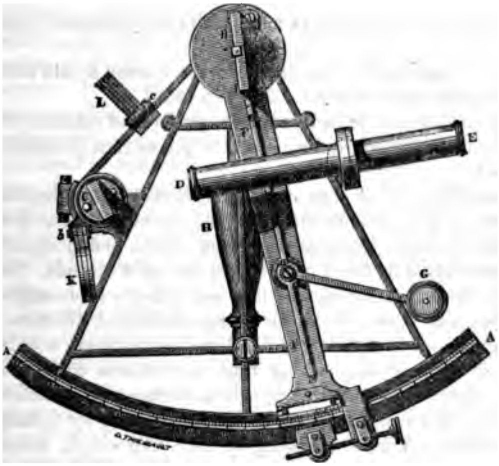
The Quadrant and Sextant.—These two instruments are the same in principle,—both are equally employed to measure angular distances; but as the distance between a celestial object and the horizon, for the purpose of determining the latitude at sea, is a measurement more frequently made than any other, the former of the two above-mentioned instruments—the quadrant—is constructed with exclusive reference to this purpose, and, being less elaborate in its fittings and workmanship, is by far the cheaper instrument of the two.

The arc of the sea quadrant is the eighth part of an entire circumference, or 45°. This arc is therefore, strictly speaking, not a quadrant but an *octant;* but as it is capable of measuring all altitudes from the horizon to the zenith,—as will presently be explained,—a greater extent of arc is unnecessary.

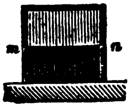
The sextant is a more delicate instrument. Its arc is the sixth part of an entire circumference, or 60°, and it is capable of measuring angular distances up to 120°. The arc of the more common quadrants is divided—and that by means of an auxiliary scale attached to the index-limb—into minutes only: those of a superior kind are thus divided into half minutes; but the arc of a sextant is frequently sub-divided—by aid of the Vernier scale just alluded to, and hereafter explained—to every 10."*

Artists generally extend the arc of a quadrant to a few degrees beyond 45°, and the arc of a sextant to a few degrees beyond 60°. With either of these instruments an altitude may be taken; but usually with more precision with the sextant than with the quadrant, on account of the more minute subdivisions of the arc. The manner of holding the instrument in taking an altitude is figured in the margin.

The sextant, however, is almost exclusively used for taking a lunar distance. All the essential parts of this valuable instrument are represented in the accompanying figure. H is the handle by which the sextant is held while taking the angular distance; DE is a small telescope fixed to the frame of the instrument, and directed towards the plane reflector or mirror, C. This reflector is immoveable; the other plane reflector or speculum, B, is fixed to the moveable radius, or moveable limb, F, at the extremity of which is the index for pointing out the angle measured on the graduated limb, A A.



The two reflectors B, C are perpendicular to the plane of the instrument,—that is, to the plane in which the arc, or graduated limb, lies. The immoveable reflector C is called *the horizon glass;* the upper half of it is transparent and unsilvered, and it is through this part that the horizon is viewed in taking an altitude; the lower half only, strictly speaking, is a reflector, being the only part coated with quicksilver. The moveable reflector B is called *the index glass;* it turns with the radius, or limb carrying the index, round the centre of the graduated limb.



Suppose this radius to be turned into such a position that the planes of the two reflectors B, C may be exactly parallel to one another: in this position of the index-limb, the point on the graduated arc shown by the index is to be marked 0. It is the perfection of the instrument that, when the index points to 0, the two reflectors should be accurately parallel to one another. And here we pause for a moment to explain the cause of what the learner may have hitherto considered as the result of indifferent workmanship in the instruments employed in the preceding problems of Nautical Astronomy:—we have repeatedly spoken of the *index error*, or the *error of the instrument*.

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* Subdivisions of less extent than this must be estimated by the eye. A skillful observer can usually make this estimate within two or three seconds of the truth. Such an error, in the measurement of a lunar distance, would not occasion an error of more than about a mile in the resulting longitude. (See page 158)

These terms do not imply faulty workmanship; all instruments whatever—the most elaborately-finished specimens in the most richly-furnished observatories—have, without exception, their *instrumental errors*, which are very different things, however, from errors of workmanship.

In the mathematical volume of the Circle of the Sciences (Commentary on Euclid) the reader has been frequently reminded that it is beyond the reach of art to draw a line of a prescribed length *accurately*, to raise a perpendicular, or to draw a pair of lines that shall be accurately parallel. The perfect perpendicularity and parallelism, therefore, of the reflectors B, C are things that cannot be practically brought about; some minute departure from strict geometrical precision always exists, and hence what is called "the index error." How this index error may be discovered and allowed for, will be seen presently; it is always furnished to the purchaser by the maker of the instrument.

In reference to this subject, it may not be unprofitable to the learner to add to the present digression the following observations of Sir John Herschel:

"Astronomical instrument making may be justly regarded as the most refined of the mechanical arts, and that in which the nearest approach to geometrical precision is required, and has been attained.

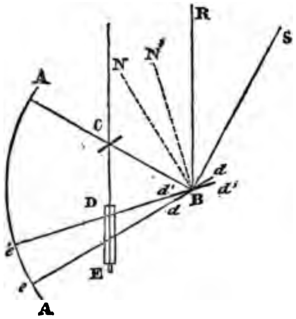
"It may be thought an easy thing, by one unacquainted with the niceties required, to turn a circle in metal, to divide its circumference into 360 equal parts, and these again into smaller subdivisions,—to place it accurately on its centre, and to adjust it in a given position; but, practically, it is found to be one of the most difficult. Nor will this appear extraordinary when it is considered that, owing to the application of telescopes to the purposes of angular measurement, every imperfection of structure or division becomes magnified by the whole optical power of that instrument, and that thus not only direct errors of workmanship, arising from unsteadiness of hand or imperfection of tools, but those inaccuracies which originate in far more uncontrollable causes, such as the unequal expansion and contraction of metallic masses, by a change of temperature, and their unavoidable flexure or bending by their own weight, become perceptible and measurable.

"An angle of one minute occupies, on the circumference of a circle of ten inches in radius, only about $\frac{1}{350}$th part of an inch,—a quantity too small to be *certainly* dealt with without the use of magnifying glasses; yet one minute is a gross quantity in the astronomical measurement of an angle. With the instruments now employed in observatories, a single second, or the 60th part of a minute, is rendered a distinctly visible and appreciable quantity. Now the arc of a circle, subtended by one second, is less than the 200,000th part of the radius; so that on a circle of six feet in diameter it would occupy no greater linear extent than $\frac{1}{5700}$th part of an inch,—a quantity requiring a powerful microscope to be *discerned* at all.

"Let any one figure to himself, therefore, the difficulty of placing on the circumference of a metallic circle of such dimensions (supposing the difficulty of its construction surmounted), 360 marks, dots, or cognizable divisions, which shall be true to their places within such minute limits, to say nothing of the subdivision of the degrees so marked off into minutes, and of these again into seconds. Such a work has probably baffled, and will probably for ever continue to baffle the utmost stretch of human skill and industry; nor, if executed, could it endure. The ever varying fluctuations of heat and cold have a tendency to produce not merely temporary and transient, but permanent, uncompensated changes of form in all considerable masses of those metals which alone are applicable to such uses; and their own weight, however symmetrically formed, must always be unequally sustained, since it is impossible to apply the sustaining power to *every part* separately; even could this be done, at all events force must be used to move and to fix them, which can never be done without producing temporary, and risking permanent, change of form. It is true, by dividing them on their centres, and in the identical places they are destined to occupy, and by a thousand ingenious and delicate contrivances, wonders have been accomplished in this department of art, and a degree of perfection has been given, not merely to *chefs d'œuvre*, but to instruments of moderate prices and dimensions, and in ordinary use, which, on due consideration, must appear very surprising. But though we are entitled to look for *wonders* at the hands of scientific artists, we are not to expect *miracles*. The demands of the astronomer will always surpass the power of the artist; and it must therefore, be constantly the aim of the former to make himself, as far as possible, independent of the imperfections incident to every work the latter can place in his hands. He must, therefore, endeavour so to combine his observations, so to choose his opportunities, and so to familiarize himself with all the causes which may produce instrumental derangement, and with all the peculiarities of structure and material of each instrument he possesses, as not to allow himself to be misled by their errors, but to extract from their indications, as far as possible, all that is *true*, and reject all that is erroneous. It is in this that the art of the practical astronomer consists,—an art of itself of a curious and intricate nature, and of which we can here only notice some of the leading and general features." (See Herschel's Astronomy, Lardner's Cyclopædia.)

Returning now to the sextant:—To understand the way in which an angular distance is measured by this instrument, we must first assent to the following simple optical principle, namely, when a ray of light from a luminous body falls upon a reflecting surface, and is thence received by the eye, the *incident* ray—or that direct from the object to the reflector—makes, with the perpendicular to the surface of the reflector, drawn from the point where it impinges on it, an angle equal to the angle made with the same perpendicular and the *reflected* ray, or that received by the eye. This property is briefly expressed thus:—the angle of incidence is equal to the angle of reflection. It is the same with an elastic sphere striking a smooth hard surface, as, for instance, a common marble shot against a smooth wall; if the marble be shot perpendicularly, it will rebound along the same path, and return to the hand, the angle of incidence and the angle of reflection being *nothing;* but if the marble be projected obliquely, it will rebound on the other side of the perpendicular, and the oblique incident path and the oblique reflected path will make equal angles with that perpendicular.

This principle being admitted, conceive the limb F to be moved so that the attached index points to zero on the graduated arc (the point A on the right in the foregoing figure). In this position of the index and of the reflector B, if the eye at E looks through the upper or unsilvered part of the horizon glass at C, and perceives a celestial object, such as a star, it will at the same time also perceive the *image* of that star reflected from the silvered part of C. For as the reflectors are, by hypothesis, parallel, and the star so distant that two rays from it falling, the one on the glass B and the other on C, must differ only insensibly from parallelism, it follows that the ray from the star, reflected at B and thence proceeding to C, from which it is again reflected at E, must proceed to the eye E in the same direction as the direct ray from the star through the unsilvered part of C. But this will be better illustrated by a distinct diagram.



Let *d d* be the position of the index glass when the index points to zero, which in the diagram is marked by the letter *e*. By hypothesis the surface of the half-silvered glass C is parallel to that of *dd*. An object, R, so remote as one of the heavenly bodies, would be equally seen to an eye at E in the direction of E D, parallel to B R, as to an eye at B, in the direction B R. If the dotted line B N be perpendicular to *d d*, R B N will be the angle of incidence, and C B N the angle of reflection: the reflected ray B C is now incident on C,—a surface *parallel* to *d d;* therefore the reflected ray C D must be parallel to R B, so that an eye at E will see the object directly through the transparent part of C, and the image of it, after two reflections, in the same direction; or rather the object and its image would become confounded and superposed. Suppose now, while the eye is still looking at the object R through the telescope E D, the index-limb be moved from *e* to *e'*; the reflector *d d* turning round with this limb will take the new position *d' d'*, and the *image* of R will disappear, and that of some other object S, in reference to a ray from which, B C will still be the reflection, will take its place.

In this way two luminous points, R, S, as, for instance, two stars, or a star and an edge of the moon, or the edges of the sun and moon, may be brought together; one of the two, R, being seen by direct vision, the other, S, after two reflections at the mirrors.

The movement of the index from *e* to *e'*, necessary to bring the two objects R, S into contact, moves the reflector *dd* into the position *d' d'*, and the perpendicular BN into the position BN'. Now BN, by the above-mentioned optical principle, bisects the angle RBC, and BN', in like manner, bisects the angle SBC. Hence

$$NBN' = \tfrac{1}{2}\,(SBC - RBC) = \tfrac{1}{2}\,SBR$$

And since the angles NBN', *eBe'*, measured by the arc *ee'*, are obviously equal, it follows that the arc *ee'* measures half the angle SBR, formed by the two incident rays SB, RB, that is to say, it measures half the angular distance of the two objects S, R.

If, therefore, the arc AA, supposed to be 60°, be divided into twice that number, that is, 120 equal parts, then, by considering each part as a whole degree, the index at *e'* will show the number of degrees in the angular distance of S and R. In like manner, the degrees and minutes will be shown if each division be subdivided into 60 equal parts. This is the important principle in the construction of the sextant.

Besides the two reflectors, B and C, in the figure at page 173, several other glasses,—called *screens* or *shades*,—are attached to the framework of the instrument, as shown at K and L. These are merely stained or coloured glasses to be interposed in the path of the rays from luminous objects, as the sun, and sometimes the moon, to reduce the intensity of the glare, which might be too strong for the sight, and, in the case of the sun, could not be endured with a clear sky. The principal use of these glasses, therefore, is to screen the eye; but they also serve to distinguish the object from its image by difference of colour. The moveable glass, G, is a microscope supported by a slip of metal turning about its extremity *a*, so as to allow of its being brought over the divisions of the *Vernier*,—a small and important scale attached to the index, for the purpose of marking subdivisions too minute to be engraved on the circular limb of the instrument. We shall explain the Vernier presently.

To use the Instrument.—The plane of the quadrant or sextant must always be held in the plane of the two objects, of which the angular distance is to be taken; it is grasped by the handle H, usually with the right hand (see the figure at page 173), the other hand being employed in moving and adjusting the index,—when, as in common quadrants, there is no handle, the instrument is held by the frame-work itself. If the sun's altitude is to be taken, the instrument is to be held in a vertical position, the index set to 0 on the limb, and the eye applied to the telescope or—removing this—to the sight-vane, which supplies its place, and directed through the horizon-glass to that part of the horizon which is vertically under the sun. The index is now to be moved forward till the image of the sun, which we shall see to be gradually descending, till the limb just touches the horizon: the observed altitude of that limb will thus be obtained.

The sight-vane may now be turned down and the telescope introduced, which, by magnifying the image, will render the contact more distinct. It is, in general, more easy to get a contact, though with less precision, without the telescope than with it, as the telescope greatly limits the field of view; but after the index is adjusted to the approximate contact, the telescope, previously set to distinct vision, will at once show the object more clearly defined, and give the contact more accurately,—of course, whatever shades may be necessary to protect the eye and to distinguish the object from the image are to be put down.

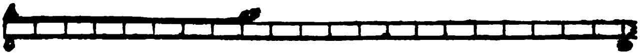
If the altitude of a star is to be taken the operation is just the same, care being taken to keep the star's image in view during the whole of its descent to the horizon, to avoid the mistake of bringing down the wrong star. The moon's altitude is taken in the same way as the sun's, using such shades as may be found necessary.

When a lunar distance is to be observed, the plane of the sextant must be held so that both objects may lie in that plane, and the sight is to be directed, through the horizon-glass, to the fainter of the two, so that when the brighter object is to the left, the instrument must be held face downwards.

The practical management of the instrument, in making observations at sea, can be efficiently acquired only on ship-board; the movements of the body must be accommodated to the motion of the vessel, and peculiar attitudes and positions will be necessary in peculiar circumstances: the observer sometimes stands erect, sometimes reclines against a support, and sometimes lies on his back on the deck, when taking a lunar distance. It is plain that nothing but experience can dictate to him the best way of handling his instrument on the various occasions that may require its use. Supposing the observation to have been made, it remains to *read off* the angular measure; this is done by aid of the *Vernier*, a contrivance so called from the name of its inventor.

The Vernier.—This is a small scale attached to the index-limb, F, of the instrument; it is slightly inclined to the face of the divided limb AA, and moves, with the index-limb, in close contact with the divided arc AA. It is attached to many other scales—as, for instance, to those of the barometer and thermometer—as well as to the scales of the quadrant and sextant, and is, in fact, an appendage to many astronomical instruments used for angular measurement; its object and utility may be explained as follows:—

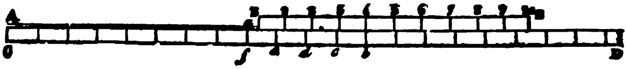
Let CD represent any graduated scale, and AB a line which we wish to measure by it; the scale and line must of course be of the same character—both straight or both circular. If, upon applying the scale to the line, as in the following figure, we



find the extremity B to fall accurately upon one of the divisions of the scale, we, of course, obtain the measure without any fractional parts of a division: we may, for illustration, call the divisions degrees, and we shall conclude the measure to be so many degrees exactly.

But if, as would be most likely, the extremity, B, of the line projects beyond the boundary of a division without reaching the next, the length would be so many degrees and some fractional parts of a degree, which the scale affords us no efficient means of measuring. In the figure, the measure of AB is eight degrees, with a fractional part, *a*B, of the ninth degree, the exact amount of which can only be guessed at. The object of the Vernier is to make known the value of this fractional part.

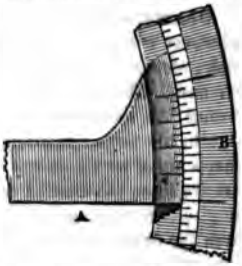
Imagine a second scale, BE, with its commencement placed in contact with the extremity, B, of the proposed line, to be applied to the scale CD; suppose the whole



length of this second scale to be 8 degrees, but that it is divided, as in the figure, into 9 equal parts, and let us assume, moreover, that *a*B may be expressed in tenths of degrees.

Among the divisional marks of the second scale BE, thus placed, there will necessarily be found one which exactly corresponds to a divisional mark on the first scale, CD; in the figure above it is the *fourth* mark, and we accordingly conclude that *a*B measures four-tenths of a degree, so that the whole measure of AB is 8$\frac{2}{5}$. That such is the case will be readily seen from considering that one of the divisions of BE is only $\frac{9}{10}$ths of one of the divisions of CD, so that one of the latter divisions exceeds one of the former by $\frac{1}{10}$th of a degree, two of the latter exceed two of the former by $\frac{2}{10}$ths, and so on.

Now from 4 to B, on BE, there are four divisions, and from *b* to *f*, on CD, there are also four divisions; the latter four, in their whole length, exceed the former four by $\frac{4}{10}$ths of a degree; but this excess is the length *a*B, consequently *a*B = $\frac{4}{10}$ths. It follows, therefore, that if *a*B be only an exact number of tenths of a degree, we shall be able to measure those tenths by this contrivance; and the error of measurement, if *a*B be not an exact number of tenths, and therefore the mark 4 not strictly the continuation of the mark *b*, must be less than $\frac{1}{10}$th. In like manner, if the scale BE had *the length of* 19 degrees of CD, and were divided into 20 equal parts, *a*B could have been measured accurately to within $\frac{1}{20}$th of a degree, and so on. The scale BE is the *Vernier*.



The annexed figure will give an idea how the Vernier attached to the index-limb of the quadrant or sextant adapts itself to the circular graduated limb of the instrument. The point marked *a* on the Vernier is the *index* of the graduated arc of the limb, and is that which marks out the integral part of the measure of the angle, the fractional part being indicated by the Vernier divisions, as already explained. It is the mark *a* which ought to correspond with the mark 0 on the graduated limb, when the index and horizon glasses of the instrument are parallel: it is common, however, to speak of the whole moveable limb A as the *index*.

Suppose each of the divisions on the graduated limb to denote n minutes, and let m be the number of those divisions which make up the whole extent of the Vernier scale, then the Vernier will contain $m\,n$ of the minutes of the graduated limb. If this extent be divided into $m + 1$ equal parts, then the difference between one division on the graduated limb and one division on the Vernier will be

$$n - \frac{m\,n}{m+1} = \frac{n}{m+1}$$

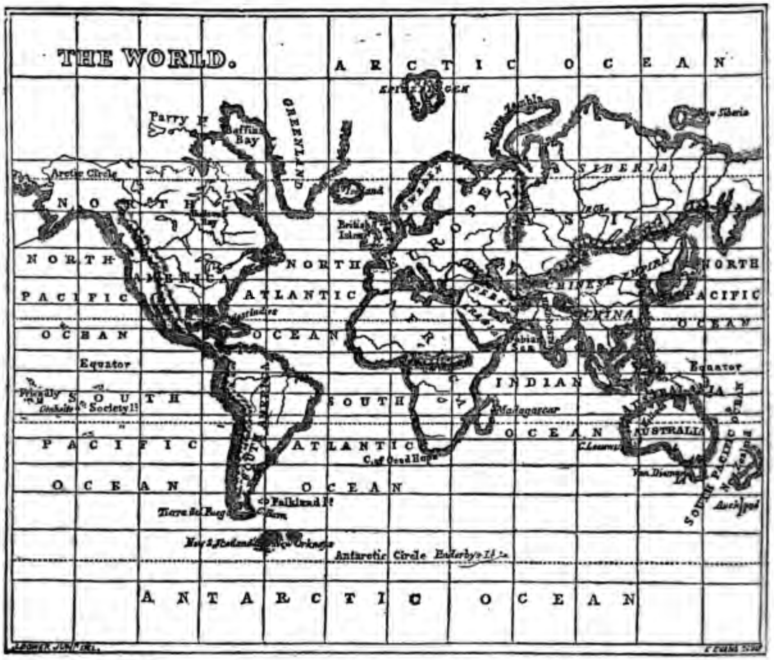
If $n = 20'$, and $m = 19'$, $\therefore \frac{n}{m+1} = 1'$. If $n = 10'$, and $m = 59'$, then $\frac{n}{m+1} = 10''$, &c.

The Index Error.—If when the index and horizon glasses are parallel, the beginning *a* of the divisions on the Vernier, does not coincide with the mark 0 on the graduated limb, the distance between them is the *index error*—subtractive when the Vernier mark is to the left of the 0, and additive when it is to the right. To discover the amount of index error, move the index till a point of the horizon, or some more distant object, coincides with its image: the distance of the index mark *a*, from 0 on the graduated limb, is the amount of index error.

The sextant just described is only a modification of the quadrant; both instruments are in principle the same. The first published description of the quadrant appeared in the Philosophical Transactions, No. 420, for the year 1738, the paper communicating it having been laid before the Society in May, 1731. It was the production of John Hadley, and the instrument he described was consequently called Hadley's Quadrant; there is little reason to suspect that the invention was not his own. But a similar instrument had long previously been invented by Newton, and communicated to Halley, who kept the contrivance to himself. "The description of the instrument was found, after the death of Halley, among his papers, in Newton's own handwriting, by his executor, who communicated his papers to the Royal Society, twenty-five years after Newton's death, and eleven after the publication of Hadley's invention, which might be, and probably was, independent of any knowledge of Newton's, though Hutton insinuates the contrary." "But," adds Sir John Herschel, "the priority of invention belongs undoubtedly to Newton, whose claims to the gratitude of the navigator are thus doubled, by *his* having furnished at once the only theory by which his vessel can be securely guided, and the only instrument which has ever been found to avail in applying that theory to its nautical uses."—(Astronomy, p. 102.)

Charts.—A chart is a map upon a plane surface of a portion of the sea, including whatever within its limits it may be useful to the mariner to have exhibited, such as rocks, shoals, &c., and in some the directions of currents, and the variations of the compass. Charts are of two kinds—the Plane Chart and Mercator's Chart. As in the former the meridians are all parallel lines, and the degrees of latitude all of equal length, the proper relations of latitude and longitude are grossly violated, and such charts are of no use except for mere coasting purposes. At sea, the only charts of any use are those constructed on Mercator's principles, described at page 60, the degrees of longitude remaining invariable, while those of latitude are enlarged more and more from the equator to the pole, agreeably to the law investigated and explained at the page referred to.

The following is a representation of the globe projected upon Mercator's plan, but on a scale far too minute, of course, to serve any other purpose than mere illustration, for which alone it is here introduced. On the actual sea chart, representations of the compass are placed at intervals, for the purpose of setting off courses.



The meridians which limit the chart employed at sea are graduated, as also the parallel of latitude which bounds the extent of the chart north or south. A point on the chart being marked, we find its latitude by simply observing where the edge of a straight ruler applied to the point, and held parallel to any parallel of latitude, cuts the graduated meridian; and the longitude is found by placing the edge, while on the point parallel to any of the meridians, and observing where the edge cuts the graduated parallel.

To find the course between two places on the chart, apply the edge of a parallel ruler to the two places, and, holding it there, move the other part of the ruler till an edge passes through the centre of one of the compasses: the course will thus be indicated.

To lay down on the chart the position of the ship by dead reckoning—that is, from her course and distance from a given point of departure, as the preceding noon, place the ruler on the point of departure, and parallel to the given course: from the graduated meridian at the side of the chart, and in the latitude of the ship, take the distance, in degrees, &c., in the compasses: this distance applied from the point of departure along the edge of the ruler in the direction of the course will mark out the position of the ship by dead reckoning. To lay down the position as given by the latitude and longitude is sufficiently obvious: the intersection of two pencil lines, through the given points of latitude and longitude, and parallel to the boundaries of the chart, will be the position sought.

It will be perceived by the reader who has gone over what has been delivered in the Navigation respecting Mercator's sailing, that although positions are correctly exhibited on the chart, as regards latitude, longitude, and course, yet *distances* are exaggerated (see page 61). Distances which are the same on the globe become more and more elongated on the chart as we approach the pole.

The Ship's Journal.—A Sea Journal is a record of the daily transactions and occurrences in connexion with the navigation of the ship, including whatever observations and remarks that may be necessary to give a brief but connected professional history of the voyage.

The entries in the Journal are made hourly after the departure is taken: the ship shapes her course towards a definite point, and to do this either a chart is consulted or the angle determined by computation; allowance then being made for the variation of the compass, and the local deviation, the compass or steering course for the first stage is obtained. The ship, however, is usually considered to depart from the point of land or other conspicuous object last seen, and the bearing opposite to that of this point of departure is regarded as the first course, and the distance of it as the first distance. The ship is not, however, considered as having fairly commenced her voyage till her final departure has been taken.

Time is generally recorded as in the affairs of civil life, and not according to astronomical reckoning; noon and midnight equally divide the twenty-four hours as on land. From the hourly registry of the course and distance, the leeway and the variation of the compass being properly allowed for, the ship's position is determined every day at *noon*. If no astronomical observations have been made, the position thus determined is the place of the ship by *dead reckoning;* but if the latitude or longitude, one or both, have been computed from observations, a distinct entry to that effect is made, although the day's account by dead reckoning is still preserved.

This clearing up the ship's account every day at noon, so as to enable her to take a fresh departure daily *at that hour from* a known position, just as she took her departure at first, is called a *day's work*. As noon is the time invariably fixed upon for ascertaining the resultant of all the preceding twenty-four hours' sailings, whatever latitude or longitude may have been determined by astronomical observations in the interval is brought up to that hour by help of the dead reckoning. Hence the entries "Latitude by Observation" and "Longitude by Observation," frequently inserted in the noon results, are in general made up in some small part of the latitude and longitude by account.

In keeping the ship's reckoning, the position departed from at each noon is considered to be that which nautical astronomy assigns; so that when observations have been made in the interim, the dead reckoning commences afresh, and is not a continuation of the yesterday's account. The record is then carried uninterruptedly on till a noon arrives, immediately before which the true position of the ship has again been settled by observations, and so on. The meridian observation for latitude is of course recorded for the noon on which it is made.

The working of a day's work may always be effected by the Traverse Table, after the manner shown at page 48; and as the twenty-four hours' sailings should be regarded only as furnishing data for finding the position of the ship at the end of that time approximately, it is not considered as, in general, necessary to attend to minutes in the courses. "It is mere waste of time," observes Lieut. Raper, "to work the course nearer than to the whole degree; for even if the compass could be depended upon, as it cannot be, to 1°, the ship cannot be steered to twice that quantity." We shall now give a very short specimen of a Ship's Journal; there is no settled and uniform plan of arranging all the entries; but there are certain prominent features in which all sea journals are alike. The specimen here offered is, with some slight modification, extracted from that given by Mr. Riddle in the work before referred to.

The reader is already aware that the principal entries in the Journal are, in the first instance, written in chalk on the black board called the *log-board*, from which they are afterwards transferred to the *log-book*. The correction of the several courses for leeway and variation being made, and the position of the ship at noon, as deduced from the sailings, and as determined by observation when observations are taken, completes the day's work, and renders the log-book a *journal*.

From what has already been said as to the unavoidable imperfections in even the most careful measurements of a ship's course and distance, the difficulty of making the proper allowances for the leeway, &c., and of estimating the effects of squalls, currents, &c., it will readily be inferred that a ship's journal would soon become so erroneous, as a registry of actual facts, as to be quite valueless, unless repeatedly rectified by astronomical observations. In the absence of these, such a journal would describe little other than the imaginary route of an imaginary ship, which, after a voyage of any length, might terminate at almost any point of the globe as likely as at the point reached by the real ship. No opportunity, therefore, should be lost to check this increasing tendency to error in the dead reckoning, by carefully determining the position of the ship from the safe principles of nautical astronomy; and the ship's account should be regarded as of value only in so far as it can be made auxiliary to the application of those principles.

The letters H, K, F, at the heads of the first three columns in the following specimen of a journal, stand for hours, knots, and fathoms, or tenths of a nautical mile. The entries between noon and midnight are marked P.M. (*post meridiem*), and those between midnight and noon are marked A.M. (*ante meridiem*).

The result of each day's work is inserted at the bottom of the page; the courses being corrected for leeway and variation, the result exhibits the true course and distance from the point of departure; together with the diff. lat. and diff. long. made in the twenty-four hours, or rather the latitude and longitude reached.

Extract from a Journal of a Voyage from St. Michael towards England.

H	K	F	Courses.	Winds.	Lee-way.	Remarks Sunday, Sept. 11, 18—
1				W.		Moderate and clear weather. P.M.
2						At 3 p.m. the eastern end of St. Michael's, lat. 37° 48′ N., long. 25° 13′ W., bore W.S.W., dist. 6 leagues, from which I take my departure.
3				N. N.W.		
4	5	8	E.N.E. ½ E.			Ditto weather.
5	5	7		N.		Fresh breezes and cloudy weather. In top gallant sails.
6	5	4				Ditto weather.
7	4	2	E. *b* N.	N. *b* E.	½	In second reef topsails, hauled down jib, and set fore topmast staysail.
8	4	1				Strong breezes and hazy weather; brailed up spanker, and set mizen staysail.
9	3	0	E. *b* S.	N.E. *b* N.	2	
10	3	0				In third reef topsail, and set trysail.
11	2	8				
mid-night	2	6				Ditto weather.
						Monday, Sept. 12. A.M.
1	3	4	E. S.E.	N.E.	3½	Fresh gales and hazy weather.
2	3	6			3	More moderate.
3	3	9				
4	4	2				Out fourth reef of topsails.
5	4	3	N. N.E.	E. *b* S.	2¼	Tacked; strong breezes and cloudy weather.
6	4	0			2	Out third reef topsail; set jib and spanker.
7	4	5				Set top gal. sails. Fresh breezes and cloudy weather.
8	5	0			1½	
9	4	7				Out second reef top sails.
10	4	2				Lat. by doub. alts. at 11 a.m. 38° 15′ N.
						Diff. lat. up to noon . . 5′ N.
						Latitude at noon 38° 20′ N.
11	4	8				Moderate and clear weather.
noon	4	8				Variation by amplitude 1¼ points W.

Course.	Dist.	Lat. acct.	Lat. obs.	Long. acct.	Long. obs.	Bearing and dist. of Lizard at noon.
N. 58°E.	57m.	38° 19′ N.	38° 20′ N.	24° 11′ W.		N. 49°½ E. Dist. 1074m.

Extract from a Journal of a Voyage from St. Michael towards England.

H	K	F	Courses.	Winds.	Lee-way.	Remarks Monday, Sept. 12, 18—.
1	4	6	N. N.E.	E.	1¼	Moderate and clear weather. P.M.
2	5	0				Out first reef topsails, set royals, and flying jib.
3	5	3	N. *b* E.¾E.	E. ¼ N.	½	Light breezes and clear weather.
4	5	6				Ditto weather. Swell from E. from 4 p.m. till 8, for which allow a drift of 24 miles.
5	6	0				
6	6	1				In royals and flying jib.
7	5	8	E. S.E.	N.E.	1	Tacked.
8	5	7				
9	5	0			½	Ditto weather.
10	5	3				
11	5	8			0	
mid-night	6	2				Ditto weather.
						Tuesday, Sept. 13th. A.M.
1	5	9	E. S.E.	N.E.	1¼	Moderate and clear weather.
2	5	7				
3	5	3			1	Fresh breezes. In topgallant sail.
4	5	4				
5	5	0	E. N.E.	N.	1½	In first reef topsails.
6	5	0				
7	4	8				Strong breezes and cloudy.
8	4	3				In second reef topsails.
9	3	9			2	Long. by chron. at 8 a.m. 23° 2′ W. Diff. long. up to noon . 17′ E. Long. at noon . . 22° 45′ W.
10	3	4				Flying clouds, with light showers.
11	3	3			2½	Fresh gales and squally: down jib and in spanker.
noon	3	5				Lat. at noon by mer. alt. 38° 46′ N. Var. 20° W. by azimuth.

Course.	Dist.	Lat. acc.	Lat. obs.	Long. acct.	Long obs.	Bearing and dist. at noon, Lizard.
N. 69° E.	63m.	38° 43′ N.	38° 46′ N.	22° 56′ W.	22° 45′ W.	N. 48°¼ E. Dist. 1013m.

In reference to the two preceding days' works, it will be observed that as the variation of the compass is westerly, it must be allowed to the left of the compass courses; and, therefore, when the ship makes leeway on the larboard tack, the difference between the leeway and variation is the correction to be applied to the course,—to the *left*, if the variation is the greater, but to the *right* if the leeway is the greater.

When the ship makes leeway on the starboard tack, the allowance for it, as well as that for variation, being to the left, their sum will be the correction to be applied to the compass course; and when no leeway is made, the only correction is for the variation. Now E. N. E., the opposite point to the bearing of the land from which the departure is taken, is the first course; and this, and the drift, being corrected for variation, and the other courses for both variation and leeway (when there is any), and the distances on each of these courses added up, we have the following traverse table for the first day's work:

TRAVERSE TABLE.

Courses.	Dist.	Diff. Lat. N.	Diff. Lat. S.	Departure. E.	Departure. W.
N.E. ¼ E.	18	12·1		13·3	
N.E. ¾ E.	16·9	10·1		13·6	
N.E. *b*. E. ¾ E.	8·3	3·5		7·5	
E.S.E. ¾ E.	11·4		2·8	11·1	
S.E. ¼ E.	7 0		4·7	5·2	
S.E. ¾ E.	8·1		4·8	6·5	
N.N.W.	4·3	4·0			1·6
N. *b*. W. ¾ W.	8·5	8·0			2·9
N. *b*. W. ¼ W.	9·7	9·4			2.4
N. ¾ W.	4·2	4·2			·6
N. ¼ W.	9·6	9·6			·5
S. ¼ W.	18·0		18·0		·9
		60·9	30·3	57·2	8·9
		30·3		8·9	
Course, N. 58° E.					
Distance, 57 miles.		30·6		48·3	

Lat. left . .	37° 48′	Mer. parts. . .	2453
Diff. lat. . .	31′		
Lat. in. . .	38° 19′	. . . .	2492
		M. diff. . . .	39

Hence, by Mercator's sailing, page 64, the difference of longitude is 1° 2′ E.:—

Long. left. . . . 25° 13′ W.
Diff. long. . . . 1° 2′ E.

24° 11′ W.

Therefore the ship's place by account is

Lat. 38° 19′ N. Long. 24° 11′ W.

From 11 A. M. till noon the ship's true course was N. ¼ W., 5 miles nearly: hence the difference of latitude also is 5 miles nearly; and this added to the latitude, as determined by double altitudes, at 11 A. M. gives 38° 20′ N. for the latitude by observation at noon.

With this latitude, and the longitude by account, the bearing and distance of the Lizard are found to be N. 49° ½ E., 1074 miles.

The courses for the second day's work, being first corrected for leeway only, as the variation is given degrees, we have the following traverse table:—

TRAVERSE TABLE.

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
N. ¾ E.	9·6	9·5		1·4	
N. *b*. E. ¼ E.	23·0	22·3		5·6	
S.E. *b*. E.	11·5		6·4	9·6	
S.E. *b*. E. ½ E.	10·3		4·8	9·1	
E.S.E.	12·0		4·6	11·1	
S.E. ¾ E.	11·6		6·9	9·3	
S.E. *b*. E.	15·7		8·7	13·0	
E. ½ N.	14·1	1·4		14·0	
E.	7·3			7·3	
E. ½ S.	6·8		·7	6·8	
W. (swell).	24·0				24
		33·2	32·1	87·2	24
		32·1		24·0	
Compass Co., N. 89° E.					
Distance, 63 miles.		1·1		63·2	

Compass course	N. 89° E.
Variation	20° W.
True course	N. 69° E.
Distance	63 miles.

With this course and distance 68 miles, found from the Traverse Table, the diff. lat. and departure are found to be 22·8 N. and 58·8 E.

Lat. left	38° 20′ N.
Diff. lat.	23′ N.
Lat. by account	38° 43′ N.
Long. left	24° 11′ W.
Diff. long.	1° 15′ E.
Long. by account	22° 56′ W.

The departure made, from 9 A. M. till noon, is nearly 14 miles, with which, and the mid-latitude, about 38½°, the difference of longitude is found to be 17 miles; which, taken from 23° 2′, the longitude by the chronometer at 9 A. M., gives 22° 45′ for the longitude by the chronometer at noon.

Conclusion.—In here terminating our treatise on the Principles of Navigation and Nautical Astronomy, we would remind the reader of the objects proposed in undertaking it, as sufficiently declared in the INTRODUCTION. This object was two-fold: first, to furnish to the mathematical student a convincing proof of the great practical value of the abstract sciences which he cultivates; and, second, to supply the professional navigator with the theoretical principles on which his rules of operation are founded. It did not enter into our plan to go into all the practical details of navigating a ship, nor to dwell upon those facilitating expedients which could be rendered intelligible only by the aid of extensive nautical tables. The present work is offered to the notice of the mariner more in the character of a companion or supplement to the books of rules and tables in every-day use at sea, and as serving to show him the scientific theory on which his practice is based, But as far as could be done, without special tables, we have fully gone into the calculations necessary for determining the position of a ship on the ocean; and have shown that all the practical demands of navigation may be amply satisfied by help of only the common logarithmic tables, and that knowledge of turning them to account which the elementary principles of plane and spherical trigonometry supplies.

The remaining portion of the present volume will, in an especial manner, prove acceptable to the mariner. It will supply an extensive amount of valuable instruction, the result of long and varied experience, digested and methodized by one of the most distinguished scientific navigators of the present day, Lieut. Maury of the United States navy. Even to the non-professional reader, the philosophical exposition given by this eloquent writer of the physical geography of the ocean will offer attractions of no common kind; and we consider ourselves fortunate in being enabled to incorporate a performance of so much merit in the CIRCLE OF THE SCIENCES.

To the treatise just concluded, we shall now add a few general remarks by way of comment, subjoining some interesting and valuable information respecting the action of iron ships on the compass.

It cannot fail to strike an attentive reader, that the subject of Navigation presents a forcible example of the value of abstract science, even in circumstances where the practical application of its principles would seem to be almost precluded, on account of the unavoidable imperfections of our observations and experiments; of the instruments with which we work, and of the materials upon which we operate.

The mechanical tools or implements of the navigator are the log, the compass, the chronometer, and the sextant; these are to furnish him with the materials upon which his science is to work, and from which he is to extract his all of information in situations where no external aid can reach him, and where to err may involve life and property in sudden destruction. Yet the mechanical means upon which he thus depends for guidance and safety, are all confessedly imperfect; he can measure with accuracy neither the rate at which he sails, nor the course upon which he steers; and even if the log and compass were perfect, hidden and unsuspected agencies may vitiate, and falsify the indications of both. The sextant, fortunately is beyond the operation of these disturbing causes; it is, moreover, the least imperfect of all his nautical appliances, and accomplishes the important end of rectifying and adjusting, to a very close approach to accuracy, what the other instruments may have done amiss. It is among the most valuable gifts that science has ever presented to man to aid him in his necessities; Nautical Astronomy could not exist without it: and to say that it is not perfect, is only to repeat what has been applied to every work of man's hands. The best sextants, however, are sub-divided to no smaller arc than 10″, so that fewer seconds than 10 must be estimated, by help of the microscope, entirely by the eye.*

As just noticed, the sextant—including of course in this term the quadrant—is of the utmost use in correcting the results of the dead reckoning. But the ship's account continuously accumulates, and its errors must run on till the weather and the sky furnish opportunities for celestial observations. In the interim, the vessel is trusted almost entirely to the guidance of the compass; and it most unfortunately happens that, from the local attraction, the ship may often be said rather to direct the compass than the compass to direct the ship. It is most important, therefore, that the intervals between observations at sea be shortened as much as possible by seizing every occasion that offers for making them. From what is taught in the preceding pages the reader will easily perceive how it happens that even very gross errors in the dead reckoning become comparatively inoperative in the results deduced from astronomical observation, although the calculations founded upon these observations do virtually involve the data furnished by the ship's account. These data, however, do not directly enter into the work; it is the *time* which corresponds to them that is employed; and, fortunately, the astronomical elements, taken from the Nautical Almanac, in reference to this time—semidiameter—declination—right ascension—horizontal parallax, &c., vary so little, even in a large interval, that an error in the ship's place to the extent of a quarter of the globe would not, in general, entail an error of a quarter of a degree in the adjustment of that place by the lunar observations, provided, at least, that the time be not deduced from the moon.

As already remarked in the volume on the MATHEMATICAL SCIENCES, page 71, the reasonings of pure geometry tolerate no errors in the premises; but practice can never satisfy these rigorous conditions, and in proportion as they are departed from will be the geometrical shortcoming of our conclusions. At first sight, therefore, it would seem chimerical to hope for any close approach to accuracy from data so widely erroneous; but when it is considered that these data connect themselves with other dependent data, which are incapable of error beyond a very limited range, we at once perceive that these latter may be very near the truth, though the former may greatly depart from it; and that if the inquiry involve the dependent data only, and not in a direct manner the original, the inaccuracies of *these* need give us but comparatively little concern.

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* The error of this estimation can never reach *five* seconds, so that the corresponding error in longitude cannot be more than about two miles; an error that is quite compatible with perfect safety, except in very extraordinary circumstances (see page 158).

It is thus that a very close approximation may be made to the true position of a ship, though the dead reckoning may displace her many degrees, and though, at the same time, we employ this reckoning in the operation, as if it involved no error at all. But in the intervals between these adjusting observations the safety of the ship is often wholly dependent on the trustworthiness of the compasses, and of late much mischief has arisen from placing too implicit a confidence in them in certain circumstances.

Since the prevalence of iron vessels, the disturbances of the compass have been seriously forced upon the attention of scientific men; and the subject of local deviation, still involved in considerable obscurity, is becoming more and more a matter of anxious scrutiny and investigation. The importance of the inquiry was strongly urged by Dr. Scoresby, at the meeting of the British Association, at Liverpool, in 1854. "There were certain principles," observes the Rev. Dr., "connected with the navigation of iron ships, which were universally admitted. Those principles were, that iron, being more especially disposed to the magnetic condition, was a material, of course, calculated above all others to disturb the action of the compass on board the ship. Again, it was admitted that there were difficulties in the navigation of iron ships, arising not merely out of the original or primarily magnetic condition and disturbing influence of the iron, but also in respect of certain changes which had been held as mysterious—changes which took place not unfrequently in regard to ships whose magnetic condition had been supposed to be very well ascertained." Dr. Scoresby then adverts to the circumstances connected with the melancholy wreck of the "Tayleur," the fate of which must be in the recollection of all our readers. The ship "Tayleur," a new vessel, bound to Australia, sailed from Liverpool on Thursday, January 19, 1844. She was 1,979 tons burthen, new measurement, and she had on board about 458 passengers,—the crew and passengers altogether making a total of 528 persons. She left the Mersey about noon on the above-named day. The pilot left her between seven and eight o'clock in the evening, in a position between Point Lynas and the Skerries. On Friday she encountered very heavy weather; and about eight o'clock on the following morning (Saturday), it was for the first time ascertained that there was any material difference between the compasses.

There were three compasses on board. Dr. Scoresby makes special reference to two of these. One of the two was near the helmsman, and was the one by which he steered; and the other was near the mizen-mast. Both of these compasses had been what is called adjusted, by permanent magnets; so that if the principle of adjustment had been correct, they should not either of them have changed or differed from the other.

Trusting to the compass near the helmsman, the captain had the idea firmly impressed upon his mind that he was sailing fairly down almost mid-channel; at all events, in a good position for navigating the Irish Channel. The other compass indicated a difference of about two points. The captain, however, judging from certain indications which he had noticed previously, assumed that the wheel compass was the correct one.

In the course of a few hours, about half-past eleven o'clock on the same morning, the wind having increased, and a heavy sea setting up the channel, the ship made rather a rapid progress, when they suddenly came in sight of land on the lee beam, in such a position that there was necessarily a great difficulty—in this case (according to the measures pursued) an insurmountable difficulty—in avoiding the land. An attempt was made to wear the ship round. This failed; and then an attempt was made to use the anchors to bring her up. Both the cables snapped on the occasion, and the ship was then left helpless, driving broadside upon the rocks of Lambay Island. The result was the fearful catastrophe with which most persons are acquainted—namely, the loss of about 290 lives: out of 100 females who were on the ship only three escaped upon that melancholy occasion.

Investigations into the cause of the calamity were undertaken, and the Marine Board of Liverpool, after stating that Captain Noble had given very great attention to the ascertaining of the correctness of his compasses, and verifying their action on different occasions, report that "notwithstanding these precautions, it appears to this Board that the "Tayleur" was brought into the dangerous position in which the wreck took place through the deviation of the compasses, the cause of which they (the Marine Board) had been unable to determine."

To these important matters Dr. Scoresby has given much thought and attention, and he finds from numerous experiments, some of which are very simple, that mechanical violence has a very considerable influence upon the magnetic condition of iron. Thus, an iron bar, entirely neutral as to its molecular magnetism, if held in an upright position, or inclined in the axial direction of the earth's magnetism, were subjected to percussion or other mechanical violence, not only did its magnetism become much more powerful than that of simple induction, but it strongly exhibits its augmented polarity when placed in the east and west equatorial position; and, however it might be moved about and swung round, its polarity remained the same. Dr. Scoresby applies these facts to iron ships, and points out that, in consequence of the percussive action to which the material is exposed while the ships are in course of construction, it became as intensely magnetic as it is possible for malleable iron to be. This augmented magnetism, however, is not permanent or fixed, but, under different circumstances, as to the relative directions of the ship's magnetism and that of the earth, is easily changeable, and liable necessarily to be changed. The magnetism developed by mechanical violence can be readily neutralized or changed, under a proper change of conditions, by other processes of mechanical violence.

The general result of his experiments went to the establishing of the fact, that besides the two denominations of magnetism ordinarily received—that of simple terrestrial induction, and that of permanent independent magnetism—there is another denomination corresponding with neither; not being absolutely controllable, like the former, by terrestrial influences, nor capable, like the latter, of resisting all kinds of mechanical violence. To this third denomination he gives the name of Retentive Magnetism, and which he proves to be a fluctuating quality, though hitherto considered as permanent. On the contrary, the long-continued vibration of a ship under steam, and much more so the straining of the vessel in a heavy sea, under the circumstances when the terrestrial induction might be acting in a very different direction from the original axial polarities of the ship, would be sufficient to change the direction of the magnetism originally developed in the course of her construction. Hence, he observes much would depend, in respect of the mechanical action of the sea, on the position in which the ship had been built. In the case of the "Tayleur," when he first heard of the catastrophe and had read the evidence, he stated to some friends at Torquay that he would venture to affirm that she had been built with her head *northward*; he found on inquiry that she had been built with her head nearly north-east. Here then, he adds, were the precise circumstances for expecting a change in the ship's magnetic distribution. Having been built with her head to the north-east, she had a certain magnetic distribution accordingly, and when she began to strain, with her head to the south-west, that distribution was necessarily changed, and the first effect of it had been to produce a great difference in the two compasses adjusted by fixed magnets. If the captain had been aware of the changes which might, and most probably would, take place when the ship began to strain in a different position from that in which she had been built; if he had known that the compasses, having so large an original deviation as 60°, might vary as much as two, three, or even four, points, he would have known, of course, that he must place no reliance upon them.

It is most important, therefore, continues Dr. Scoresby, for safety in navigating these vessels, that captains should be made aware of the liability of the compasses to change, and so to mislead them; that they should know the circumstances under which, in accordance with natural laws regulating and applying the earth's inductive action, changes were most likely to occur; that they should be always watchful of opportunities for determining the true magnetic direction, with reference to their compasses, by observations of the sun and stars; and that by providing a place for a standard compass aloft, as far from the deviating influence of the body of the ship as possible, they might have guidance sufficient, with some small allowances, for steering a correct magnetic course. With such precautions, Dr. Scoresby did not doubt that the difficulties in respect of compass guidance, in the navigation of iron ships, might be mainly and practically overcome.

These remarks and suggestions from an experienced navigator so well acqnainted with his subject deserves the serious attention of mariners; and there is no doubt that, even in wooden ships, such a locality for a standard compass, as he here recommends, would prove of service. In iron ships, as sufficiently shown above, any compass-adjusting apparatus, applied at the outset of a voyage, becomes of little or no avail when the vibration and strain of the vessel are thus known to change its magnetic condition, and any confidence placed in such adjustments is likely to beget a feeling of security, and to allay apprehension, even in situations of the most imminent peril.

For a full account of Dr. Scoresby's views, and of those of Mr. Towson, another very competent authority, the reader is referred to "The Proceedings of the Twenty-fourth Meeting of the British Association for the Advancement of Science."

NOTE ON THE PENDULUM EXPERIMENT.—PAGE 82.

The interesting conclusion arrived at, at page 82, in reference to the time in which the horizontal meridian line performs a complete revolution is rendered somewhat obscure by a clerical error, which the small space at our disposal here enables us to correct. Instead of "angle of deviation" at line 16, it should have been "angle at the base; and "(x)," a little lower down, should be "or." The general conclusion arrived at in the text is that the angle of deviation of the horizontal meridian from its first position, is to the corresponding angle of revolution of the earth about its axis, as the sine of the latitude of the place is to unity; that is

$$\frac{\text{angle of deviation}}{\text{angle of revolution}} = \text{sin latitude}$$

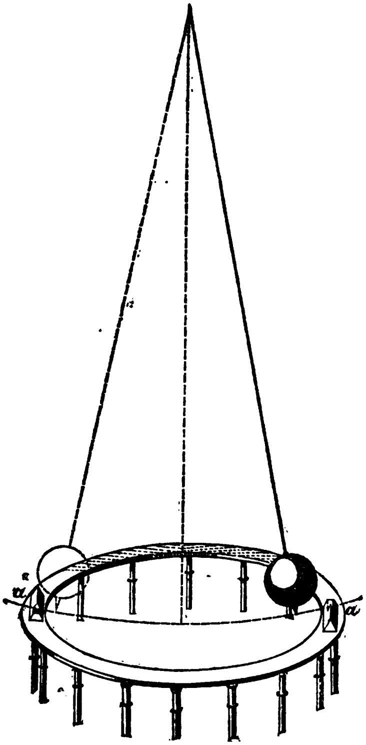
Hence the angle of revolution of the earth in any time being represented by x, we have, generally,

$$x = \frac{\text{angle of deviation}}{\text{sin latitude}}$$

So that when the horizontal meridian has completed an entire circuit, that is, when the angle of deviation is 360°, we have, as in the text

$$x = \frac{360°}{\text{sin lat}} = \frac{24 \text{ hours}}{\text{sin lat}}, \text{ in time.}$$

In the article here referred to, the name of Foucault has been inadvertently spelt Faucault.



The annexed figure will give a sufficient idea of the manner in which the pendulum experiment may be exhibited.

J. R. YOUNG.

PRACTICAL ASTRONOMY.

◆

ɪᴍʏ is that science which teaches the distribution and arrangement of the
r bodies, their true and apparent motions in space, their magnitude, distance,
sical condition; and its history presents some of the most brilliant examples of
lopment of the human mind.

nomy essentially owes its existence as a science, as well as its present
f perfection, to continuous observations of the heavenly bodies, from the
things of the Chaldean shepherd to the refined instrumental measurements of
nomers of the nineteenth century. But if the accumulated observations of
, which the genius of man would naturally endeavour to preserve, had not
le subservient for the prediction of recurrences of the same phenomena, Astro-
uld exist only in name, and cease to hold its high position among the accurate

Its course, however, has not always been progressive: it is only within the
centuries that the errors of antiquity have been completely eradicated. To the
of the telescope, and its application to graduated instruments, we are in a great
indebted for this progress, which, in a physical point of view, it is impossible to
mate, for it has permitted great and permanent advances to be made in the accu-
rvations of the heavenly bodies. By the latest improvements in these instruments,
ers have been enabled to note, with an incredible degree of precision, the appa-
es of the fixed stars and *planets*. *In* this manner, as the accuracy of observation

increased, the errors of the ancient theories exhibited themselves more clearly, and the mind of man has been enabled to explain, on sound and natural principles, the new phenomena which they have revealed. The discovery of the aberration of light and of the nutation of the earth's axis, two of the greatest discoveries of any age, have thus resulted, having afforded to Dr. Bradley the means of investigating, by accurate observation of the places of the stars, the preceding discovery by Roemer of the successive transmission of light, which he had determined by observations of the eclipses of Jupiter's satellites.

The total overthrow of the Ptolemaic system is owing to the same cause—the observations of Tycho Brahé having furnished means of clearly exhibiting to Kepler the errors of a circular hypothesis, in the same manner as the errors of the Alphonsine tables, based also on the Ptolemaic theory, pointed out to Copernicus the errors of the ancient philosophy, when it was discovered, about the epoch of 1500, that they differed more than two degrees of the truth. Calculation has also lent its aid in reducing to law, and in grappling with difficulties, problems which the happy invention of logarithms have rendered accessible, and which in other respects, without this assistance, would be almost insurmountable.

We find that, amongst the most ancient nations of antiquity, the appearance of the heavens was attentively watched. And, in the first place, that the motions of the sun and moon commanded attention, in regard both to the duties of the husbandman and appointments for the assemblage of large masses of the people. The Egyptians and Chaldeans discovered that the motions of these bodies and the planets were all performed within a certain compass of the heavens, which was termed the Zodiac; the names of the constellations through which this belt passed having especial reference to the motions of the sun. We also find that eclipses of the sun and moon were recorded with superstitious veneration; whilst, in connection with the foolish study of judicial astrology, the positions of the planets, with reference to bright stars, were also carefully noted. The Egyptians and Chaldeans were acquainted with the time of the revolutions of the moon's nodes, an epoch which they termed the *Saros*, which would give them the means of predicting eclipses, recurring, as they naturally would, in the same eras. The Chaldeans were also acquainted with the lunar-solar period of 600 years, which Josephus attributed to the ancient patriarchs, although Pliny cites Hipparchus as its author. Their primitive observations appear to have consisted in the heliacal rising of bright stars, and probably in the use of the gnomon or sun-dial.

Thales, of Miletus, in Asia Minor, who had studied the astronomy of the Egyptians, is supposed to have predicted, by a knowledge of the period of 585 days, or the Saros, the famous eclipse which occurred about 585 B.C., and which is recorded by Herodotus.

Thales, on his return from Egypt, founded the Ionian school (B.C. 640). This school had for its followers Anaximander (B.C. 610), Anaximenes (B.C. 530), and Anaxagoras (B.C. 500). The first is known by a most useful invention, viz., that of geographical charts. The others added some fanciful notions, in addition to those of Thales, on the construction of the universe; and they all appear to have believed in the plurality of worlds. The most enlightened disciple of this school, however, was Pythagoras (B.C. 580), who, having been advised by Thales to travel into Egypt to perfect his studies, became initiated in the secret mysteries of the priests in a greater degree than any of his predecessors. He founded a school in Italy on his return, where the doctrines of the *Ionian school* were promulgated in a greater degree. He appears to have been *acquainted* with the earth's rotation on its axis, its annual revolution, and its spherical figure, doctrines which, in imitation of the Egyptian priests, were kept secret from the multitude.

But the first approach to a systematic system of astronomy was that of the Alexandrian school, where we find observations carried on regularly, and a theory which, though founded in error, rudely satisfied the observations of the period. The most distinguished members of this school were Aristarchus of Samos (B.C. 281), Eratosthenes of Cyrene (B.C. 276), and Hipparchus of Bythinia (B.C. 140). Aristarchus is famous for having made an approximation to the distance of the sun from the earth, by the angular measurement of the distance of the sun and moon at the time at which the moon is half illuminated. Having found that the moon's angular distance at this period was 87°, he thence concluded that the sun was distant eighteen or twenty times that of the moon. This method, however ingenious, fails in consequence of the roughness of the moon's surface. By careful observations of the sun's path, he also made an approximation to the diameter of this body, which he considered to be $\frac{1}{720}$th part of the whole daily motion—an observation not very far from the truth. His tract on the magnitude and distances of the sun and moon has been preserved. In addition to the tenets of the Alexandrian school, he was of opinion that the stars were at distances immensely greater than the sun, on which account this astronomer must hold a high rank among those of antiquity.

Eratosthenes of Cyrene, who lived B.C. 276, is known for a determination of the obliquity of the ecliptic, by observations of the altitude of the sun at the summer and winter solstices—an observation of great value, as showing the progressive diminution of this element when compared with modern observations. He also made an approximation to the magnitude of the earth on correct principles by the measure of the celestial arc, included between Syene and Alexandria, combined with the known distance between the two places. Syene was one of the most southern cities of ancient Egypt, where it happened that at the summer solstice the sun was exactly vertical, which he determined by the fact that a deep well was wholly illuminated. At Alexandria, which was situated in the same meridian, and at the same period of time, the sun's zenith distance was 7° 12′, or one-fiftieth part of the whole circumference. The distance of the two places was known to be 5,000 stadia, whence he concluded that the circumference of the earth was 250,000 stadia. The great uncertainty of the length of the stadium does not permit us to make use of this determination, or to compare it with the method pursued with more accurate instrumental means by modern astronomers.

Apollonius of Perga deserves also to be mentioned as having invented the system of deferents and epicycles, for the purpose of accounting for the direct, stationary, and retrograde appearances of the planets.

But one of the greatest astronomers of antiquity, Hipparchus (B.C. 140), now appeared, famous alike as an accurate observer and geometer. To this distinguished man we are indebted for the discovery of the "precession of the equinoxes." The appearance of a new star led him to form a star catalogue, by a comparison of which with a former catalogue of the same school this important element became evident. He was also acquainted with the unequal motion of the sun in its orbit, and the inequality of the solar days. The former he endeavoured to explain by a modification of the preceding system of deferents and epicycles, which will be explained in its proper place.

The revolution of the moon's nodes, and the inclination of its orbit to the ecliptic, were also known to Hipparchus, as well as the eccentricity of the sun's orbit and the motion of the apsides. *By a comparison* of his observations of the sun with the former astronomers, he determined the length of a tropical year. His solar tables were held in high estimation by Ptolemy three centuries after their construction.

Ptolemy (B.C. 130) collected the observations of the Alexandrian school in his Almagest, a work which existed as a text-book among astronomers for centuries. He is famous for having discovered the *evection* of the moon, imperfectly known to Hipparchus; and by a comparison of observations of eclipses, and at the other parts of the moon's orbit, he empirically determined its amount with a great degree of accuracy. He confirmed the value of the precession of the equinoxes, previously discovered by Hipparchus. He also proportioned the magnitudes of the deferents and epicycles, for the purpose of explaining the planetary motions, which had been previously invented by Apollonius and Hipparchus, whence the name of the "Ptolemaic System." In his Geography, he collected the longitudes and latitudes of all known places. He also left behind him many other works, testifying to his skill and genius as a philosopher.

After the death of Ptolemy, the Alexandrian school existed only in name. During five centuries the discoveries of Hipparchus and Ptolemy were not extended, and the labours of its followers consisted principally in commenting on their works, and in noting extraordinary phenomena.

After the dissolution of the Alexandrian school, the science of astronomy in Europe had almost disappeared; but among the Arabs it had been cultivated with some success under the auspices of the more enlightened caliphs. The Almagest of Ptolemy was translated into their language, and their annals transmit to us many observations of the sun, moon, and planets, as well as the measure of a degree of latitude. Among the Arabian astronomers, Albategnius deserves honourable mention as an industrious and accurate observer. He confirmed and corrected the rate of the precession of the equinoxes, and also determined the obliquity of the ecliptic, which, from several collateral circumstances, is entitled to great confidence. He also paid considerable attention to the theory of the sun, and determined the eccentricity of its orbit with accuracy. He also found that the apogee of the sun was subject to a small annual displacement, according to the order of the signs, and determined its position at that period to be in the constellation Gemini, which, when compared with the modern elements, does not differ more than 40′ from the truth. The astronomy of the Arabians does not open to us any new theories, but is merely an extension of the system of Ptolemy. In the art of observation they had, however, improved; larger instruments and more refined calculations had been introduced by them, on which account their investigations are entitled to hold a high rank in the history of astronomy.

After the Persians had thrown off the yoke of the Arabians, Holeg Ilecoukhan, one of the Persian sovereigns, founded an academy of astronomers, where they formed new tables based on the Ptolemaic system, with trifling changes in their elements. But the greatest praise is due to Ulugh Beigh, one of their princes, who was a great patron of this science, as well as an observer, and who formed a catalogue of stars, and improved the solar and planetary tables. He also determined the obliquity of the ecliptic, and the precession of the equinoxes.

The next epoch of the history of astronomy commences with its revival in Europe, when, after the overthrow of the Arabian Empire, it again became cultivated in Spain. Alphonso, the tenth king of Castile, collected a body of astronomers at Toledo, and having made use of the observations of the Arabians for the purpose of correcting the planetary motions, published the Alphonsine tables, which are more correct than any which preceeded them in some respects, but their accuracy was not commensurate with the great trouble and expense they occasioned. The great confusion of the Ptolemaic system, caused Alphonso to say, "that if he had been consulted at the creation, he could have devised a better arrangement." In Austria, the study of astronomy was cultivated under the auspices of Albert, the third duke, and Frederick the second emperor. In the 15th century, Purbach, Regiomontanus, and Waltho lived, known as assiduous cultivators of this science. Ephemerides of the planets and eclipses were published for the meridian of Vienna from 1475 to 1505, by Regiomontanus, who, being also an observer, was well acquainted with the errors of the Alphonsine tables, the place of Mars being above 2° in error. At this period, the art of measuring time received an important improvement by the substitution of clock-work motion for the clepsydræ of the ancients, which also increased the accuracy of observations. About this period the true system of the universe forced itself on the attention of Copernicus.

This distinguished astronomer, who saw clearly the errors of the Ptolemaic system, and who, after thirty-six years of study, published, in a clear and connected form, the present system of the world, was born at Thorn in Prussia, in 1473, four years before the death of Regiomontanus. The ancient systems of astronomy, as related by Plutarch and Cicero, had, by their extreme simplicity, captivated his mind, and to this circumstance we owe his explanation of the celestial motions; an explanation which, however imperfect in several respects, shows his originality. In his system he adopted the hypothesis of the earth's rotation on its axis, its annual revolution round the sun, in common with the other five known planets, whose distances from the great central body he proportioned. As regards the "precession of the equinoxes," he conceived it to arise from a small motion of the earth's axis. In consequence, however, of his ignorance of the elliptical motions of the planets, he was unable totally to abolish the system of epicycles. The work which contained his explanations was dedicated to Pope Paul III., and it was owing only to the urgent representations of his scientific friends, that he was induced to publish it in the form of an hypothesis. He had only revised the last proof sheet when death put an end to his labours in his seventieth year.

One of the most important inventions ever introduced by the genius of man was afterwards applied to the observations of the heavenly bodies by Galileo. This was the telescope, which has enabled the astronomer to overcome the feebleness of his natural vision, and to examine leisurely and attentively the physical constitution of the celestial bodies. Galileo, an ardent follower of Copernicus, was immediately convinced of the truth of the system, when, having observed the phases of Venus, he traced their connection to their elongation from the sun, and found that the orbit of this planet was included within the earth's orbit. Galileo also discovered the four satellites of Jupiter, and was acquainted with the existence of spots on the sun, from which he concluded the rotation of that body on its axis. He appears also to have first applied the pendulum to the clock.

The overthrow of the Ptolemaic system followed quickly on the footsteps of Copernicus and Galileo, but not till the new system had encountered much opposition. Tycho Brahé, who was born in 1546, is celebrated in having supplied observations of greater excellence than any of his predecessors. The cultivation of practical astronomy had made but little progress since the time of the Arabs, and the new theory required accurate observations to test its correctness. Tycho thus paved the way for Kepler to make his splendid discoveries. Personally, however, he resisted the Copernican theory, and invented an hypothesis of his own, retaining the earth as the centre of the universe, but making the sun the centre of the planetary motions. This system had few admirers; but the accuracy and number of his planetary observations, his star catalogue, his discovery of the "variation" of the moon, and his table of refractions, give him a high position among the astronomers of his period. From the planetary observations of Mars, the immortal discovery of the elliptic motions of the planets has resulted. Kepler placed such confidence in the observations of Tycho, that he affirmed his belief that the true system of the universe might be founded on them. Taking at first the Copernican doctrine of a circular hypothesis, he found that a difference of 8′ existed between the theory and observation of Mars, which difference, from the accuracy of the observations, he had no doubt was attributive to an erroneous hypothesis. After labour infinitely great, extending over a period of twenty years, he discovered his three famous laws which serve as the foundation of modern astronomy. Kepler appears also to have had some notions on gravity; and he left as well some tracts on optics. The laws which he had determined for the sun and planets, were found also to extend to the satellites of Jupiter, they having been just discovered by Galileo, Huygens, and Cassini.

Among the distinguished men who followed Kepler, mention must be made of Huygens, Hevelius, Cassini, and Roemer. The first, in addition to his optical and mechanical improvements, described the ring surrounding Saturn as well as one of its satellites, and is said to have applied the pendulum to clocks—a discovery also claimed for Galileo. Hevelius is known as a persevering observer. But all his observations were made with the naked eye; and although he was able to estimate angular spaces by these means to a wonderful degree of accuracy, which Dr. Halley confirmed by direct comparison, his observations cannot be made use of in any delicate inquiry. His researches on comets, and his tables of the sun, rank him in a high position among practical astronomers. Cassini formed tables of Jupiter's satellites from observation, and determined the rotation of Jupiter and Mars. He also occupied himself wiih refractions, and made a theory of the libration of the moon, which had been left in an imperfect state by Hevelius. Roemer is known as the inventor of the transit instrument and meridian circle, with which instruments he carried on a series of observations of the heavenly bodies for several years; but only three days have been preserved, the others having been destroyed in the great fire which occurred at Hafnia. Roemer also discovered, by comparison of Cassini's tables of Jupiter's first satellite, the successive transmission of light. The discovery of universal gravitation by Newton, at the latter end of the seventeenth century, produced a total revolution in astronomy, and enabled its illustrious author to account for the previous empirical deductions of Kepler.

Since the discovery of gravitation by Sir Isaac Newton, astronomy has been much advanced by the establishment of observatories. And in this respect England has great reason to be proud of her position, her astronomers having followed up this immortal discovery by a series of observations unexampled for their extent—an object which is essentially necessary in order to continue the comparison of theory and observation for the correction of the elements of the system. The accuracy of the science has been considerably increased by the institution of the Royal Observatory at Greenwich, both with regard to the eminent men who have directed this establishment, and to the continuous mass of observations which it has furnished. Founded by King Charles II, in 1675, it has had for its directors successively Flamsteed, Halley, Bradley, Bliss, Maskelyne, Pond, and Airy.

Flamsteed is known as the greatest observer of the age in which he lived. Patronised by *Sir Jonas Moore*, to whom the foundation of the Greenwich Observatory is due, he carried on for many years continuous observations of the planets and fixed stars, and contributed materially to furnish Sir Isaac Newton with the observations on which his system of the world is based. He was succeeded by Dr. Halley, an astronomer to whom we are indebted for many useful investigations, but whose province did not so much lie in practical astronomy as his predecessor. This distinguished man has the merit of adding considerably to our knowledge of cometary astronomy and the lunar theory. But one of his greatest triumphs, which will always render his name illustrious, is his proposition of determining the sun's distance by the transits of Venus over its disc.

Dr. Bradley, the next in succession, enriched this science by two important discoveries—viz., the aberration of light and more exact knowledge of the nutation of the earth's axis. Distinguished as the most accurate astronomer of his age, he re-observed, with better instruments, all the stars of Flamsteed, as well as the sun, moon, and planets, the former of which have been incorporated in the *Fundamenta Astronomiæ* of Bessel. His observations of the moon and planets are included in the Greenwich planetary and lunar reductions.

The same important class of observations was continued by Dr. Maskelyne, from 1765 to 1811, extending over an interval of forty-six years. In addition to other important works, this astronomer has the credit of establishing the *Nautical Almanac* for the use of mariners, and of introducing in that work tabular elements for the improved method of finding the longitude at sea.

On the same regular system have been carried on, at the Royal Observatory, observations of the sun, moon, and planets, under the superintendence successively of Mr. Pond and Professor Airy. If it were necessary to discuss the importance of the immediate comparison of theory and observation, it would be merely necessary to mention that the discovery of the planet Neptune has thus resulted—a discovery which, in the opinion of all competent judges, deserves to be ranked among the most brilliant triumphs of the human mind.

In the field of telescopic research, the labours of Sir W. Herschel deserve particular mention. His discovery of the planet Uranus and its satellites, as well as his investigation on Saturn's rings—his theory of the motion of the solar system in space and its direction, which have been verified recently by several independent researches, are sufficient monuments of his skill and industry.

In France, the cultivation of practical astronomy in the eighteenth century was well sustained by the celebrated astronomers Lalande and La Caille, the latter having observed the stars of the southern hemisphere about the epoch 1750, the publication and reduction of which have since been undertaken by the British Association. He is also known as having determined the *constant* of lunar parallax by corresponding meridional observations at the Cape of Good Hope and at European stations. Besides these important undertakings, he was an energetic calculator of ephemerides, and other useful investigations on refraction, figure of the earth, &c.

Lalande is known as an industrious and indefatigable observer. His immense catalogue of stars bear sufficient evidence to his unremitting industry for many years. The catalogue of stars formed from his observations have been published and reduced by the British Association.

In Sicily, the labours of Piazzi have been principally occupied in an extensive catalogue of stars, the epoch of which is the beginning of the present century, and which is now one of the *standard works of the day.* His zeal was rewarded by the discovery of

Ceres among the asteroids, in 1801. The discoveries of these bodies, during the last ten years, are too numerous to be mentioned here.

The progressive improvements of instruments during the last century have been rewarded by increased accuracy of results. At Greenwich, the mural arc of Flamsteed is now replaced by the magnificent transit circle; and the aid of galvanism has been brought to bear on an essential and delicate element of time—viz., that of right ascensions. All these circumstances produce a corresponding improvement in theory. Successive comparisons of theory and observation engage the attention of our analysts, and to this we are in a great measure indebted for the important discoveries of the various inequalities, which have enriched the science of astronomy.

ASTRONOMICAL DEFINITIONS.

Having thus shown how the investigating disposition of man has been occupied in its endeavour to obtain a more correct and perfect knowledge of the universe, we proceed to explain the laws which have been established.

Astronomy is usually divided into three parts:—1. SPHERICAL ASTRONOMY, which teaches the knowledge of the various points and circles of the celestial sphere, the constellations, the position of the stars with respect to these points and circles, and the phenomena occurring in the sphere of the heavens. 2. THEORETICAL ASTRONOMY, which enables us to determine, from observation, the path of the heavenly bodies. 3. PHYSICAL ASTRONOMY, which gives the laws by which the heavenly bodies are regulated, teaches how their motions are to be calculated according to the rules of mechanics, and combines all that is known of their physical characters. Without the formality of so dividing our subject, which our space does not permit, we shall endeavour to combine all the useful and practical portions of the subject.

In order to facilitate the study of the heavens, artificial representations have been made, similar to those of the surface of the earth; or celestial globes, on which the stars are depicted in their natural positions, the observer being supposed to be in the centre, viewing them in the concave surface.

To represent the apparent diurnal motion of the heavenly bodies, the celestial globe must be turned from east to west.

Circles.—To designate with precision the situation of the sun, moon, and stars, imaginary circles have been considered as drawn in the heavens, most of which correspond to, and are in the same plane with, similar circles supposed to be drawn for similar purposes on the surface of the earth. If a line be drawn on the sphere (Fig. 1) of the earth P Q, the plane cuts the surface of the sphere, and forms a great circle E F, which is the celestial equator, the sphere being divided in this circle into two hemispheres, in which one of the poles forms a central position; these form the northern and southern hemispheres.

A plane is that which has surface, but not thickness. The plane of a circle is that imaginary surface which the circle bounds.

The axis of the earth, P Q, is an imaginary line passing through its centre, north and south, about which its diurnal revolution is performed; the poles of the earth are the two extremities of the axis, where it is supposed to cut the surface. The axis of *the heavens is* the earth's axis produced both ways to the concave of the sky; the *poles of the heavens* are two imaginary points exactly above the terrestrial poles.

Great circles are those which divide the globe into two equal parts, as the equator, the ecliptic, and the colures. R R, S S, and T T (Fig. 1) are small circles which divide the globe into two unequal parts, as the tropics, polar circles, and parallels of latitude.

Every circle is supposed to be divided into 360 equal parts or degrees. A degree is further subdivided into 60 equal parts or minutes; and a minute into 60 seconds. Degrees are marked °, minutes ′, seconds ″. The space included by a degree of a great circle in the heavens, is equal to nearly twice the apparent diameter of the sun and moon, when considerably above the horizon.

The equator of the earth, E E (Fig. 1), is an imaginary great circle passing round the globe, east and west, everywhere equidistant from the poles, dividing it into northern and southern hemispheres. The equator of the heavens, or the equinoctial, is the plane of the terrestrial equator extended to the concave surface of the heavens, and called the equinoctial, because, when the sun appears in it, the days and nights are equal all over the world.

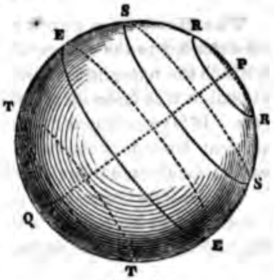


Fig. 1.

The Ecliptic is the *via solis*, or sun's path, the great circle which he appears annually to describe among the fixed stars; though, more properly, it is the track which the earth actually describes among the stars, as viewed from the sun. The ecliptic is so called, because solar and lunar eclipses only can happen when the moon is in or very near this circle. It cuts the equinoctial obliquely at two opposite points, making an angle with it of 23½°, which is called the obliquity of the ecliptic. One half lies on the north side of the equinoctial; the other half on the south side. The points of crossing are the equinoctial points. A zone or girdle extending 8° on each side of the ecliptic, or 16° in breadth, is the zodiac, in which are the orbits of all the planets, with the exception of three of the asteroids. The ecliptic and zodiac are divided into twelve equal parts, called signs, each containing 30°. Their names, with the days on which the sun enters them, are as follows:—

Northern signs, being north of the equinoctial.

Spring Signs.	*Summer Signs.*
Aries, the Ram, March 21.	Cancer, the Crab, June 2.
Taurus, the Bull, April 19.	Leo, the Lion, July 2.
Gemini, the Twins, May 20.	Virgo, the Virgin, August 22.

Southern signs, being south of the equinoctial.

Autumnal Signs.	*Winter Signs.*
Libra, the Balance, Sept. 23.	Capricornus, the Goat, Dec. 21.
Scorpio, the Scorpion, Oct. 23.	Aquarius, the Water-bearer, Jan. 20.
Sagittarius, the Archer, Nov. 22.	Pisces, the Fishes, Feb. 19.

The Colures are two great circles passing through the poles of the heavens, dividing the ecliptic into four equal parts, and marking the seasons of the year.

One passes through the equinoctial points, Aries and Libra, and is therefore called the equinoctial colure. When the sun is in either of these points, the days and nights on every part of the globe are equal to each other. The other passes through the solstitial points, *Cancer and Capricorn,* which mark the sun's greatest declination, north and south of the equator, and is thence called the solstitial colure. When the sun is in or near these points, his meridian altitude undergoes scarcely any sensible variation for several days; and hence the term solstitial applied to them.

The Horizon is a great circle, whose plane passing through the centre of the earth, and extended to the sphere of the fixed stars, divides the heavens into two hemispheres, of which the upper is the visible, and the lower the invisible hemisphere. This is the rational or true horizon, which determines the rising and setting of the sun, planets, and stars. It is represented by the wooden horizon of the artificial globe. The sensible or apparent horizon is the circle which bounds our view, where the land or water and sky seem to touch each other, more or less extensive according to the position of an observer.

The sensible horizon of a person changes as he moves, and in an open country enlarges or contracts as his station is high or low. Standing on a plain, the eye having an elevation of 5 feet above the surface, the radius of the sensible horizon will be less than 2¾ miles. At an elevation of 6 feet it will be just 3 miles.

Rule, to find the distance when the height is known—Increase the height in feet one half, and extract the square root for the distance in miles.

Thus, in the preceding case the eye is supposed to have an elevation of 6 feet above the surface of a plain, and 6 with its half is 9, the square root of which is 3, which gives the distance in miles which a person will be able to see in a right line upon that surface.

Again—a tower, 32 yards above the level of the ocean, may be seen along that level from a distance of 12 miles. For 32 yards = 96 feet, increased one half = 144, the square root of which is 12.

The poles of the horizon are the zenith and the nadir. The zenith is the point in the heavens which is directly over our heads; the nadir that which is exactly under our feet. The zenith to us is the nadir to our antipodes, and the nadir to us is their zenith. Circles drawn through the zenith and nadir of any place, cutting the horizon at right angles, are called azimuth or vertical circles; and that which passes through the east and west points of the horizon, is the prime vertical.

Meridians are imaginary great circles passing through the terrestrial and celestial poles, cutting the equator and equinoctial at right angles.

A meridian is supposed to pass through every place on the earth, and every point in the heavens, but only 24 are drawn on the globes through every 15° of the equator and equinoctial, including altogether 360°. These meridians mark the space which, in consequence of the earth's diurnal rotation, the heavenly bodies appear to describe every hour through the 24 in the day. They are sometimes called, therefore, hour or horary circles. As 15° answer to an hour, 1° answers to four minutes of time, ½ to two minutes, and ¼ to one minute.

Longitude on the earth is distance east or west from a fixed meridian measured on the equator. The Fortunate Islands, supposed to be the Canaries, supplied the ancients with their first meridian. The western extremity of Africa, as then known, was taken by Abulfeda, the Arabian geographer. The meridian of Terceira was used by the Spanish and Portuguese in the sixteenth century; and that of Ferro by all nations in the seventeenth and eighteenth centuries. We now adopt the meridian of the Greenwich, and the French that of the Paris, observatories.

Longitude in the heavens is distance east from the great meridian which passes through the first point of Aries, or the equinoctial colure, measured on the ecliptic.

Right ascension is distance east from the same meridian measured on the *equinoctial.*

Terrestrial longitude being reckoned in two directions from a fixed point, east and west, can only extend to 180°. Celestial longitude, and right ascension, are only reckoned in one direction, east from the prime meridian, and may, therefore, extend to 360°.

Parallels of Latitude are small circles supposed to be drawn on the surface of the earth, north and south of the equator, and parallel to it, dividing the globe into two unequal parts. Let us suppose A (Fig. 2) to be placed under consideration, and P, E, Q, E' its meridian, E E' the line intersecting the equator, and P Q the line of the poles, it is here the arc A E, or, which is the same thing, the angle A, O, E, which represents the latitude sought. P, O, E being a right angle, the latitude is the complement of the angle A, O, P; but the angle A, O, P, is only another thing for the zenith distance Z, A, P, of the pole of the celestial sphere. For to an observer placed at A, P Q is a parallel to the earth's axis; the latitude of the point A then is the complement of the zenith distance from the pole at that point, to the height P', A, H of the pole above the horizon, A H being equal to the distance Z, A, P. We can, therefore, say that the latitude of a place is equal to the beight of the pole above the horizon of that place. Parallels of declination are such circles produced in the heavens, north and south of the equinoctial, and parallel to it.

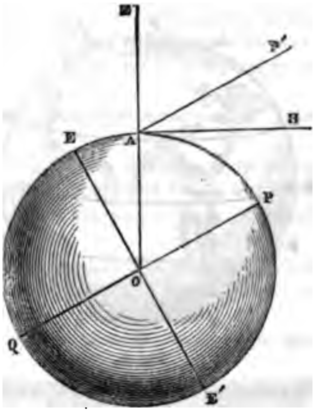


Fig. 2.

Latitude on the earth is the distance of a place from the equator, measured on a meridian, north or south.

Declination is the distance of the heavenly bodies from the equinoctial, measured on a meridian, north or south.

Latitude in the heavens is distance from the ecliptic, at a right angle, north or south.

Terrestrial latitude and declination may extend to 90°. The sun has no declination when in the equinoctial. His greatest declination is 23½° north or south. He has no latitude, being always in the ecliptic. The greatest declination of a planet is 30½°, and latitude 8° north or south, with the exception of the asteroids. It is more convenient to describe the position of the heavenly bodies by their declination and right ascension, than by their latitude and longitude, the former corresponding to terrestrial latitude and longitude.

The Tropic of Cancer is a small circle 23½° north of the equator, and parallel to it; and the tropic of Capricorn is a similar circle, at the same distance, on the south. The polar circles are also small circles, each 66½° from the equator, and at the same distance from the poles as the tropics from the equator.

The tropics on the celestial sphere mark the limits of the sun's farthest declination north and south.

The tropics on the terrestrial sphere divide the torrid from the two temperate zones, and the polar circles divide the temperate from the two frigid zones.

Zones.—Twice in the year the sun is vertical to those who dwell in the torrid zone. Consequently, *at noon they deflect no* shadow, and are hence styled *ascii*, meaning shadowless. At other times their shadows fall at noon, north or south, according as the sun is north or south of them. They are then called *amphiscii*, signifying that their shadows fall both ways.

The variation in the temperature has led to the division of the earth into zones; between the latitude 66′ 32″ north and south. The sun rises and sets every day. In all higher latitudes, however, there are certain periods of the year when the sun never rises, and others where it never sets. The two parallels of latitude A A, B B′ (Fig. 3), which correspond to the latitude 66′ 32″, divide the surface of the earth in three parts. The two zones at A P A′, B Q B′ are called the frozen zones. The circles A A′, B B′ are the polar circles, or arctic pole, while the other is the antarctic circle; in the points between these two are the several zones, in which the sun sets and rises daily.

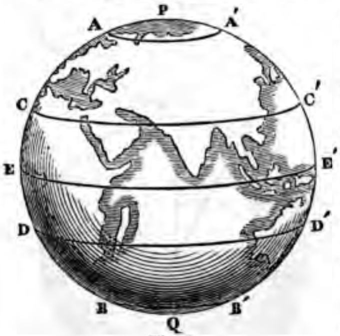


Fig. 3.

Those who dwell in the temperate zones have their shadows at noon always cast towards the north in the north temperate zone, and towards the south in the south temperate zone. They are styled *heteroscii*, meaning, that they have their shadows at noon, either north or south.

The inhabitants of the frigid zones are called *periscii*, signifying, a shadow turning about, as, during a revolution of the earth on its axis, their shadows are projected towards every point of the compass.

The Disc of a planet is its apparent face, seemingly perfectly flat though spherical bodies. The planets exhibit discs as seen with the telescope. The diameter of the disc of the sun and moon is considered to be divided into twelve parts, called digits.

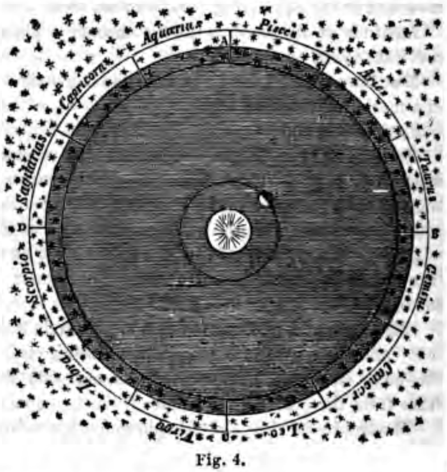


Fig. 4.

The Orbit of a planet is the path described by it in revolution round the sun.

The Plane of a planet's orbit is an imaginary surface cutting through the centre of the sun and the planet, and reaching out to the stars.

Figure 4 shows the plane of the earth's orbit. The stars to which it extends form the constellations of the zodiac. The circle A, B, C, D, is the ecliptic, the sun's place in the heavens, as seen from the earth, and the earth's place as seen from the sun.

The Inclination of the orbit of a planet is its plane referred to the plane of the *earth's orbit, to* which it is inclined.

If we suppose the shaded part of the diagram to be a surface of water, a ring or hoop held inclined so as to be half immersed will describe the relation of the planetary orbits to that of the earth. They are all inclined towards it, one half being below and the other half above it. The angle of inclination varies, but exhibits at the greatest only a slight divergence.

Mercury . . .	7° 0′ 9″	Jupiter	1°. 18′ 51″
Venus	3 23 28	Saturn	2 29 35
Mars	1 51 6	Uranus	0 46 28

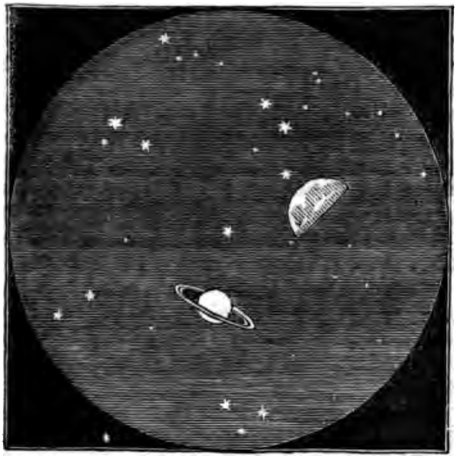
The Nodes of a planet are the two opposite points where its orbit appears to cut the orbit of the earth, or the ecliptic. Where the planet appears to rise above the orbit of the earth it is called the ascending node; the opposite point, where it appears to go below it, is called the descending node.

Conjunction and Opposition are terms used to denote certain situations of the planets with respect to the sun and to each other.

A planet directly between the earth and the sun is said to be in inferior conjunction with the sun. This can only take place in the case of Mercury and Venus, but along with them the earth may be in inferior conjunction with the sun to Mars, and with Mars to Jupiter, &c.

A planet with the sun directly between it and the earth is said to be in superior conjunction with the sun. The terms inferior and superior refer to the smaller or greater distance of the planet from the earth.

A planet with the earth directly between it and the sun is said to be in opposition. This can never take place in the case of Mercury and Venus, whose orbits are included in that of the earth.



Planets are said to be in conjunction with each other when they are in the same sign and degree. This is a common occurrence in the case of two; but the very rare phenomenon of Mercury, Venus, Mars, Jupiter, and Saturn, being in conjunction between the wheat ear of Virgo and Libra, took place September 15, 1186. It will be ages before they cluster again in that part of the heavens. Venus, Jupiter, and the moon, were in conjunction in Leo when the peace of 1801 was proclaimed. A telescope view of the conjunction of Venus and Saturn, on Dec. 19, 1845, is annexed.

The aspect of the planets to each other is said to be sextile, when they are two signs apart, the sixth part of the zodiac; quartile, when they are three signs distant, the fourth part of the zodiac; trine, when they are four signs distant, the third of the zodiac; and in opposition, when they are six signs, or half the zodiac, from each other.

Apogee is that point of the moon or a planet's orbit which is farthest from the earth.

Perigee is that which is nearest.

Aphelion is that point of the orbit of the earth, or of any planet and comet, which is farthest from the sun.

Perihelion is that which is nearest. A straight line joining the points of aphelion and perihelion, is called the line of the apsides.

Culmination is the act of coming to the meridian in the case of any star or planet, when it attains on any given day its greatest altitude in the heavens.

Daily Acceleration is the interval of time that the stars rise, culminate, and set, sooner every succeeding day than on the one preceding. It amounts to about four minutes daily, or two hours a month.

Rotation.—Besides the apparent diurnal motion of the stars caused by the earth's rotation upon its axis, they appear to have a motion westward, in consequence of the earth's orbital course eastward. They gain, therefore, on the sun, rising, culminating, and setting sooner, day after day. Thus those stars and constellations that on any given evening rise at ten o'clock, will, at the same hour, a month afterwards, be 30° above the horizon; and three months afterwards, they will be advanced over our heads; and six months afterwards be setting in the west, having accomplished half of their apparent annual revolution.

Hence the same constellations are not always visible to us through the year. Some, not visible before, successively rise to view in the east, while others sink in the west and are not seen again, until, having passed through the lower hemisphere, they reappear in the east.

Rising and Setting.—Cosmical, achronical, and heliacal rising and setting of the stars and planets, are phrases of the old poets, who spoke of the phenomena in reference to the rising and setting of the sun. A star or planet rising and setting with the sun was said to rise and set cosmically. A star or planet rising at sunset, or setting at sunrise, was said to rise and set achronically. A star or planet appearing a little before the sun, in the morning, after having been so near him as to be hid by his effulgence, was said to rise heliacally, and to set heliacally when it ceased to be visible after him in the evening, on account of its proximity to his orb.

Apparent Solar Day is the time included between the centre of the sun leaving the meridian of any place to its return to the same meridian again.

It varies continually in length, owing to the unequal motion of the earth in its orbit, and the obliquity of the ecliptic, being sometimes more and sometimes less than twenty-four hours. The greatest variation occurs about Nov. 1, when the solar day is 16′ 17″ less than twenty-four hours, as shown by a well-regulated clock.

Mean Solar Day is the time which would elapse between consecutive returns of the sun to the meridian of any place, if moving in the plane of the equator with an equable motion. It is the mean of the true solar days throughout the year, and consists of twenty-four hours as measured by a time-piece, which, on some days of the year, is as much faster than the sun-dial, as on other days the sun-dial is faster than the time-piece.

Sidereal Day is the time which elapses between consecutive returns of any fixed *stars* to the same meridian, or, in other words, the period which the earth takes to *accomplish one* rotation on its axis. This period is unvarying and immutable—

23 hours, 56 minutes, 4 seconds—which would always be the length of the solar day, if the earth stood still in space, and only turned upon its axis.

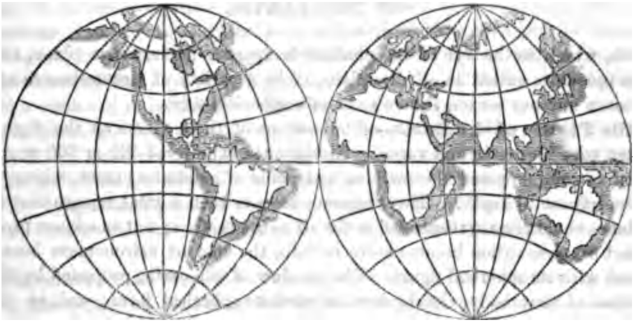
In comparison with the immense distance of the stars, the diameter of the earth's orbit is but a point; and, consequently, in relation to them the diurnal rotation is performed precisely the same as if our globe had no translation in space. This is not the case in relation to the sun, a nearer neighbour; and owing to the earth's change of place, somewhat more than one diurnal revolution, or twenty-four hours, is required to bring the sun round again to the same meridian.

Astronomical Day is reckoned from noon to noon; and, consisting of the same length of twenty-four hours in all latitudes, is called a natural day.

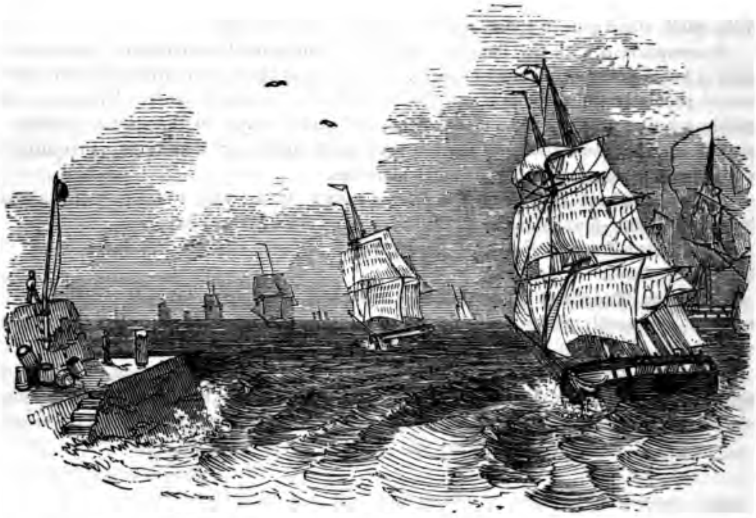
Artificial Day is the time between sunrise and sunset, and varies with the latitude of places.

Solar or Tropical Year is the time which the earth takes in moving in its orbit, or apparently the sun in the ecliptic, from one equinox or tropic to the same again, consisting of 365 days, 5 hours, 48 minutes, 49 seconds.

Sidereal Year is the time occupied by the earth in moving in its orbit, or apparently the sun in the ecliptic, from a determinate point in relation to any fixed star to the same point again, and consists of 365 days, 6 hours, 9 minutes, 12 seconds.



PARALLEL AND MERIDIAN LINES ON THE STEREOGRAPHIC PROJECTION.



ON THE EARTH.

The earth, which to the eye of its inhabitants appears an immense plane, stretching out to an indefinite extent in all directions, is, by a variety of circumstances appealing to our reason and our senses, shown to be of a spherical form.

On the Figure of the Earth.—The notions of the ancients on the figure of the earth were very uncertain and vague. Xenophanes, who lived 500 or 600 years before the Christian era, supposed the earth to be a plane of indefinite extent, whose foundations were infinite in depth. There appears to have been a great repugnance to admit that a planet could remain suspended in the air as the earth is; but as science progressed, and the art of observation became more certain, the ancient astronomers became well acquainted with its spherical figure. The shadow of the earth, projected by the moon in an eclipse of that body, clearly demonstrated its spherical form; and, on the other hand, the measures of its circumference, which the Grecian and Arabian astronomers have recorded, indicate clearly that, when reduced to a common modulus, their agreement is sufficiently striking to show that they possessed some knowledge of its dimensions. The natural pride of man, however, placed it in the centre of the universe; and according to the doctrines of the Ionian school, the solid crystalline orb, to which the stars were supposed to be attached, revolved around it in the space of 24 hours. The celebrated Aristotle, whose philosophy reigned for many centuries, was of this opinion, but he supposed that the motions of the sun, moon, and planets were performed with solid heavens, but at a nearer distance; and to explain their proper motions, he considered that a presiding genius was placed in each planet.

The usual arguments brought forward in favour of the spherical figure of the earth, *are, that navigators* sail around it, setting out in an easterly, and returning in a westerly direction; but a happy illustration is given in the appearance of a vessel as it approaches the shore, or in leaving a harbour, or a succession of steam-vessels at sea, as shown in our engraving. In the first case, we see at a distance the upper parts of the vessel, and gradually the lower parts, till finally the hull; the most conspicuous parts of the vessel, in other circumstances, appear from beneath the waters. In the case of leaving a harbour, the contrary appeaance takes place.

These appearances arise from the convexity of the water between the eye and the object; for if the surface of the sea were a dead level, the largest objects would be visible the longest.

Other arguments are adduced:—Upon the bosom of the ocean, or in the midst of an extensive plain, the boundary of vision is a well-defined circle, and this circular horizon is a certain indication of the circular figure of the body to which it relates.

Navigators proceeding in the same general direction, east or west, have arrived at the same point from whence they have started. This enterprise, now so common, was first undertaken by Ferdinand Magellan, who sailed westerly from Seville, in Spain, August 10, 1519, passed the extremity of the South American continent, entered the Pacific, reached the Philippine islands, where he was killed in a skirmish, but one of his ships arrived at St. Lucar, near Seville, September 7th, 1522.

Voyages of circumnavigation demonstrate the convexity of the earth, east or west, or that its form must be either globular or cylindrical. The convexity of the surface, north and south, is shown by the gradual declination and rise of the north and south circumpolar stars as the equator is approached and receded from, which proves the figure of the earth not to be that of a cylinder but of a sphere.

Though spherical, like the other planets whose round discs are defined by the telescope, yet the earth is not a perfect sphere, whose circumference is everywhere at an equal distance from the centre. It is more convex within the tropics than towards the poles, the equatorial diameter being longer than the polar; so that its general shape is that of an oblate spheroid, bulging out in the middle and flattened at the two opposite sides.

This proposition of Newton, the result alone of theory, has been amply confirmed by accurate measurements conducted by the most eminent mathematicians in various places, from the equator to the polar circle. He conceived that the velocity of the earth's daily rotation upon its axis being the greatest at the equator, the consequent greater action there of the centrifugal force would produce a bulging out of the surface in the equatorial regions, and a flattening at the poles. But as one of the first fundamental principles of astronomy consists in an accurate determination of the figure and dimensions of the earth, we shall briefly describe the methods adopted by astronomers for this purpose. The problem requires that all our operations must be carried on at the surface, as we cannot remove ourselves from the earth. In shifting our positions on the surface, we will take as a point of reference the successive appearances of the fixed stars. These bodies are so immensely distant, that at all positions on the earth the rays emitted by them may be considered as parallel. If we shift our latitude 10° more southerly, other objects invisible at the former station *will come* under our notice near the south horizon, whilst those near the north horizon will vanish from our view. If the difference of distance on the same parallel of longitude be measured to produce a change of 10° in the apparent altitude of a star, we shall find, roughly speaking, that for this variation there will be a corresponding distance on the earth's surface of nearly 695 miles. And this is the principle on which the first recorded attempt of one of the ancient astronomers was founded.

Modern astronomers proceed on the same principle in a more accurate manner, and with every possible precaution to insure a correct result. In England the first measurement of a degree was that by Norwood, in 1635, who found its length at the mean latitude of London and York, or 52° 45′, equal to 367,196 English feet, or 69 miles 288 yards. This determination, however, is not entitled to much weight, as it appears that his latitudes were obtained by the solstitial zenith distances of the sun observed with a five-feet sextant, and it is feared, from other circumstances, that he was not sufficiently careful in his reductions to the meridian. Snell, Picard, and Cassini determined the length of a degree of latitude with a considerable accordance. It does not, however, appear that the deviation of the earth from a strictly spherical form was noticed till 1672, when Picard found that the pendulum of his transit clock, which beat seconds at Paris, required to be made shorter to beat seconds at the station at the island of Cayenne (Lat. 4½° N.). This deviation of the earth from a spherical form was that which Newton and Huygens predicted, from the theoretical considerations of a revolving body, would be the case. But it is not from theoretical considerations alone that the form of the earth is adduced. In 1735, the French Academy fitted out an expedition for the purpose of determining its figure, with better instruments and methods than had been previously in use. Their proceedings became celebrated for the additions made to our astronomical knowledge. One of the stations fixed upon was near the equator at Peru, under the direction of Bouguer; the other at Lapland, under the superintendence of Maupertius, Clairault, and other celebrated men. All the measures taken by these astronomers denoted a decided ellipticity, but still the observations were not sufficiently numerous to infer its amount. At the Cape of Good Hope, in latitude 33° 18′ south, the celebrated La Caille measured an arc of the meridian; and in North America, in the plains of Pennsylvania, near the Alleghany Mountains, Mason and Dixon also measured a degree of the meridian in north latitude 39° 12′.

In Italy we have two measures of degrees—the first we owe to Boscovid and Le Maire, the other to Beccaria.

The surveys undertaken in places near the neighbourhood of mountains are very often affected by the attractions of those mountains. Thus, in the South American surveys the attraction of the Chimborazo Mountains affected the deviation of the plumb-line by a quantity equal to 7½″. In latter times, in a survey undertaken by Plana and Castini, in Italy, the plumb-line was affected in a very perceptible manner, so much so that the resulting value of 1° in measure was 674 toises too large, from the known ellipticity of the earth deduced from a combination of observations at France and Peru. From all recorded measures, the length of a degree was greatest at the poles, and diminished gradually at the equator. If we lay down graphically the length of a degree from the different observations, we shall find that the intersections of these radii will form a curve, which will differ from the centre of the sphere in the following manner:—At the pole, the intersection or radius will touch the axis; it will afterwards recede from it, the convexity being averted to the polar axis; till, finally, at the equator it will be perpendicular to that at the poles. The curve thus traced is termed the evolute.

In the annexed (Fig. 5), P Q and E E are the minor and major axes of the ellipsoid of revolution; *m m'* and *n n* are measurements of a degree of latitude at the equator and the pole, which show that a longer arc, *n n'*, is required at the latter to form an equivalent angle (deduced from observations of stars) than at the equator. The effect of this flattening or compression is $\frac{1}{300}$.

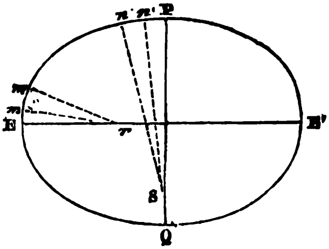


Fig. 5.

The practical method, then, of determining the length of a degree at the different positions of the earth's surface is to find, first of all, the angle included between the verticals of the two stations, by the differences of zenith distances of certain selected stars. This is performed by means of a zenith sector, the stars being chosen near the zenith, in order to eliminate any uncertainty with regard to atmospherical refraction. But the adjustments of a zenith sector essentially depend on the accurate verticality of the plumb-line, which has been, as already stated, considerably affected by the attraction of neighbouring mountains. In Fig. 6, C M A is a surface of the earth, M a mountain, A B the direction of the plumb-line if the mountain did not exist, A B' the observed direction of the plumb-line. In a similar manner, C D should be the real direction of the plumb-line, and C D' the observed direction of the plumb-line. The effect of this attraction will be easily seen to affect observations by the zenith sector, and we may infer, from this circumstance, that the irreconcilable differences in the results of some surveys have been thus occasioned.

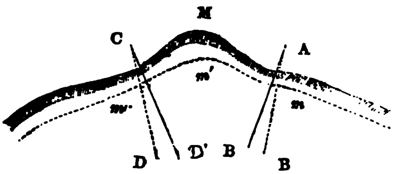


Fig. 6.

The measurement of the distance between the two stations, extending the whole length of a kingdom, is effected by a series of triangulations in the following manner:—In the figure (Fig. 7), the length of the meridian A *r* is required; and for this purpose we select certain stations A, B, C, D, &c., as clock-towers or elevated objects. Conceive these to be joined as in the figure, and thus to form a series of triangles, as A B C, B C D, C D E, &c.

If we know all the sides and all the angles of these different triangles, as well as the angle formed with the meridian A *m n*, with the side A B, we can conclude easily the lengths of the different parts, A *m*, *m n*, *n p*, of this meridian. In fact, in the triangle A B *m* we know the side A B, and the two adjacent angles A B *m*, B A *m*, whence we can readily find the side A *m*, which forms the first portion of the meridian, as well as B *m*, and the angle B *m* A. In the triangle *m* C *n* we throw the side C *m*, which is the difference between B C and B *m*, and the two adjacent angles *m* C *n*, C *m n*, the second being equal to B *m* A, previously determined. We arrive at the same conclusion with the side *m n*, which forms the second portion of the meridian,

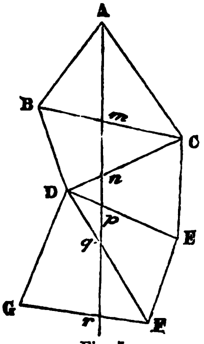
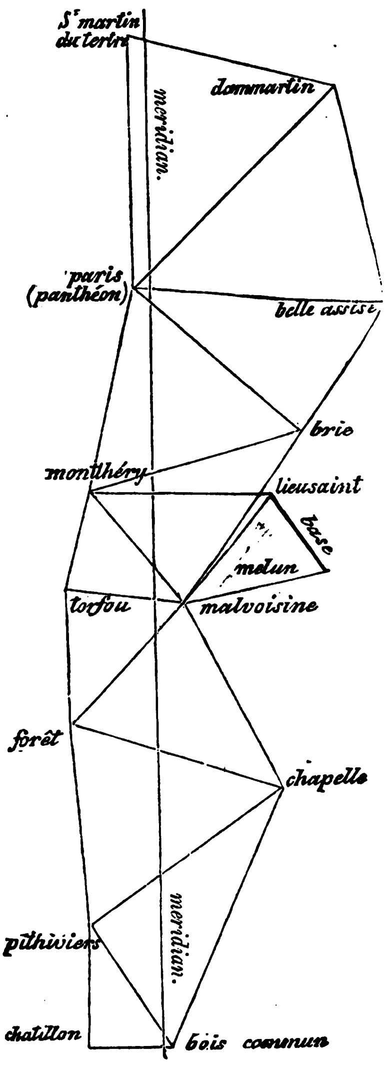


Fig. 7.

and at the same time the side C *n*, and the angle C *n m*. In the same manner the triangle D *n p* will show us the third part of the meridian, and by this proceeding we have determined all the parts of the meridian of the point A.



Measurement of the Arc of a Meridian in France.

But we do not require a measurement of all the sides—it will be necessary only to measure a certain part termed the *base*. Suppose that the side A B be the part measured, then the triangle A B C is entirely known, since we know one side and the three angles, and we can find the lengths of the two sides, A C, B C. In the same manner, the knowledge of the three angles of the triangle B C D, and of the side B C, that we find, permits the determination of the length of each of the two sides B D and C D; and proceeding in this manner, we find all the sides and angles of all the triangles as well as if we had measured these lines directly.

In determining an arc of the meridian, we know well the point of departure, but we do not know where the second extremity is situated. We can find, it is true, after having determined conformably to what precedes, the length of the portion F *r* of the side F G, by measuring the distance F *r*; but besides that this often presents great practical difficulties, it will frequently happen that the point *r* would not be favourably placed for erecting an instrument such as a repeating circle or theodolite. We have also sometimes occasion to know the latitude of the point *r*, as well as that of the point A, in order to deduce the angle included between the verticals drawn through the two points. To attain this, we observe the latitudes of the two extremities F G of the side on which the point *r* is situated; we can easily compute the latitude of the point *r*, by the knowledge that we have ascertained of the distances comprised between this point *r* and the two points F G. Having the length of the line *F A, relatively* to the dimensions of the earth, we can admit that in going from

F to G, along the line F G, the latitude varies proportionally to the distance passed over on this line.

Meridian of France.—One of the greatest measurements of an arc of the meridian was that performed at the end of the last century, by the celebrated astronomers Delambre and Méchain. The arc which they measured took its departure at Dunkirk and across France at its greatest length, terminating in Spain, near Barcelona, the Pantheon at Paris forming the summit level. Part of this survey is contained in the preceding figure; the base line, on which the success of the whole measurement depends, was measured with every possible precaution by means of rods of platinum, and was found to be 6075·98 metres. The angles of the triangles were measured by means of the repeating circle, and the lengths of the different sides of the triangles were successively determined by the method before mentioned.

In order to have some independent check on the result, a second base was measured near Perpignan, that is, near the southern extremity of the series of triangles. The length of this second base, reduced to the level of the sea, was found to be 6006·25 metres. In comparing the length thus obtained to that of this same base, deduced from the successive calculations, we have not found a greater difference between the two results than 10 inches 8 lines (cent. metres, 288). So small a difference on a length of more than 6000 toises is surprising, especially when we consider the immense distance which separates the base of Melun and Perpignan, a distance of more than 450,000 toises. This certainly shows that the operations had been executed with great care.

The results arrived at by these measurements showed at once that the value of a degree was different at the equator and the poles, being least at the equator and greatest near the poles.

From a judicious combination of the observations at all the stations, Bessel deduces the following elements of the earth's figure :—

Equatorial diameter, 41847199·9966 feet, or 7925·606 miles.
Polar diameter . . . 41707314·3324 feet, or 7399·113 miles.
Which shows an ellipticity of $\frac{1}{300}$.

The following are the principal results on which the above elements are founded, viz :—

Name of Place.	Mean Latitude.	Length of a Degree in English feet.
Peru	1° 31′ 1″	362,808
India	12° 32′ 21″	363,013
France and Spain	46° 8′ 6″	364,649
England	52° 2′ 20″	364,914
Lapland	66° 20′ 10″	365,782

In European latitudes, actual observation gives :—

Stations.	Mean Lat.	English Feet.
Formentera	40° 0′ 50″	364,206
Montjouy	42° 17′ 29″	364,239
Carcassonne	44° 41′ 49″	364,347
Evaux	47° 30′ 46″	364,935
Pantheon	49° 56′ 29″	365,052
Dunkirk	51° 15′ 25″	365,116
Greenwich.		

From Bessel's Elements the next table is formed :—

Lat.	Length of a Degree of Meridian. English Feet.	Length of a Degree of Latitude. English Feet.	Radius Vector.	Angle of Vertical.
°				′ ″
0	367,749	365,186	1·000000	0 0·0
5	362,776	363,805	0·999975	1 59·5
10	362,858	359,674	0·999899	3 55·5
15	362,992	352,821	0·999778	5 44·3
20	363,174	343,296	0·999612	7 22·7
25	363,398	331,168	0·999407	8 47·9
30	363,658	316,524	0·999170	9 57·1
35	363,946	299,472	0·998907	10 48·3
40	364,254	280,135	0·998626	11 19·8
45	365,572	258,657	0·998336	11 30·5
50	364,590	435,198	0·998045	11 20·5
55	365,199	209,933	0·997763	10 49·7
60	365,489	183,052	0·997499	9 59·1
65	365,752	154,759	0·997259	8 50·2
70	365,979	125,271	0·997052	7 25·1
75	366,163	94,812	0·996884	5 46·3
80	366,300	63,620	0·996759	3 57·0
85	366,382	31,568	0·996683	2 0·3
90	366,411	0	0·996657	0 0·0

The elements of the earth's figure, deduced by Professor Airy (Encyclopæ Metrop., Art. *Figure of the Earth*), are—

Equatorial diameter in miles . . .	7925·648.
Polar diameter in miles	7899·170.

The equatorial circumference being a little less than 25,000—accurately, 24,899.

To illustrate the very trifling proportion which subsists between the inequalit of the earth's surface and its entire volume, we may suppose an artificial ball eight inches in diameter to represent our globe, when the proper proportionate elevation be assigned to its highest mountains would be $\frac{1}{80}$th of an inch.

Rotation of the Earth.—Two principal motions belong to our planet; one rotation upon its axis, called its diurnal motion, producing the succession of day night; and another of progression in space, or revolution round the sun, called annual motion, causing the vicissitude of the seasons. Both of these motions are be understood of the whole earth, its interior substance, its superficial masses of l and water, the surrounding atmosphere, and the clouds in suspension over it; both motions are in the same direction from west to east.

The exact time occupied by the diurnal rotation is 23 hours, 56 minutes, 4·09 seconds. This forms a sidereal day; so called, because in that time the st appear to complete one revolution round the earth. A star which is on the merid of a place at a given period, will be on the meridian again after that interval. Bu while the earth rotates upon its axis it is also moving in its orbit round the sun, it require twenty-four hours, upon an average through the year, for the sun to p *from the meridian* of a place to the same meridian again. This forms a solar d *longer thana sidereal*; and consequently, in the course of the earth's annual re lution round the sun, while we have 365 of the former we have 366 of the latter, or that number of complete rotations of the earth upon its axis. Hence the well-known fact, that, in travelling round the globe, a person finds, on arriving at the point whence he set out, that he has gained or lost a day in his reckoning of time, according as he has travelled east or west, as compared with the reckoning of those who have remained at rest.

The earth's motion upon its axis is perfectly uniform and equable. Sidereal days, therefore, are always of the same length, every rotation being accomplished in the same time; but the motion of our planet in its orbit being unequal, sometimes faster, sometimes slower, solar days vary in length at different times of the year. Hence the hour shown by a well-regulated clock and a true sun-dial is scarcely ever the same; the difference between them, sometimes amounting to 16¼ minutes, being called the equation of time. About the 21st of December, the solar day is half a minute longer, and about the 21st of September nearly as much shorter than 24 hours; which is an average of all the solar days throughout the year.

The rotation of the earth upon its axis is not susceptible of ocular evidence like that which the observation of spots upon the sun, and some of the planets, affords of the same fact in relation to those bodies. Nevertheless, the truth of the doctrine is established by various considerations.

Either the globe revolves upon an axis every twenty-four hours, or the whole universe, including the sun, moon, comets, and fixed stars, accomplishes a revolution round the earth in the same time. No third opinion upon the subject can possibly be held. In the latter case it is evident, from the distance of the celestial bodies, that their diurnal revolution around our planet must involve a rate of motion that is utterly inconceivable. The sun must travel at the rate of 400,000 miles a minute, the nearer stars with the velocity of upwards of 1,000,000,000 of miles a second, and the more distant with a rapidity which no numbers can express. It is absurd to suppose this, when the end to be gained requires only our little globe to revolve upon itself.

The following considerations present themselves. If the diurnal movement which we see, were attributable to a movement of the stars, E, E', E'' (Fig. 8) would describe uniformly a circle situated in a plane, T P, perpendicular to the plane of the poles and the centres of the circle, C'', C, C', for the reduced perpendiculars of the stars upon this line; that is to say, the most distant points from the earth T. But we know that when a body describes a circle with a uniform motion, it is attracted towards the centre of the circle by a constantly acting force, of which the magnitude depends at once upon its rate of motion and the radius of the circle which it describes. The stars E, E' E'' cannot move in the circle of which we speak without being attracted towards the points *C, C', C'', situated* upon the line of the poles.

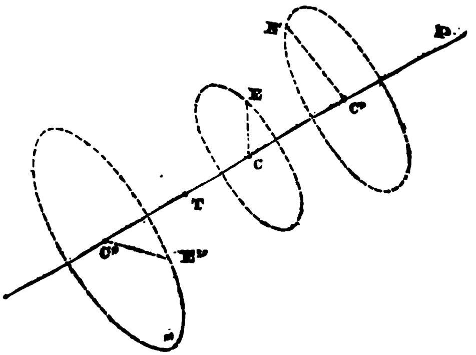


Fig. 8.

But something like demonstrative proof of real circumstances may be adduced. If the earth rotates, the summit of a high tower, having a larger circle of rotation to describe in the same time than the base, must obviously move with greater rapidity; a stone, therefore, dropped from the summit, leaving it with a greater momentum, will move faster through the whole of its descent than the base, and reach the ground a little in advance, or easterly, of the foot of the perpendicular, the direction of the earth's rotation. Owing to the small height of buildings suitable for the purpose, this is a very difficult matter to test; but experiments have been conducted with this result, in 1804, in St. Michael's Tower, at Hamburg, and, in 1805, in a coal-pit at Schlebusch, in the county of Mark. Let the circle E (Fig. 9) be the equatorial circumference of the earth, the line T a tower perpendicular to C the centre, the circle M will then be the circumference described by the summit of the tower S, in the course of one rotation of the earth upon its axis. If we suppose the base of the tower *b* to pass to *c*, the summit S will in the same time pass to *a*, and this being the larger arc, it follows that the summit must travel faster than the base. If then the earth rotates eastward, a ball dropped from the summit will leave it with its momentum, and move faster eastward through the whole of its descent than the base. The result will be that it will deviate a little from the plumb-line, and fall a little to the east of *c*.

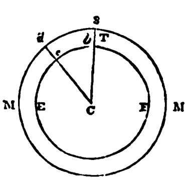


Fig. 9.

Again: a pendulum of a given length, which makes 86,535 vibrations in a day at London, will make only 86,400 in the same time if transported to the equator. This shows that the force of gravity which produces its oscillations must be least where the movement is the slowest, or less at the equator than at London. Now assuming the earth's rotation, its equatorial regions, where the circle of the circumference is the greatest, must revolve with greater velocity than those which are situated towards the poles; consequently, the tendency to fly off from the centre is greater there, which proportionably neutralizes the force of gravity, and accounts for the unequal action of the pendulum.

At the equator, the rate of the rotation is about 1042 miles an hour, or 17 miles a minute; at 30° of north latitude it is 14 miles a minute; at 45°, or about the centre of France, it is 11.

Annual Motion of the Earth.—The annual motion of the earth, or its orbital movement round the sun, occupies a period of 365 days, 5 hours, 48 minutes, 49·7 seconds. This forms the solar year, or the period which the sun appears to take, through the actual procession of our planet, in passing from a particular point of the ecliptic, say the first point of Aries, where the ecliptic and the equator intersect, to the same point again. It is also called the tropical year, because the interval occupied by the sun in visiting the tropics and returning to the equator. The sidereal year, or the space of time which the sun takes in apparently passing from any fixed star till it returns to it again, is 365 days, 6 hours, 9 minutes, 9·7 seconds; rather more than 20 minutes longer than the tropical year, which is due to a slow annual displacement of the equinoctial points.

As the mean distance of the earth from the sun is 95,000,000 of miles, the diameter of the orbit is 190,000,000 of miles, and its linear extent near 600,000,000 of miles. This enormous distance is traversed at the rate of 68,000 miles an hour, or 19 miles in a second. This is the mean velocity for the year; but in January the earth travels at the rate of 69,600 miles an hour; which is more than 3,000 miles an hour its rate of motion in July, when it is only 66,400 miles an hour. This results from the second law of Kepler, to which we shall afterwards return.

Hipparchus, two thousand years ago, was the first who closely approximated to the true length of the solar or tropical year. His determination of 365 days, 5 hours, 55 minutes, 12 seconds, exhibits a value but slightly in excess of the truth.

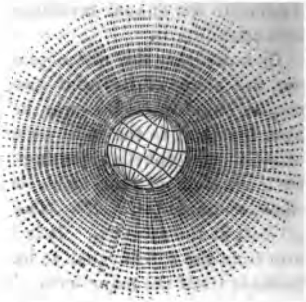
The earth's motion of translation in space, like that of its rotation, is imperceptible by us; resembling that we experience in calmly floating down a stream, when surrounding stationary objects appear to be in movement, and our senses are lulled into complete forgetfulness of our own progression. It appeals not to the eye, as in the case of the other planets, which are seen to be constantly changing their place; but, besides the evidence of strong probability in its favour, it has received sensible confirmation from the discovery of the aberration of the stars.

In its annual revolution round the sun, the axis of the earth, inclined 23½° from a line perpendicular to the plane of the orbit, maintains invariably the same position, causing the phenomena of the seasons. An attempt is made in the engraving at page 219 to delineate the annual revolutions of the earth about the sun, and the other phenomena attending it.

By measuring the distances of the earth from the sun, at different times of the year, the shape of its orbit has been ascertained. These distances, as they were unequal, could not, of course, be semi-diameters of a circle, but they corresponded, taken together, to the radii vectores of an ellipse. The straight line connecting the perihelion and aphelion, passing through the centre of the sun, is the line of *apsides.* The inclination of the earth's orbit to its equator, or the so-called *obliquity of the ecliptic*, amounts to 23° 27′. The velocity of the earth is greatest at the perihelion and least at the aphelion. It is further to be observed, that the mean distance of the earth from the sun is equal to half the major axis of the earth's orbit, and the line of apsides is itself the major axis. There are four noteworthy points in the earth's orbit in the engraving, viz., those which mark the beginning of the four seasons. Two of these points are called the *solstices*—they mark the beginning of winter and summer. The straight line uniting them, passing through the centre of the sun, is called the *solstitial colure.* The two other points are the *equinoxes*, vernal and autumnal, marking the commencement of spring and autumn. The straight line cutting them at right angles, and passing through the centre of the sun, is the *equinoctial colure.* This engraving also represents—1. Group of stars in Aquarius.—2. Mars, as seen Aug. 16, 1830, by Sir John Herschel at Slough.—3. Group of stars in the constellation of Hercules.—4. Great Comet, as seen Sept. 10, 1811.—5. Groups in Cancer.—6. A star in the middle of the elliptical nebula.—7. Comet of 1811.—8. Nebula in Ursa Major. —9. Bright elliptical nebula in Sagittarius.—10. Three stars in Auriga.—11. View of Saturn with his rings.—12. Nebula in Gemini.—13, 14. Nebula in Andromeda.—15. Nebula in Monoceros.—16. The comet of 1819.—18. Streaks in Jupiter, as observed Sept. 23, 1832, by Sir J. Herschel.—19. Group of stars in Cancer.

The earth is represented on the first day of each of the twelve months of the year, the solar distances corresponding to these twelve positions, and the shape of the earth's orbit. The deeper circle surrounding the pole at a short distance, is intended to represent the parallel of latitude of Europe, or the hour circle of that place divided into twenty-four hours. Although at the end of December the earth is nearest the sun, yet, at that time in the northern hemisphere, the heat is less than at any other. The reason of this lies in the fact of the short days and long nights, as well as that the sun's rays fall very obliquely on the earth, traversing a longer path through the atmosphere, and consequently losing much of their heating power. At the beginning of July, on the contrary, although then the earth is at its greatest distance, the temperature of the northern hemisphere is greatest, on account of the long days and short nights, and the great altitude of the sun at noon. It must not be forgotten, however, that owing to the precession of the equinoxes, these signs no longer correspond to the constellations of the same name, so that now the sign Pisces corresponds to the constellation Aries, the sign Aries to the constellation Taurus, &c. It is further evident, that if the earth at the beginning of spring, summer, autumn, and winter, should be in the signs, Aries, Cancer, Libra, and Capricornus, respectively, then the sun, as being always directly opposite in the ecliptic, will be in the signs Libra, Capricornus, Aries, and Cancer.

An atmosphere—the region of the winds, lightning, and meteoric corruscations—upon which respiring beings depend for vitality, surrounds our globe, and is one of its most important attributes; chiefly interesting to the astronomer, on account of its effect in displaying to us the heavenly bodies and diffusing the rays of light in every direction around us. The extent of this wonderful and benign envelope is not precisely known; but its density diminishes as we ascend from the surface, and, at a very inconsiderable elevation, it becomes so rare as to interfere with the functions of existence. The diagram represents the engirdling atmosphere of our planet, various strata of air resting upon it, the upper pressing upon the lower, and causing the interior to be more dense than the exterior strata.

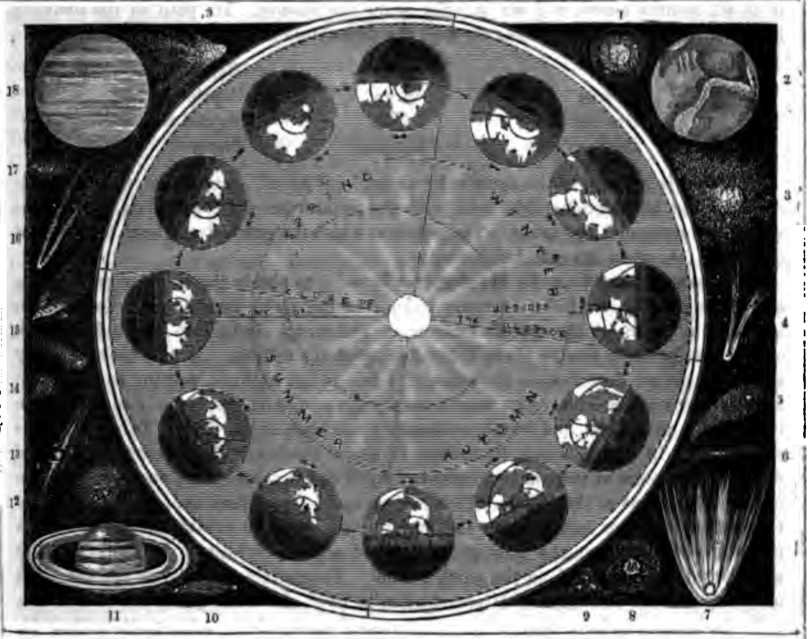


The density of the earth is $4\frac{8}{10}$ that of water, so that our globe would counterpoise $4\frac{8}{10}$ globes of the same size, composed of materials of the same specific gravity as water. Yet, abandoned to the solar attraction, it would require 64 days, 13 hours, to fall upon the sun.

The varying intensity of the force of gravity at the surface of the earth, as shown by the unequal action of the pendulum, which vibrates slower at the equator than in other places, is as 1 at the equator to $1\frac{1}{194}$ at the poles. A body, therefore, weighing 194 pounds at the equator, would weigh 195 pounds at the north pole.

The place of the earth in the system is a favoured one, where nearly all the planets are visible to the naked eye. While they appear in our heavens, Jupiter, Venus, and occasionally Mars, shining with great splendour, our planet may be presumed to return the compliment, exhibiting to Venus, at the time of her inferior conjunction, when she is nearest to us, a full orb resplendent through her whole night.

◆



ON THE SUN.

This stupendous luminary, to which we are indebted for many of the blessings we enjoy—the source of light and heat, and which also contributes materially to the development of vegetation—will now claim our attention. It also serves as a standard for the regulation of our calendar. The orbit which it appears to describe about the earth fixes our year, whilst its displacements in the ecliptic regulate our seasons. A little attention will shew us, that its motions in the heavens from day to day differ from that of a fixed star. We have only to mount firmly any line of sight, properly protected by coloured glasses from the glare of sun-light, and we can readily determine, by means of a watch or chronometer, that the interval between the successive returns of the sun will be different at different times of the year; whilst the invariable constancy of the returns of the fixed stars, on the other hand, impress us at once with the notion that the earth rotates on its axis. The apparent path of the sun, however, in the heavens is not so easily traced as that of the moon, or planets whose position can be readily compared with the fixed stars. For any accurate investigations we are obliged to make use of a transit instrument and mural circle, which will be described hereafter. By these instruments we find two elements necessary to define its path, namely, its right ascension and declination. Its diameter can also be readily known. We thus find, that, in addition to its diurnal motion, it partakes of a proper motion always in the same direction from west to east, and that it is six months above, and six months below the *equator*. Its path in the heavens is termed the ecliptic, and passes through the following constellations:—

Aries.
Taurus.
Gemini.
Cancer.
Leo.
Virgo.
Libra.
Scorpio.
Sagittarius.
Capricorn.
Aquarius.
Pisces.

Its orbit is inclined to the equator by an angle of 23° 27′, termed *the obliquity of the ecliptic.* The intersections of its orbit with the equator occur at two points, termed the vernal and autumnal equinoxes. The points at which the greatest and least declinations of the sun take place are called the solstices.

Motion of the Sun in its Orbit.—In order to obtain the accurate path of the sun in the ecliptic, it will be necessary to convert the right ascension and declination, obtained by means of the transit instrument and mural circle, into longitudes, which can be readily done, having given the obliquity of the ecliptic. It will then be easily seen that the arc described in the space of a day is not uniform, varying in amount at different parts of his orbit. The maximum change occurs on the first of January, when it amounts to 61′ 10″, and which gradually diminishes to the first of July, when the value is 57′ 12″, after which its motion is again quicker. The average velocity of 59′ 11″ takes place at the commencement of April and October. The observed diameter obtained by means of the passage of his eastern and western limb over the meridian, or the vertical diameter measured by the mural circle, will also exhibit fluctuations; the maximum occurring at the time of its greatest angular velocity. It would by this appear that its distance from the earth varied. The variable velocity of the sun in its orbit engaged the attention of Hipparchus, who lived about 140 years before our era, and who appears to have been a more accurate observer of the solar motions than his predecessors. This celebrated philosopher, who was of opinion that the motion in a circular orbit was the more natural, invented an *eccentric hypothesis*, which explained the greater angular velocity, and the smaller angular velocity, but which failed at the intermediate positions. He considered that the earth was removed some distance from the centre of the sun's motion, as in the annexed diagram. If the earth be supposed to

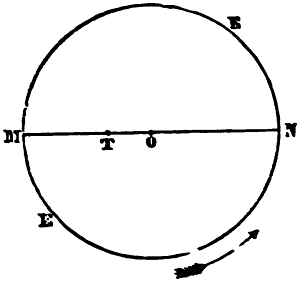


Fig. 10.

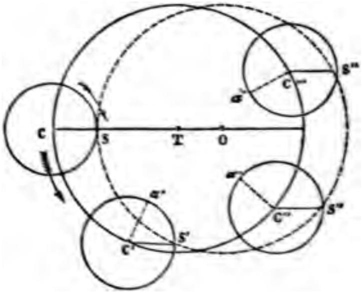


Fig. 11.

be placed at T (Fig. 10), instead of at O, the centre of the circle M E N E, it will follow that the equal arcs described by the sun in equal times will not appear uniform, and thus the motion will be slower at N than at M, and will gradually increase from the former to the latter point. Hipparchus explained this change in the angular velocity of the sun, by means of the epicycle. Supposing that the earth is placed at T (Fig. 11), with a radius T C, describe the circle C C′ C″. If the sun be fixed at S, in the smaller circle C S (which is called the epicycle), and makes a complete revolution in the direction of the arrow, in the same time that the circle itself passes round the earth; it would follow that the sun would always remain at equal distances from the point T, as at *a*′, *a*″, *a*‴. But if the sun be supposed to travel uniformly, and always preserve the same direction C S, C′ S′, C″ S″ &c., it is plain that its distance will vary from the point T. If we make T O equal to C S, and describe a circle from the centre O, this circle will pass through all the points S S′ S″ S‴. This agrees with the preceding explanation, for whilst the sun describes a circle round O, the earth T is placed eccentrically in this circle. The amount of this eccentricity may be determined by comparing the angular velocities of the sun when at apogee and perigee; and we thus find T M : T N :: 3431·5; 3670·1; whence it would result that the greatest, the mean, and the least, distance, supposing the earth's orbit to be circular and the motion uniform, would be 1·0338 : 1·0000 : and 0·9662 respectively; the eccentricity of the earth's orbit would be 0·0338, or $\frac{1}{30}$. We may also arrive at a knowledge of the eccentricity *e*, by comparing the diameters of the sun at different epochs.

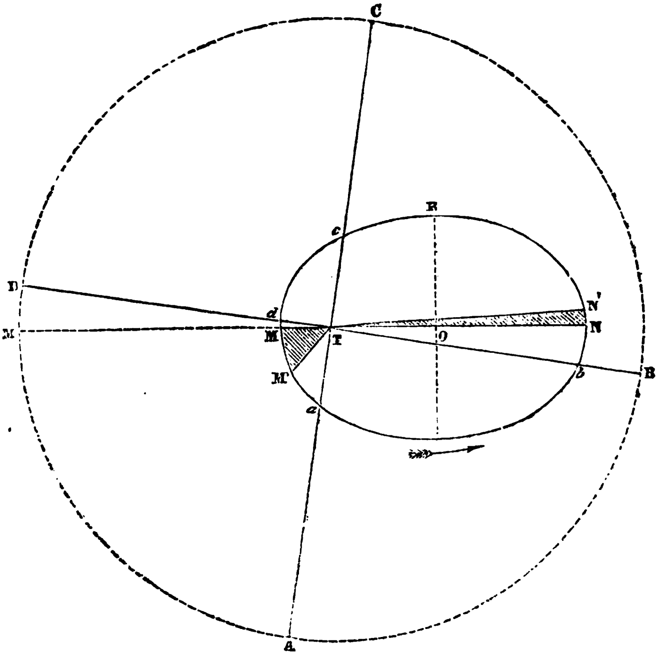
Thus, let *d* be the least diameter, D the greatest diameter, and δ the mean diameter, then

$$d=\frac{\delta}{1+e},\quad D=\frac{\delta}{1-e},\quad \frac{\delta}{D}=\frac{1-e}{1+e},\quad \text{and } e=\frac{D-d}{D+d}$$

In this case D = 1955″·6, *d* = 1891″, and the resulting eccentricity is 0·0168; consequently the greatest and least distances would be 1·0168 and 0·9832. It would follow from this, that the motion of the sun in its orbit cannot be uniform, but that it must move really more rapid when at perigee than at apogee. The variation of its angular velocity is about twice as great as that of its distance. We may arrive at a knowledge of the true orbit of the earth round the sun by marking off the longitudes daily, and the proportional radii (estimated from the observed diameters). It will thus be found that the figure, roughly traced, will differ considerably from a circle, and will be an ellipse, having the earth in one of its foci. It was not, however, by these simple means that Kepler made the great discovery of the elliptic motion of the earth and planets, but by a long and laborious discussion of the observations of Mars, made by his master and patron Tycho Brahé.

By further consideration on the motions of the planets, Kepler discovered that their angular velocity diminishes in the same proportion as the square of the distance increases, and that the areas described by the radius vector are proportionate to the times of description. If we suppose a planet to pass through M M′ and N N′ in the same space of time, it would follow from this that the areas M, T, M′, and N, T, N′, are equal, and that the angular velocities are greatest at perigee and least at apogee. The form of the ellipse which the earth describes about the sun differs but little from that of a circle, as the distance O T is only one-sixtieth part of the semi-major axis O M; and if such an ellipse were described of a yard in diameter, the difference between the major and minor axis would be almost invisible.

The direction of the line of the apsides of the earth being known, its position in respect to the fixed stars has next to be determined. The line to which this is referred is that of D B of the equinoxes. At the present time, the inclination between the major axis M N of the earth's orbit, and the line D B of the solstices, is nearly 10°, and is gradually increasing—the major axis increasing in the same direction as the sun's motion, at the rate of 62″ annually. The duration of the seasons is consequently unequal, as may be seen by comparing the areas, *a* T *b*, *b* T *c*, *c* T *d*, and *d* T *a*, which being proportionate to the times, the sun will consequently remain longer in the quadrant *b* T *c*, than in *b* T *a*, &c. Whilst, therefore, the length of the year is constant, the duration



of the seasons is subject to a slight change from year to year. At the present time, the lengths of the different seasons are as follows:—

	d.	*h.*	*m.*
Spring	92	20	59
Summer	93	14	13
Autumn	89	18	35
Winter	89	0	2

The sun, therefore, remains nearly eight days longer in the northern than in the southern hemisphere. By taking into account the rate of the progressive motion of the apsides, M N, which is 61″·9 per annum, according both to theory and observation, the time when the line of solstices, D B, coincided with the line of apsides may be determined; and it has been concluded that this occurred in the year 1250. At that time the autumn and winter quarters were of equal length, and likewise the spring and summer; but the summer was longer than the winter.

Distance of the Sun.—Before we make any very accurate progress in the motion of the sun in its orbit, we must first discover the sun's actual distance, or the angle to which the earth subtends at the sun.

Whilst the relative distances of the planets from the sun, expressed in values of the earth's distance, can be determined by means of Kepler's laws, the actual distance of the earth from the sun, expressed in some known measure, can only be arrived at by a long and tedious process; and that only on some rare occasions. The micrometrical apparatus of the present day has, indeed, successfully detected changes in the positions of double stars, &c., of one-fiftieth the actual amount of the solar parallax; but the objects under those circumstances are much more favourably situated for observation, and considerably better defined than the sun; and the irregularities of refraction scarcely, if at all, enter into the question. If these causes of error could be removed from the description and definition of parallax which have been previously given, it is plain that the parallax of the sun might be deduced in the manner hitherto described, viz., by observing its zenith distance in the northern and southern hemispheres, and in the same meridian; or, if the meridians are different, the change in the sun's declination may be calculated, and applied in order to reduce the one position to the other for the purpose of comparison. Such a method has not, however, succeeded with the sun, in consequence of the smallness of its parallax, which is also mixed up with the uncertainty of the observations and the errors of refraction. In modern times, when increased instrumental accuracy and skill in their use have been brought to bear on this question, the result arrived at by this means may be more approximate. The method by which the parallax, as constantly made use of at the present time, has been determined, is that deduced by means of the passage of Venus over the sun's disc—or the *transit* of Venus, as it is more commonly called—the value of which was first pointed out by Dr. Halley. This method, although indirect, gives the solar parallax with great accuracy; and so well was it determined at the last passage of 1769, that most astronomers are of opinion that it can scarcely be amended by future determinations.

The manner in which this phenomenon is applied to the determination of the solar parallax will be seen by the diagram (Fig. 12). The relative distances of the earth and Venus from the sun's centre are accurately known. Let us suppose that when Venus is situated between the sun and the earth, its distance from the former is 0·73, the earth's distance being 1·00. The earth being situated at T, and two observers stationed at the opposite diameters A and B, the observer at B will perceive Venus, V, crossing the sun's disc from *c* to *d*, whilst the observer at B will see it passing from *e* to *f*. The breadth of this zone, *a b*, will be considerably greater than the diameter of the earth, A B, in consequence of the greater proximity of the planet to the earth, V S being 2·7 times greater than the line V T, and consequently the line *a b* is 2·7 greater than the diameter of the earth, A B. If the angular measure of *a b* could be obtained, this would therefore be 2·7 times greater than the earth's apparent diameter as seen from the sun, or to 5·4 times the sun's horizontal parallax. This angular measure can be found by the observers A and B taking the distance of the limb or centre of Venus from the sun's limb. The line *a b*, as here represented, is drawn much greater than it really is, for the resulting diameter of the earth is only 8½ seconds of arc; and consequently the breadth of this zone is only 46″·4, or about three-quarters of the diameter of Venus at the time of inferior conjunction, or $\frac{1}{42}$ of the mean diameter of the sun.

It is not only, however, by observing micrometrically the distance of Venus from the sun's limb that the relative distance *a b* may be deduced; it can likewise be determined by the *time* which the planet remains on the sun's disc at the several stations, or the interval elapsed between its entry at *c* and *e* to its disappearance at *d* and *f*. The motions of Venus are accurately known by means of the tables, and thus the lengths of the chords *c d* and *e f*, and therefore the distance *a b* also. It is usual, in these cases of the passage of the planets Venus and Mercury over the sun's disc, to note the time when the limbs of the planet and sun come in contact, and also, after the ingress has occurred, to note the time when they are last in contact. The same must be done at its egress. The times of interior contact can be perceived much more exactly than the exterior. If the transit is central, the duration of its passage may extend from 7h. 52m. to 7h. 54m.; and according as it is more distant from the centre, the chord will be shorter and the time in the same proportion. In the passage of 1769, the duration at Wardhus was 6h. 29m.; at Hudson's Bay, 6h. 22m.; at California, 6h. 14m. 3s.; and by Captain Cook, at Tahiti, at 6h. 6¾m. From these measures the horizontal parallax of the sun was calculated by the astronomers of the time at 8″·5693; but the results differ considerably, the entrances being 8″·2 and 9″·2. The small altitude of the sun in the northern latitude, and its consequently bad definition, made it difficult to estimate the exact moment of entrance on the solar disc. Some observers saw it as much as twenty seconds sooner than others, but the effect of this would only entail an error of one-sixtieth, or probably of only one-half of that amount, on the total parallax; so that instead of 8″·5693, we should have 8″·640 or 8″·498. These calculations have been performed anew with the utmost accuracy, by Professor Encke of Berlin. From the first transit of June 5, 1761, that celebrated astronomer has obtained the value 8″·490525 comprised between the limits of 8″·429813 and 8″·551237. The discussion of the second passage of June 3, 1769, gives 8″·5776, not greatly different from the above; but, being made under more favourable circumstances, is the value now generally adopted by astronomers. The resulting mean distance of the earth from the sun would, consequently, be $\frac{1}{\sin 8''\cdot5776}$, or 24046·9 radii of the terrestrial equator, which corresponds to 82,667,200 miles, of 60 to a degree. To pass through this distance light employs 8m. 13·15s., and in consequence we see the sun 20″·252 behind its real position. The radius of the sun at its mean distance being 960″·9, the real diameters of the two bodies will be in proportion to their apparent diameters, or as 8·578 to 960·9, or the diameter of the sun is 112·024 times greater than that of the earth. Their volumes being proportionally as the cubes of their radii, it would follow that the bulk of the sun is 1·405·845 times that of the earth.

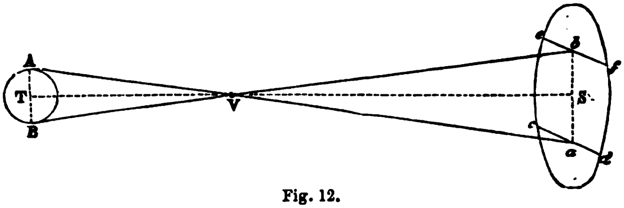


Fig. 12.

Passage of Venus across the Sun's Disc.—This phenomenon can only take place at intervals of eight years and 105 years, and the two next times will occur on Dec. 8, 1874, and Dec. 6, 1882. Since Venus comes nearly to the same point of the heavens every eight years, it may be expected that if it transits over the sun at one given epoch, it will pass over it on the eighth following year, and this generally takes place. As Venus, however, changes her latitudes during this period of eight years, it is impossible that three can follow each other in succession, as it must then pass beyond the disc of the sun, which is only 32′ in diameter; and as they can only take place when the plane is very near its nodes, it follows that for some centuries this will occur either in the months of June or December.

The mass of the sun is found to be 354936 times greater than that of the earth; and, comparing this with its volume, we find that the latter is only equal to one-fourth of that of the sun. The gravity on the surface of the sun is 28·36 times greater than that on the surface of the earth, or a body which weighs one pound on the surface of our planet, weighs twenty-eight one-third pounds on the surface of the sun, and consequently a body will fall with twenty-eight one-third times the velocity during the first second of time.

Form of Sun's Disc.—In measuring the disc of the sun with the heliometer, or any graduated instrument, it is necessary to take precautions that it is favourably situated, and not too near the horizon, where the refraction has a very sensible effect in elevating the lower part of the disc in a greater degree than the upper. When the sun is 45° above the horizon, the difference between the vertical and horizontal diameters only amounts to 1″; but when it touches the horizon, the vertical diameter is one-sixth part of the whole diameter less than the horizontal. The figure which the sun takes under those circumstances will be seen by Figs. 13 and 14, which have been constructed with exact proportions. The refraction causes the sun when really at S to appear at S′, with its apparent lower limb touching the horizon. This is the usual effect of refraction, but at the horizon many irregular local causes tend to produce changes, and it is generally found to be serrated, and with a continual whirling motion, due to the irregular motions of the atmosphere. It appears strange that the discs of the sun and moon, which, if anything, are smaller under those circumstances than when at a great altitude, are commonly remarked to be larger at the horizon than

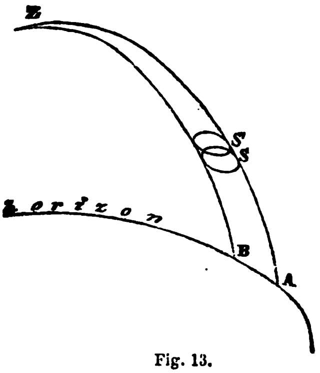


Fig. 13.

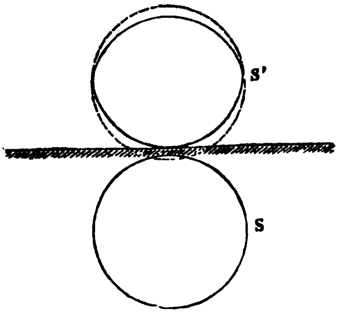


Fig. 14.

elsewhere. The ancient astronomers sought to explain this optical illusion, by the effect which the atmosphere had upon the luminous rays of the sun. It is, at the present time, more naturally attributed to the effect which a comparison with the various objects seen at the horizon produces; and, as we suppose the sun to be removed an immense distance beyond those bodies, its diameter appears greater than it really is. A similar effect takes place when the apparent areas of the constellations are compared at the zenith and the horizon; in the latter position, they appear considerably more extensive than they really are. It is, probably, due to a similar cause, that if we attempt to measure an altitude of 45° from the horizon, or the point half way between the zenith and horizon, it frequently happens that when we come to measure it with an instrument, that which we consider to be the central point is situated some degrees nearer to the horizon than it really is.

Telescopic Appearance of the Sun.—An observer, viewing the sun's surface with an instrument of moderate power, would probably not perceive any marked difference of light and shade over its general surface; but with higher powers, and a better telescope, a steady mottled appearance would become apparent, dark and light specks being softly intermingled, but not strikingly apparent when seen for the first time, or under unfavourable circumstances. Whilst the *ground* is not uniformly bright, there are very frequent portions which are decidedly brighter than the general surface, and others, which are much more remarkably darker, being as dark and black as the surrounding heavens, when viewed with the coloured glasses made use of. It is by means of these fleeting *maculæ* and *faculæ*, as they have been called, that nearly all our knowledge of the solar nature, its rotation, and the position of its axis, as well as the structure of its lurid and changeable atmosphere, is derived. These spots have constantly been perceived ever since the discovery of the telescope, and there is little doubt but that they were frequently seen before that time with the unaided eye, as they have occasionally been seen since. They sometimes cover large portions of the sun's surface, and are strikingly apparent from the contrast they afford with the bright surface surrounding them.

By whom these spots were first seen after the invention of the telescope, is a matter which has found its way into the debateable land of scientific history. It would be most natural to suppose that Galileo, who had first scanned the heavens with the optic tube of his invention, and who was so capable of distinguishing every phenomenon which was apparent, would have been the first to notice these irregular appearances; but the first publication of them is due to Fabricius, who perceived them early in the year 1611, and his work, published in the same year, has a dedication bearing date June 13, 1611. In so far, however, as the mere observation of the spots is taken into account, they were seen by our countryman, Harriot, some time previous to any date fixed upon by the regular claimants to the discovery. Gallileo appears to have seen them in April or May, 1611. Scheiner informs us that he perceived them during the same months, but he did not take them into consideration before the October of that year.

In the annals of China it would appear that a large spot was visible on the sun in the year 321 of our era. In the year 807, a large spot was visible on the sun for the space of eight days, which was supposed by many to be a passage of Mercury over the sun's disc, but the length of time during which it remained visible is, of course, quite incompatible with such a supposition. Large spots which were seen by Averrhoes, Scaliger, and Kepler, were likewise supposed to be passages of this planet; but when it is remembered that a spot of the dimensions of Venus, with a diameter of five times that of Mercury, could not be detected on the sun's disc, we may be quite certain of their nature. Since that period they have been frequently seen with the naked eye. It was, however, from telescopic observation of these spots, that the fact of the rotation of the sun on its axis was made apparent, and Fabricius was the first who surmised that they adhered to the sun, judging from their slow motion when they arrived at the edges of that luminary. This rotation, and its duration, were subsequently confirmed by Galileo. The inclination of the solar equator to the plane of the ecliptic was determined by Scheiner to be seven degrees.

It has been remarked that the spots are confined to a certain equatorial zone, and that they rarely, if at all, extend beyond it. Their disposition in this respect may be seen from the accompanying diagram. By Galileo they were seen as far as 29 degrees of latitude north and south, but Scheiner extended this to a zone of 60 degrees in breadth, called by him the *royal zone.* They have frequently been seen, however, even beyond this limit, Messier having seen one of 31⅓ degrees, and Mechain one of 40⅓ degrees of north latitude, whilst another observer has detected one of 60 degrees of latitude. This, however, may have been one of the dark *pores*, as Herschel calls them, with which the whole surface of the sun is dotted, which tends to produce that mottled appearance which he compared to the skin of an orange. The rarity of the spots seen beyond 30 degrees of latitude tend to confirm the limited nature of the equatorial zone, which the multitude of spots constantly seen on the sun renders more remarkable. It was surmised by Cassini that more spots were seen in the southern than in the northern hemisphere of the sun; but there does not appear to be any foundation for this, as they are generally equally disposed on each side of the equator.

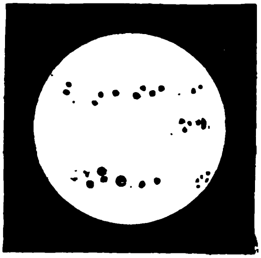


Fig. 15.

In the large dark spots, or *maculæ*, properly so-called, it has constantly been noticed that the very dark central portion, termed the nucleus, is fringed and surrounded on all sides with a shade of less intensity (Fig. 16), known by the name of the penumbra (pene-umbra), and which is nearly of the same degree of darkness throughout. Herschel found that even the largest of these spots commenced with one of these minute dark points, or pores, in the bright surface of the sun, which became larger and larger by degrees. When two spots were within a short distance of each other, they appeared to have a tendency to unite, and continued to expand until the moment of reunion. When the nucleus of a spot was going to disappear, it was noticed by Scheiner that the penumbra encroached gradually, but irregularly, upon it, so that the nucleus vanished before the penumbra. In modern times, and with better instruments, this breaking up of a spot in the manner here specified has been repeatedly noticed; the interior edge of the penumbra becomes ragged and irregular, the nucleus breaking up irregularly, and separating into many distinct nuclei, and presenting the appearance as if the luminous matter of the sun flowed in upon the dark spot. The exterior edges of the penumbra, although taking all shapes, are most frequently rounded off, and seldom appear with sharp projecting promontories. This, however, is not always the case, as was believed by Scheiner, since Herschel on one or two occasions perceived some prominent

Fig. 16.

branches in the nucleus, which were equally apparent in the penumbra. Whilst observing it with his usual attention, he perceived six branches suddenly appear instead of the two at first observed, whilst a corresponding change took place in the penumbra. The same cause, therefore, may be regarded as having affected both equally. In regard to the proportional dimensions of nucleus, and penumbra, it is generally noticed that the latter is about three times the breadth of the former. It sometimes, though very rarely, happens, that even large nuclii are unaccompanied with any penumbra, and this was confirmed by Herschel on one or two occasions. A large path of penumbra was, on another occasion, perceived by him without any nucleus. It is but rarely that these phenomena take place in large spots; the smaller ones, on the contrary, seldom have any penumbra about them. Occasionally, the spot covers such a large space on the surface of the sun, that when, by virtue of the rotation, it comes near the edges, and is about to disappear behind the disc, it forms a dark notch on its limb. This was seen in 1703, 1719, 1800, and 1846. It has been determined by Herschel, in respect to the relative brightness of the nucleus and penumbra, that the former may be estimated at 7, and the latter at 469; the intensity of the solar light being 1000. Many observers have noticed that the interior edge of the penumbra, or that immediately contiguous to the nucleus, is sometimes fainter than the exterior portion. At times, this is so apparent that the nucleus appears quite

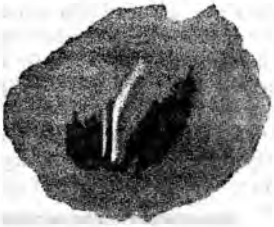


Fig. 17.

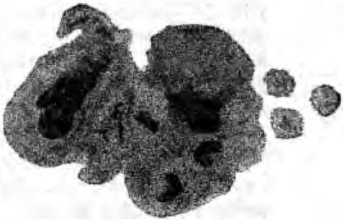


Fig. 18.

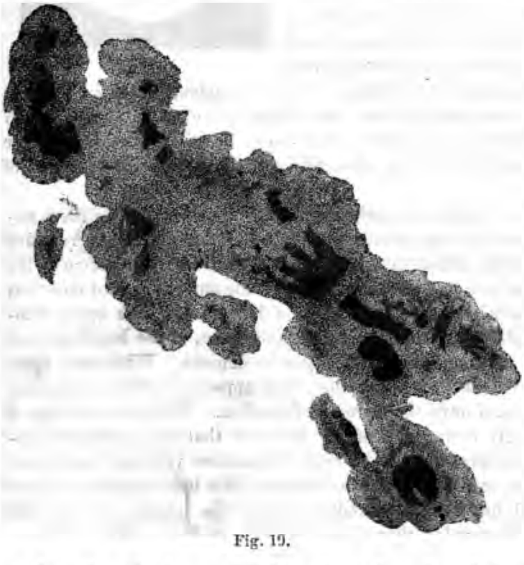


Fig. 19.

detached from the penumbra, which has the appearance of a separate armulus. In many regularly round spots, the penumbra has been noticed to take a decidedly radiating appearance, like the iris of the eye, and the nucleus itself observed to be riddled with minute pores, presenting the appearance of wire gauze. Mr. Dawes has made still further observations on the telescopic appearance of the spots, and by means of eye pieces, having a very small field of view, by which he gets rid of the surrounding glare of the sun's light, he has detected within the part generally called the nucleus a smaller spot, which may be considered as the nucleus proper. From his observations it would appear as if what has been termed' the nucleus and penumbra had a rotatory motion around the nucleus proper.

Figs. 17, 18, 19, are representations of spots really seen on the sun's disc, and it will be perceived under what irregular forms they appear. The long train of spots represent it as much broken up, with the nuclei and penumbra intermingled, and it is in this manner they are noticed when about to disappear. In Fig. 17, a bright streak of the surface of the sun is perceived in the central part. The spots are very capricious in their appearance, being sometimes very plentiful, and at other times the disc of the sun is entirely free from the slightest trace. Schroeter has recorded 81 separate spots perceived on the sun's disc at the same moment. For many years together no spot of any considerable size has been seen. Their dimensions, as already noticed, are frequently very considerable; and Schroeter saw one whose superficial extent was sixteen times greater than that of the earth. It does not appear that the great number or size of the spots have at any time been remarked to produce any degree of cold on the earth; for numerous and large as they seem to be, they bear but a small proportion to the total extent of the surface of the sun. The contrary opinion is much more generally regarded as true, and seems more accordant with the observed facts—viz., that the warmest seasons are those in which the sun is most plentifully covered with spots.

The bright spots which appear on the solar surface either occur under the forms of long irregular veins, or minute specks, and are both perceptibly brighter than the general surface of the sun; the former being known as *faculæ*, and the latter as the *luculi*. The *luculi*, like the *pores*, are situated at all parts of the sun's disc, and assist in giving it that mottled appearance which it has. The faculæ are mostly confined to a zone of 60° in breadth, in which the spots make their appearance, and generally accompany the latter as invariably as the surrounding penumbra. The faculæ are only visible at the edges of the sun, as, when they are carried by the rotation of the sun to the centre, they are seldom, if ever, visible. When the faculæ appear in great numbers and brightness on the limb of the sun, they are certain forerunners of large spots, which are almost sure to make their appearance in a few days afterwards. Cassini states that faculæ ordinarily show themselves in those places where spots have previously appeared, and that they have subsequently again become spots. Darham noticed a similar appearance, and, on one occasion, relates that he perceived changes in the spots with his eye at the telescope—a black spot appearing and disappearing successively in the centre of a brilliant faculæ. Herschel has likewise perceived extraordinary rapid changes with elongated faculæ; although Scheiner did not give credit to the existence of the *faculæ* or *luculi*, or anything on the sun brighter than its surface, yet they were detected with much less powerful instruments than he possessed by earlier observers. Galileo first remarked the existence of the *faculæ*, and Scheiner that of the *luculi*.

Physical Constitution of the Sun.—Until comparatively recent times, those spots were held to be the dark smoke or vapour which floated over the solar surface, although this did not explain the penumbra, or the shape which it takes when seen at different parts of the disc, when the penumbra surrounds a spot pretty equally at all sides near the centre of the sun, as at Fig. 20; when it passes to the margin and is received obliquely, the portion most distant would appear the narrowest. This, however, is directly contrary to observation, as it has been noticed that the nucleus and penumbra, when seen at the edge of the sun, appear as at Fig. 21, the part of the penumbra most distant from the sun's limb disappearing entirely, whilst the opposite side is only slightly diminished in breadth. Taking these facts into consideration, Dr. Wilson became convinced that the nucleus was in reality a deep hollow in the surface of the sun, and that the penumbra was the shelving sides surrounding it. If such a hollow as this be received obliquely, it is evident that it will appear as in Figure 20. Dr. Herschel confirmed this theory in many respects; but, instead of holding the opinion that they were indentations on the surface, he considered, rather, that they were openings in the luminous atmosphere of the sun. The sun itself he considered to be a dark body surrounded by two envelopes, the interior one being formed of very luminous clouds and very bright, whilst the one lying between the photosphere and the body of the sun, is formed of clouds, very little, if at all luminous.

Fig. 20.

If we suppose an opening is formed in these envelopes, by a gas ascending from the body of the sun, and driving the atmosphere away, the opening at the centre of the sun (Fig. 21), or to an observer at A, the nucleus will appear of the breadth *a a'*, surrounded equally on all sides by a penumbra, whose breadth is *b b'*. If, however, the observer is at B, he views the spot obliquely, the sides of the two openings, *a* and *b*, will coincide, and will lie in the same direction. On the opposite sides, however, at *a'* and *b'*, they will still be fully apparent. If the gas drives away the clouds of the two atmospheres, they will, of course, accumulate about the opening, and this may account for the faculæ which are to be perceived about the nucleus and penumbra. In regard to the luculi or points of light which cover the surface of the sun, they may be due to the roughness which would result from such a cloudy and irregular mass as the photosphere is imagined to be. An observation which tends to prove the unstable and cloudy nature of the outer envelope is, that they shift their positions from day to day; and it is very difficult to determine the exact period of rotation of the sun from these observations, some showing much longer periods than others.

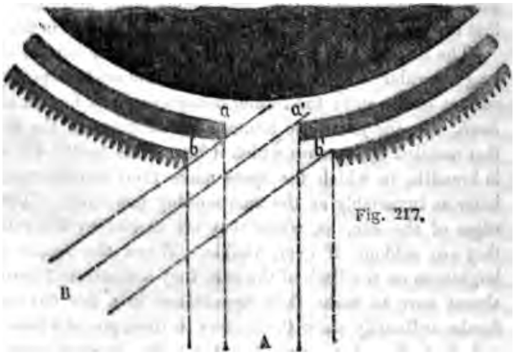


Fig. 21.

From observations on a great number of spots, M. Langier, of the Paris Observatory, has recently determined the exact period of rotation to be 25·34 days. Since, however, the earth is moving around the sun during this interval, we must wait nearly two days longer before we perceive the spot again at the centre of the sun. To explain this (Fig. 22): If T be the earth, S the sun, and *a* the spot as seen in its centre, whilst the spot appears to make a complete revolution, and to arrive again at the centre, the sun passes from S to S′; and when it arrives at S′, the spot appears at *a*′. If the sun had made exactly one revolution on its axis in the direction of the arrow, the radius S *a* would have taken the position S *b* parallel to its first position. When it arrives in *a*′, it must therefore have made more than one revolution by the angle C S′ *a*′. To pass through 360° *plus* the angle C S′ *a*′, it requires 27·3 days; whilst to make a rotation on its own axis it only requires 25·34 days.

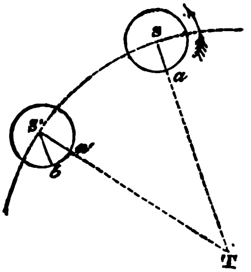


Fig. 22.

As the solar equator is inclined 7° 9′ 12″ to the ecliptic (according to M. Langier's investigations), the path which the spots appear to describe on the sun will vary at the different seasons of the year, the concavity being sometimes turned towards the north, and at other times to the south. In the beginning of December, the spots will appear to describe straight lines with reference to the ecliptic *e e*, the lines being inclined to it by an angle of 7° (Fig. 23). From the 1st of December to the 1st of June, they described curved lines, the convexity being turned to the north (Fig. 24). At the commencement of June they again describe straight lines, but in a contrary direction to the 1st of December. From June to December they describe curved lines, with the concavity turned to the north.

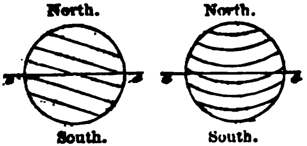


Fig. 23.

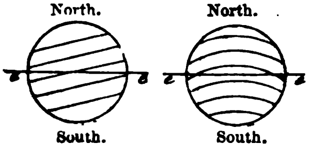


Fig. 24.

Zodiacal Light.—In the evenings of March and April, a cone of faint light is occasionally seen immediately after twilight in the western horizon, pointing in the direction, and sometimes reaching to the Pleiades, from the quarter in which the sun sets, and nearly along the ecliptic. The breadth at the base is from 20° to 30°, and its height occasionally fifty degrees. This is known as the *Zodiacal Light*, a name which was given to it by Cassini. It is very transparent, since the faintest stars can be perceived through it, equally as well as any other portion of the sky, although the glare of its bright light tends to extinguish very faint objects. Its nature is altogether unknown. It was formerly considered to be the atmosphere of the sun, but it has been proved by Laplace that this could not extend to such a distance as the zodiacal light has been observed to pass. An atmosphere to the sun could only extend to such a distance as the orbit of a planet could be situated, whose period of revolution would be equal to that of the sun on its axis, or 25 days, which would be placed considerably within the orbit of Mercury. Neither would the form of such an atmosphere agree with

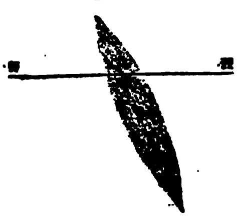


Fig. 25.

that observed with zodiacal light, as the equatorial and polar axes could not be beyond the ratio of three to two. Its appearance in the heavens may, however, be explained in this manner, although the cause is unknown; and it would seem as if a lenticular envelope surrounded the sun, as a portion of which we see above the horizon (Fig. 25).

On the Seasons.—If we admit the annual revolution of the earth around the sun, its diurnal rotation on its axis, and the inclination of this axis to the ecliptic, we can readily account for the seasons, and the derivation of day and night. The annual revolution of the earth in its orbit is forced upon us by its extreme simplicity; we cannot imagine that the sun would revolve around a body 1,400,000 times smaller, as the earth is. We thus rank the earth as a planet, obeying all the laws of the other planets, describing an elliptic orbit around the sun in the same manner as Mercury, Venus, Mars, &c. Another proof of the revolution of the earth around the sun is by the well-known existence of a phenomenon, termed the "Aberration of Light," arising from the velocity of light emanating from a star, combined with the orbital motions of the earth. The amount of this angular displacement has been determined with great accuracy by astronomers.

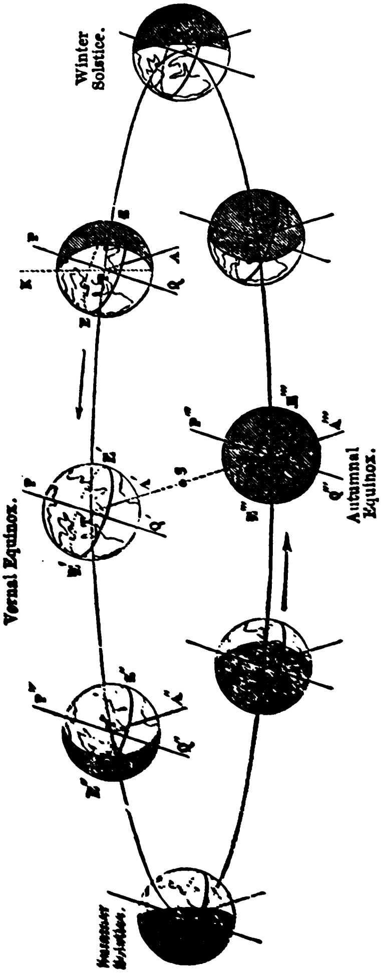


Fig. 26.

The annexed diagram (Fig. 26) will elucidate the phenomena of the seasons. Let S be the sun; T, T′, T″, &c., the different positions of the earth in its orbit; P Q its axis of rotation invariably parallel to itself; T A the line of the equinoxes: then at the vernal and autumnal equinoxes the sun is in the earth's equator, and the days and nights are of equal length, the sun illuminating at once one-half of the convex hemisphere. When the sun is at the winter solstice, the south pole of the earth is turned towards the sun, and the equator is above his path by an angle of 23° 27′. In our latitude, the cold northern hemisphere, the nights are longer than the days. At the summer solstice, the north pole is turned towards the sun, which is above the equator by its greatest inclination. In this latitude the days are longer than the nights. By an attentive examination of the figure, it will be seen that contrary appearances will take place in the southern hemisphere. Thus, if we consider the earth in its diurnal rotation at the summer solstice, the south pole will be constantly in darkness. At the vernal and autumnal equinoxes, the sun will shine on both poles of the earth.

Had the equator coincided with the ecliptic, the days and nights would be always equal, and had the inclination of the ecliptic to the equator been greater than at present, there would have been a corresponding difference in the seasons. Providence has wisely ordained that this inclination cannot exceed certain limits; the seasons will not, on this account, be sensibly different in occurrence and temperature.

The Earth's Equator.—In all the preceding investigations we have supposed the pole of the earth's equator to remain fixed, but this is not the case; and, although it retains the same inclination to the ecliptic, it describes in the course of a lapse of years a complete revolution around the pole of the ecliptic. A very simple experiment will shew, in the spinning top, that a motion of rotation around an axis may exist, without at the same time affecting its inclination, the axis of which will describe any figure. This phenomenon is termed the "precession of the equinoxes," and was discovered by Hipparchus, the astronomer, to whom we are indebted for one of our most ancient catalogues of stars. He found, by comparing the longitudes of his catalogue with those of some ancient catalogues, that whilst the latitudes of the stars were not changed, their longitudes were increased in the proportion, as he considered, of 1° in 72 years. The physical cause of the precession of the equinoxes was partially explained by Sir Isaac Newton, and afterwards more fully by the celebrated D'Alambert, and by La Place, in his *Mechaniqus Celeste*, on the theory of gravitation being produced by the effect of the attraction of the sun and moon on the excess of matter at the earth's equator, which produces a slow angular motion at the plane of the equator in a contrary direction to the earth's rotation.

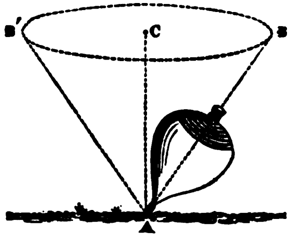


Fig. 27.

Precession of the Equinoxes.—This motion, though slow, being always in the same direction, and therefore continually accumulating, was, as we have seen, early remarked, and was one of the celestial appearances that suggested the idea of the *Annus Magnus*, or great astronomical period by which so many days and years are circumscribed. As it affects the whole heavens, and as the changes produced are spread over a vast extent of years, it has proved a valuable guide amid the darkness of antiquity, and has enabled the astronomer to steer his course with tolerable certainty, and here and there to discover a truth in the midst of the traditions and fables of the heroic ages. The accurate analysis of the complicated effect thus produced was a work that surpassed the power either of geometry or mechanics at the time when Newton wrote. His investigation accordingly was founded on the assumption that, though not destitute of probability, it could not be shewn to be perfectly conformable to truth; and it even involved a mechanical principle which was taken up without due consideration. The first who solved this difficult problem was D'Alembert He employed the principle of the equilibrium among the forces destroyed, when any change of motion is produced, and by means of the equation this proportion furnished, this great mathematician was enabled to proceed with a solution that has never been surpassed for accuracy or depth of reasoning. Laplace proceeded on a more general principle, and with broader conclusions. He has shown that the phenomena of the precession and nutation must be the same; that, whatever may be the irregularity of the depth or currents of the sea, that nothing can effect an alteration in the earth's rotation on its axis.

Wherever the sun in his apparent annual course crosses the equinoctial in spring, there is the vernal equinox; and wherever he crosses it in autumn, there is the autumnal equinox. The two points of intersection are not, however, the same year after year, but are subject to a slow annual displacement westward, so that the sun does not cross the equinoctial, spring and autumn, exactly in the same points, but every year a little behind those of the preceding year. This effect is termed the precession of the equinoxes, because it accelerates their time, though it is really their retrocession. It amounts to about 50¼″ in a year, or to 1° in 70½ years, to 30°, or a whole sign, in 2140 years; so that in somewhat more than 25,000 years, the equinoctial points will complete a revolution westward along the ecliptic. and return to the same position.

One obvious effect of the falling back of the equinoctial points, is a progressive increase of longitude in all the heavenly bodies. Hence those stars which in the time of Hipparchus were in conjunction with the sun when he was in the equinox, are now 30°, or a whole sign, eastward of it; and the constellations and signs of the zodiac no longer correspond, as may be seen by reference to a celestial globe.

The annual precession of the equinoxes, apparently a change in the sun's passage across the equinoctial, is really a change in the point of the earth's orbit at which its two hemisphere's are equally exposed to the sun. The cause of this remained unknown till the age of Newton, who showed that it resulted from the form of the earth, and the unequal attraction of the sun and moon on the unequal masses of matter at the equator and the poles, producing a slow reeling motion of the earth's axis from east to west, and the recession westward of the equinoctial points. Fig. 28 illustrates its effect.

Let S be the sun, T K the pole of the ecliptic, T P the pole of the earth revolving around T K, in the direction of the arrow. In the space of one year, the revolution will change from T P, to T *p*, in the second year to T *p*′, and so on. Let E E be the earth's equator, then, when the pole of the earth moves from T P, to T *p*, the line of the equinoxes, or the intersection of the ecliptic and equator, will move from T A, to T *a*, and so on, turning slowly around the centre of the earth. It is evident that this progressive change of the direction of the poles of the earth will have a corresponding influence on the planes of the fixed stars, and on the commencements of the seasons.

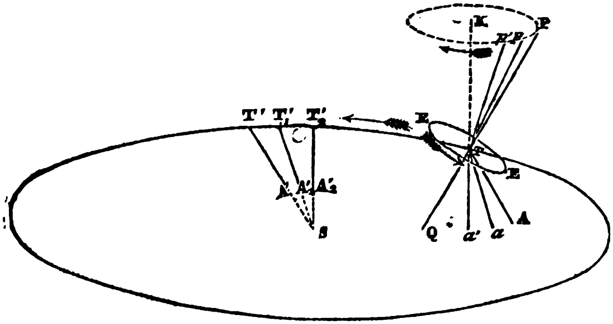


Fig. 28.

Thus, spring commences when the line of equinoxes is in T′ A. The succeeding year would have the same commencement, but, in the meantime, the line of nodes has moved to T_1, A_1; in the following year will have moved to T_2', A_2',—all passing through the sun S. But as the direction of the arrow shows direct motions, it is easily to be seen that the intersections *precede* at every year—for this reason this phenomenon is termed, as we have already seen, the precession of the equinoxes.

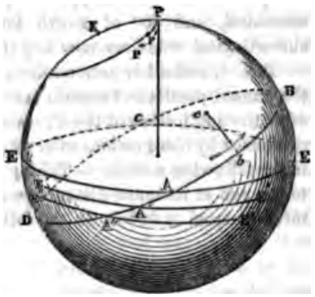


Fig. 29.

In order to exhibit the effect of this phenomenon on the planes of the stars, let E E (Fig. 29) be the earth's equator, A B C D the ecliptic, and let the intersections of the ecliptic and equator occur successively at A′, A″, &c., in an opposite direction to the sun's path. These successive changes of the equinoxes will cause corresponding changes in the pole P to P′ and P″, the right ascensions. Declinations of all bodies will be changed, as also their longitudes; but it will not have any effect on the position of the stars, the longitudes of which will be constantly the same.

On the Aberration of Light.—In ancient times it was supposed that the velocity of light was infinite and immeasurable. Roemer, a Danish astronomer, pointed out, in the year 1675, that in comparing Cassini's tables of Jupiter's first satellite with observation, he discovered the following fact:—When Jupiter was near opposition, the eclipses always happened earlier than the predicted time, and when the planet was near conjunction, the eclipses happened later, the whole difference amounting to upwards of sixteen minutes. The tables of Cassini were founded on an extensive series of observations at all parts of the orbit of the planet, and would therefore give a result free from aberration at the mean distance of the planet from the earth. Thus, in the figure, T T′ T″ represents the earth's orbit, S the sun, and J J′ J″ the orbit of Jupiter. Between T and T″ the planet is at opposition, and is at its least distance from the earth—the eclipses would, at this position, happen earlier—at T′ the planet is at conjunction, or at its greatest distance, when the eclipses would, of course, happen later about this period. Huygens and others confirmed this assertion of Roemer, but the subject does not appear to have been further attended to till Dr. Bradley, in the year 1728, communicated to the Royal Society the theoretical cause of the dis-

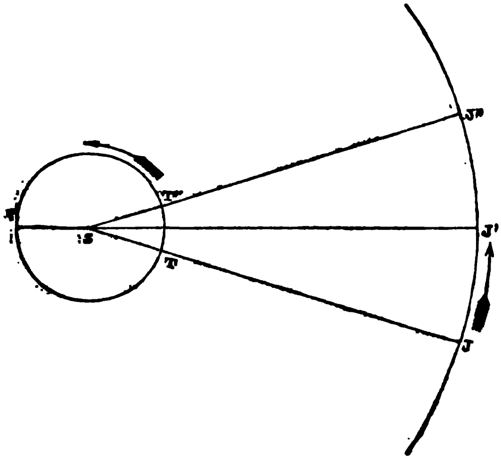


Fig. 30.

placement of a star, termed the "Aberration of Light." To the same astronomer we are likewise indebted for the discovery of the Nutation of the earth's axis. Without these two corrections of Aberration and Nutation, there would be a discordance in the prediction of the apparent plane of a star to the amount of 1′ nearly of right ascension, and 30″ of North Polar distance. To the first of these discoveries, in chronological order, we now beg the reader's attention.

Picard, and other astronomers, in observations made for the purpose of determining the annual parallax of certain stars, found an unaccountable difference of 40″ (annually) after the application of the Prussian and all other known corrections. Dr. Bradley, who confirmed by observations of several stars this difference, explained the theoretical cause in the following manner:—Taking for granted that light took 18′ 3″ to pass from the earth to the sun at its mean distance, we have the motion of the earth in its orbit during this interval equal to 20″·5. In the following figure, when the earth is at T, the star being in the direction E T, it is evident that the telescope will not be in this position to see the star; it must have a certain inclination, T A or T′ A′, such that the cross of wires placed in T′ describes the distance T′ T by virtue of the motion of the earth, whilst that light describes the distance, A′ T. We see, in fact, that the light which passes the optical centre of the object glass A′, when the telescope occupies the position T′ A′, arrives in T when the telescope has taken the position T A, and can, consequently, meet the cross wires, which are then found at the point T.

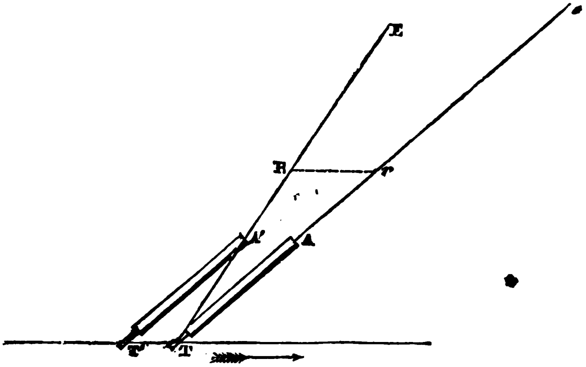


Fig. 31.

In order that the reader may see clearly the effect of the aberration of light on a star's position, we shall commence with the most simple appearances. And, in the first place, we will consider the effect of the varying position of the earth in its orbit on the place of a fixed star. For this purpose we shall take as a point of reference the place that a star would successively take, when seen from the centre of the sun, in the positions of the earth in its orbit. In the next place, we shall treat of the effect of the velocity of light combined with that of the earth in its orbit on a star's position.

In the annexed figure (Fig. 31) S represents the sun; T, T′, T″, &c., the earth in its orbit; E the place of a star. When the earth is at T, an observer at T will see it in the direction of T E; the corresponding direction that an observer at the sun would see it, would be found by making *e*E parallel and equal to T S, the radius of the earth's orbit; and drawing S *e* equal and parallel to T E, the star's place would then be to an observer at S in the position *e*; similarly when the earth has moved to T′, making *e*′ E equal and parallel to T′ S′, and completing the parallelogram, the star's place would be found in *e*′. In the same manner, tracing the position of the star throughout the year, it will be found to describe a curve equal as $e e' e'' e'''$ is to the orbit of the earth, and parallel to its plane.

Now, to transfer these appearances to an example, let us take, in the next figure (Fig. 33), a sphere of which O is the centre, A B C D the ecliptic, K its pole, K S a circle of latitude passing through a star as γ Draconis; then, according to the preceding example, when the star is at S, the star will appear, with reference to the centre of the sphere, in a line E e, equal and parallel to O S, which cuts the cone in the direction p of the small ellipse $m p n q$. It may be remarked that each star will appear to describe an ellipse in the heavens, which will, for a star situated at the pole of the ecliptic, have its major and minor axes equal, or will become a circle, and will gradually become more and more elongated as it approaches the ecliptic, when it becomes merely a right line. The major axis, $m n$, of this ellipse is parallel to the ecliptic; and the minor axis, $p q$, is perpendicular to it. We have thus found that when the sun is at S', or at the foot of the circle of latitude, the star should appear at p; and as the sun progresses in its orbit, it should successively be found in u, q, and m.

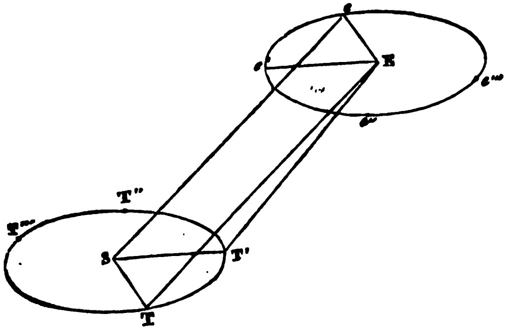


Fig. 32.

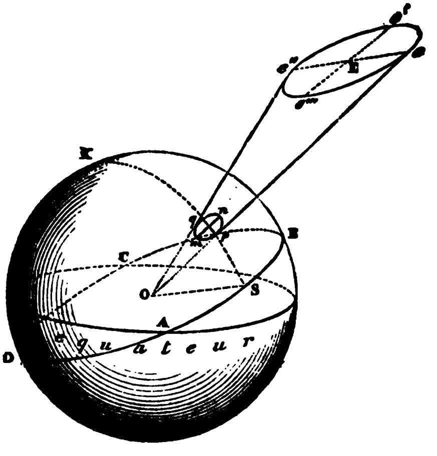


Fig. 33.

Such are the positions that Dr. Bradley calculated that the star should occupy in the annual revolution of the earth in its orbit. He was surprised to find that the most distant position of the star from the pole of the ecliptic did not occur when the sun was at S, but when it was distant 90° from this point. His observations of γ Draconis were commenced at December, 1725, when the sun would be about the position S of our figure, and the star in the position p; but he found that it gradually went towards the south, till it attained a position 20″ more southerly than at December. This occurred at the beginning of March, 1726. It afterwards went northerly, being in September more northerly by 39″ than it was in March. In the ensuing December the star was found in the position of the previous December, after making a proper allowance for the precession of the equinoxes.

Dr. Bradley did not, however, rest satisfied till he had repeated the observations with another instrument, made by one of the most celebrated artists of the day, Graham, which included a larger range of zenith distance. The same results as before mentioned were arrived at; the last instrument gave results identical with the three foot quadrant of Picard, and the twenty-four foot secter of Molyneux. He endeavoured to trace the origin of the difference to the effect of refraction, or a *nutation* of the *earth's axis*, the latter of which corrections was then unknown, with the exception of the trifling effect of *solar nutation*, whose period was six months, and which at its maximum, as determined by Newton, only amounted to a fraction of a second. The cause was certainly proved by Bradley not to be owing to nutation, since a star, opposite in right ascension to γ Draconis, and at the same distance from the pole, observed at the same time of the year, exhibited differences only half the amount of γ Draconis. Had the cause been the effect of nutation of the earth's axis, the same result of fluctuation would have affected both stars by exactly the same quantity.

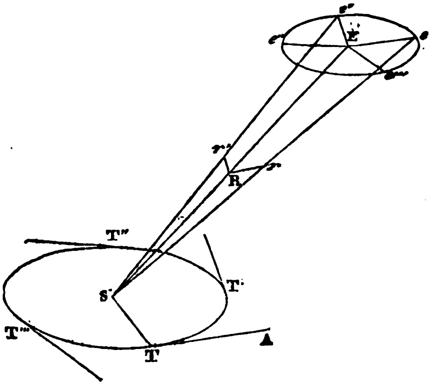


Fig. 34.

The effect of aberration may be thus explained. In the figure, T T' T'', &c. (Fig. 33), represent, as before, E the plane of the star. If we take with line S E, a part S R, and another line R *r*, parallel to the tangent to the earth's orbit at T, which would be the direction of the motion of the earth unless restrained by gravitation, the proportion of R *r* to S R is that of the motion of the earth in its orbit to the velocity of light. Thus when the earth is at T, the star, to an observer immoveable at the centre of the sun would, by the effect of the "aberration of light," be seen in the position S *e*, looking at the star as if he had been on the earth at T. Thus, by analogous reasoning, when the earth is at T', by drawing R *r'* parallel to the direction of the earth's motion, an observer at the sun will see the star in the direction S *e'*; and so on. The star would then appear to describe a curve in the heavens, *e*, *e'*, *e''*, *e'''*.

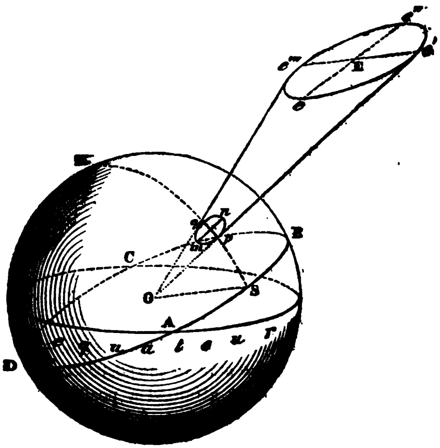


Fig. 35.

There is a great difference, therefore, between the two positions of the star, by taking into account the effect of aberration. In the former case, the direction of the visual ray was, for an observer, immoveable at the sun, in a line parallel and equal to that joining the star with the earth in its orbit, and distant by the radius of the earth's orbit. In the latter case, the effect of aberration is to make the star appear in a line parallel to a tangent of the earth in its orbit, and distant by a quantity in proportion to the velocity of light combined with that of the earth in its orbit.

Fig. 34 will show the real effect of aberration on the position of γ Draconis. By the preceding reasoning, when the sun is at S, the star should appear at *m*, in the small eclipse *m p n q*. The effect of aberration will therefore be to *retard* the position of the star by 90°, which will agree with Bradley's observation of December, 1725. At March, 1726, the star had increased its polar distance by $39''$, or was found in the position *p*, and so on, the succesive positions of the star agreeing precisely with observation.

Nutation of the Earth's Axis.—After the discovery of the Aberration of Light, the indefatigable astronomer, Dr. Bradley, prosecuted his observations at Oxford and Greenwich, when his zeal and care were rewarded by the discovery of another important inequality—viz., Lunar Nutation. After applying Precession and Aberration, he found that, after a series of observations carried on from 1727 to 1745, that an inequality existed, depending on the longitude of the nodes of the moon, whose period was 18 years, the existence of which had been previously mentioned by Roëmer, but of which no published account had been given. Dr. Bradley communicated this "Nutation of the Earth's Axis," in a Memoir, to the Royal Society, in 1748, in which he mentioned that in order to reconcile it with his observations, it would be necessary to assume that the pole of the equator described a small ellipse about its mean plane, whose major and minor axes were respectively $18''$ and $16''$. It was afterwards explained on Sir Isaac Newton's Theory of Gravitation, as a necessary effect of the retrograde motion of the moon's nodes, by which the moon's inclination to the equator varies from 18° to 28°. By this it is evident that this variation of inclination will cause a considerable difference in the moon's attraction on the protuberant parts of the earth's equator. Nutation is, then, an irregularity of *Lunar Precession*, the mean value of which is $35''\cdot 9$ annually—the remaining $14''\cdot 4$ being principally due to the sun—or the whole amount of the precession of the equinoxes being equal to $50''\cdot 3$.

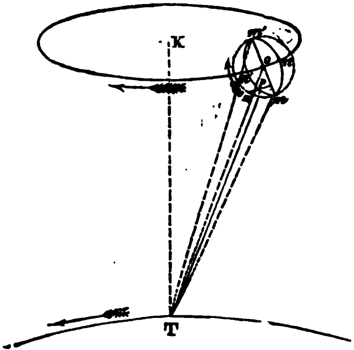
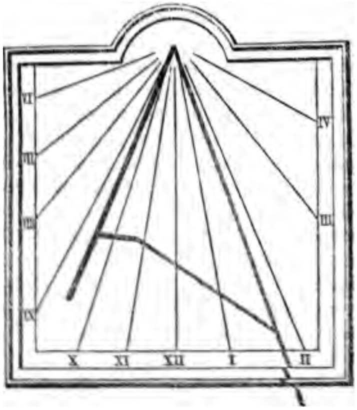
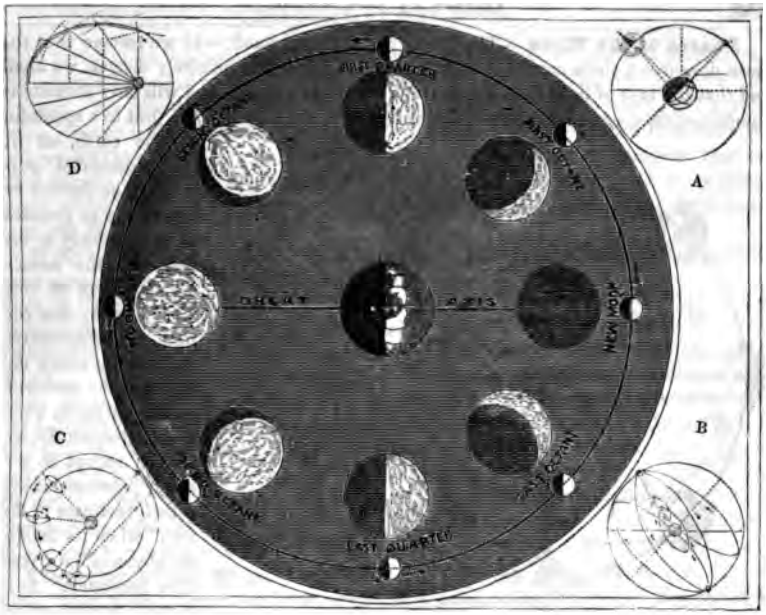


Fig. 36.

The following explanation will clearly show the effect of a nutation of the earth's axis on the position of a star:—

It has been previously explained that the pole of the equator makes a complete revolution around the pole of the ecliptic in a term of about 25,000 years, and that it is termed the "Precession of the Equinoxes." In Fig. 33 (which is not in exact proportion to the phenomena), T K is the pole of the ecliptic, and T O the pole of the equator, the angle K T O, or the obliquity of the ecliptic, being 23° 28′. In consequence, however, of *nutation*, the pole of the equator, T O, does not preserve the same inclination to the pole of the ecliptic, but moves on the surface of a little cone at the elliptic base, T *m n m′ n′*; still, however, the cone itself describing its revolution around the pole of the ecliptic, which is the "Precession of the Equinoxes," as before stated. The motion in the smaller cone, around its centre, O, will evidently cause the pole of the equator to be alternately on one side and another of its mean plane. The major axis of this ellipse, *m m'*, is directed in the plane which passes through its axis, T O, and through the perpendicular, T K, to the plane of the ecliptic—its amplitude is 19″·3. The smaller axis of the ellipse is 14″·4. The pole, P, makes a revolution of this ellipse in the space of 18 years, returning to *m* each time that the ascending node of the moon is found at the spring equinox. To know, at any time whatever, what is the position of the pole on the ellipse, we must imagine a circle described on the major axis, *m m'*, as a diameter, and to suppose that a point, Z, describes uniformly this circle in the direction of the arrow, in a manner to come back again always to *m* at the times at which the pole ought to be found there. At any time whatever, the pole, P, is always situated at the point of meeting the ellipse, *m n m' n'*, with a perpendicular to its major axis, drawn through the position that the point Z occupies at this instant.





ON THE MOON.

THIS object, the most valuable in relation to mankind :—which serves as a guide to the mariner in the trackless path of the ocean, and whose influence, combined with the sun, produces the tides, will next claim our attention. Viewing its path in the heavens, we find its motion to be the most rapid of any other body; the earth's changes of position which, with reference to the sun, amounted in the course of a day to 1°, will, for the moon, be more than twelve times that quantity. In consequence of this quick displacement with reference to the stars, its motions have become a most valuable means of determining, at any instant of time, the position of a vessel at sea.

Her appearance, and the breadth of her illuminated disc, vary every moment, showing that she is an opaque body which receives light from the sun. The disc, however, during each successive lunation, always presents the same physical appearances; and we have never been able to perceive the other hemisphere, which can only be seen when in conjunction with the sun, or at that time when the darkened hemisphere is directed to the earth. This arises from her rotation on an axis, in a period nearly coincident with the time of her sidereal revolu tion.

A most striking proof of the opacity of the body of the moon is shown by a phenomenon frequently visible—viz., the occultation of a star by the moon. In consequence of the *proper* motion of the moon from west to east, she will pass over or occult in her march all the fixed stars in her path. Before full moon, we see the star disappear at her unenlightened hemisphere and reappear at the illuminated limb. The time of the occultation will depend on the motion of the moon in her orbit, which can thus be calculated with great precision, and is found to agree exactly with observation.

Phases of the Moon.—The phases are thus explained:—If we assume that the moon describes a circle around the earth, and that the sun is infinitely distant, we shall find that the rays of light from the sun will always be parallel, and will only illuminate one-half of the convex hemisphere limited by the line *m n* (Fig. 37). But to a spectator on the earth, at T, the illuminated portion will be limited by the line *p q*, directed perpendicularly to the radius of the moon's orbit. Thus at new moon, or A, the enlightened hemisphere is turned towards the sun, and the darkened part to the earth, when it is, consequently, invisible, except in peculiar circumstances, called "eclipses of the sun," when its diameter obscures for a time that luminary. At B a small portion of the enlightened hemisphere is seen from the earth. At C it is at its "first quarter." The enlightened portion increases gradually till it becomes a maximum, or is in opposition to the sun. At G it is again on the wane, or at its last quarter, decreasing gradually, but in a direction opposite to the first appearances. At H it is again found in conjunction. The above successive appearances are shown by the annexed diagrams. When the moon passes into the region of the heavens where the sun is, it is invisible. At the end of a few days, a careful observer will see the moon after sun-set in the form of a crescent, with its convex side to the right, as it is represented in A (Fig. 38); this disappears after a

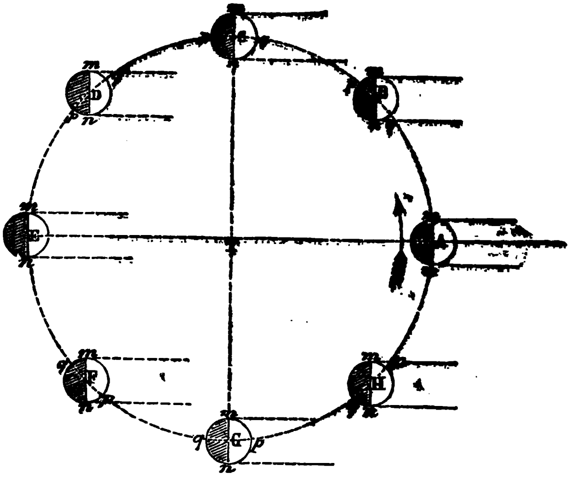


Fig. 37.

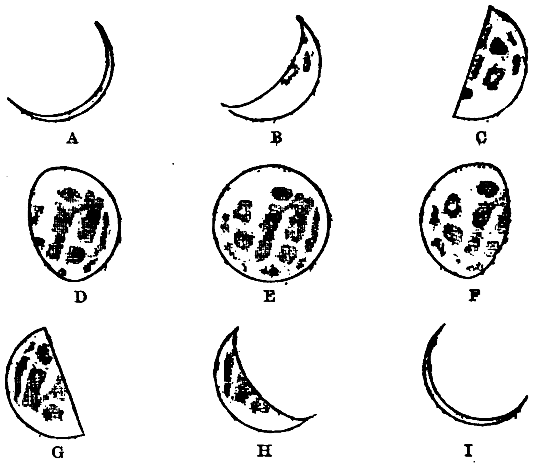


Fig. 38.

period under the horizon. For some days the same appearances present themselves,
escent becoming broader at its centre, as in B, and remaining longer above the
m. Six or seven days more and the crescent has nearly disappeared, and a half
presents itself to us, as in C. At this period it is sufficiently distant from the sun,
osses the meridian six hours after it. The moon continues to increase, first to
fourths, and finally to a full circle, as in D and E. In another week the waning
which is seen in the morning, assumes the appearance of F, G, H and I, the con-
ide of the crescent being now to the left.

the moon did not revolve on its axis during this progress, we should not have the
spots visible on its surface at each position in its
in nearly the same direction. We are thus led
er that the moon makes a corresponding turn
axis, in the interval from conjunction to con-
on, Thus, if the surface was without motion,
she moved in her orbit from L to L′ (Fig. 39),
sition L *a* would take a position L′ *b*; but during
me she has revolved on her axis through an
b L′ *a*, equal to the angle L T L′ described in
bit. Similarly, we find that when she has
ed an orbit of 180° or 270° she has rotated
axis by an equivalent angle.

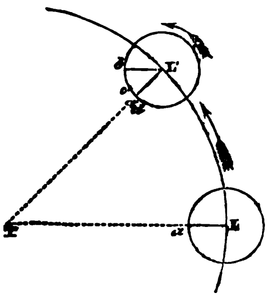


Fig. 39.

bration of the Moon.—Seen through a tele-
there is another phenomenon called the *libration*, which enables us to perceive a
portion of the disc, otherwise invisible. The first, or the *libration* in longitude,
from the two motions of rotation and translation not being exactly equal, and
he appearance of the moon's borders will likewise partake of this inequality.
cond arises from the circumstance that the axis of rotation is not exactly per-
ular to the plane of its orbit, but is inclined to it by an angle of 6° 37′.
s called the *libration in latitude*. Thus, when the moon is at L (Fig. 40), we
thout difficulty the pole *q*, but cannot perceive the opposite pole *p*. But when
on is at L′ the pole *p* becomes visible, while the pole *q* vanishes from our view.
sition of a spot near the moon's equator at *a*, would, therefore, to a spectator
earth, appear in different positions; this libration is directed perpendicularly
to the plane of its orbit,
or nearly to the plane of
the ecliptic.

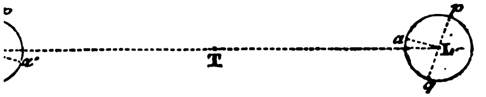


Fig. 40.

Finally, there is a
third libration, called the
libration, which depends on the positions of the moon to an observer on the
surface, in the interval from rising to setting. In consequence of the proximity
moon, the spots will not retain the same positions at rising and passage across
ridian, but the effect at its maximum is only 32″, and is scarcely worth
n.

allax of the Moon.—Before attempting to deduce the orbit that the moon
s around the earth, we shall find it imperatively necessary to correct the results
vation for "Parallax." Parallax is the angle which the radius of the earth sub-
the moon. It is felt entirely in a vertical direction on the meridian, and can
mined in the following manner:—

If the zenith distance of the moon's centre be observed by two astronomers at two stations, differing considerably in latitude north and south, as Greenwich and the Cape of Good Hope, as (B and C) in Fig. 41, we shall have the angles Z B L and Z′ C L. The latitudes of the two stations, B O E and C O E, are well-known, and their difference of longitude is also accurately known. The motion of the moon in zenith distance is known to a great certainty, and thus the data are perfectly comparable. If the moon were as distant as the fixed stars, the sum of the zenith distances thus found would be precisely equal to the sum of the latitudes, north and south, of the two observatories, when proper allowance has been made for refraction. But the effect of parallax will be in both cases to increase the apparent zenith distances, and the observed sum will be greater than the sum of the latitudes by the whole amount of the two parallaxes. In this manner, by corresponding observations of the moon at her meridian passage, the constant of the horizontal parallax has been determined.

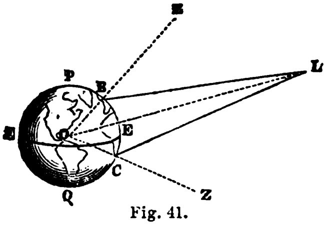


Fig. 41.

In its practical application to the results of observation, it is customary to correct the zenith distances for the angle made by the direction of the plumb-line at each station, and a line drawn to the centre of the earth. This is termed the angle of the vertical, and its value, as well as the radius of the earth at the two stations, are known from the "figure of the earth." The point of reference to which horizontal parallaxes are referred in the case of the moon is the radius of the equator, and in this manner the individual results of observation are corrected for the effect of parallax in altitude, which varies as the line of the zenith distance.

The Moon's Path in the Heavens.—By the transit instrument and mural circle, we are now able to trace the path of the moon in the heavens, after properly applying the corrections for parallax and semi-diameter before mentioned. We thus determine its angular displacements with respect to the equator, and find that it describes a revolution in a period of about 27 days, returning nearly to the point of departure; but if, during several lunations, its progress is watched, we shall find that its inclination to the equator differs by an angle varying from 18° to 20°. Instead, however, of referring to the equator, if we use the ecliptic, we shall find that it constantly preserves the same relation—viz., an inclination of about 5° 9′.

The figure (Fig. 42) will show the orbit of the moon in its progress through two lunations. E′ E represents the earth's equator, A B C D the ecliptic, and A B C D the orbit of the moon, inclined to the ecliptic by an angle of 5° 9′. The intersections of the orbit N′ are termed the *nodes*. Supposing N to represent the node at one lunation, at the next it will be found to have retrograded to N′, and at each succeeding lunation in the same manner, always preserving the same inclination to the ecliptic, till, at the end of about 19 years, it returns to the

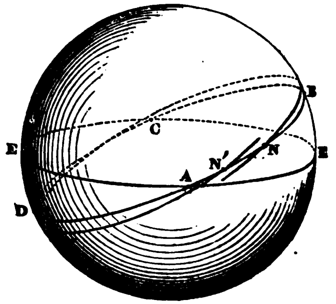


Fig. 42.

position N. This will naturally cause a variation in its inclination with regard to the equator, which is shown by the figure (Fig. 43).

Let A B C D be the ecliptic, E E the equator, N L N′ L′ the orbit of the moon, O P the axis of the earth, O K the axis of the ecliptic, and O R the axis of the moon's orbit. In the motion of the axis of the moon's orbit around the pole of the ecliptic, it will describe a small circle, R R′ R″, around the point K. The axis O R describes a cone of revolution of which the axis of the figure is the line O K. In this motion the angle of O R with O K is always equal to the inclination of the moon's orbit, or 5° 9′. The inclination that the equator will make with the moon's orbit will be successively the angles P O R′, P O R, and P O R″, and as the angle P O K is the obliquity of the ecliptic, or 23° 28′, the angles P O R′ and P O R″ will be the minima and maxima inclinations, being 18° 19′ and 28° 37′ respectively.

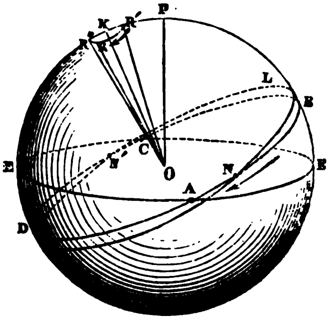


Fig. 43.

Motion of the Apsides.—The line of the apsides of the moon has been found to have a very rapid, though not uniform motion, amounting to 6′ 41″ daily, which is direct, or according to the order of the signs, and the major axis of the ellipse thus makes a complete revolution in 3,232½ days. In one hundred Julian years, or 36,525 days, the line of the apsides makes eleven complete revolutions + 109° 2′ 46″·6, and at the present time this motion decreases 50″·4203 in one hundred years. The longitude of the perigee may be calculated for any period—that in 1801, January 1 (Paris time), being 266° 10′ 7″·5. The true longitude of the moon, supposing it to move in an elliptic orbit, whose eccentricity was 0·0548442 in parts of the semi-major axis, or the greatest equation of the centre was 6° 17′ 12″·7, might thus be calculated for any period in the ordinary manner; but a great number of minute corrections must be applied to it before the tabular plane, thus obtained, would be found to agree with the observed plane of the moon.

Evection, Variation, and Annual Equation.—These are the three greatest periodical fluctuations of the moon in longitude. The effect of the evection can be explained by a diminution of the equation of the centre, when the moon is at opposition and conjunction, and by an increase in its value when at quadratures; the amount of the correction in the latter case not, however, being so considerable as in the former. If we suppose the line of apsides to be in syzigy—that is, new or full moon—the observed longitude will be found to be 80′ greater than the calculation; and if the apsides are in quadratures, or first and last quarters, the observed will be found to be smaller by the same amount. The evection depends on the double of the elongation of the moon from the sun, the mean anomaly being subtracted from the product, and the coefficient of this quantity, amounting to 1° 20′ 30″. Hence, when the apsides are in syzigies, at which period the eccentricity is greatest, the greatest equation of the centre is 7° 39′; and when in quadrature, at which it is smallest, it is only 4° 58′. At the latter position of the apsides, the gravitation is greatest at apogee, and least at perigee; and in the former case the gravitation is greatest at perigee, and least at apogee, and, consequently, the eccentricity of the orbit is increased. The discovery of the equation of the centre and evection is due to Hipparchus and Ptolemy. The more accurate observations of

Tycho proved the existence of a second irregularity, which is termed the *variation*. The observed planes disagreeing with the computed act of syzigies and quadratures, sometimes as much as 37′ when the line of apsides was in the octants, Tycho found that it depended on the elongation of the moon from the sun. The cause of this is the position and distance of the sun and the earth; for when the sun's disturbing force is at right angles with the radius vector, the moon's motion is accelerated from the quadratures to syzigies, and retarded in the contrary direction. The coefficient is 35′ 42″, and the argument is double the elongation of the moon and sun. The *annual equation* follows nearly the same law as the equation of the centre of the earth's orbit, only with opposite signs. This is greatest in the months of March and September, but almost vanishes in June and December. From this it derives its name—the period being an anomalistic solar year, and the coefficient 11′ 13″. The motion of the moon, in consequence, is slowest in winter and quickest in summer. It is due to the disturbing force of the sun in different parts of its orbit, being greatest at its least distance, and least at aphelion.

The greatest of the periodical disturbances of the moon in latitude depends on twice the elongation of the moon from the sun, from which the distance of the moon from its nodes being subtracted, the coefficient is 8′ 48″. In addition to this, there is another remarkable inequality, depending principally on the compression of the earth at the poles, and which may be termed the spheroidal inequality. The theory of universal gravitation has come to the aid of observation in detecting numerous inequalities which observation alone would scarcely ever be able to discover, and has explained all those which were previously known to exist.

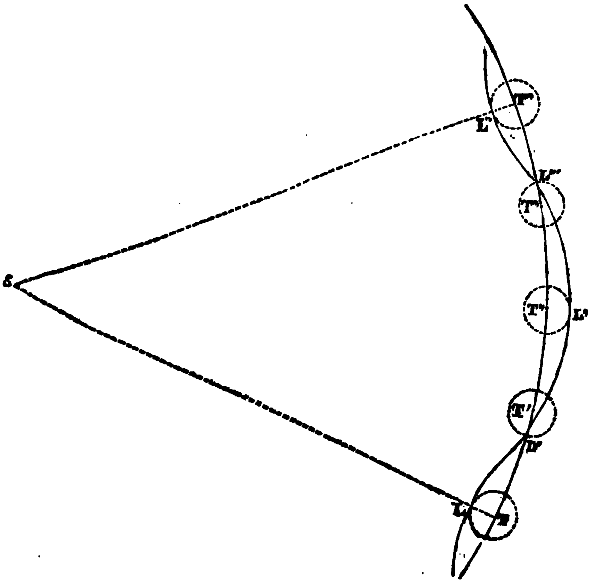


Fig. 44.

Motion of the Moon in Space. — The curve which the moon describes in space may be laid down in the following simple manner:—Let the earth, T (Fig. 44), pass round the sun, S′ in the orbit T, T′, T″, &c. An observer on the earth will see the moon in various positions in respect to the sun. When the earth is at T, the moon will be at L or in *conjunction; when at* T′, the moon will be a L′, or at its first quarter; at L″ at full; at

L''' at last quarter; and as the earth describes its path around the sun, it follows that the curve described by the moon in space will pass through the points L, L', L'', L''', &c. This line is represented in the diagram as considerably more curved than it really is, the distance T L being only $\frac{1}{400}$th of the distance T S.

On the Harvest Moon.—The phenomenon of the harvest moon, when for some nights together at that period of the year the moon rises nearly at the same time, depends on the inclination of the ecliptic with the horizon. That part of the ecliptic in which this inclination makes the least possible angle lies in the constellation Aries (in north latitude), and when the sun is in Libra, as at the time of the autumnal equinox, the moon, when at full, will be near the first point of Aries and but little distant from the ecliptic. It is clear that, when at this part of its orbit, as it travels from west to east, the times of successive rising must be within a short period of each other; and if the ecliptic were wholly parallel to the horizon, then it would rise exactly at the same time on each night. As the moon, however, is a little inclined to the ecliptic at some periods, this will make the difference even less. In the latitude of London, the least possible difference between the times of successive rising of the moon has been calculated at seventeen minutes. When the constellation *Libra* rises, the ecliptic makes the greatest possible angle with the horizon, and the differences between the times of successive risings of the moon is then greatest. This takes place with the full moon at the time of the vernal equinox, and the greatest possible difference in the latitude of London is 1h. 17m.

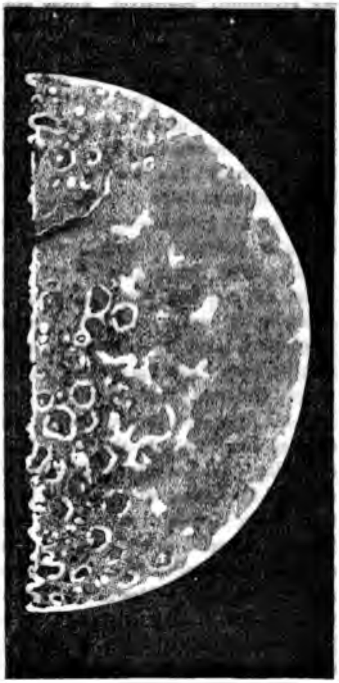


Fig. 45.

Physical Constitution and Telescopic Appearance of the Moon.—No celestial object is better known to us, by means of the telescope, than the moon; and instruments have been constructed which shorten its distance by a thousand times, and thus brings it optically within less than two hundred and fifty miles of the earth. In viewing it, the observer is immediately struck with the roughness of its surface, and the numerous circular formations, which appear as if surrounded by a steep and high wall; whilst those are by far the most numerous, there are also caves and hilly prominences. There exist also immense mountain chains, which for extent and height appear to surpass even those of the earth. Immense ridges, rising up from the open plains, are remarkable for their height and brightness, as well as for the numerous peaks and eminences into which the surface is separated. One of these is very visible in the interior of the crescent of the moon when about half-full, and can even be seen as a *notch* in that portion, with the naked eye, from the shadow *which it casts upon* the open plain. This part is seen in the accompanying representation of the moon when half-full (Fig. 45), and the mountain chain to which it belongs is a part of the *Lunar Appenines*, some of the peaks of which are calculated to be upwards of five thousand feet in height. Although, however, the general surface of the moon is rough and uneven, and the large circular mountains are extraordinarily plentiful, especially in the southern hemisphere, yet some portions are nearly level and of great extent. When the moon is full these portions appear as dark grayish spots, and give it that appearance which has been likened to the human face; and as the moon cannot cast any shadows at this time, it follows that this must be their natural colour. These are known by the name of *seas*, and still retain that appellation; although, when examined by means of the telescope, they have not any appearance of being sheets of water, but seem to be large level alluvial tracts of land on which craters, peaks, cavities, and long banks are plentifully scattered. These seas take up a large portion of the lunar surface, although it is only the most extensive which are seen with the naked eye. The smaller ones are known by the names of bays, lakes, and marshes. They vary considerably in intensity of colour, and some of them have been even noticed of a grayish tinge. The greater number of these seas were named by Riccioli, who, according to the spirit of the age, called them, in the astrological manner, Mare Imbrium, Humorum, Serenitatis, &c. If we view the other portion of the moon when at the full, they might be mistaken for a crude, chaotic mass of crystallized or frozen fragments, without shape or termination. It is only when examined in its various crescent aspects, with good telescopes, that the various formations are seen, as it were, raised above its surface, which to all appearance they are when viewed in this bold relief.

The most remarkable of the lunar mountains are those of the circular shape, which are not, however, equally regularly defined all over its surface, some of them being quite perfect, and others misshapen and broken. The larger of the annular mountains, taken in general, do not possess the same regularity of structure as the smaller ones. For the greater part they have a level interior, which in many instances is very irregular; and the wall is but seldom equally high or broad at all parts, which will not, however, be perceived at first: and it is only when the sun has just commenced to illuminate the tops, that the shadows are noticed to be so rough and irregular. The exterior is frequently surrounded by high rocks, which are often more elevated than the wall. In some few instances the interior surface is perfectly level, as in Plato, Archimedes, and the eastern portion of Stoflis; but for the most part they are covered with mountains, mountain chains, and craters of various depths and sizes. In several instances, the chain of mountains in the interior is quite straight and regular, and divides this surface into two parts more or less equal. These mountains are not in general so elevated as those which surround them. It is but rarely that those which are level in the interior, and may be denominated the *walled plains*, are quite circular in form. The wall which surrounds them is generally uneven, and in many parts is completely broken away; sometimes these appear as gaps, and in other cases the broken portion is completely levelled with the soil. In some cases the wall is wanting for a sixth part of the circumference; in others for one-fourth, or even one-third; but in these instances there is mostly some trace of a continuation of the real wall by a difference of brightness, or a succession of small hills. The walled plains may thus be considered to bear some affinity to the bays and some neighbouring portions of the great seas, as to the splendid *Sinus Iridum*, which, for a considerable portion of its boundary, bears a resemblance to some of the larger walled plains.

By far the greater number of the *walled plains* are seen in the southern hemisphere, where they are interlinked among each other in great plenty, and where it is a difficult matter to distinguish them properly, or to map them with the requisite distinctness.

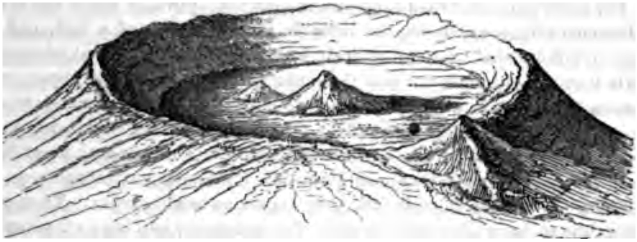


Fig. 46.

In the northern hemisphere they are less common, but appear to greater advantage from being more isolated; and it is here we see the more regular specimens of this class, as *Plato* and *Archimedes*. It has been remarked that, in numerous instances, the walled plains appear ranged in a row in a north and south direction; and if the southern part is viewed at the first and last quarter, in that portion which separates the *Mare Nubium* from the hilly parts of the south-west quadrant, a row of these formations become plainly visible, some of which are of considerable magnitude and curiously entwined with one another. Similar to the circular-walled plains in shape, but mostly of smaller dimensions, are the circular-walled concavities, whose diameter varies considerably; and which, like the latter, are best seen in the northern part of the moon, though they are plentifully distributed in other parts, in the mountain chains, in the level seas, and even in the interior of the walled plains. The interior is mostly regularly concave, and in general they are furnished with a central peak, which occurs likewise, though not so frequently, in several of the walled plains, as may be seen in the accompanying representation (Fig. 46). The interior portion of the wall is much steeper and considerably smoother than the outer. The central peak has no connection with the surrounding wall, and never attains to the same height, and frequently not even the same elevation, as the surrounding country, while in many cases they are so small as to be seen with the utmost difficulty. The smaller walled concavities are almost always without them. The central peak is in general very steep, but it sometimes happens that it takes the form of a mountain of gradual ascent, and, in some instances, a mass of mountains, as in Gassendi. In the great annular mountain *Tycho*, which presents such a fine appearance at the time of full moon, the central peak is very steep and high. In some of the annular mountains, instead of the central part being occupied by a peak, it is frequently the locality of a deep crater, although this is not so exactly situated in the centre, as the peak generally is. It has been remarked that the very dark walled plains and concavities are mostly without any central mountain or peak, as is the case in Julius Cæsar, Plato, Endymion, and many others of the same class. The central masses of mountains are, in general, very steep towards the summit, but are surrounded by low scattered mountains at the base. Several of the walled concavities are deficient in the same manner as the walled plains, a greater portion of the outer wall being broken down, and several gaps apparent in the circumference. The *pits* of the walled concavities are deeper than the surrounding country, the exterior height of the wall being generally one-half or one-third of the interior. But there is no proportion between the diameter and depth of the annular mountains, the smaller ones being sometimes deeper than the larger ones.

The walled mountains form but a small portion of the circular formations on the moon. Far more numerous than those are the *craters*, by which name are designated the smaller concavities which present little or no appearance of a wall, and some of which are so minute that they can scarcely be perceived with the best telescopes. In some parts they are so numerous, that the lunar surface presents the appearance of a spongy mass covered with innumerable minute pores. Some of these craters, small as they appear, are furnished, like those of the former class, with a miniature peak in the centre. These are found at all portions of the moon, and are very plentiful in the interior of the walled concavities and plains, in the rocky chains and seas, and the level surfaces. When the moon is full, they appear as bright specks. They require a favourable time to be seen at all properly, for, unless the sun is at a favourable altitude, they can scarcely be perceived.

In viewing the moon when at full, several radiating bright streams will be noticed proceeding from many of the mountains, which pass to considerable distances from their apparent centres. None of the mountains, however, are surrounded by those bright radiating streaks to the same extent as Tycho, and many of them proceed to a distance of 500 miles. It is scarcely possible to imagine that these could be streams of lava, which flowed from the central eruption, and it is more plausibly conjectured that they must have flowed upwards through fissures, produced by the central explosion of the volcano. In either case, however, it may serve to give an idea of the tremendous force of action required in order to cause these appearances.

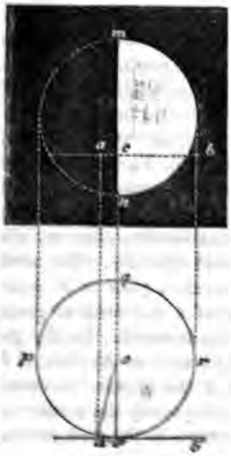


Fig. 47.

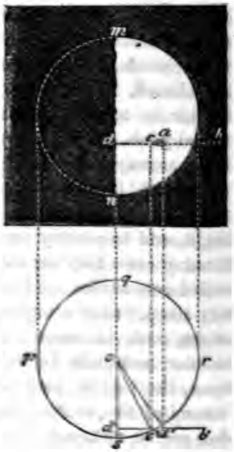


Fig. 48.

Height of the Lunar Mountains.—The height of the lunar mountains is found either by observing the distance of the illumined summit of a mountain from the generally illuminated portion of its surface; or, otherwise, by the length of the shadow which it casts upon the plane. Both these methods will be easily understood from the following considerations. In determining their height by the former method, we find the distance of the illumined summit *a* (Fig. 47) from the moon's illumined edge *c a*, and we perceive that it is a tangent to the circle *p q r c'*. Knowing the two sides *o c'* and *o a'* of the right angled triangles, we calculate $o\,a' = \sqrt{oc'^2 + a'c'^2}$, which is the distance of the summit of the mountain from the centre of the moon, and hence, subtracting the radius *o c'*, the remainder gives the height of the moon above the moon's surface. By this method we find its height in seconds of arcs, and this is nothing else than if we observe it resting on the limb of the moon. Its height may easily be observed in feet. In general this method will give too small a value of the height, as the sun may have shone on the mountain for too great a length of time previously. It was by this means that Hevelius and Galileo ascertained the heights of many of the lunar mountains, some of which appeared to be as high as five English miles. More recent observations have shown them to be very high, though not of such altitude as this.

This method, however, is not applicable unless the moon is in quadrature, or half-full. The second method has been more commonly made use of on that account. As the mountain a (Fig. 48) casts the shadow $a\ c$, the length of this is measured by means of the micrometer. Describing the circle p, q, r, s, the solar ray directed towards $b\ c'$ will cut the circle in c'. The distance $c'd' = od$ being measured, and the radius $o\ c$ known, and the two sides of the right-angled triangle $o\ c'\ d$, the angle $o\ c'\ d$ is known, and consequently the angle $o\ c'\ a'$. In the triangle $o\ c'\ a'$ we thus know the sides $o\ c$, and $c'\ a$, and the included angle, and consequently the side $o\ a'$. Subtracting the radius $o\ c'$, the height of the mountain casting the shadow $a\ c'$ is determined.

Many observers have perceived bright spots on the *dark portion* of the lunar disc. Sir W. Herschel describes three which he once saw upon it, two of which appeared almost extinct, but the third was more luminous, and resembled a piece of burning charcoal covered with white ashes. A similar appearance was once noticed with the *naked eye* by two observers at Norwich and London. When the positions of those spots have been examined, it has been found that they are almost always situated at the brightest, and consequently the most reflective, portions of the lunar surface. When the disc of the moon is illumined by *earth-light*, as is the case when slightly crescent, or, as it is commonly called, the "old moon in the new moon's arms" (Fig. 49). Those bright parts, which return most sun light, also reflect the earth's light stronger than any others. The mountains Kepler, Tycho, and Copernicus are thus frequently visible in this manner, as also the bright mountainous districts. But, above all, *Aristarchus* is most commonly visible at those times. This is the brightest part of the moon, and was thought by Hevelius to be a burning volcano of sulphur and saltpetre.

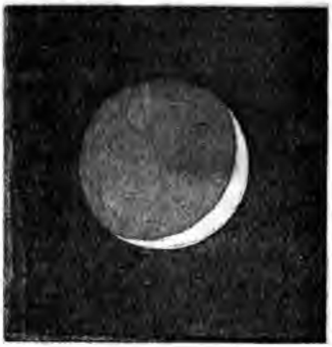
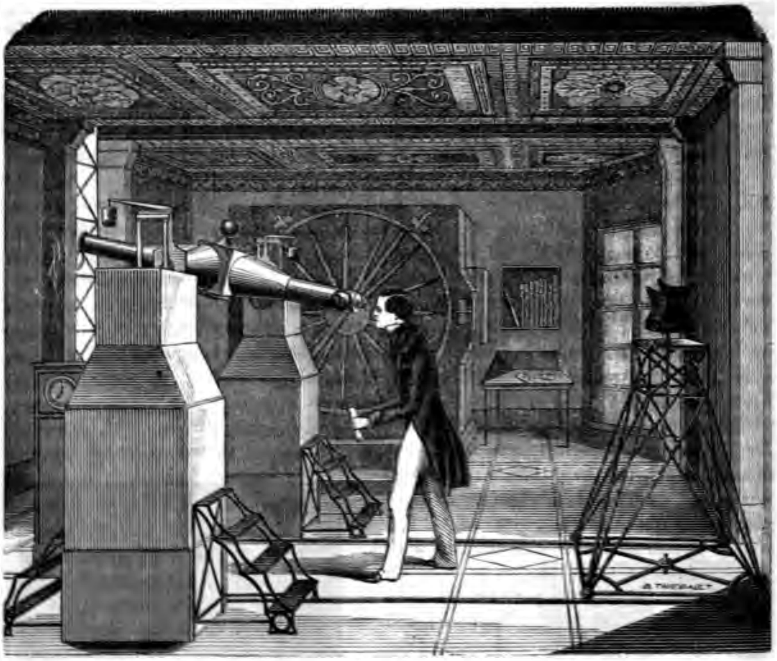


Fig. 49.

In the engraving at the head of this chapter, let us suppose B, the earth, to be opposite the sun in the tropic of Capricorn on the 21st of December, the longest night in the year, and moon full in the tropic of Cancer, the horizontal line to be our horizon, showing that the diurnal arc of the sun is small, while the moon has a very large one to traverse. The contrary of this takes place in the summer solstice. This explains why the moon in summer seems to describe a very small, and in winter a very large arc. C represents the moon in various positions in reference to the earth and the sun. In the first and second figures she is in conjunction, or directly between the earth and the sun. In the next figures the earth is interposed between the sun and the moon. D shows how the elliptical orbit of planets is produced by the co-operation of attractive and tangential forces.



ON THE PLANETS.

Among the more lustrous objects which arrest our attention in the heavens are those which, since the earliest times, have been known by the name of *Planets*, from their erratic course among the fixed stars. The variable brightness of Venus, the morning and evening star, as well as those which appear at all times of the night, claim our attention equally with the sun and moon. Five of those objects were known to the ancients—Mercury, Venus, Mars, Jupiter, and Saturn; the first of which is but rarely seen with the naked eye in our latitudes, in consequence of its constant proximity to the sun, although a sparkling object, and much brighter than any of the fixed stars. These bodies, it will be perceived, do not, like the sun and moon, move in one constant direction from west to east, but sometimes direct, sometimes retrograde, and at other times stationary. It was noticed, however, by the ancients, that whatever might be their movements in longitude, their latitudes did not depart much from the ecliptic; and a zone of 16° in breadth, or 8° north and south of the ecliptic, B D, contained all the ancient planets. This was termed by them the zodiac, and was divided into twelve equal portions, the signs at which it cut the equator, or the line of the equinoxes, being in the constellations Aries and Capricornus.

If Venus, the most brilliant of all the planets, be observed in its passage through the heavens, it will be seen that, in the course of a few months, it describes in the sky a very irregular circle, and that it oscillates to a certain distance on each side of the sun, sometimes rising before it in great brilliancy in the morning, and subsequently setting after it in the evening, and then for a length of time being altogether invisible to the naked eye (Fig. 50). It thus appears at an epoch nearly in the direction of the sun S, it then passes from S to A, and then returns from A to S; and having passed the sun, it goes from S to B, and finally returns to it in the contrary direction, or from B to S. When an evening star, it is seen between S and A, and between S and B in the morning. Its latitude varies in the same manner. The greatest elongation from the sun never exceeds 47¾°, and is never less than 45°; and at those points its motion is considerably slower than when near the sun. In order to explain the oscillatory motion of Venus in respect to the sun, the ancients imagined an epicycle similar to that already mentioned in the case of the sun. If T be the position of the earth, it was supposed that V the planet described an orbit round the centre C, which had a motion itself around the earth, the three points T C S being always in a straight line. At the times of greatest elongation, the planet was in the direction of T A and T B; and in the intermediate times its apparent motion was sometimes from west to east, and at others from east to west, or retrograde. If Venus be, however, examined with

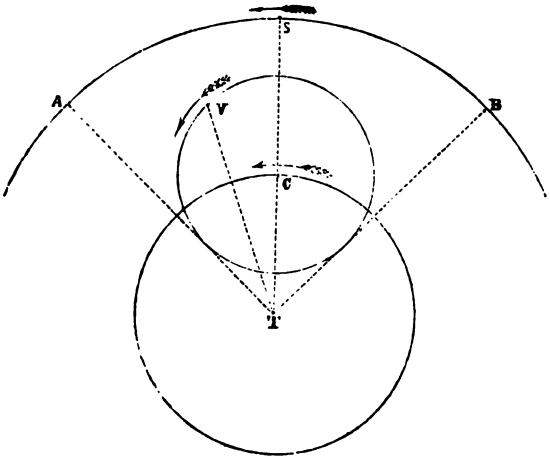


Fig. 50.

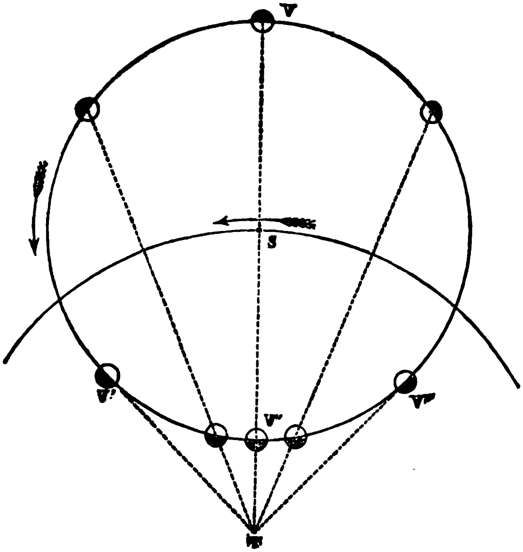


Fig. 51.

the telescope at its various distances from the sun, it will be seen that its diameter varies greatly, and that its disc undergoes a series of changes similar to those of the moon. By observing those in its different positions, it will be perfectly apparent that they agree with a motion of the planet round the the sun, it being supposed that the planet is an opaque globular body, of which one hemisphere is illuminated by the sun. If the sun S (Fig. 51) be supposed to revolve round the earth T in the direction of the arrow, and the Planet V around the sun, at the point V, where it is in a line with the earth and sun, and the hemisphere turned towards both illumined by the latter body, its disc will appear wholly illuminated by the latter. At the points V′ and V‴, where the angles S V′ T and S V‴ T are right angles, the planet will appear as half-full. At the point V″ the illuminated portion will be completely turned away from the east, and it will therefore be invisible, but would appear on the sun as a dark spot, and at its greatest diameter. In the positions between V and V′, and V and V‴, it will appear more or less gibbous; and between V′ and V″, and V″ and V‴, as crescent. The appearances of the planet and comparative sizes of its disc, under these circumstances, will be seen in the accompanying representation (Fig. 52).

The magnitude of the illuminated part can readily be obtained for any epoch by the following simple construction :—The plane of projection being perpendicular to a line joining the earth and Venus, the boundary of the illuminated hemisphere will be projected into an ellipse more or less eccentric upon it, and the minor axis will be in the same proportion to the major, as the radius is to the cosine of inclination between the two planes. The inclination is the angle at the centre of Venus, formed by lines drawn to the sun and earth.

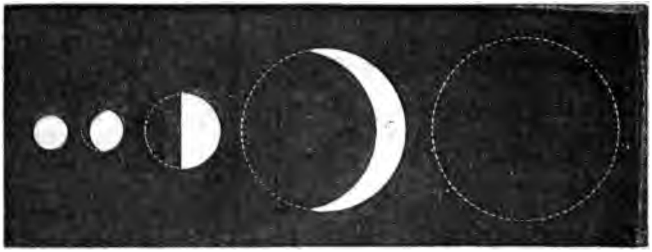


Fig. 52.

The greatest and least diameters of Venus are about 60″ and 10″, as seen from the earth; and it follows that if r be the distance between the sun and Venus, that the greatest and least distances will be $1 + r$, and $1 - r$ between Venus and the earth. The relative distances of Venus and the earth from the sun may therefore be found, as the extreme distances will be in proportion to the extreme diameters, or

$$1 + r : 1 - r :: 60 : 10, \text{ or } r = \frac{50}{70} + 0{\cdot}72.$$

The distance of Venus from the sun is therefore $\frac{72}{100}$ of that of the earth.

Whilst the planets Mercury and Venus can only be separated by a certain distance from the sun, and only perceived in the mornings and evenings, the other planets which are known at the present time can be seen at all parts of the sky, and pass the an at midnight when it is in opposition. The first are called the inferior, the the superior planets. The apparent course of the latter in the sky are, however, y irregular as the former, for the greater part of their revolution they are direct, r a short time retrograde; their motion is sometimes slow and sometimes fast, heir latitude is equally variable. Mars returns to the same position in the s in 687 days, but preserves its direct motion for 707 days, whilst its retrograde ı varies between 61 days and 81 days; but the direct motion, combined with the rade motion which follows, comprises about 780 days on the average. When in ıction with the sun, its motion is always direct; but when in opposition, it es retrograde, and the arc of the latter may vary between 10° and 19½°. Jupiter, ı, Uranus, &c., are in a similar manner retrograde at opposition, and direct at ıction. Its di- otion continues ›out 278 days, the retrograde 116½ to 122½ the arc of the ›eing constantly ut 10°. In a r manner, the during which ı moves in the of the signs is ys, and the du- of retrograda- may vary be- 135½ and 138½ he arc described g between 6° 40′ ' 55′. The manner in which these various motions were accounted for by the ts was the same system of epicycle already described for Venus, but some slight cation was introduced. The planet M (Fig. 53) was supposed to describe an le in such a manner that the radius C M, which joined it to the centre, was ıl to T S at all parts of its orbit, as C′ M′ to T S′, &c. If the different positions rs in reference to direct and retrograde motion be examined, it will be found to closely with this explanation.

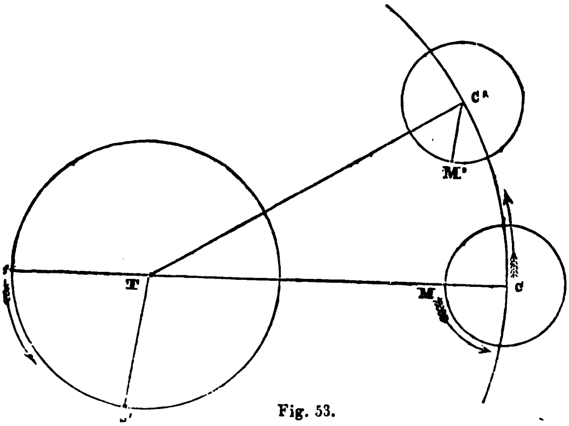


Fig. 53.

was by this system of epicycle and deferent, that Ptolemy accounted for the nt motions of the planets in their paths through the heavens, and he endeavoured nge the radii and motions of the epicycles in such a manner that they would agree he observed appearances. As, however, new inequalities were brought to light, found to be impossible to explain them by those single epicycles, and, in most a compound system of "wheel within wheel" had to be constructed in order ain a more approximate similarity between the calculated and observed places. ore simple Ptolemaic system will be perceived from the following diagram (Fig. The earth T is placed at the centre, next to which is the moon, L, which per- a revolution in 29½ days. Next to the moon is placed Mercury, *m*, whose period days; beyond which Venus, V, is situated, with a period of 584 days. The sun, t occurs in the order of distances, then Mars, Jupiter, and Saturn successively, as their distances from the earth were judged to be greater or lesser, according t length of time which elapsed between their successive returns to the same point heavens. It will be seen that the radii drawn from Mars, Jupiter, and Saturn, centres of their respective epicycles always remain parallel to the line which joi sun to the earth, and that the centres of the epicycles of Mercury and Ven always in the same line with the earth and sun. Plato considered that the sun

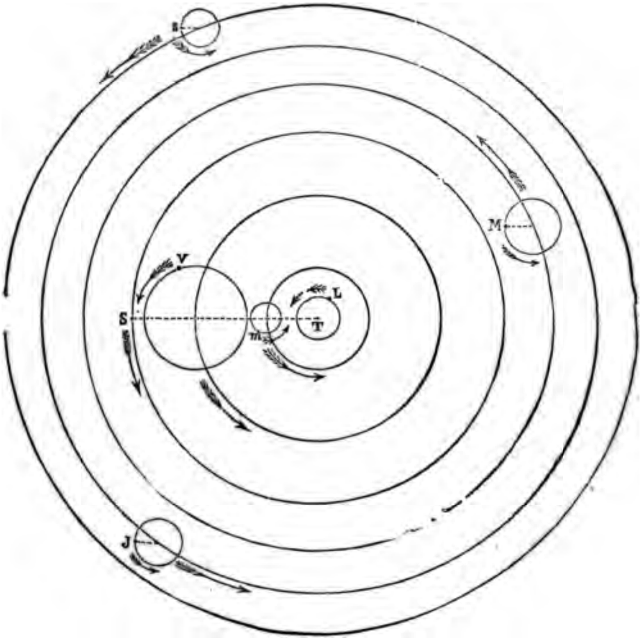


Fig. 54.

be placed nearer to the earth than Mercury and Venus, as these two latter planets never seen upon the disc of the sun; but it does not interfere with any of the fore explanations or appearances on which side it is supposed to be placed. At this pr time, this could not remain arbitrary, as the phases and variable diameters o planets would prove. It would have been much more natural for him to have the planets Venus and Mercury revolve round the sun, but as it was not possib him to do this in respect to the superior planets, it was likely that he would not it on that account.

Copernican System.—After the Ptolemaic system had been received among learned for so many ages, who were educated in the belief of the heavens "with and epicycle scribbled o'er," it was a difficult task which Copernicus had to co with, when, with all those motions partly accounted for by the Ptolemaic system endeavoured to reconcile all by far more simple machinery. How far the supposi of former astronomers, however true they were found to be, essentially agreed

ubsequently enunciated by him, is of little consequence, as the idea of placing
at the centre of the system is but a small portion of the glory of his discoveries,
onsisted in explaining all the apparent motions of the sun and planets, and the
of seasons, on the most simple grounds. The Egyptians had conjectured that
ets Mercury and Venus might revolve about the sun, and others supposed that
h was itself in motion around that body, but no further progress was made in
jecture; and, although it might have arrested the attention of Copernicus,
ith many other of the dreams and speculations, it could not afford him any
to the system he developed.

system of Copernicus is represented in Fig. *55*. The sun, S, is here the centre
great orb, and the planets Mercury, Venus,
th, Mars, Jupiter, and Saturn, *m*, V, T, M,
volve about it in regular order, and all in
e direction. The moon was held, as in the
ic system, to have a direct motion around
th, but was now made to accompany it in
e manner as a subsidiary planet; and,
to explain the rising and setting of bodies,
diurnal motion, the earth was supposed to
motion on its axis in twenty-four hours.
which have already been explained, all tend
rm it as the true system of the heavens,
he phases of Venus and Mercury, and many
roofs hereafter to be given, tended further
it beyond doubt, while its simplicity and
hical spirit led to its being generally ac-
lged. The motion of the earth was,
r, greatly opposed by many of the learned at
e, and, among others, Tycho Brahé endea-
to form a system, in which all the apparent
would be explained on the supposition of
nobility of the earth, which he considered
ly apparent from the absence of a sensible
parallax of the fixed stars, as well as from
e judged to be the improbability that an
e body, like the earth, would move at so
rate in space.

system framed by Tycho, which, at the
time, is preserved only as one of the curiosi-
scientific history, is a combination of the
can and Ptolemaic systems. The earth T is
pposed to be stationary, the planets revolve
e sun, but the sun, accompanied by all these
evolve around the earth, in exactly a similar
to the moon. Whilst it explains the vari-
ions of the sun, moon, and planets, it must at the same time be regarded as
philosophical, and as only fitted for the prejudices of the age.

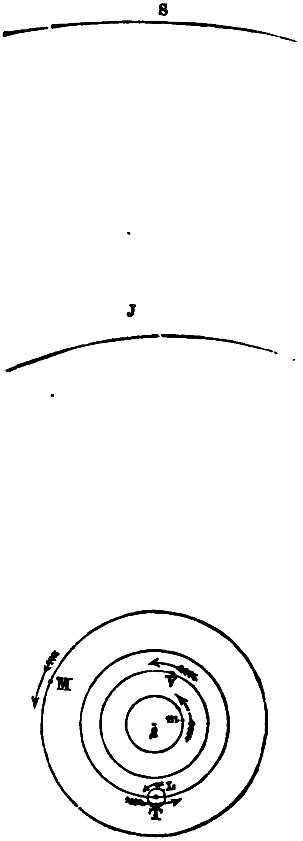


Fig. 55.

tions and Retrogradations.—The various progressive and retrograde motions of the planets, explained by the cumbrous machinery of the Ptolemaic system, on the supposition of the earth being stationary, follow naturally from the circular motions of the planets themselves, combined with that of the earth, in the true system of the universe, and, indeed, led the illustrious Copernicus to his great discovery. The two inferior planets, Mercury and Venus, move round the sun in less time than the earth, and if S be the sun (Fig. 56), V Venus at inferior conjunction, and T the earth, the planet will pass through an arc V V′ of its orbit, in the same time that the earth passes through the smaller arc T T′ of its orbit. The planet, therefore, has appeared to move quicker than the earth in the direction of the arrow, which is in a retrograde direction. If, however, Venus is found at superior conjunction with the sun, and it passes from V″ to V‴ in the same time that the Earth moves from T to T′, it will appear to move in a contrary sense to the arrow, and, therefore, direct. Between these two points, the planet will, of course, be animated with different rates of motion, and at certain times will appear quite stationary in the heavens. If the earth was fixed at T, the planet would appear stationary at the point of greatest elongation. As it is, however, the planet and earth must be both moving at the same rate, and in the same direction, in order that the former may appear stationary. The duration of the retrograde movement of Venus is only three weeks before and after the time of inferior conjunction.

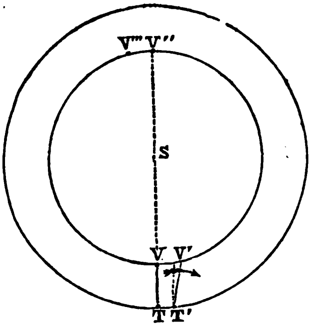


Fig. 56.

The superior planets bear the same relation to the earth as the latter does to Venus and Mercury, so that when the earth is stationary for any of those planets, the planets appear in the same manner to be stationary to the earth. The progressive and retrograde motions of the superior planets may, therefore, be explained in nearly the same manner. If M be Mars in opposition (Fig. 57), as the earth moves more rapidly in its orbit than the planet, it will pass from T to T′, whilst Mars only moves from M to M′, so that whilst it was first seen in the direction T M, it will afterwards appear in the direction T′ M; or, in other words, the planet will *appear to have* fallen back in the manner indicated by the arrow, or in retrograde *direction*. When the planet is in conjunction with the sun, or at M″, whilst it passes

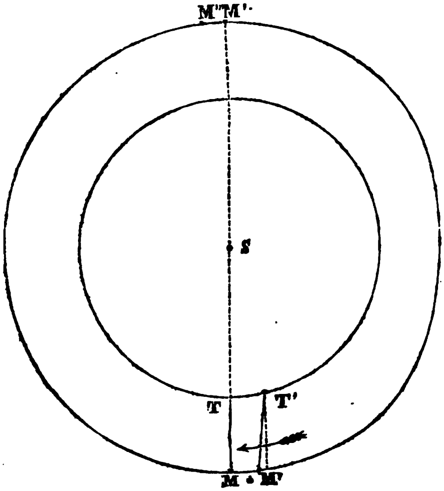


Fig. 57.

to M‴, the earth will have moved from T to T′ in the contrary direction, so planet will appear to move in a direct sense, or according to the order of the Thus all the superior planets are retrograde at the time of opposition, because tion is slower than that of the earth; and at the time of conjunction they are direct, because moving in the contrary direction. Between those points the ill appear as stationary at that part of its orbit, when the earth, passing from will be so oblique in regard to M M′ that the lines T M and T′ M′ will be ; and as the distances of the stars are immense in comparison with any of ets, the lines T M and T′ M′ will be directed to the same point of the heavens, planet will appear exactly in the same position in respect to any fixed point or

etermine the Elongation of a Planet when Stationary is a difficult when the inclinations and ellipticity of the orbit are taken into consideration; the present time it is more a matter of than interest, the orbit may be cons circular without much error, and situ- ie same place as the ecliptic. Let S (Fig. ie sun, T the earth, and M the position near its stationary point, then, when h passes from T to T′, the planet ve from M to M′; and the lines M′ T, ay be considered as parallel, as before d. Consequently M′ T′ S — M T S = M T S = T S T′; and in the same man- r — S M T′ = S r′ T′ — S M′ T′ = Whence it follows that the angular s at T and M are proportional to the S T′ and M S M′; or as the angular s are inversely as the periodic times, then T $\frac{3}{2}$ = putting a = S M, and S T = unity, the sines of the angles M and T n the ratio of their opposite sides, or as a : 1, and the cotemporary variations gles will be as their tangents; or,

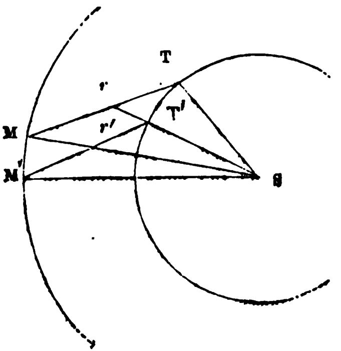


Fig. 58.

$$\frac{\text{n T}}{\text{s S}} : \frac{\sin M}{\cos M} :: a^{\frac{3}{2}} : 1\text{, whence } \frac{\sin T}{\sqrt{1-(\sin^2 T}} : \frac{\sin M}{\sqrt{(1-\sin^2 M}} :: a^{\frac{3}{2}} : 1$$

$$\text{And } \sin^2 T = \frac{a^3-a^2}{a^3-1} = \frac{a^3}{a^2+a+1}\text{, and } \sin T = \frac{a}{\sqrt{(a^2+a+1)}}$$

n inferior planet the M and M may represent the place of the earth, and T and sitions of Mercury and Venus. Thus if the mean distances of the earth and taken at 100,000 and 72,333, then sin T = 0·48264, the sine of 28° 51′, or at e of elongation from the sun the planet is stationary.

termine the time when a planet is stationary, the time of conjunction or oppo- ust be known. If m and n are the daily motions of the earth and planet, then r $n-m$ is the daily variation at the angle T S M, as it is a superior or inferior then,

$$\left.\begin{matrix} m-n \\ n-m \end{matrix}\right\} : \angle TSM :: 1 \text{ day} : \frac{\angle TSM}{m \backsim n}$$

Phases of the Superior Planets.—None of the superior planets show the same succession of phases as Mercury and Venus; and only one of them, **Mars**, is near enough to the earth to show any appearance of departure from a full disc, the illuminated hemisphere being turned nearly always towards the earth. The point of its orbit, at which Mars will appear most gibbous, will be at M (Fig. 59); at the points M′ and M″, when at opposition and conjunction, its illuminated hemisphere will be wholly turned towards the earth. The diminution of its disc from a complete circle is, however, very sensible; and a small table of its defective illumination is given in the Nautical Almanac for every month in the year. It is wholly insensible in Jupiter and Saturn, and to reduce the observation of the limb to the centre, it is only requisite to apply the semi-diameter.

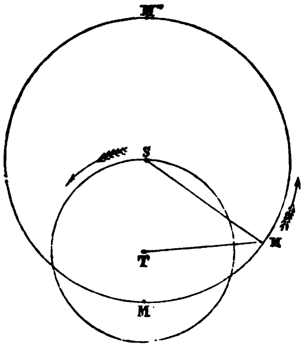


Fig. 59.

Kepler's Laws.—Copernicus, in explaining the apparent motions of the planets in the heavens, confined himself, it will be seen, to circular motion, and all the planets were supposed to revolve round the sun, as a centre in circular orbits, which he considered the most natural of any. It has already, however, been seen that the sun's motion in its orbit is not equable; nor will it be explained by supposing the earth to revolve in a circle, of which the sun is placed eccentrically to the orbit, and that it can only be accounted for by supposing that our planet moves in an ellipse, of which the sun is placed in one of the foci, whilst rejecting all the other cumbrous hypotheses of the Ptolemaic system. In order to explain the various inequalities caused by this elliptic motion, Copernicus was forced to retain the supposition of epicycle and eccentric. The observations of Tycho Brahé, in the hands of the immortal Kepler, however, threw off this last obstruction, and discovered not only the true figure of the orbits of the planets, but likewise other wonderful laws in relation to them.

In examining the orbit of Mars, he endeavoured to determine the eccentricity in respect to the sun, supposing it to be a circle, in the following manner:—Supposing that D (Fig. 06) was the point round which the motion of the planet was uniform, C the centre of the circle, and S the position of the sun, M, M′, M″, M‴, being four places of Mars at opposition, he endeavoured to determine the angles A D M″, A S M″, in such a manner that the four points, M, M′, M″, M‴, were situated on the circumference of the circle with the centre C. By assuming the distance S D, and the two angles, *A D M″*, and A S M″, he calculated trigonometrically all the other parts of

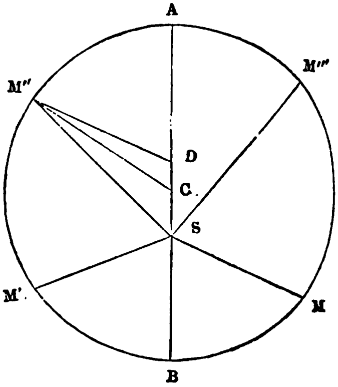


Fig. 60.

gure, in order to determine if the four angles at S made up 360°, and the points, and D, on the same straight line. After seventy most laborious trials, he at h arrived at one which agreed so well with observation, that out of twelve itions, none of them differed more than 1′ 47″ in longitude; and he thought Tycho's observations might be in error by that, or even a quantity of 2′. n compared with observations of the planet out of opposition, it, however, ne apparent that this orbit would not answer, the errors of longitude sometimes nting to 8′, which he was persuaded could not be due to the observation of Tycho. as consequently led to doubt observations could be satisfied y circular hypothesis, and further computation served to confirm ubts. Supposing S to be the Fig. 61), M, Mars, and T and T′, positions of the earth, when returned to the same part of bit, when the earth was at T, termined by observation the M T S, and when at T′, the M T′ S. The distances T S, being known, and the angle ′, the side T T′, and the angles ′ and S T′ T will be known, and consequently the angles M T T′ and M T′ T. side M T may be found in the triangle M T T′, and finally the side M S e triangle M T S, or the distance of Mars from the sun. By this method Kepler mined the distance M S, at perihelion and aphelion, the former of which he found 138500, and the latter 166780, the mean distance being 152640, that of the earth being sed to be 100000. In the same manner Kepler determined three other distances ars, from observation at different parts of his orbit, which he found to be 166255; 0; 147750. But by calculating these distances on the supposition that the mean ce was 152640, and the eccentricity 14140, the distances were found to be tively 166605; 163883; 148539. Whence the errors were found to be 350, nd 789. The true distances of Mars from the sun were, therefore, shorter than calculated; and as the line of apsides and the perihelion and aphelion distance truly known, it followed that the orbit was of an oval form, and the ellipse being mplest of all ovals, he found that this was the true curve. By examining Mars ny other parts of its orbit, he found that it agreed closely with this supposition.

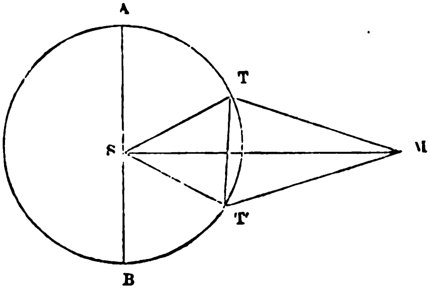


Fig. 61.

the first law of Kepler was discovered by infinite labour and sagacity, viz., *that anets revolve about the sun in elliptic orbits, the sun being situated in one of the foci.*

he second great law of Kepler was likewise discovered by him from observation, mparing the velocity of the planets in their orbit with their distances from the r rather the areas of the sectors, and the arcs included between the radii vectors ding them. He found that when at their apsides, the velocity of their motion nversely as their distances from the sun, or that the *planets describe equal areas in times* at those points; and he was of opinion that this was true at all parts of the , although he could not prove such to be the case. This, however, has since been d to be a necessary law, and to follow consequently from the doctrine of rsal *gravitation.*

The third great law of Kepler connects the distances of the planets from the sun, with the time in which they make a complete revolution about that luminary. The further a planet is removed from the sun, the slower he found its motion on comparing it with the absolute motion of the earth. The distance of Saturn from the sun being nine and a half times that of the earth, and consequently if their rates of motion were alike, the period in which Saturn would perform a complete revolution would be nine and a half years. It is apparent that the periodic times do not, however, increase according as the squares of the distances, as in that case the periodic time of Saturn would be upwards of ninety years, whilst in reality it is about thirty. After numerous and laborious trials, Kepler at length found out the remarkable analogy that *the squares of the numbers representing the periodic times were exactly proportional to the cubes of their mean distances from the sun.* Comparing the earth and Mars for example, we find that $(365{\cdot}2564)^2 : (686{\cdot}9796)^2 :: 100000^3 : 152369^3$. All of those remarkable laws were afterwards found by Sir Isaac Newton to be necessary consequences of the laws of gravitation, and thus what was deduced by Kepler from observations on one particular planet, and at one particular point of its orbit, was verified, and found to be universally applicable to the whole solar system, and to form a connecting link between all its members.

From the third law of Kepler the mean distance of a planet from the sun can be readily calculated, and with much less difficulty and more accuracy than by direct observation. The periodic time may be found by observing the time of the planet's passage through the nodes, and by observing the intervals between the ascending and descending nodes, and the descending and ascending (which, if the orbit is an ellipse, will not be exactly alike); a value of the eccentricity may be obtained from the greatest difference between the true and mean anomalies, or the equation of the centre. To determine the mean distance—supposing t to be the periodic time of the earth, and τ that of the planet, D being the mean distance of the earth, and Δ that of the planet, we have

$$t^2 : \tau^2 :: D^3 : \Delta^3, \text{ whence } \Delta = D \times \left(\frac{\tau}{t}\right)^{\frac{2}{3}}$$

The longitude of the planet in the nodes, or the longitude of the node, may be determined from two heliocentric latitudes and longitudes.

Telescopic Appearance of the Planets.—The planet Mercury appears under all the phases of the moon and Venus, but is seldom visible in our latitudes to the naked eye, as, when most favourable for that purpose, there is always a strong twilight, and it is situated too near the mists of the horizon. At its nearest approach to the earth, or at the time of inferior conjunction, its apparent diameter amounts to 12″, and at its superior conjunction this decreases to 4″; at its mean distance from the earth its apparent diameter is 6·7″. Its diameter is upwards of 3000 English miles, but observers differ considerably as to its apparent diameter as measured by the micrometer. In such a small body as this, it is almost impossible to perceive anything beyond the mere form of the disc by means of the most perfect and powerful telescopes; and even those are sometimes rendered useless by the scintillation and bad definition of the planet. The only observer who has followed this planet with the requisite attention, is Schroeter, who, by means of powerful reflecting telescopes, was able to perceive that the crescent was not always regular, but that sometimes one horn was blunter than another. This, he naturally considered, was due to their irregularities on the surface of the planet, a mountain or chain of mountains, situated in the southern hemisphere, intercepting the rays of the sun from proceeding onwards. Schroeter endeavoured to make use of this appearance in determining the time of rotation of the planet. He attentively observed the form of the disc, and the degree of bluntness of the southern horn at one particular time; and the changes which it underwent in a few hours were found to be plainly perceptible. When the planet reappeared on the following day, or any number of days afterwards, the time at which its disc presented the same form was again recorded, and so on for any length of time. Schroeter concluded from these observation that the time of rotation of the planet on its axis must be 24h. 5m., and by taking the extreme times of observation, and dividing the interval by the number of revolutions, he concluded it must have one rotation on its axis in the space of 24h. His assistant, M. Harding, on one occasion perceived a faint spot on the disc, and, by following it attentively, he arrived at the same conclusion as Schroeter. The latter attempted to arrive at a knowledge of the height of the mountains by measuring the deficient part of the horn; and his observations go to prove that some of those were upwards of twelve miles in altitude, which is three times that of any on the earth, and, compared with the actual size of the two planets, is out of all proportion. During the passage of Mercury over the sun's disc on November 9, 1802, Sir W. Herschel could not perceive any indications of an atmosphere, nor the least departure from the circular shape in its form. Even if it were as much compressed at the poles as the earth, the ellipticity of its outline would not, however, be visible in such a small body. Many observers have perceived a dusky ring of considerable extent surrounding the planet at those times, and others have noticed a bright spot on its disc on the same occasions, but this has been explained on optical grounds. During the passage, Mercury appeared as a very dark spot, and considerably more so than any of the spots which peared on the sun at the time. The transits of Mercury over the sun's disc, which will occur during the remainder of the present century, are as follows:—

1861, November 11. 1870, May 6. 1891, May, 9.
1868, November 4. 1881, November 7. 1894, November 10.

Those in 1881, November 7, and 1891, May 9, are invisible in the northern parts of Europe.

Venus.—The phases which Mercury undergoes are seen in a much more perceptible manner in those of Venus, from the great size and brilliancy of the planet at those times. The same appearance of the different sharpness of the horns has likewise been noticed, and from these circumstances Schroeter endeavoured to determine the period of rotation in the same manner as pursued by him in Mercury, and apparently with more chance of success. Not only was the southern horn noticed to be very blunt, but a detached point of light was perceived by him, which he concluded to be the summit of a mountain illumined by the setting rays of the sun. By numerous consecutive observations on this planet (continued for many years), Schroeter came to the conclusion that it performed a rotation on its axis in the space of 23h. 20m. 59·04s., or, in round numbers, 23h. 21m. Previous to this time, Cassini had perceived in the clear sky of Italy a small bright spot on the surface of the planet, by the observation of which, on several consecutive mornings, in the summer of the year 1667, he arrived at the conclusion that its period of rotation was 23h. 21m. or 22m. A far more extensive series of observations on several *dusky* spots was made during 1726-27 by Bianchini, at Rome, with excellent telescopes. The period of rotation, as determined by this observer, from these numerous spots, was very different from those of Cassini and Schroeter, amounting to 24d. 8h. It had been suspected by recent observers that a mistake arose from the spots so closely *resembling each* other; and as they were only observed during the evenings, it was impossible to recognize them by their appearance alone. The late Devico has reobserved all the effects perceived by Bianchini (from which it is certain that they were not of the fleeting, cloudy nature, suspected by Herschel and Schroeter), and from some thousands of micrometrical observations, made both during the day and evening, he concluded that it performs its rotation on its axis in 23h. 21m. 21·9s. The axis of rotation was supposed by Cassini, Schroeter, and Bianchini to be inclined as much as 70° to the pole of the ecliptic. The latter determination of Devico, although it does not show an inclination so considerable as this would prove to be, shows one nearly twice as great as that of the earth, viz., 53° 11′. Sir W. Herschel, on one or two occasions, perceived spots, but they were too faint and uncertain to give any idea of the time of rotation. He was never, however, able to perceive the bluntness of the southern horn observed by Schroeter, which is the more remarkable as it has plainly been seen by most observers who have examined it; and Maedler has perceived the quick changes which they undergo. Herschel, however, confirmed other appearances detected by Schroeter, viz., the brightness of the outer or circular edge of the crescent, and the dimness of the elliptical boundary. Herschel considered that the atmosphere of Venus was like our own, refracting and reflecting light; and as we view the circular part more obliquely than the elliptical boundary, and consequently a greater thickness of the atmosphere comes into view, this explanation would be more in accordance with the appearances observed by Schroeter, where the outer bright portion gradually melted into the interior dusky region that was observed by Herschel himself—the outer bright part being distinctly separated from the inner, and appearing as a bright colour.

In observing Venus when near its inferior conjunction, Schroeter perceived, in the very slender crescent seen on such occasions, a very faint light stretching beyond the pointed horns, which, as seen in an ordinary telescope, would be the extreme points of the illuminated semicircle; and Herschel confirmed this curious fact. This twilight affords further proof of the atmosphere of Venus, which Schroeter supposed to be of about the same density as that of the earth, having concluded that the horizontal refraction was 30½′. In the inferior conjunction of 1849, Professor Maedler plainly perceived this faint extension of the horns, and he found, for the horizontal refractions, quantities varying from 39 to 48 minutes, but whose most probable value was 43′·7. It is consequently about one-sixth greater than that of the earth's atmosphere. From appearances noted during the transits of Venus across the sun's disc, it would appear probable that the atmosphere was very considerable. The two next transits of Venus will occur on December 8, 1874, and December 6, 1882.

The apparent diameter of Venus, at its mean distance, is 16″·9; but at the time of inferior conjunction this increases to 62″, and at the time of superior conjunction decreases to 9½″. It resembles the earth in volume and density more than any other of the planets. Its diameter is 0·985 of that of the earth, its volume 0·957, and its density 0·923. A body which would weigh one pound on the earth, would weigh 0·91 on the surface of Venus. Light and heat would be nearly twice as great at Venus as on the earth. No compression at the poles has been perceived in Venus.

Mars.—This planet shines with a ruddy and dusky light quite different from any of the other heavenly objects. When at favourable opposition, or when near its perihelion and opposition at the same time, it is a very bright object, and só different from its ordinary appearance, that it has been frequently mistaken for a new star. Its *apparent diameter* at its mean distance is 5″·8, but at the time of opposition this can increase to 23″, and at conjunction decrease to 3″·3. The true diameter of this planet is 0·519 of that of the earth, and consequently its volume is only 0·140 of that of this earth. Its density, compared with that of the earth, is 0·948. A body which would weigh one pound on the earth, would only weigh half a pound in Mars. The light and heat which it receives at its mean distance from the sun is 0·43 of the earth.

When examined with powerful telescopes, many dark spots are perceived on the surface of this planet, which then loses much of that red colour so apparent to the naked eye. These dusky portions, it has been found, are quite fixed and constant in their positions, and they have thus been supposed to be the seas and continents of this planet. But not only have the land and water become visible on its surface, but likewise its climate; for we perceive, at its north and south poles, bright white patches of light which are naturally held to be the snows of Mars collected in an immense mass. This conjecture is considerably strengthened by the fact, that when, by the position of the axis of Mars in respect to the sun, those luminous spots are turned towards that luminary, they diminish in size rapidly. In 1781 Sir W. Herschel noticed that the southern white spots were extremely brilliant and extended; this was after a winter of a year's duration on this part of the planet. In 1783 this same spot was very small, but the sun had continued for nearly eight months above the horizon of this part, and melted it away. The existence of an atmosphere to this planet is apparent from other considerations, as at various times dark, extraneous, cloudy patches have been noticed, which have obscured the planet and hid the spots on its surface. When the snow melts away, a dusky and densely-clouded atmosphere appears to hang over the planet at those parts.

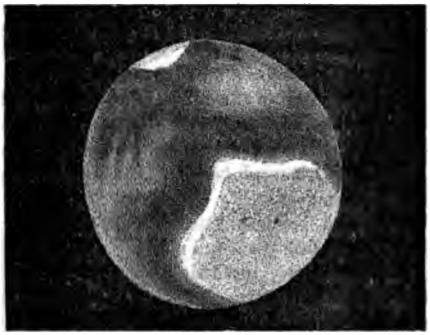


Fig. 62.

The rotation of the planet has been determined by several observers; and from the fixed nature of the spots observed, and the good definition with which they appear under favourable circumstances, it has been found with considerable accuracy. Cassini, in 1666, determined it to be performed in 24h. 40m.; and, in 1704, Maraldi repeated the observation, and found the period of rotation as 23h. 39m. Herschel made many observations relative to the telescopic appearances of this planet. He determined the sidereal rotation as 24h. 39m. 21·7s., and the synodical at 24h. 38m. 20·3s. The equator of Mars he found was inclined at an angle of 28° 42′ to his orbit, and the node was directed to the constellation Sagittarius. The seasons on this planet would not, therefore, be much different from those on the earth; but, on account of his great eccentricity, their duration was very different.

The compression of Mars at the poles appears to be considerably greater than that of the earth, which, considering his small diameter, and that its rotation and density are nearly similar to those of the earth, is rather remarkable. It does not appear, however, to be so considerable, as determined by Sir W. Herschel, according to whom the ratio of the polar and equatorial axes was as 98 to 103, or as 15 to 16. Schroeter estimates them at 80 to 81. According to M. Arago, it is more than $\frac{1}{30}$. Harding has noticed that the equatorial sides of Mars sometimes appear very bright, and he thinks that the irradiation caused by this circumstance produces discrepancies in the measurement of the equatorial and polar diameters.

The Asteroids.—The present century opened with the discovery of four small planets, Vesta, Juno, Ceres, and Pallas, situate between Mars and Jupiter, called asteroids (the appearance of stars), because of their stellar aspect under telescopic examination. The order of their discovery, with the names of their discoverers, is as follows:—

Ceres . . .	January 1, 1801, by M. Piazzi, of Palermo, in Sicily.
Pallas . . .	March 28, 1802, by M. Olbers, of Bremen, in Saxony.
Juno . . .	September 2, 1804, by M. Harding, of Lilienthal, in Hanover.
Vesta . . .	March 29, 1807, by M. Olbers, of Bremen.

These bodies are sometimes styled planetoids, as more expressive of their character, and extra-zodiacal planets, because their orbits are not confined within the zodiac like those of other planets. They are exclusively telescopic objects, and require the very best instruments to be caught, with the exception of Vesta, which, under favourable circumstances, has been seen by the naked eye.

Vesta, the first of the group, following the order of succession in the system, is at the mean distance of 225,000,000 of miles from the sun, and performs a revolution in 1325 days, somewhat more than three years and a half.

This planet is much brighter than its compeers, and appears like a star of the fourth or fifth magnitude.

Juno is at the mean distance of 254,000,000 of miles, and accomplishes an orbital revolution in four years and a hundred and twenty-eight days.

Juno has a very eccentric orbit, varying in her distance from the sun to the extent of 130,000,000 of miles. This eccentricity has a remarkable effect upon her motion, for she goes through that half of her orbit which is nearest the sun, in nearly half the time that she travels through the remainder. The planet has a reddish colour, and appears like a star of the eighth magnitude.

Ceres, the next in order, revolves about the sun in about four years and two-thirds, at the mean distance of 263,000,000 of miles.

The telescope reveals Ceres as a ruddy star of the eighth magnitude, under circumstances which leave the impression of an extensive atmosphere.

Pallas is at nearly the same mean distance from the sun as Ceres, and has nearly the same period of revolution.

The discovery of the asteroids was not an accidental circumstance, but the result of a search conducted upon the presumption that the harmony of the solar system required the presence of a planetary body between Mars and Jupiter. It was observed, that the distance between the orbits of Mars and the earth is about double that between those of the earth and Venus, or 50,000,000 of miles; whereas between the orbits of Mars and Jupiter there is the tremendous interval of 349,000,000 of miles, nearly two and a half times the whole distance of the former from the sun. Astronomers, therefore, became thoroughly imbued with the notion that an undiscovered planet existed in the gap; and commencing an active search of the heavens, the asteroids were met with in the vacancy.

On the ground of these peculiarities, it has been surmised that these four small bodies have diverged from a common node, and therefore originally formed a single *large planet, which* some mighty convulsion shattered. This bold hypothesis, first started by Olbers, has received a very general sanction. It obtains evidence from the powerful explosive forces in action in the interior of our own globe, to which its volcanic vents act as safety-valves; from the phenomena of meteoric showers, respecting which we have no supposition better than that they are *debris* which the earth encounters in its orbit, fusing upon contact with its atmosphere.

Of the telescopic appearance of this group of planets, now increased to the number of thirty-seven by the addition of Atalanta and Fides, both of which were discovered on the same day, viz., October 5, 1855, it would be difficult to say anything. A keen eye can detect, in a good telescope, the difference between one of these asteroids (when not less than the eighth or ninth magnitude) and a star of the same degree of brightness, but this perhaps is the extent of the power of the telescope. Their diameter is very doubtful. Some observers consider them as a few hundred miles in circumference, others as many thousands. The hazy appearance surrounding Ceres, which was noticed by Herschel and Schroeter, whence they surmised that the asteroids were partly of a cometary nature, has not been noticed by modern observers, and may be due to the imperfection of their reflecting telescopes.

Jupiter.—This is one of the finest planetary objects to which the telescope can be directed, and even with a small instrument its disc, band, and satellites, can be plainly seen, and at each opposition there is but little difference in the brightness. Whenever viewed with a telescope, the singular appearance which its disc presents, being always surrounded by two or more dark belts or bands, cannot fail to strike the beholder, and the changes to which they are subject are still more curious. The equatorial portions are generally the brightest, and it is at this part the bands are most distinctly visible. They are generally directed parallel to the equator, but this is not always the case. The polar portions are for the most part darker than any other, but it is but on rare occasions they are equally so, and indeed the whole cloudy covering of the planet is of such a shifting and unsteady character, that it may almost be said to vary from night to night. In addition to the dark bands, there are occasionally to be seen on the disc of the planets dark spots of irregular form, which are subject to the same changes as the bands, though they are frequently of long duration, and remain for months together at nearly the same place. A telescopic view of Jupiter, and the appearance of its surface under favourable circumstances, will be seen at Fig. 63.

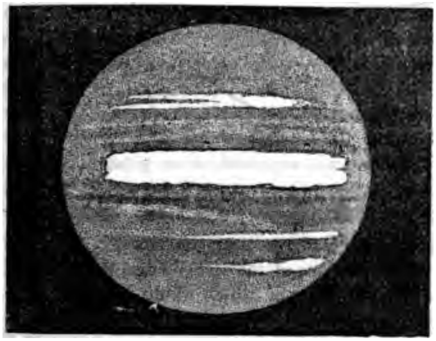


Fig. 63.

By attending to the motions of those spots, it has been found that Jupiter, like the planets hitherto described, rotates upon its axis; but the length of its day is much shorter than that of the planets in the vicinity of the earth. It is rather difficult to tell the exact duration, for the spots themselves have got a motion on the disc; and Herschel found that the time of rotation appeared to be sometimes 9h. 55m. 40s., at other times 9h. 54m. 53s., from observations of the same spot; whilst by a different spot it was sometimes 9h. 51m. 45s., and at other times 9h. 50m. 48s. As the equator of the planet is but very slightly inclined to the plane of its orbit round the sun, it follows that there is but very little variation of the seasons, so that during its long year the length of the day and night varies very slightly, either being at any latitude about five hours in length.

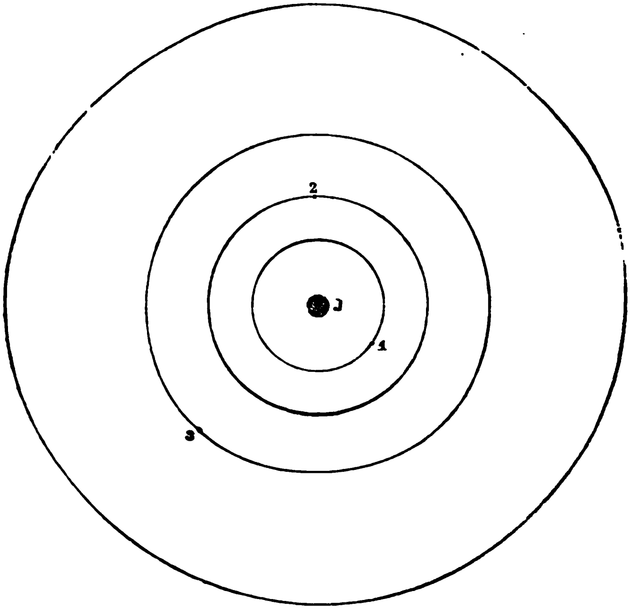


Fig. 64.

From the rapid motion which those spots undergo near the equator, Herschel thought it was not improbable that at these portions of the planet's disc there existed currents of air similar to the trade winds, the effect of which would likewise be to form the loose vapours on its surface into the parallel belts. From Herschel's observations on these accidental clouds—as he was of opinion the spots were—he considered that they were occasionally driven along at the rate of ninety-six leagues per hour, a velocity far exceeding that of our most violent hurricanes.

It might be thought, at first sight, that the dark bands represented in Fig. 63, were the clouds, the overcast portion of the planet's disc, and the brighter belts were parts of its surface. Herschel, however, considered that the contrary was the case, and in this he is followed by all modern observers. He supposed that the more brilliant portions were the zones, in which the atmosphere was charged with clouds, the darker parts those on which it was quite clear and serene. The latter allow the solar rays to pass through to the surface of the planet, where the reflection being less powerful than from the clouds, less light was consequently returned.

The disc of Jupiter is considerably flattened at the poles, and bulged out at the equator. This is immediately seen when the eye is directed through a telescope to the planet; and the extent to which it occurs will be seen by the diagram (Fig. 63). The belts appear always parallel to the greater axis.

The four satellites of Jupiter are constantly seen in the same direction, and almost in the same straight line; and the motions of the two inner ones round their primaries are very rapid. They almost describe circles around the planet, the orbits of the third and fourth satellites are, however, slightly eccentric. The following table shows their mean distances from the centre of the planet expressed in the equatorial diameter of Jupiter. The diagram (Fig. 64) represents the relative orbits of the four satellites in exact proportions:—

	Mean distance.	Period of revolution.
1st satellite . . .	3·03 . .	1·77
2nd „ . . .	4·81 . .	3·55
3rd „ . . .	7·68 . .	7·15
4th „ . . .	13·50 . .	16·69

The laws of Kepler are as strictly fulfilled in the passage of the satellites round their primary, as in the case of the motions of the planets round the sun. They are subject to eclipses and occultations, and are sometimes seen projected on the limb in the form of small white spots. As the cone of the shadow is of considerable length, Jupiter being at a great distance from the sun, and the satellites comparatively near it, it follows that at every revolution the first three satellites are immersed in the shadow, and it is only the fourth which occasionally escapes. The use made of the eclipses of Jupiter's satellites is represented by A of the engraving at page 241. The centre globe being our earth, two lines are drawn from places on its surface, forming an angle and meeting at Jupiter, an observer at each place marks the time at which the satellite enters the cone of the shadow. Assuming the time at the nearest line to be ten and the farthest eight o'clock, the two hours = 30° shows the one place to be distant 30° from the other. These observations of the eclipses gives the readiest means of determining the longitude of any place at which they are observed. This accurate observation of the moment of disappearance depends partly on the excellence of the telescope used, and partly on the keenness of sight and experience of the observer. When the satellites pass between the sun and Jupiter, they produce solar eclipses; and the shadow of the satellite, in addition to the satellite itself, may be seen slowly traversing the disc of the planet in a powerful instrument.

Saturn.—The most wonderfully constituted body of the solar system is Saturn.

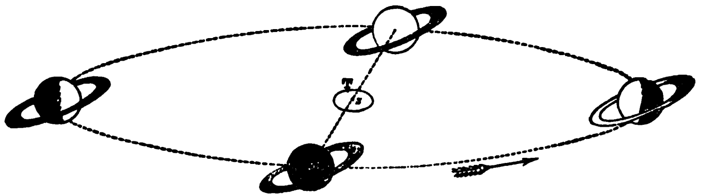


Fig. 65.

Surrounded by numerous rings, of different form and brilliancy, and eight satellites, with the varied bands which, as in the case of Jupiter, cross the ball of the planet, it forms the most curious of telescopic objects. The ring was first detected by Huygens by means of his powerful telescopes, and he was likewise the first to perceive the brightest of the satellites. He explained the various appearances at different times by taking into account the obliquity of the ring, and the parallelism which it retains at all parts of its orbit; which is, indeed, similar in all respects to the change of seasons on the earth. This will easily be seen by the diagram (Fig. 65), where S is the sun and T the earth, and the outer circle the extent of Saturn. It is plain that to an observer at T, the upper side of the ring and the northern pole of Saturn will be visible, as at Fig. 66. At the opposite part of the orbit of Saturn, the *southern* pole of the planet and the *under* side of the ring is that which will be illuminated by the sun, and seen by an observer on the earth. As Saturn passes from the one to the other of these points, the ring will become gradually less open; and at the points between the two it will only be visible by the illuminated edge, as in Figs. 67 and 68; whilst at other times the unillumined plane of the ring will be turned to an observer at T, and the outer edges of the ring will not be visible, as at Fig. 65. This disappearance of the ring will take place twice during every revolution of Saturn, and it may be compared to the position of the earth's axis at the times of the equinoxes in respect to the sun, both poles coming into sight, for the ring of Saturn surrounds the equator of the plane.

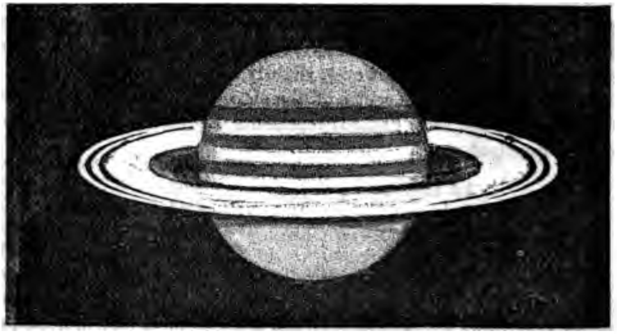


Fig. 66.

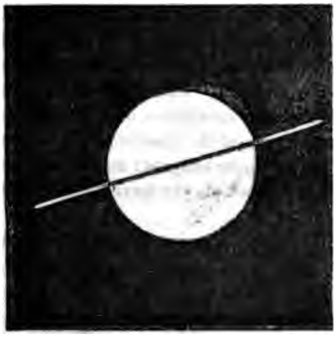


Fig. 67.

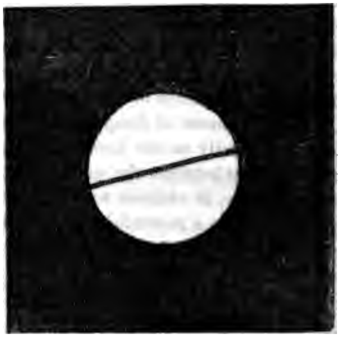


Fig. 68.

The ring of Saturn is seen, with an instrument of moderate power, to be divided into two parts; but when a stronger telescope is made use of, under favourable circumstances, the outer portion is found to be divided into two likewise, thus making three in all, as seen in the diagram. Some observers have been able to detect other divisions in the bright ring when it is most open. The most remarkable appearance detected of late years is, however, the dusky ring, situated between the ball and the bright ring, and adjoining the latter. The existence of this curious appendage is now placed beyond doubt by the testimony of different observers, and it appears to be of a misty, semi-transparent nature, as the ball of the planet can be seen through it. From the observations of the elder Herschel, it would follow that the globe of Saturn revolves on its axis once in 10h. 16m., and that the ring makes a rotation in its plane once in 10h. 32m. 15s. Both move in the direction of the other planetary bodies, viz., from west to east.

The system of satellites which surround Saturn and his ring will be seen from the accompanying diagram (Fig. 69).

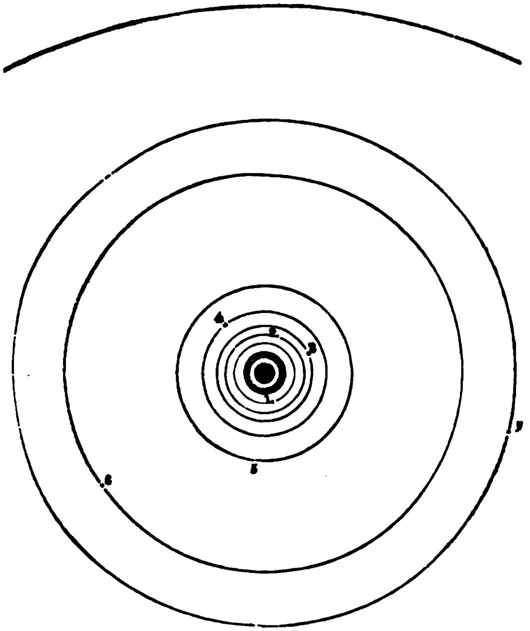


Fig. 69.

Of these the brightest is that discovered by Huygens, and resembles a star of the ninth magnitude. It is the sixth in the order of distance counting from the planet, and its orbit is the best determined of any of them. The third, fourth, fifth, and eighth, discovered by Dominique Cassini, are likewise comparatively bright; but the first and second, discovered by Herschel, and the seventh, discovered by Mr. Sassel in 1848, are the faintest of telescopic objects, and require both great light and high power to be seen at all. The orbit of the eighth satellite is considerably inclined to the equator of the planet; those of the other seven lie nearly in the same plane as the ring. The mean distances, expressed in equatorial radii of the planet, and the periods of their revolutions, is seen from the following table :—

	Mean Distance.	Time of Revolutions.
1st Satellite	3·35	0·94
2nd „	4·30	1·37
3rd „	5·28	1 89
4th „	6·82	2·74
5th „	9·52	4·52
6th „	22·08	15·94
7th „	27·78	22·50
8th „	64·36	79·33

Uranus and Neptune.—Of the telescopic appearance of the two exterior planets, Uranus and Neptune, very little can be said. The former appears in the telescope as a star of the sixth magnitude, of a planetary aspect, and with the appearance of a well defined disc. The latter is of the same faint blue colour, but the disc is more difficult to be perceived; but, if compared with a neighbouring star of the same magnitude, the difference in their definition is more apparent. Herschel considered that he was able to detect a slight ellipticity in the figure of Uranus, from which it appeared that its axis is very little inclined to the plane of the elliptic, and that consequently the sun is hid for many years from its poles.

The satellite system of Uranus (according to Herschel) is represented in Fig. 70, but only four of those satellites, two of which do not agree in their periods with that set down by Herschel, have been perceived by Mr. Sassel, and he seems inclined to believe that only that number exists. The second and fourth have frequently been seen, and their orbits are pretty well determined by the observations of Sir J. Herschel and Mr. Lamont of Munich. The orbits of those satellites are very greatly inclined to the plane of the elliptic, the angle between the two being nearly 80°. The periods of revolutions and mean distances of the several satellites is seen from the following table:—

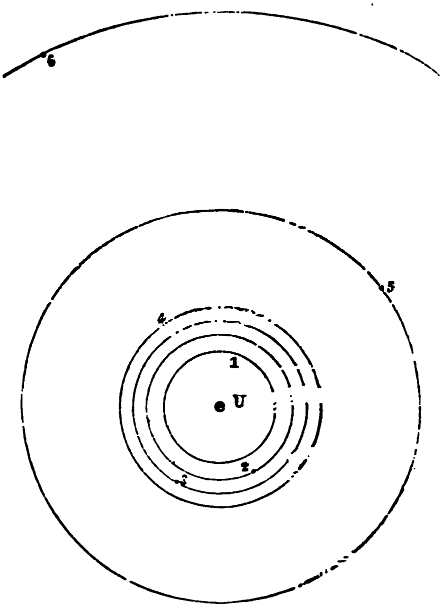


Fig. 70.

	Mean Distances.	Time of Revolutions.
1 Satellite	13·12	5·89
2 „	17·02	8·71
3 „	19·85	10·96
4 „	22·75	13·46
5 „	45·51	38·07
6 „	91·01	107·69

The only satellite to Neptune which has yet been perceived is that discovered by Mr. Sassel, and it is represented in Fig. 71, on the same scale as that of the satellite systems of Jupiter, Saturn, and Uranus. It will be seen that it differs but slightly from that of the moon to the earth (Fig. 72) in the dimensions of its orbit. The period of revolution is 5·87 days, and its orbit is inclined at an angle of 35° on the elliptic. It is excessively minute, and has hitherto been seen only by Messrs. Sassel and Bond.

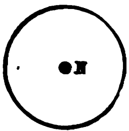


Fig. 71.

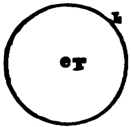
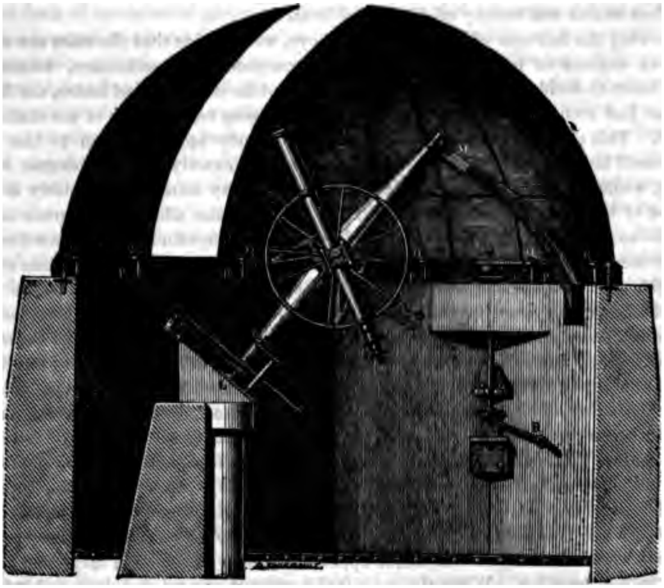


Fig. 72.



GREAT EQUATORIAL INSTRUMENT.

STELLAR ASTRONOMY.

r no other subject to which the mind of man can be directed, is the grandeur so iposing, or the subject so inexhaustible, as in those distant regions of space known as e sidereal heavens. The countless stars, which reach far beyond the limits which ur most powerful instruments can furnish evidence respecting them, reaching even to finity. This is not merely made manifest by the help of gigantic telescopes, and other tificial applications which the intellect of man has devised; even with the naked e we can perceive how multitudinous are the brilliant specks of light so lavishly attered in every direction, and congregated in such numbers as in the constellations of rion or Taurus, or so densely that the eye can no longer detect them individually, in the Hyades, the Presepe of Cancer, and, above all, in the great southern Magellanic Clouds, or the wonderful zone of the Milky Way. But although the prospect is so agnificent and apparently so boundless, even in the most simple and universal view, becomes infinitely more so when the depths of the heavens are sounded by means the telescope; when it becomes probable that those stars which are seen by the assisted vision form one cluster, and that not a very rich one, out of many thousands. hen we consider, furthermore, that our sun is but one out of those myriads of objects, ining by their own light, and that each of these is performing in its sphere the same important functions to the worlds revolving around its concentrated heat and light, which the sun performs for us, we become more and more deeply impressed with the boundless extent and variety of celestial objects.

Viewing the heavens with the unassisted eye, we perceive that the stars are of very different degrees of brightness; and, for the purposes of classification, astronomers have chosen to divide those visible to the naked eye into six degrees of lustre, the faintest or those just seen by persons of ordinary eyesight being termed stars of the sixth magnitude. This classification, however, gives but a rude approximation to the truth, as it would be difficult to find a number of stars of exactly the same degree of brilliancy, while it would be easy to count twenty or thirty stars of perceptibly different degrees of brightness. With thehelp of the telescope ten additional degrees of magnitude—each ranging from the seventh to the sixteenth magnitude—have been described by astronomers. Stars of the seventh magnitude can be perceived by persons of keen eyesight, but those of the fifteenth and sixteenth magnitudes are only visible in the largest telescopes; even in the gigantic reflectors of Herschel, of twenty feet focal length and two feet aperture, they are very faint objects. The satellites of Uranus and the faintest of Saturn's satellites are estimated to be of this degree of brightness. When the magnitudes of the fixed stars are expressed in this manner, it should be premised that their apparent magnitudes are understood by the expression and not their intrinsic and absolute value. The relative intensity of the light of the stars is still a matter of considerable doubt; they have not yet been determined photometrically with any exactitude. Sir W. Herschel endeavoured to compare the light of a star of the sixth magnitude with that emitted by Sirius, by covering the speculum of the telescope when pointed at the latter object with a disc having a circular opening; and the aperture was so diminished that Sirius appeared as a star of the sixth magnitude, as seen with the full opening. When the magnitudes were thus rendered artificially equal, he found that Sirius, viewed with a circular aperture of one inch in diameter, was reduced to the same intensity of light as a star of the sixth magnitude when viewed with the full aperture of eighteen inches; and he concluded from this, that if the light of the latter be supposed equal to unity, that of Sirius would be eighteen times eighteen, or three hundred and twenty-four. As the light of Sirius is fully three times that of a star of the ordinary first-class magnitude, he considered that, in general, the light of a star of the first, in proportion to that of one of the sixth, magnitude was as a hundred to one. If in the intermediate classes the brightness is supposed to be inversely as the squares of the distances, we have—

1st magnitude .. the brightness $= 100$
2nd „ „ $\frac{100}{4} = 25$
3rd „ „ $\frac{100}{8} = 12$
4th „ „ $\frac{100}{16} = 6$

And in the two remaining classes he concluded, without reference to this law,

5th magnitude . . the brightness $= 2$
6th „ „ $= 1$

This determination of the relative intensities of the brightness of stars rests, however, on too narrow a basis to be regarded as more than an approximation; and it is, perhaps, impossible to determine the brightness of one class of magnitudes in fractional parts of that of another. All that astronomical observers have hitherto accomplished on this subject has been to arrange and catalogue them in order of brightness, which, for the first six magnitudes, is best done with the naked eye. When they are estimated by means of the telescope great discrepancies have occurred between different observers, and the fault of our celestial globes and charts has been that a number of faint telescopic stars are inserted as among those visible to the naked eye, whilst others equally bright are completely omitted, their magnitudes being set down at the time their positions were first determined, and, perhaps, wrongly estimated from the state of the atmosphere, position of the moon, or other causes. Those visible to the naked eye in the northern hemisphere have been thoroughly revised by Argelander, who has subdivided the six classes formerly reckoned into sixteen, inserting two new divisions between each of the ancient classes; thus, for instance, between the fourth and fifth magnitudes he finds many stars a little fainter than the fourth, which he terms the 4·5 magnitude, and others a little brighter than the fifth (yet not so bright as those of the 4·5 class), which he denominates the 5·4 magnitude. Sir John Herschel, during his sojourn at the Cape, has instituted a further approximation to correctness in the relative brightness of stars, which will be of the utmost importance to future ages in arriving at a knowledge of the constancy of their brightness. This method of *sequences*, as he terms it, consists in choosing one of the brightest stars in any region of the heavens; secondly, one just inferior to it in lustre; thirdly, one immediately inferior to that of the suppositious *second* magnitude, and so on for twenty or thirty times, until a catalogue is formed for this particular region, with the stars ranged merely in regard to their brightness. On any subsequent opportunity this is repeated with a new set of stars, introducing, however, as many as possible of the former series. By this means, the relative intensity of all such stars as are visible to the naked eye may be determined with the most rigorous accuracy. A valuable catalogue of such objects, and various proofs of the almost absolute certainty of the process, may be seen in Sir J. Herschel's "Result of Observations" made at the Cape. The want of such a class of observations has led many astronomers to conjecture that great changes have taken place in the brightness of stars since the time of Bayer, at the commencement of the seventeenth century. Among others, Sir W. Herschel was of opinion that at least one out of every thirty stars observed and mapped by Bayer, had diminished or increased in brightness in the two centuries which intervened between the end of the sixteenth and commencement of the nineteenth century. It was supposed that Bayer's practice was to call the brightest star of *any* constellation by the first letter of the Greek alphabet α, the next brightest β, the third γ, and so on. But Sir W. Herschel found that, instead of the stars in the constellation of cygnus preserving this order, α, β, γ, δ, ϵ, &c., they now appeared according to the order α, γ, ϵ, β, and δ; in Aquila they had changed to the order α, γ, δ, β, ϵ; in Draco to γ, β, δ, α; in Leo to α, γ, β, δ, ϵ; and this change was apparent in numerous other instances. But it would now appear as if the magnitudes of the stars had been carelessly inserted by Bayer, no such change being due to the objects themselves.

Number of Stars.—In our estimation of the number of the fixed stars, it would appear that we are liable to an illusion; and that the generality of individuals suppose a much greater number to be visible than is found to be the case. The attention is probably directed to the richest portions of the heavens, and it may not unfrequently happen that the judgment is biased by what we have heard or read, rather than by what we see. In this case it may naturally be supposed that other parts of the surface of the heavens are equally crowded with stars, though they are not so bright or apparent as the part which we really have in view, it may thus be erroneously concluded that thousands of stars are visible at the same moment, which only exist in imagination. When we come to test this conclusion by absolute proof, it is found to be vastly out of proportion to these impressions. The *whole number* of stars visible to the naked eye in the central parts of Europe—viz., those included in the whole surface of the heavens north of the equator, and including a zone of 30° of south declination, comprising nearly eight-tenths of the whole sky—only amounts to 3,256, so that scarcely more than 2,000 stars can be visible to the naked eye and above the horizon at the same moment.

An attempt has been made to ascertain the number of stars of different magnitudes by supposing them to be situated at equal distances from one another, and that they are all of the same absolute magnitude, but appearing differently in consequence of their various distances. There being fourteen stars of the first magnitude, we are to suppose them arranged at equal distances upon a sphere: supposing the stars of the second magnitude to be twice the distance of those of the first, the surface over which they would be scattered would be four times that of the former; and if placed at the same distances from each other, it would take fifty-six stars to cover this area. The sphere of stars of the third magnitude would be nine times the area of that of the first, and it would consequently take one hundred and twenty-six stars to fill that surface; and in a similar manner there would be two hundred and twenty-four on the fourth, three hundred and fifty on the fifth, and so on. This law, however, does not correspond with the observed number of stars of different magnitudes, there being seventy stars of the second, and three hundred of the third—a much greater number than would exist on this supposition. Various other hypotheses have been formed to show the probable number of stars, of different magnitudes, visible on a given portion of the sky; but the data are too inconclusive and vague to secure any degree of accuracy in the result.

It would be difficult to determine the number of stars fainter than those of the sixth magnitude. The catalogue of Lalande (called the *Histoire Céléste*, and published at the latter part of the last century) contains the places of about 50,000 stars visible from the north pole to 25° of south declination, and including those from the first to the ninth magnitude. The zone included between 15° of north, and 15° of south declination, contains, according to the more modern observations of the illustrious Bessel, 31,085 stars, viz:—

664 bright stars from the 1st to the 6th magnitude.
2,500 of the 7th magnitude.
8,183 „ 8th „
19,738 „ 9th „

Stars fainter than those of the ninth magnitude increase in number in a wonderful degree, and Struve concludes that the number of stars visible in the twenty-feet telescope of Herschel, in the same zone of 30° in breadth observed by Bessel, amounts to the enormous number of 5,819,000, by far the greater number of which are situated at those parts where the milky way intersects the equator at six and eighteen hours of right ascension. Nor will this appear overrated, when we recollect that on one occasion Herschel perceived nearly 120,000 stars, which passed through the field of view of his telescope (15′ in diameter) in a quarter of an hour. The stars observed by Herschel appear to have been situated in the following order in respect to their right ascension, and their density as they approach the milky way becomes immediately apparent.

From 1h. to 5h. of right ascension . .	391,700 stars.
„ 5h. „ 9h. „ „ . . .	1,984,200 „
„ 9h. „ 13h. „ „ . . .	235,400 „
„ 13h. „ 17h. „ „ . . .	387,000 „
„ 17h. „ 21h. „ „ . . .	2,365,100 „
„ 21h. „ 1h. „ „ . . .	455,600 „
	5,819,000

Thus, in respect to the distribution of stars of different magnitudes in the heavens, we perceive that the least crowded regions lie between nine and seventeen hours, and twenty-one, and five hours of right ascension. If we take the most brilliant stars, or those of the three first magnitudes only into account, we find them to be pretty evenly distributed over the surface of the heavens; but those of the fourth, fifth, and sixth magnitudes are congregated more densely as they approach the milky way. At every succeeding class this becomes more and more apparent, and the faintest stars are most thickly crowded in and near this zone. This would naturally lead us to imagine that there was some connection between the great galactic circle and the other portions of the heavens, and that they might form one great system.

The milky way extends completely round the heavens, and makes almost a great circle upon its surface. The breadth is very unequal—in some regions it is not more than 5°, in other parts it is two or three times that breadth. For one third of its extent, viz., between Serpentarius and Antinous, it is divided into two branches; but for this distance of about 120° the dark opening is of no great breadth. The resolvability of the milky way into distinct individual stars, which was proved immediately on the invention of the telescope, had been long previously conjectured by some of the ancient philosophers, whilst the absurd suppositions of others on its structure—if they were ever supposed worthy of examination—were dissipated by the same discovery. No other theory was started, however, to supply their place, or to explain this phenomenon, until about the middle of the eighteenth century, when Thomas Wright, of Durham, author of the *Clavis Cœlestis*, endeavoured, in his "Theory of the Universe," to account for this appearance by supposing that the stars were ranged in regular *strata*, and not dispersed fortuitously throughout space, as was previously supposed. By this arrangement, although the individual stars composing the stratum were at vast distances apart, yet supposing our sun and its attendant satellites to be situated near the centre of this plane, we should witness such an appearance as the milky way presents. In the direction of the plane, the stars would be seen in such vast numbers, although the more distant ones would be so extremely small, that it would appear as a white and confused zone of light projected on the dark space surrounding us on every side. This idea received further development from the celebrated Immanuel Kant, who considered that it was rendered probable from the arrangement of the three or four thousand stars visible to the naked eye, the greater number of which were contained in a zone within a short distance of the galactic circle. Lambert was likewise of opinion that all the stars visible through the best telescopes lay in one vast stratum, but he considered that many of the clusters in the milky way were separate and individual systems, but nevertheless subsidiary members of one great system. Those immense clusters, each containing millions of stars, were connected and held together by the same power which predominates in inferior and more simple systems, and Lambert went so far as to imagine that the separate clusters performed revolutions round a great central body in the same manner as our planet and satellite moves round its primary.

The attention of Sir W. Herschel was particularly directed to the subject of the construction of the heavens, and his labours in this field must be regarded as among the most important of his works. In order to bring the theories started by his predecessors to a test, and to obtain an idea of the form and dimensions of the stellar universe, Herschel had recourse to a long and laborious method, which he properly termed "gauging the heavens." This was done by directing a powerful telescope (the twenty-foot reflector) to different parts of the sky, and counting the number of stars in each field of view. In order to insure greater accuracy, he counted the number of stars in ten contiguous fields, and took the average to express the comparative riches or poverty of the district. In some portions of the sky only three or four stars of all magnitudes were seen *per* field for a considerable distance round, whilst at other times the field was crowded with many hundreds, and those latter portions were always found in or near the milky way. By combining the numerous "gauges" which he made, he endeavoured to determine the various depths (from the different degrees of obliquity in which the stars were viewed) of the milky way; supposing the individuals composing it to be placed at pretty equal distances, he concluded the whole visible heavens to be of a lenticular form, and not a stratum of stars inclosed by plane surfaces. The proportion the thickness bears to the diameter of this *lens* he considered was as one to five and a half, and he further concluded that the sun was removed but little from the centre of the group. Subsequently, when the motion of various double stars, consequent on their physical connection, was discovered by Herschel, his ideas were considerably modified; he now imagined that he perceived evidences of this physical connection in the great groups of the milky way, and that this "clustering power," as he termed it, tended to break it up into fragments.

Nor was this the only evidence of the motion of the stars in space, many of them were endowed with an indubitable motion, as was apparent from their positions, compared with neighbouring stars; and, whether due to their own proper motion or to that of the sun, it must be considered as an absolute proof of the instability of these bodies in space. Among the stars which are thus known to have considerable proper motion, we may mention μ Cassiopeiæ, which has a proper motion of 5″·82 annually, in right ascension, and 1″·55 in north polar distance. The star 61 Cygnis, whose distance has been determined with some exactness, is one of those which has a large proper motion, and it was in consequence of this circumstance that it was chosen by Bessel for the determination of the annual parallax. The proper motion in right ascension amounts to 5″·39, and in north polar distance to 3″·30. In 40 Eridani the proper motion in right ascension amounts to 2″·16, and in north polar distance to 3″·45. The star 1830 of Groombridge's, which was considered to show the greatest amount of parallax, is one of those whose proper motion is the most considerable, amounting to 5″·16 in right ascension, and 5″·70 in north polar distance. Other stars in almost every constellation have large proper motions, but the quantities are as yet somewhat doubtful, as it is only by the comparison of accurate modern observations with ancient authorites nearly as accurate, that these small quantities can be deduced; and it is only since the times of Bradley that the places of the objects can be depended upon.

Enough, however, has been effected to show the reality of their movement and its direction. If this were due to the motion of the stars themselves, it might be supposed that they would move in all directions, north or south, east or west; and though this is found to be the case with some, yet with far the greater number the direction taken is much more regular. It has hence been conjectured that it is the sun itself which is in motion—the consequence of which would be, that those stars which are situated in that part of the sky which we approach would appear to be separated more and more, as the angular distance would increase the nearer we approach to the objects. In that part of the celestial regions which we are leaving behind us, the stars would appear to be falling closer together for the contrary reason. Although this supposition, explaining the proper motions of stars as being due to the simple effect of parallax, was made by both Mayer and Lambert, it was Sir W. Herschel who first attempted to show the direction of the solar motion, which he concluded was towards a point in the constellation of Hercules, whose position, in 1783, was at 257° of right ascension and 65° of north polar distance. An attempt has since been made by Argelander to solve this problem, and that celebrated astronomer arrives nearly at the same result as Herschel, finding, from an examination of the proper motions of three hundred and ninety stars, that the part of the heavens towards which the sun is progressing, is probably situated at 260° 51′ of right ascension, and 58° 43′ of north polar distance.

Nature and Different Species of Stars.—It would, of course, be impossible to form any idea of the stars; but analogy would lead us to imagine that the heat and light which they emit is in every respect similar to that of our own sun; and some photometrical experiments, which have been made on the latter object, seem to show that, if it were transported to the same distances as some of the stars whose distances are pretty well known, its magnitude would not be so great as many which we see around us. From the distance and orbit of 61 Cygni, Bessel was able to arrive at a rough idea of the masses of those stars, which he found were not greatly under that of the sun; and if this was the case with a star of that magnitude, we may conclude that the brighter objects are considerably larger than our luminary. It is certain that reflected light could not reach us from such immense distances, and we must conclude that, like our own sun, they are self-luminous, although we know nothing of the agency at work to produce those extraordinary effects. It would be rash to imagine that, like it, they are all accompanied by a cortege of planets; for some of the latter bodies in the solar system have numerous satellites, our own earth but one, and others none whatever. But it would be equally rash to conclude that their heat and light expend themselves in the unprofitable and dark voids of the celestial spaces; and, in addition to the slight evidence which analogy affords, we have the further proofs of their being the centres of great systems from the remarkable phenomena of *double* and *changeable* stars, the latter of which, in a different point of view, may be regarded as evidence of their rotation on their axes, as in the case of our own sun.

Double Stars.—A few of those curious objects have been known ever since the invention of the telescope; and in a cursory examination of some of the brighter stars, as Castor, ζ Ursæ Minoris, α Herculis, γ Virginis, an observer could not help detecting the strange appearance of two stars close together and almost blending their light, forming, apparently, but one star to the naked eye. In the middle of the seventeenth century further importance was attached to those curious objects, and several were closely examined, although without any result, for the purpose of determining their annual parallax, for which they offered peculiar advantages. It was then supposed that all these objects were fortuitously or optically double, one of the components—most probably the fainter—being situated at a much greater distance from us than the brighter, but both appearing in the same direction. It was for this purpose, likewise, that Sir W. Herschel commenced, in 1779, to apply his powerful telescopes and delicate micrometers to the task of recording their distances and positions in respect to one another. But in looking for one thing, as sometimes happens, another was found; and it became apparent that the components were not only at the same distance from the sun, but that the smaller body, in many instances, described an orbit round the larger star. It should not be forgotten, however, and it tends much to the credit of philosophical conjecture, that this remarkable law was previously surmised by the celebrated Lambert, who considered it possible that there were some groups in which the stars might make complete revolutions round a common centre of gravity in a comparatively short period of time. Mitchell conjectured the same law to apply to the more simple case of a double star, and in 1784 he supposes that in a few years this question would be resolved by the stars whose positions and distances were ascertained by Herschel. The great discovery was first published by Herschel in 1803, who had then perceived a decided change in the positions and distances of several stars, as ξ Ursæ Majoris, Castor, ξ Bootis, 70 Ophinchi, ξ Cancri, ξ Herculis, &c., &c. The labours of the elder Herschel were resumed in more modern times by his celebrated son, who, in conjunction with Sir J. South, reobserved all the stars in the northern hemisphere, and who has also observed independently and measured all those discovered by him in the southern hemisphere. Struve, at Dorpat and St. Petersburg, also made the subject his constant care since the year 1814, and with the powerful means at his command has observed and reobserved their positions with the greatest possible accuracy; indeed the publication of his great work, "Mensuræ Micrometricæ Stellarum Duplicium," forms an era in this subject. This work contains observations of three thousand one hundred and twelve double stars, nearly three-fourths of which were discovered by himself.

The combinations of the components of double stars take every variety, both in regard to magnitude, distance, and, we might add, colour. The components of Castor are each of the third magnitude and of the same ashy white colour, and close together, as are likewise those of γ Arietis in the same respects; whilst the stars Polaris, α in Herculis, γ in Delphini, are very different either in magnitude distance, or colour. Out of the 3,000 stars detected by Struve, only a small portion have as yet been determined to be physically connected or to form *binary systems*, as it has been termed; and the numbers whose orbits and periods have been even approximately determined are fewer still. The time which has elapsed since the discovery has been too inconsiderable to determine their periods, some of which, as that of Epsilon Lyræ, cannot be less than 2000 years; and in others, as in 61 Cygni or γ Leonis, the period is several hundred years. Among those whose orbits have been determined with more or less exactness, we may mention the following:—

Name of the Star.	Mean Distance of the Components.	Eccentricity.	Period.	Direction of Motion.
	″		Years.	
ξ Herculis . . .	1·25	0·448	36·4	
η Ursæ Minoris .	2·44	0 431	61·6	Retrograde.
η Cor. Bor. . . .	1·20	0·404	67·3	Direct.
70 Ophinchi . .	4·97	0·444	92·3	Retrograde.
σ Coronæ . . .	2·93	0 577	199·9	Direct.
γ Virginis . . .	5·35	0 868	157·6	Retrograde.
Castor	7 01	0·797	230·3	Retrograde.

M. Savary was the first who determined that the revolution of a star round its primary was performed in the same manner as that of a planet round the sun, and conformably to the laws of gravitation; and the same principles applied to other stars have shown that the first two laws of Kepler, founded upon the motions of Mars, extend to the revolution of sun around sun. In determining the orbit, supposing it to be an ellipse, the same elements have to be deduced as in a planetary orbit, with the exception that the period of revolution and the semi-major axis are here two distinct elements, and that the latter is not expressed in linear measure, but in an arc of a great circle. From four observations of the angles of position and the distances, knowing the intervals and taking the Keplerian laws as the basis of calculation, the seven elements of period, semi-major axis, eccentricity, nodes, inclination, position of peri-astre, and epoch of the peri-astre, can be determined. In the first place, however, the *apparent* ellipse must be determined before the *real* one can be arrived at. The first is that which the star describes around its primary on the plane of the sky; but the *orbit* may be inclined at any angle to this plane, and it is only when the plane of the orbit is perpendicular to the line of sight that the true and apparent ellipses are identical. The projection of the circular or elliptic orbit on the plane of the heavens remains always an ellipse, but the projection of the focus and the major and minor axes will take different positions; and the proportions of the latter to one another, and consequently the eccentricity of the orbit, will be different. In consequence of this difference between the apparent and true orbits, we sometimes see the one star projected on, or occulted by, its companion. Such was the case with γ Virginis, which, from 1834 to 1836, appeared as a well-defined single star. The plane of the orbit of the companion of 44 Bootis, is almost perpendicular to the plane of the heavens; and between 1802 and 1819 an *almost central occultation* must have taken place. The companion of ξ Herculis has twice undergone an eclipse, in consequence of the great inclination of the orbit to the plane of vision—once in 1802, and again in 1831.

In addition to the double stars, triple and quadruple stars are sometimes met with; and if the fortuitous combination of the former is so little to be expected, that of the latter is much less probable. Among the more conspicuous of the triple stars, that of ξ Cancri holds the most prominent position, the three stars, all of which are nearly of the same magnitude, being physically connected. The close double star has made upwards of a revolution in a retrograde direction since 1782, and the more distant one has moved fifty degrees in the same time. In the list of quadruple stars, the most remarkable is that of ε and 5 Lyræ, which can be detected as *double* by a keen eye without the aid of any instrument; but each of which, when examined with a power of two hundred, appear as a double star, the four components being nearly of the same magnitude and colour. This system likewise seems to be physically connected. Among the multiple stars, that of θ Orionis, situated in the centre of the great nebula, is the most apparent, six stars being situated in a circle of twenty-three seconds in diameter.

The colours seen in the components of many double stars have been described by some as an optical illusion, on the ground that it is always the complementary colours which are thus perceived, as in a similar manner a white spot seen on a red ground will appear green. It is, however, impossible to view the components of α Herculis or γ Andromeda, without coming to the conclusion that they have a proper colour of their own, and that the blue and orange so vividly distinct is something more than the effect of contrast. In those stars which show high and brilliant colours, the larger star is always of a golden orange, the smaller greenish or bluish. In many, however, the components are of the same colour; but it seldom happens that the two colours are of a bluish or greenish tint, most frequently they are white and yellow.

Temporary and Variable Stars.—Those stars which vary in brilliancy from time to time, and others which suddenly seem to start into existence and disappear as precipitately as they have become visible, must be regarded as the most curious objects in the heavens, for they prove manifestations of life and motion in the immeasurably distant regions of space; they form one of the subjects, too, in which even a lover of science who does not possess a telescope can, by simple observation of the relative brightness of the various stars, recorded on favourable opportunities, confer considerable benefit on this branch of sidereal astronomy. A telescope in this case is of little value; the field of view is much too small to include the stars proper for comparison with the object whose light is suspected to be variable, and the photometer has hitherto been of very little service. The most accurate results which have been obtained as yet are due to observations with the naked eye, compared with stars in its vicinity; by this means the interval between the faintest and brightest phases have been found with far more accuracy than at first sight would be imagined.

There are numerous instances of the appearances of temporary stars recorded in the Chinese annals, but the two most remarkable of those phenomena have occurred in comparatively modern times, and have been minutely described by the two great contemporaries, Tycho Brahé and Kepler. The first appeared in the year 1572, in the constellation of Cassiopeia. On the 11th of November of that year, when seen for the first time by Tycho, it surpassed in brilliancy both Sirius and Jupiter, the most lustrous objects in the heavens. In the following month it had diminished slightly in brightness, but was still equal to Jupiter. At the beginning of 1573 it was inferior to that planet in brightness, and by the end of March it was not brighter than the principal star in Taurus, although still a good first magnitude. It continued gradually to decrease until the end of the year, but remained visible to the naked eye until March, 1574. If its period of increase was equal to its period of decrease, it must have been visible to the naked eye for nearly three years, as it was not noticed until it had attained its maximum brightness. For a description of the second instance we are indebted to Kepler. It was perceived suddenly in the constellation Serpentarius on October 10, 1604, and appears to have been nearly as brilliant as the former one, though not so favourably situated for observation. It remained visible to the naked eye (the telescope had not yet been invented) for upwards of a year. A star of the third magnitude also appeared suddenly in the constellation of Cygnus in the year 1670, which soon afterwards disappeared; it again made its appearance, and again disappeared, and has not been since seen. It underwent several changes during the two years in which it was observed.

Variable Periodic Stars.—The appearance of temporary stars are as rare as the phenomena observed are extraordinary. Of a similar nature, however, are the class of periodic variable stars, whose changes of lustre are equally as decided and curious as those of the objects just mentioned; being more known, however, they excite less attention. The most singular of those objects is Omicron Ceti, whose variability was first discovered by Holwarda in 1639, and which has a period of about eleven months. Although at its maximum brightness it reaches to the second magnitude, it does not appear to have been at all noticed by any observer previous to 1596; but this may be partly in consequence of the length of its period, for if the maximum intensity of light falls in the summer months, it may sometimes remain invisible for three or four years together. During the winter months this is not the case; but, as happened in February and December, 1847, two maxima may take place during the same year. There is another circumstance which has been noticed, and which may tend to explain the silence of ancient authors and observers. This is that the star, when at its maximum brightness, does not always reach the same brilliancy, sometimes approaching in brightness to stars of the first or second magnitudes at this period, whilst at other times it is not brighter than the fourth magnitude at its maximum. It has generally been supposed that it disappears entirely at the period of its minimum even in the best telescopes; but this is not always the case, for at these times it has occasionally been observed to be not fainter than stars of the eleventh magnitude. What its variations of light may be when visible only in the telescope is not very well known, as it has not been closely observed at those times; but it is certain that, whilst visible to the naked eye, its fluctuations of brightness are very remarkable. It is visible to the naked eye on an average for about two months previous to the period of its maximum brightness; but the period of its diminution of brightness is generally longer than that of its increase, and it has been visible to the naked eye for three months after its maximum, the average duration being, however, only seventy days. It sometimes, but rarely, happens that the interval which elapses between its coming into sight and its maximum brightness is greater than the time between its maximum and disappearance. Some attempt has been made to reduce the fluctuations which its light undergoes and established a law; but some of the changes are too abrupt and irregular to be dealt with in this manner or to be foretold with any accuracy.

Among the other stars of long period, which are visible to the naked eye for a length of time, and then entirely disappear from sight, is χ in the neck of Cygnus, which is almost as remarkable as the preceding. Its light varies between the fifth and eleventh magnitude. Its maximum brightness, like that of *o* Ceti, is, however, very variable; sometimes it reaches to the fourth magnitude, and at other times its maximum is not more than the 6·7 magnitude, when it is quite invisible to the naked eye. The variability of the light of this star was discovered by Gottfried Kirch, but the period was first found by Maraldi. The interval between its successive *maxima* and *minima*, as well as its intensity of light, at those times, has since been discovered to be very irregular. On some occasions it has been visible to the naked eye for a period of nearly three months; but the average duration, according to Argelander, is only fifty-two days, being twenty days of an increase and thirty-two of a decrease. The longest period which has yet been recorded in the class of variable stars occurs with the star 30 Hydræ, whose period has been determined by Maraldi at four hundred and ninety-four days, this period, however, is very irregular. At its maximum brightness it sometimes arrives at the fourth magnitude, and at other times only at the fifth, and thence it decreases in brilliancy until its entire disappearance. The star 19 Leonis is another with a very long period, the interval between its successive maxima being three hundred and eleven days. This period, however, is also somewhat irregular, and the changes in its brilliancy occasionally abrupt. At its maximum brilliancy it is equal to stars of the fifth magnitude, and is invisible at its minimum. These are the only four stars hitherto discovered which are invisible to the naked eye at their *maxima*, and vanish out of sight at their *minima*, except to the most powerful telescopes. There are a few other stars discovered by Harding, Schwerd, and Hind, with periods which appear to be upwards of a year; but even at their maxima these stars are only visible in a telescope, and not the slightest trace of them can be perceived in the best instruments at the epoch of their minima.

In others of the variable stars, the period is much shorter; but the change of light is not so considerable. The most remarkable of those is the star β Persei, in which the interval between two successive maxima is only 2d. 20h. 48m. 58s., and is otherwise a very curious object from the abrupt and variable changes of brilliancy which it exhibits. For the greater portion of its period it remains at nearly its maximum brightness, or of the two and a half magnitude. In about four hours it suddenly decreases to the fourth magnitude, which is its minimum brightness, and in a like period of four hours it regains its former brightness. There are other but less remarkable changes in its brightness; it is one of the best determined of the variable class. The majority of the periodic stars shine with a reddish light, but this shows no sign of colour, being of a pure white. The star δ Cephei has a period of 5d. 8h. 37m., with a variation of brilliancy from the third to the fifth magnitude. This star exhibits great regularity in its successive changes of brilliancy, which have been determined with considerable exactness; but the periods of increase and decrease are very dissimilar, as it takes 3d. 18h. to pass from its maximum to its minimum, and only 1d. 15h. to return from its minimum to its maximum, according to the careful observations of Argelander. During eight hours of its decrease it scarcely, if at all, changes; and for a whole day its diminution of brightness is scarcely perceptible. Two other variable stars of short period—η Aquilæ and β Lyræ—are known at present. Both of those stars change from the three and a half to the four and a half magnitude; the former in 7·18 days, the latter in 12·8 days. The period of the latter star was first stated to be 6d. 9h. by Goodricke, who detected its variation, which he at first supposed to be from the third to the fifth magnitude. In this determination of the length of the period he was partly correct; for it is one of those stars in which there is a smaller variation of brilliancy within the larger period. After its minimum brilliancy, it passes in 3d. 5h. to its first maximum, and then in 3d. 3h. to its second minimum. In 3d. 2h. it again rises to its second maximum, and after this in 3d. 12h. to its former minimum. The whole period, according to Argelander, is 12d. 21h. 46m. 40s., in which period it performs all the preceding changes. This celebrated astronomer is of opinion that the period is diminishing, and that it is now some hours shorter than when first discovered. Even from recent observations it is apparent that there is a decrease in the period.

Previous to the time of Sir W. Herschel, it was only stars of very long and very short periods which were known to be variable. The period of α Herculis, discovered by him proved, however, that there was an intermediate class. This star is remarkable for its deep orange red colour, which is very strikingly contrasted with its small dark blue companion. The period was considered by Herschel to be 63 days, but the change of lustre is very slight, being only from the third to the third and a half magnitude. There is still some doubt respecting its period, some observers considering that it is as much as ninety-five days; but from seven years' observations it would appear that a period of sixty-six days would agree better with the changes which it undergoes. The observer Heis, who has particularly attended to this species of phenomena is of opinion that the variations of light are best satisfied by a period of 184·9 days with two maxima and two minima. Since the time of Herschel, various other stars of *medium periods* have been added to that of α Herculis, nearly all of which are, like it, stars of *considerable magnitude*. Thus α Cassiopeiæ changes from the second to the third magnitude in 79d. 3h., according to Biot; but it is very difficult to determine the exact period, the change of brilliancy being very slight and apparently irregular. This is equally the case with α Orionis, which, at its minimum, becomes slightly inferior to stars of the first magnitude; and, according to Argelander, the period of increase of light takes place in ninety-one days and a half, whilst it continues to decrease in brilliancy during a hundred and four days and a half. The period of α Hydræ is supposed by some to be performed in fifty-five days; but the change of brightness being minute, as in other instances, it is somewhat doubtful. Sir J. Herschel considers it to be performed in twenty-nine or thirty days. The star commonly designated as *l* in the constellation of *Sobieski's Shield*, is remarkable for the great variation of its magnitude, particularly at its minimum, when it varies between the sixth and ninth magnitudes. At its maximum it only varies between the fifth and sixth. Among those of shorter period may be counted that of β Pegasi, which varies between the second and second and a half magnitude, but whose period is not more than forty days. At the present time, we thus know stars of almost every period from 2d. 20h. to 406d. 1h. There are ten whose period varies between 1 and 100 days; two between 100 and 200; eight from 300 to 400; and two above 400. It is curious that there are none at present known whose period is between 200 and 300 days.

Herschel considered that, by introducing a medium period between those which were very long and very short, he proved the rotation of those objects on an axis, by which means such portions of their surfaces as were dark and obscure came into sight at stated times, and hence their apparent faintness. To the astronomers of the 16th and 17th centuries—by many of whom the rotation of the earth was doubted, and that of the other planets unknown—this method of accounting for it would have appeared strange. It was attempted, however, by Bouilland to explain the changes of Omicron Ceti on this principle; the larger portion of the surface of this star was, he considered, non-luminous, which rendered it invisible for the greater part of its period. Another supposition in respect to those changes was, that they were due to the interposition of opaque bodies, by which their light was more or less eclipsed at regular intervals. Maupertius considered that, among the many stars which were scattered through space, there were some whose figure was not regularly spherical, but greatly flattened at the polar regions, and that the variations of light were due to the thin edges of those being sometimes presented to view and at other times the large surface of the flattened disc. There is yet no *experimentum crucis* known, by which it can be said whether either or any of these hypotheses are correct.

Clusters and Nebulæ.—The most striking and magnificent objects which the sidereal heavens present to view, are the clusters and nebulæ, where, in a telescopic field of view of a few minutes in diameter, we may perceive a crowd of stars collected together and forming a small patch of light, the individuals composing which must be some thousands in number. To those wherein the stars are plainly seen, have been given the name of *clusters*; but in others which are unresolved, the terms *nebulæ* and *unresolved nebulosities* have been applied. The nebulæ and clusters which are at present known are of every degree of resolvability, and though some of those which are generally called nebulæ have been resolved by the most powerful instruments, yet there are others which have baffled all the attempts of opticians to separate them into individuals. They are of every conceivable shape and size, from the regular and minute form of the planetary nebula of a few seconds in diameter, to the amorphous and large surface of the *great nebula in Orion*. Numerous clusters of stars are visible to the naked eye, as the Pleiades, Hyades, the Presepe of Cancer, and that in the sword handle of Perseus; and in the southern hemisphere we have the Magellanic Clouds, which cover a space of some square degrees.

The most magnificent of the clusters which have been resolved by the telescope, is that in the constellation of Hercules (Fig. 73), which can be detected on a clear night by the naked eye, and which, in the telescope, presents one of the finest instances of dense, isolated clustering in the whole of the heavens. The stars are from the tenth to the fifteenth magnitudes, and are so densely congregated round the centre that it presents the appearance of a blaze of light in that part. The stars are pretty evenly distributed in the interior, but are rather irregular at the edges, where the stars are formed into curvilinear branches. Sir J. Herschel is of opinion that there are some thousands of them, some of which are the faintest that the telescope can show. This object is only seven or eight minutes in diameter, and, with the telescope which Sir J. Herschel employed, no appearance of a nucleus was perceptible; although, with a smaller one, such might be suspected.

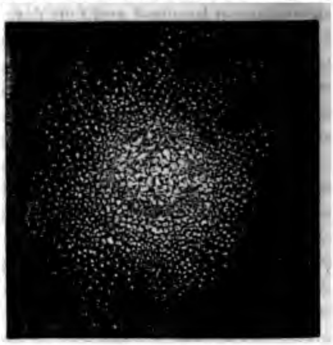


Fig. 73.

The fifth object in Messier's catalogue, is another very rich but compressed specimen of a globular cluster (right ascension 15h. 10m., north polar distance 87° 16'). In this case (Fig. 74) the density towards the centre is not gradual, the condensation commences suddenly, and the blaze of light into which it is formed appears projected on a loose irregular ground of stars. The neighbourhood of this cluster is very poor in stars—a circumstance which Sir. W. Herschel remarked in other cases, and which he conceived might be due to the stars being attracted to one point and formed into one cluster. The cluster situated at 21h. 25m. right ascension and 91° 34' of north polar distance, appears like an unresolved nebula with instruments of even considerable power; and it is only when viewed with the great light and high magnifying

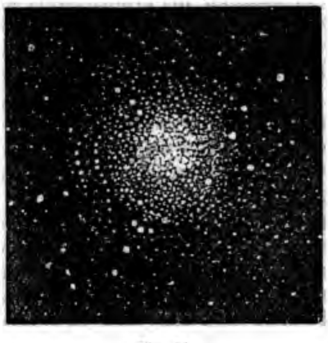


Fig. 74.

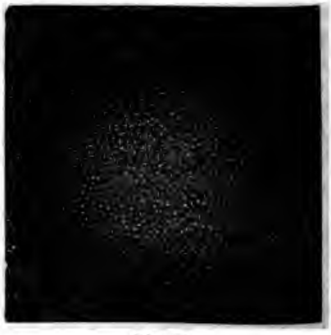


Fig. 75.

powers, that faint traces of stars become apparent on its surface (Fig. 75). *Finer* specimens of clusters, or rather nebulæ, *merely* resolved, are seen at Figs. 76 and 77,

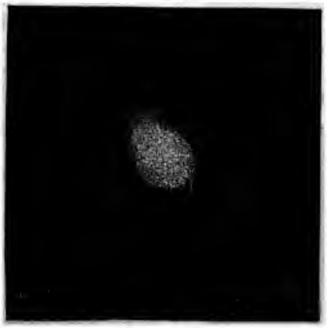


Fig. 76.

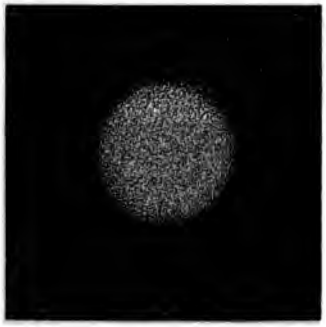


Fig. 77.

where it takes a very fine sky and splendid instrument to show the grains of star-dust faintly visible on its surface. We may consider the latter of those as the extreme boundary of the clustering species, after which the nebulæ, properly so called, begin. As the reflecting telescope advances more and more to perfection, we may, however, expect that many objects will be won from the mysterious domain of nebulæ, and placed to the account of the more explicable clusters.

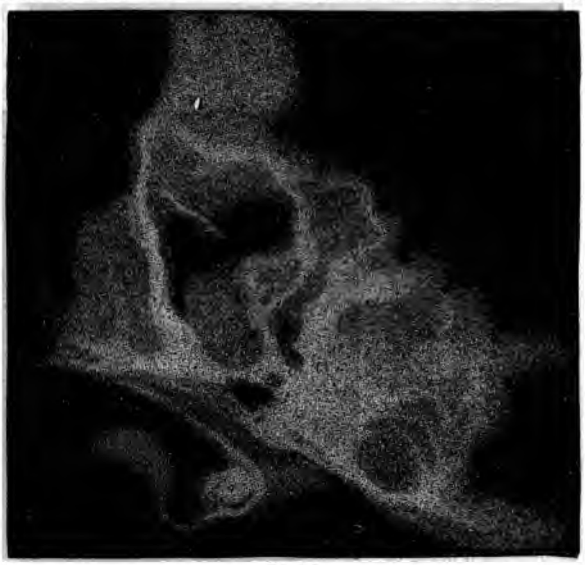


Fig. 78.

The large clusters have some approach to regularity, but this can scarcely be said of the more considerable nebulæ. The finest of those is, without doubt, that of Orion (Fig. 78), which, for its various inequalities of light and shade, its great extent, and the capricious irregularities in the form of the great promontories which jut out from the main body, constitute it one of the most noble and mysterious of those objects. When examined with the two-feet mirror of Sir John Herschel and some powerful refracting telescopes, it shows some signs of resolvability, being likened to a curdling mass, or the breaking up of a mackerel sky. But with the great mirror of Lord Rosse, the resolution is complete, and it is broken up into distinct stars, which are scattered in irregular masses through its body. The great nebula of Andromeda (Fig. 79) is another very conspicuous object of the class, and may even be perceived with the naked eye on a fine dark night. It is a long elliptical ray of light rather brighter towards the centre, and was compared, by its discoverer, Simon Marius, to the flame of a lamp shining through a piece of horn. With the improved optical means of more modern times, equally as with the imperfect telescope of Marius, no signs of resolvability have as yet appeared, though the surface has been noticed to be dotted over with innumerable stars, which are not, however, supposed to be directly connected with it, being merely casual groups interposed between it and the sun.

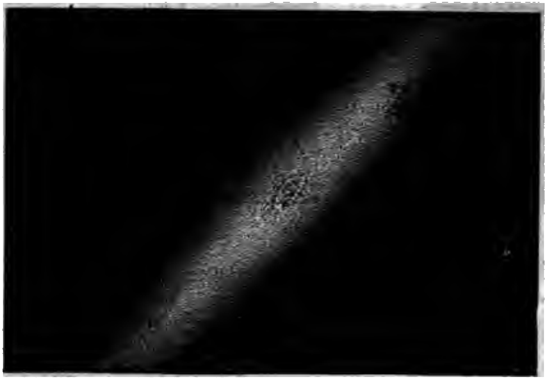


Fig. 79.

The majority of the nebulæ hitherto discovered—now upwards of 3,000 in number—are mostly of a regular form, being either circular or more or less elliptic, like that of Andromeda. But there are many others, and those among the largest and brightest, in which the nebulous matter is dispersed very capriciously, no regularity being perceptible. Among the latter, the one situated at 272° 42′ of right ascension, and 106° 15′ of north polar distance (Fig. 80), may be instanced, being of the shape of the large Greek omega, and no symptoms of resolvability being apparent. Among the more regular forms are those termed *annular* nebulæ and *cometary* nebulæ. The fine object situ-

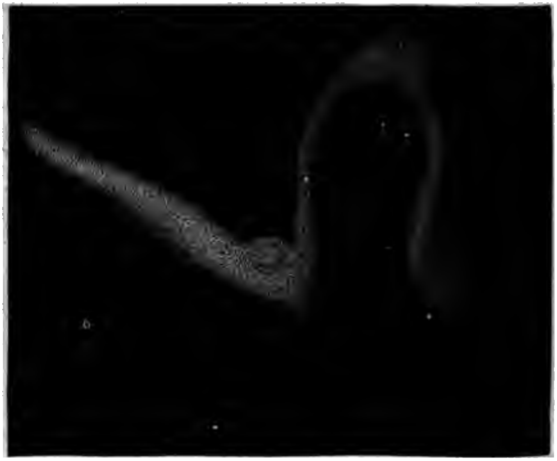


Fig. 80.

00°40′ of right ascension, and 41° 56′ of north polar distance (Fig. 81), and
s large enough
erceived as a
tch of light in
. telescope, is
the finest in-
of the former.
instance, the
ling ring ap-
n the two foot
of Sir J. Her-
be divided into
nches; but in
t telescope of
sse, the whole
e form of a
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ted as quite
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nected with
er body by a
g branch of
l. A fine spe-
the more sim-
rare form of
nebulæ is that situated in the constellation of Lyra (Fig. 82) at 281° 49′ of
ension and 57° 11′ of north polar distance. It is not quite dark in the

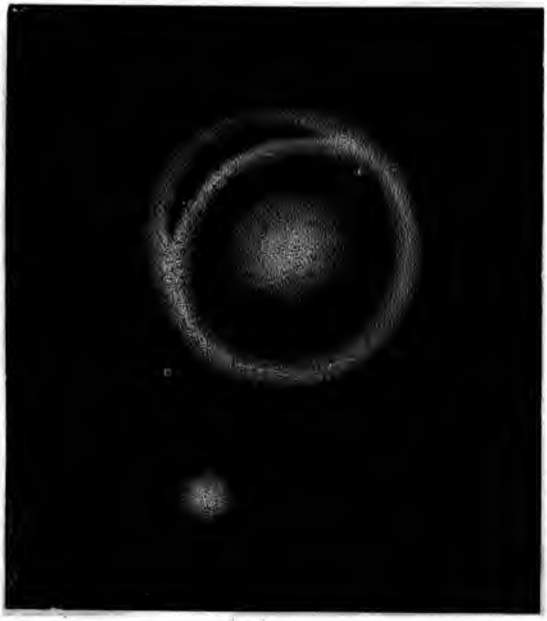


Fig. 81.

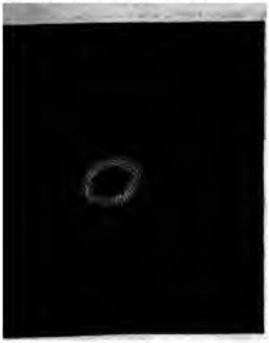


Fig. 82.

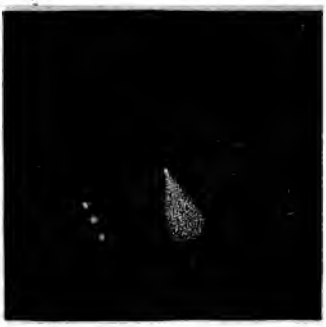


Fig. 83.

The form of cometary nebulæ will be seen by Fig. 83, the pointed end being apparently to a star which appears like a stellar nucleus.

In the circular or globular form of nebulæ, the degrees of condensation of the nebulous matter towards their centre is very different. In Fig. 84, right ascension 18° 45', north polar distance 77° 59', there is little or no condensation towards the middle.

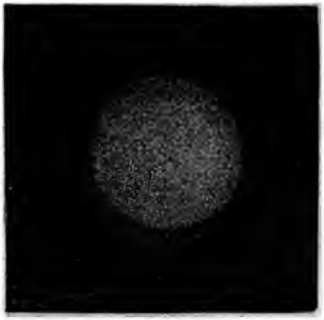


Fig. 84.

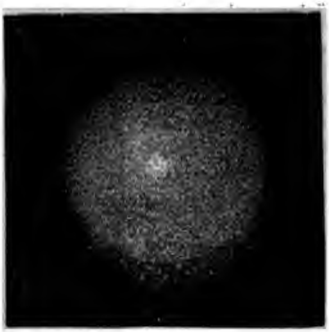


Fig. 85.

In Fig. 85, right ascension 202° 13', north polar distance 107° 1', the condensation is considerably more marked. In Fig. 86, right ascension 59° 43', north polar distance 47° 57', the central bright portion is of smaller extent than in the last instance,

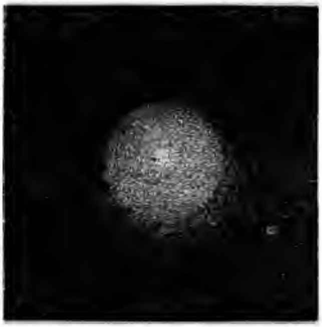


Fig. 86.

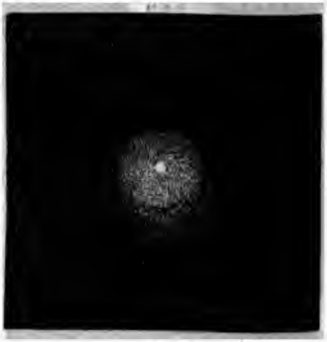


Fig. 87.

but better defined and much more lustrous. In Fig. 87, right ascension 59° 39', north polar distance 59° 40', the central portion becomes almost stellar, appearing like a well-defined star surrounded by a halo. The different degrees of condensation recognizable in these objects, led Herschel to imagine that the nebulous matter, in the course of ages, underwent considerable changes, and that, having condensed gradually, the nebulæ finally ended by becoming stars. These changes were so slow that they had not yet been perceived; but it was visible, by comparing them together, that such probably was the case. In some cases the central star or stars was distinctly visible. Such an object is perceived at right ascension 295° 5', north polar distance 39° 54' (Fig. 88), in which the nebula is evenly distributed round the star. At right

n 271° 45′, north polar distance 109° 56′ (Fig. 89), an elliptic nebula is
with a small star situated in each of the foci of the ellipse ; and at Fig. 90,

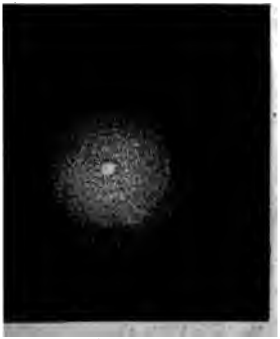


Fig. 88.

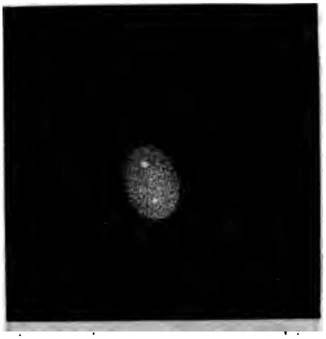


Fig. 89.

cension 80° 3′, north polar distance 55° 54′, a triangle of stars are congregated
ntre of a small nebula. In those which were equally bright throughout the
isc, Herschel conjectured that the star was not visible on account of its faint-

Fig. 90.

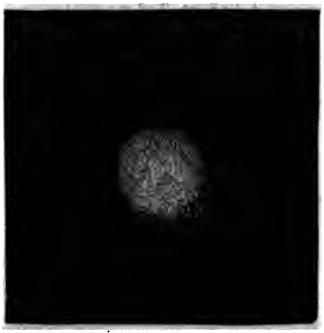


Fig. 91.

from being surrounded by a dense nebulosity. Such a one is situated at right
n 166° 12′, north polar distance 34° 4′ (Fig. 91).

ng the most curious forms of the nebulæ, those which are double take a promi-
nce. In these, both the components are sometimes of equal magnitude (Fig.
ht ascension 174° 20′, north polar distance 55° 31′; and similar in every respect,
those at Fig. 93, right ascension 140° 38′, north polar distance 67° 45′; and
4, right ascension, 183° 18′, north polar distance 84° 35′, where they are of
r size and condensation. In all these examples the components are joined
r by the surrounding nebulosity, but in some cases the two nebulæ are quite
, as in Fig. 95, right ascension 342° 48′, north polar distance 103° 43′. In
ases, Herschel imagined he perceived symptoms of the gradual formation of

a *double* star, the nebulous matter of which they are formed condensing towards two centres of attraction. It will appear from this that the *nebulæ*, properly so called, according to Herschel's ideas, were far from being such important objects as the clusters, the latter being great agglomerations of stars, the nebulæ only the material of a single, or, at most, a double star. But it would be hazardous to say that any particular nebula was irresolvable. The gigantic telescope of Lord Rosse has shown that many such, hitherto deemed of this nature, consist of innumerable stars, very different from the single individual star which Herschel thought it was probable it would finally become.

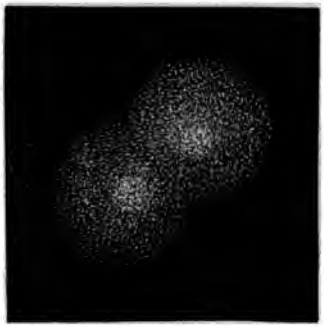


Fig. 92.

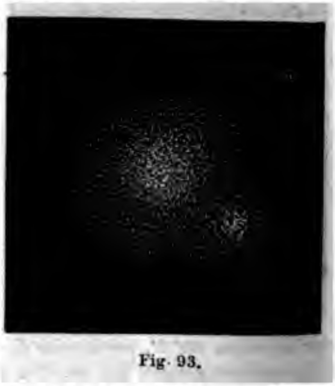


Fig. 93.

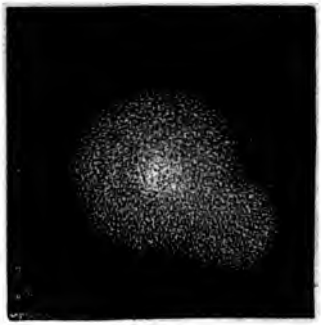


Fig. 94.

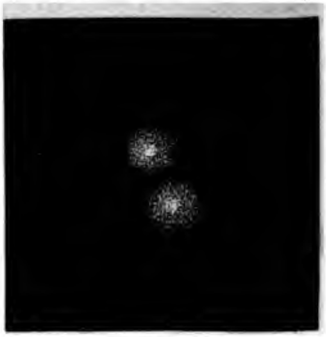


Fig. 95.

The suppositions relative to this nebulous substance, which Herschel thought to be so plentifully scattered through space, and the changes which he surmised that it underwent in the course of time, led the celebrated astronomer Laplace to form a similar hypothesis on the formation of the solar system, and the progressive development of the bodies of which it is composed, from one single and primitive mass of nebulous matter. He supposed that the sun was originally included among the nebulæ, which extended as far as the most distant planet, and was endowed with a rotatory motion

e centre; and that in the course of ages this gaseous substance became more con-
owards the centre, leaving the exterior portions in the form of immense rings, still
ng their revolving motion, precisely similar to the ring of Saturn as at present
. From many different causes it was impossible for those rings to exist in that
d they finally broke up into globular masses forming the different planets. By
ans he explains the revolution and rotation of the planetary bodies in one
, as well as of their satellites, formation of comets, &c. Even the zodiacal
s fully explained on this hypothesis. The nebulous matter, as he conjectured,

Fig. 96.

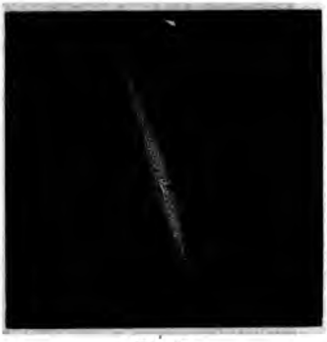


Fig. 97.

quite condensed, but still surrounded the sun in a very diffused state, and by
tion of that body on its axis took a lenticular form, as was observed to be the
r many of the nebulæ already discovered. Thus the great nebula of Andro-
of this elliptic form, and different degrees of ellipticity will be seen from the
nying figures (Figs. 95, 96, and 97), which are some of those actually observed
. Herschel.

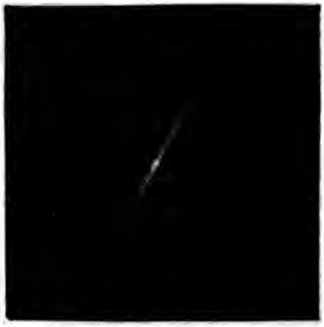
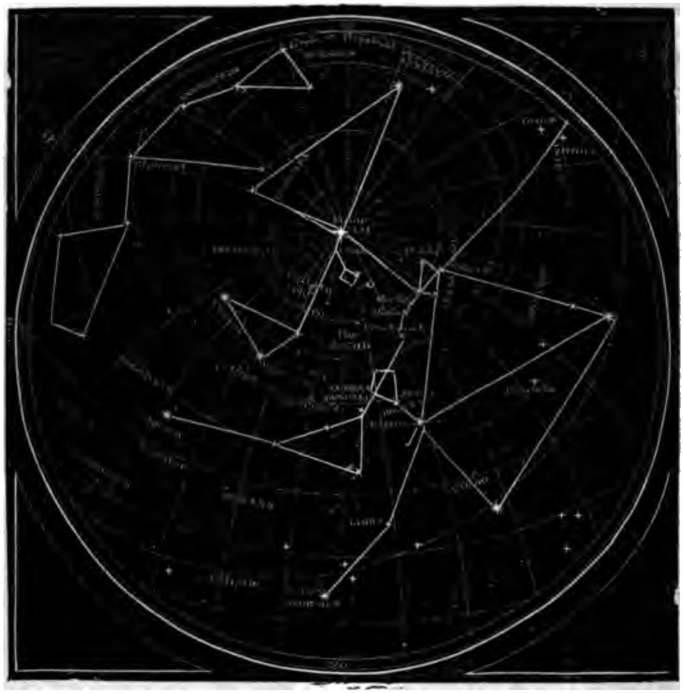


Fig. 98.



ON THE CONSTELLATIONS AND FIXED STARS.

Our present arrangement of the constellations into groups or clusters, representing figures of men and animals, dates from the most remote antiquity. Aratus, a Cilician of Eudoxus, B.C. 370, a contemporary of Plato, enumerates forty-five as being then in use, all of which still remain on our celestial globes. In the majority of instances, however, there is no similarity between these configurations and the figures to which they are supposed to bear some resemblance. Of the fixed stars, in both hemispheres, nearly 5,000 are visible to the naked eye, the greater number of which have been catalogued by Argelander, in his standard work the "Manometria Nova." They are divided into classes of magnitude, the ***first*** consisting of stars pre-eminently bright, and which amount in our latitude to about 14; the second magnitude consisting of about 50; the third of 140; the fourth of about 320; the fifth of 800; and the sixth of 2,000 (roughly speaking). Objects of a magnitude greater than the sixth are only seen by the help of telescopes, and are reckoned on to the thirteenth or fourteenth magnitude.

Bayer appears to have been the first astronomer who systematically arranged the stars of each constellation in the order of magnitude. In his catalogue, of which the epoch is 1603, he classed the brightest stars in each constellation according to the first letter of the Greek alphabet, the second by the second letter, and so on; afterwards using for smaller objects the *roman letters.* Of the first magnitude in these latitudes we have nine north of the equater and five south, viz.:—

North.

Vega, or α Lyræ.
Capella, or α Aurigæ.
Arcturus, or α Bootes.
Aldebaran, or α Tauri.
Betelgueib, or α Orionis.

Regulus, or α Leonis.
Altair, or α Aquilæ.
Deneb, or α Cygni.
Procyon, or α Canis Minoris.

South.

Sirius, or α Canis Majoris.
Rigel, or β Orionis.
Spica, or α Virginis.

Antares, or α Scorpio.
Fomalhaut, or α Piscis Australis.

Bayer's arrangement has been since used by astronomers.

The most ancient catalogues we possess of the fixed stars are those of Ptolemy, Ulugh Beigh, Tycho, and Hevelius, all of which have been formed from observations by the naked eye. The catalogue of Ptolemy is based on the observations of Hipparchus, and amounts to upwards of 1,000, which are arranged in the order of longitude and latitude. That of Ulugh Beigh contains 1,019 stars for the epoch of 1437, observed at Samarcand in Persia, north latitude 39° 52′. The epoch of Tycho's catalogue is for the end of the sixteenth century, and contains 777 stars, reduced and edited by his pupil, Kepler. The catalogue of Hevelius contains 1,564 positions of stars for the epoch 1660. All the preceding works have been re-edited by Mr. Baily, to whom this branch of astronomy is much indebted, and are given in the thirteenth volume of the "Memoirs of the Royal Astronomical Society."

Since the application of the telescope to astronomical observations, we have the catalogues of Halley and La Caille, observed in the southern hemisphere; and those of Flamsteed, Bradley, and Mayer, observed in the European observatories. The work of Flamsteed is the basis of that of Bradley, and they both, in common, contain nearly the same stars, but in the latter case with more improved instruments. The numbers in the catalogues of Flamsteed, Bradley, and Mayer are respectively 3,400, 3,222, and 998 stars.

At the commencement of the present century we possessed the catalogues of Groombridge, Piazzi, and Lalande, of which the first consists of circumpolar stars numbering upwards of 4,000, the second of 7,646 stars, and the last of 50,000 stars.

In modern times we have the catalogues of Busham and Taylor, observed at New South Wales and Madras respectively; the Greenwich catalogues of Pond and Airy; and the continental catalogues of Remiker, Bessel, and Argelander. The British Association have also published a standard catalogue of 8,377 stars.

The Zodiacal Constellations are twelve in number, all of which occur in the list prepared by Aratus.

I. **Aries** has two very conspicuous stars in the head of the Ram about 4° apart. They

are the nautical star, a Arietus, called Hamal by the Ar
and Sheratan of the third.

II. **Taurus,** one of the finest of zodiacal Asterisms,
Aries is 27° above the horizon. It includes Aldebaran,
forming with Hyades the letter Y in the face of the Bu
well-known cluster of the Pleaides.

III. **Gemini** has two principal stars, Castor of the
magnitudes, about 4½° apart, in the neighbourhood of P
magnitude. In this constellation Sir John Herschel fo
many years to guide astronomers to that planet.

IV. **Cancer** has no very conspicuous star. Two
Aselli (the Asses) of the Romans and Presepe (the Mang
distance of 2° distinguishes this constellation.

V. **Leo,** a brilliant constellation, has Regulus, of
of the Lion, and Denbola, of the second magnitude.

VI. **Virgo** has Spica Virginis, a star of th
remarkable for its solitary splendour, having on
tude, near it. This star is called *Al-simak-al-*

VII. **Libra** has four subordinate but brig
Two in the northern and two in the souther

VIII. **Scorpio** is a beautiful collection
is of the first magnitude, and is distingui

IX. **Saggitarius** has five stars of t
figure resembling a straight-handled d
situated in the Milky Way.

X. **Capricornus** has only star
formerly in this constellation, wh
declination; hence it was calle
northern. Now, owing to the
constellation till the middle of

XI. **Aquarius** is recog
distinctly to form the lette

XII. **Pisces** is a loo
large triangular space on
the zodiac, opening the

In naming and fig
been guided by the ay
parts of the heaven which
Ram, the Bull, t Arabs.
to them during t
retreat of the s ...ing produced suff-
of summer he ...ugh the bright stars
Balance indi ...Gemini, a and β
come dimi... A short distance in the

Hence ... Castor and Sirius we
... of a Canis Minoris. This
Minor ... found by prolonging the line
... of the same square prod

tip of the tail we have Benetnasch, a star of the second magnitude; Mizar, 7° distant west; and Alioth, about 4½° further off; 5½° from Alioth, at the root of the tail, is Megrez; south of it Phad, forming the shorter side of a quadrangle. On the opposite side, 8° west of Phad is Merak; and 5° north, towards the pole, is Dubhe, the brightest star of the constellation.

Dubhe and Merak are called the pointers, because a line drawn through them and carried about 29° in the same direction, passes almost over Polaris (the pole star) which is close to the north polar point in the heavens.

Ursa Minor, while it is inferior to the preceding in point of size, is more important from its position indicating the north polar point, and its utility as a guide in finding the latitude of places. Like Ursa Major, it consists of seven stars; three of the third magnitude and four of the fourth. Polaris is the important star of this group. It is between the second and third magnitudes. It is at the tip of the tail of Ursa Minor, and appears stationary, the rest of the constellations appearing to swing round it in the diurnal revolution of the sphere. All the stars appear to revolve round the pole of the ecliptic, owing to the real revolution of the pole of the earth round it; a revolution, however, which requires the long cycle of 26,000 years, or thereabout, for its performance.

Bootes appears among the stellar groups to be driving on Ursa Major, hence it has sometimes been called the Bear Driver. Bootes has Arcturus, a star of the first magnitude, long supposed by the ancients to be the nearest star to the earth.

Southern Constellations.—The constellations of the southern hemisphere are forty-six in number. The most important being Orion, which constitutes the richest part of the visible heavens, and when on the meridian (which occurs about 10 p.m., January 1), presents the most magnificent view the starry heavens offers. Orion is visible, in its turn, to all the habitable world, the equinoctial passing through the centre of it. Four principal stars, in the form of a long square or parallelogram, form its outline. Betelgueux, of the first magnitude, is 7¼° of Bellatrix of the second; Saiph of the third magnitude, and Rigel of the first, 8½° west of Saiph and 15° of Bellatrix. Canis Major, on the south-east of Orion, contains one star of the first magnitude, four of the second, and two of the third. The former, Sirius, glowing in our winter hemisphere with a lustre unequalled by any other star in the firmament. Canis Minor, east of Orion and north of Canis Major, has two brilliant stars, Procyon, of the first magnitude, and Gomelza, of the second, about 4° to the south east. A knowledge of these constellations will enable us to find our path in the heavens.

When a few particular stars have been recognized, they will serve as starting points, and by alignments, or imaginary lines, drawn from them, other stars and groups of stars will be found, and thus a general knowledge of celestial objects acquired. For this purpose one of the most conspicuous objects is the constellation of Ursa Major, which never sets in our latitudes.

The seven stars (shown in Fig. 98) are nearly all of the same magnitude; a line drawn through β and α, and produced to a distance equal to that from α to η, will point out Polaris, an object of the 2·3 magnitude. This star will be easily distinguished, since it is not surrounded by any others of the same magnitude. The alignment to find the Pole Star is as follows:—A line drawn from δ Ursæ Majoris, or from ϵ Ursæ Majoris through Polaris, and produced the same distance as that from δ Ursæ Majoris to Polaris, will reach the centre of Cassiopeiæ. This remarkable group of stars, which, in these latitudes, is circumpolar, is always on the opposite side of Polaris to

Ursa Major. When Ursa Major is near the zenith, Cassiopeiæ is near the horizon, and when Ursa Major is on the eastern side, Cassiopeiæ is on the western side of Polaris.

The right lines, which direct α and β Ursæ Majoris to Polaris, when produced in the same direction, will point out the square of Pegasus, formed of four stars of the second magnitude, the two upper of this group being respectively α Regasi and α Andromedæ. A line drawn from the two latter will pass through β and γ Andromedæ, and finally to α Persei, a star of the second magnitude nearer the pole. It will be remarked that α Persei may also be found by drawing a line from δ, α, Ursæ Majoris through Polaris.

Fig. 99.

On the opposite sides of α Persei are situated γ and δ Persei, stars of the fourth and third magnitudes respectively; a line drawn from α, δ Ursæ Majoris will meet (after passing through α Persei) Algol, or β Persei, a star remarkable for its variability. Producing the arcs γ and δ Persei, we find α Arietes. Below these, the conspicuous cluster of stars, the Pleiades, are situated. If we join Polaris and α Arietes, and produce it beyond the latter, we shall meet with the constellation Orion, which is well known by its remarkable brilliancy and form. Of this group of stars, δ, ε, and ζ are called the belt; the remaining four stars of this group form an irregular quadrilateral, α and γ being the upper, and β and δ being the lower sides. α and β Orionis are of the first magnitude, and all the others are of the second magnitude.

The most remarkable object in the heavens, Sirius, or α Canis Majoris, is pointed out by producing the belt of Orion on the eastern side. By producing it on the western side, we meet with Aldebaran, or α Tauri, a star of the first magnitude. But this star may also be found by producing the line which connects α Ursæ Majoris and α Arietes.

Fig. 100.

The diagonal of the square [α, β, γ, δ Ursæ Majoris], or δ, β, being produced sufficiently, will pass through the bright stars of the constellation Gemini, α and β, or Castor and Pollux. A short distance in the same line, between Castor and Sirius, we find Procyon, or α Canis Minoris. This may also be found by prolonging the line passing through Polaris and Pollux. The diagonal α and γ of the same square, produced on the side of γ above the pole, will point out α, or Spica Virginis, which forms an equilateral triangle with Arcturus and β Leonis.

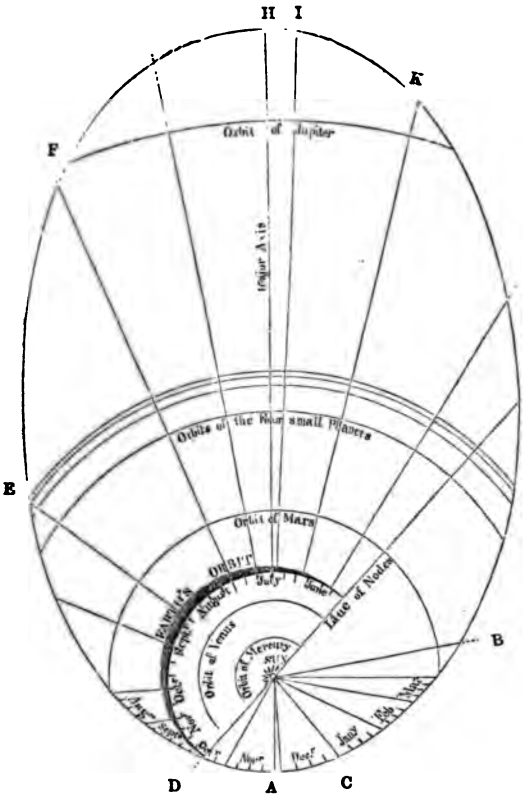
The line connecting α and β Ursæ Majoris, which points so accurately to the Pole Star, when produced, will pass through the constellation Leo. This constellation consists of four principal stars, in the form of a trapezian. The most brilliant is of the first magnitude; all the others are of the second magnitude. ζ and η Ursæ Majoris, being connected in a line, will meet with a remarkable star of the first magnitude, Arcturus or α Bootes.

At the side of Arcturus, and in the direction of the stars θ, δ, ε, ζ of Ursæ Majoris, we find the constellation of Cerara Borealis, composed of many stars, arranged in a semicircle, and of which the most brilliant is of the second magnitude.

Vega, or α Lyræ, is a conspicuous object, passing the meridian of Greenwich 13° south of the zenith. It forms a great triangle with Arcturus and Polaris, of which it occupies the summit of the right angle. At the side of α Lyræ are two stars of the third magnitude, β and γ, and three of the fourth magnitude, δ, ε, ζ. The four stars, β, γ, δ, ζ, form a parallelogram, easily distinguished.

Between Lyræ and Pegasus the constellation Cygnus is found, composed of five principal stars in the form of a cross. The line which joins Cygnus to Gemini is cut in two equal parts by Polaris. The same line, produced beyond Cygnus, passes through Altair or α Aquilæ, a star of the first magnitude. α Aquilæ is situated between γ and β Aquilæ, of about the third and fourth magnitudes respectively.

Fig. 101.



ON COMETS.

These bodies, which from the earliest ages have been the subjects of attentive ob vation, and which, even within the last few centuries, have excited the fears apprehensions of the multitude, will next claim our attention. They differ from other planetary bodies, both in their physical appearances and in the irregularitie their motions. From the investigations of Sir Isaac Newton, to whom this branch astronomy is much indebted, and which have been confirmed by the labours of successors, it has been made evident that comets describe very eccentric ellipses aro the sun, so much so that, in the majority of cases, a parabola will perfectly recon their observed path in the heavens. The inclinations of their orbits to the ecliptic also frequently very great, and the directions of their orbital motions are someti direct, and in other instances retrograde. In consequence of the great eccentricitie their orbits, they are seldom visible at the utmost more than a few months about the time of the perihelion passage. The annexed diagram (Fig. 101) will show, in a clear manner, a parabolic orbit. If with the two foci, F F′, an ellipse be described in the usual manner, we shall find that the major axis of this ellipse will be A A′, and that we shall obtain a curve differing little from an arch, and presented in a sense perpendicular to the axis A A′. or with a very slight degree of ellipticity. A second ellipse, with the foci F F″, will exhibit a greater degree of ellipticity, and will enclose the preceding one. A third ellipse, with the foci F F‴, will (still leaving A for the summit

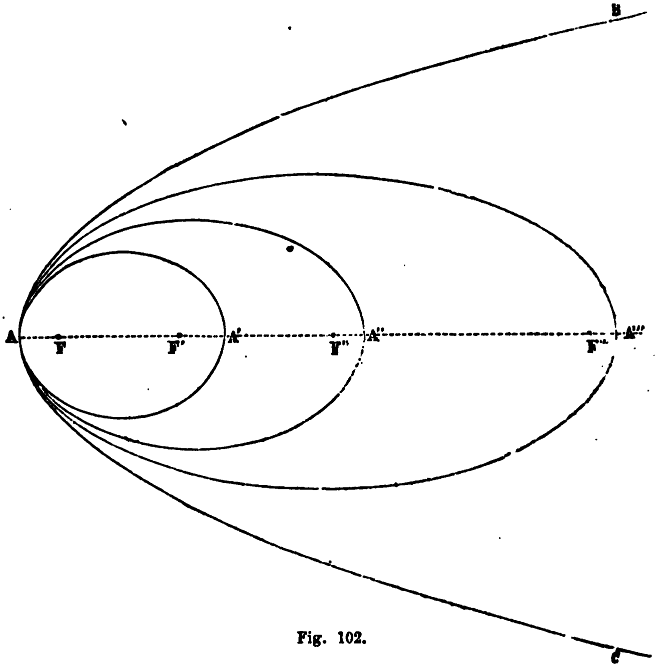


Fig. 102.

of the ellipse) show a still greater degree of ellipticity, and so on, by describing ellipses successively. We shall thus find the circumference of the curve more and more removed from the direction of the major axis A A″, till finally we shall arrive at a point beyond which there is no variation. The curve B A C, thus described, is called a parabola. The point A is called the summit of this parabola, and the point F its focus. A parabola is then formed of two parts, A B and A C, exactly alike to one another, and extending infinitely on each side of the major axis, A F F′ F″. We can easily perceive that, as the comet recedes from the sun into space, it will be necessarily lost to our view. For the determination of the elements of the orbit of a comet, three complete observations are necessary as a first approximation, which can be afterwards brought to greater accuracy by means of the other additional observations during the time of its visibility. In a parabola the following elements are required:—1st, the ongitude of the perihelion; 2nd, the longitude of the node; 3rd, the inclination of the orbit to the plane of the ecliptic; 4th, the time of perihelion passage; and 5th, the perihelion distance. The observations will also give the direction of motion. Dr. Halley alone, acting according to the method of Sir Isaac Newton, calculated twenty-four orbits, by a comparison of which he was led to infer the periodic nature of the body which bears his name, and the certainty of which has been confirmed by the well-observed and authentic observations of successive returns.

Modern astronomers have, with great zeal, applied the methods of Olbers and Gauss for this purpose, not only to those comets which have appeared in their own times, but also to others, whose paths in the heavens have been recorded by the ancient Chinese and European annalists. M. Langier has thus calculated, with a strong degree of probability, that the comets of 451 and 760, mentioned by the Chinese annalists, were returns of Halley's comet. Mr. Hind has also been led, by calculation, to consider the comets of 218 and 295 as appearances of the same body. But no certainty can be attached to any elements of this comet till the year 1378, the observations of which have been discussed by M. Langier, and of which the resulting elements are given with the others in the following table:—

		Long. of Perih.	Long. of Node.	Inclination.	Long. Perih. Dist.
1378, Nov.	8·764	299° 31′	47° 17′	17° 56′	9·7560 R.
1456, June,	8·917	301° 0′	48° 30′	17° 56′	9·7675 R.
1531, Aug.	24·888	301° 39′	49° 25′	17° 56′	9·7538 R.
1607, Oct.	26·716	301° 38′	43° 40′	17° 12′	9·7694 R.
1682, Sept.	14·795	301° 56′	51° 11′	17° 44′	9·7656 R.
1759, March,	12·552	303° 10′	53° 50′	17° 37′	9·7668 R.
1835, Nov.	15·939	304° 32′	55° 10′	17° 45′	9·7683 R.

The striking similarity of the elements of the comets of 1531, 1607, and 1682 immediately presented itself to the mind of Dr. Halley, who was also led to infer that, by the influence of the planet Jupiter, the period of its next return would be considerably retarded. This prediction was found to be accurate by the laborious calculations of Clairant, who, after minutely computing the disturbing effect of Jupiter and Saturn, found that its next return would be delayed by 618 days, as he concluded that the comet would arrive at its perihelion on April 13th, 1759.

The actual return, from the best elements, took place on March 12, 1759, and the comet was seen for the first time at Dresden, by Pazlitch, on Christmas Day, 1758. This body being thus ranked in the planetary system, the investigation of the period of its next arrival at the perihelion was made the subject of a prize in several European academies. The investigations of Damoiseau and Pentecoulant, on this question, gave only the difference of one week in the predicted time of the arrival at the perihelion —an agreement very accordant, when the great intricacy of the problem is considered. The predicted return was stated to be on November 13, 1835; and although the comet was attentively looked for during several months previously, it was not seen till August, when it was discovered at the Collegio Romano, of Rome. From the best elements, the return to perihelion took place on November 16, the predicted time being only three days in error.

With regard to the physical appearances of these bodies, it is generally admitted, at present, that comets possess a very small degree of density.

The annexed diagram (Fig. 103) will show the appearance of Halley's comet, as observed by Sir J. Herschel, at the Cape of Good Hope, on October 28, 1835. The bright spot situated in the head of the comet, is called the *nucleus*, and the luminous appendage which accompanies and surrounds this nucleus, is called the *tail*. On October 29, Sir J. Herschel observed the comet with a twenty-foot telescope, and noted its appearance, as in Fig. 104. A little later on the same evening, the appearance was as in Fig. 105. On January 25, 1836, it had an appearance as in Fig. 106; in the intermediate time the comet was not visible, in consequence of its proximity to the solar rays. On January 26, 27, and 28, 1836, it presented the appearances indicated in the Figures 107, 108, and 109.

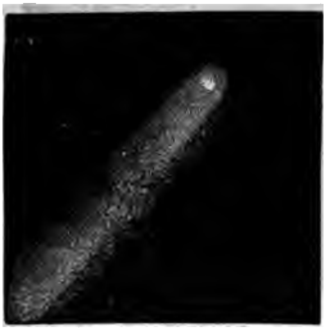


Fig. 103.

As to the light which these bodies possess, the late M. Arago inferred, from his experiments, that they shine by reflected light. But although his experiments on the comet of 1819, and that of Halley in 1835, showed that they shone by reflected light, which he satisfactorily confirmed by experiments with a polariscope on the light of stars of the same magnitude in their neighbourhood, he was rather doubtful whether they had not a proper light of their own, since, without losing their property of reflection, some independent light might at the same time reach the earth.

So long ago as the year 837 of our era the circumstance was remarked by the Chinese annalists, and again noticed by Appian in the 16th century, that the tails of comets were generally directed from the sun. The appearance of these accompaniments are variable, both in regard to magnitude and brilliancy. Of the comet of 1689, the contemporary historians state that its tail occupied an extent of 90°, and presented an appearance similar to a Turkish sabre. The tail of the comet of 1769, was observed to be 97° in length. The comet of 1744 had as many as six tails, each 4° in breadth, and 30° to 44° in length. In our own days, the appearance of the comet of 1843 cannot be forgotten. At Paris in March, its length, by observation, was found to be from 39° to 43°. The comet of 1618 had also a tail 104° in length.

In addition to Halley's comet, others of nearly similar period present themselves to our notice, and this remarkable group consists of the following:—

1st. The comet of 1812, which was discovered by Pons in that year, and only visible with the aid of a telescope. Encke, from all the observations, calculated its period to be 70·7 years.

2nd. The comet of 1815, discovered by Olbers, the period of which has been calculated by Bessel to be 74·05 years. According to the investigations of Bessel, the period of the next return will be retarded by two years, in consequence of planetary perturbations; or it would appear in 1891.

3rd. The comet discovered by De Vico in Rome, in 1846, Feb. 20, and observed till April. The calculations of De Vico and Peira, show that its orbit is decidedly elliptical, and that its period is upwards of 70 years. The only comet in the list of calculated orbits, similar to this body, is that of 1707.

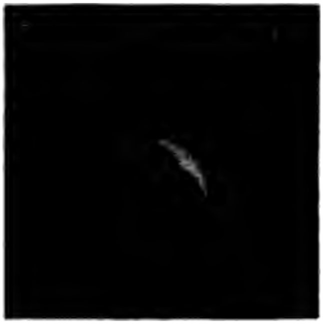


Fig. 104.

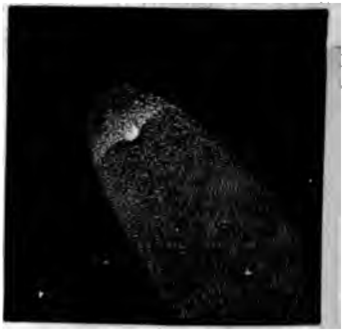


Fig. 105.

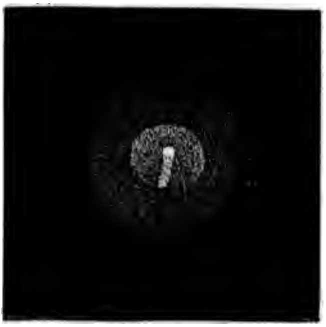


Fig. 106.

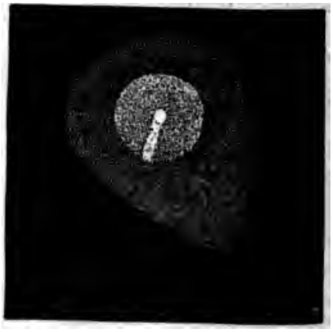


Fig. 107.

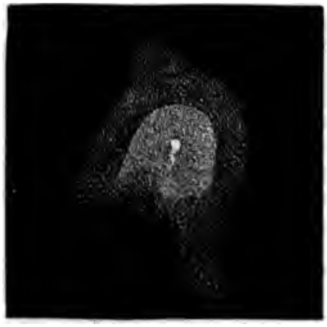


Fig. 108.

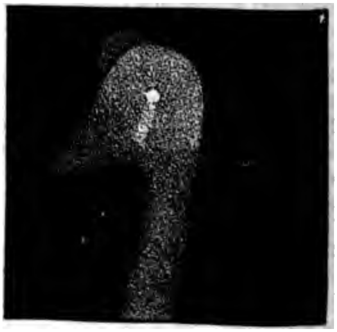


Fig. 109.

4. The next comet, nearly identical in period, is that discovered by Brossen on

July 20, in triangularus, and observed at several places for the space of one month. Elliptic elements, compiled by D'Arrest, give its period as 75 years.

Another comet of long period, but the elements of which are not so well worthy of confidence, is that discovered by Flamsteed in 1663. Elliptical elements of this body, which are, however, too uncertain to be much relied on, assign to it a period of 187·5 years.

Several other elliptical orbits have been calculated, which will appear in a table at page 309.

The most remarkable periodical comet (which there is great reason to believe to be such, from the strong identity of the elements) is that discovered by Fabricius in 1556, and which is considered to be a second appearance of the fine comet of 1264. Necessarily rude as the observations of this period were, Pingré and Dunthorne have calculated its elements from its recorded path, which agree in every particular with the comet of 1556. This subject has recently occupied the attention of M. Baume, who infers from his investigations that the comet will be retarded in its return to the perihelion by a period of some years. The last appearance in 1556 was magnificent in the extreme. The tail was more than 100° in length; and, according to the Chinese accounts, it presented a curvature similar to a sabre. It remained visible till the 2nd of October of this year, and disappeared the same night as that on which Pope Urban IV. died. There is strong probability in the inference that this body will again return to its perihelion in the period from 1858 to 1860; and, as the elements are based on such uncertain data, we must not be astonished at the circumstance of the range of the interval.

After thus mentioning two comets, whose periods are the longest on record, and the first of which has been made evident by many successive returns, we shall now proceed to others, whose revolutions around the sun are of shorter duration. The first and most celebrated is that of Encke. This body was discovered by Pons at Marseilles, on November 26, 1818; but the credit of being the first calculator of its elliptic elements is due to Encke, from which circumstance it bears his name. The similarity of the approximate parabolic elements to those of comets observed in 1786, 1795, and 1805, immediately presented itself to the mind of Encke, who likewise found that a parabolic orbit would not satisfy the observations of 1818, 1819. Its period is approximately three years and a quarter. Since 1819, no return to the perihelion has passed without observation; and for this, in a great measure, astronomy is indebted to the laborious calculations of Professor Encke, who furnished the ephemerides of its predicted places. On June 2, 1822, the comet was seen by Rümker, for the first time, at the Observatory of Paramatta, in New South Wales, which had then been recently founded by General Busham. Although it was only observed there for a short space of time, the observations were found of considerable value in determining its next return in 1825. True to prediction, the comet was discovered by Valz on July 13, within a short range of the calculated place. At this period the comet was well observed at Naples and other European observatories. On its next arrival at its perihelion, 1828, it was first seen at Dorpat by Struve, on October 13, and was observed in European observatories till December 25.

Although very unfavourably situated for observation in the northern hemisphere in 1832, the comet was observed by Harding at Göttingen on August 21. Professor Henderson also observed it at the Cape of Good Hope, but the observations of this return were so few and scattered that Professor Encke, in his investigations for its return in 1825, made no use of any of them. In 1835, the comet was observed on July 26 by Boguslawski, and previously by Kiel at Milan on July 22.

In 1838 it was again seen by Boguslawski, on August 14, as a very faint, shapeless object, which afterwards increased in brightness till November. Professor Encke, in combining all the preceding returns at this period, found it necessary to alter the received mass of the planet Mercury, and also to allow for the effect of a resisting ethereal medium, pervading all space, and which had an appreciable resistance to the cometary motions. With these two important discoveries, Professor Encke published an ephemeris for its reappearance in 1842, on the 8th of February of which year it was observed by Dr. Galle.

Although the return of 1845 was very unfavourable, the comet was seen at Rome and in America; but the observations were so few that no use has been made of them in any investigation.

In 1848 it was observed for a period of three months, extending from August to November. The comet was very faint at first, but towards November its brightness increased considerably. The observations of this year have been valuable in several important particulars.

In 1852 the perihelion passage took place on March 14, and the comet was visible on the 2nd of January.

In 1855 the return of the comet was not favourable for observation in the northern hemisphere, but it was seen at the Cape of Good Hope by Maclear.

Gambart's Comet.—This body was discovered by Beila on the 27th of February, 1826; but, as in the preceding case, it bears the name of Gambart, who first computed its orbit, and noticed its similarity with the comets of 1772 and 1805. Elliptical elements, calculated by him and other astronomers, assigned to it a period of 2,460 days. In the interval of time from 1772 to 1826, it had performed six revolutions. The return to the perihelion, in 1832, was observed generally at all the European observatories. It was first seen at Rome, on August 25, as a faint object, the light of which, however, increased afterwards. Professor Henderson likewise observed it at the Cape of Good Hope. On a calculation of the elements of its orbit—which are represented, with the relative position of the earth, in the engraving at the head of this section—in order to ascertain the precise period of its return, it was, of course, necessary to study the perturbing influence of the planets; and when their places were ascertained and compared with its course, it was supposed that it would cross the earth's orbit about a month before our planet was at the point of intersection. This announcement excited much curiosity and some alarm. It will be evident, however, from a consideration of its orbit, that it never came within sixty millions of miles. It was a small, insignificant comet, looking like a collection of nebulous matter, and having neither tail nor any appearance of a solid nucleus. In the engraving, A is its perihelion and H its aphelion, B its position, January 1, 1840; C, on January 1, 1833; D, its December node; E, its position January 1, 1839; F, January 1, 1838; G, January 1, 1837; I, January 1, 1836; K, January 1, 1835; L, January 1, 1834. It will be seen from this diagram the point where the earth's orbit and that of the comet intersect is very near the descending node. The time at which the comet reached its perihelion in 1832 was the middle of November; it crossed the earth's orbit on the 29th of October, but the earth did not reach the same point until the 30th of November. At its return, in 1839, the comet was not seen from any part of the globe. In 1845, however, by means of an ephemeris calculated by Professor Santini, the comet was discovered by

De Vico, at Rome, on November 26th. In a very short time afterwards a most singular circumstance took place; the comet appeared to have been divided into two portions. This appearance was first seen by Professor Wichmann, about the middle of December, and generally confirmed by other observers. The two nuclei were also observed till the 22nd of March, 1846.

On its return to the perihelion, in 1852, the comet was again seen by Professor Secchi at Rome, and at the Cambridge Observatory in England. At this time, although the predicted ephemeris was in error, both nuclei were again observed, but they had separated considerably. The comet at this return was exceedingly faint, and was only visible on nine different occasions. This and the comet of Encke are comprised within the limits of the solar system, and do not extend beyond the orbit of Saturn.

Faye's Comet.—The next known periodical comet, whose orbit is comprised within that of the planet Neptune, is that discovered by M. Faye, at the Paris Observatory, on the 22nd of November, 1843. At its first appearance it had a brilliant nucleus and tail. Faye and others (Gauss included) discovered that its orbit was elliptical, and that its period was 7·2 years. At first it was imagined by Argelander and Professor Henderson to be the lost comet of 1770, or that of Levell; but Leverrier has since proved that this could not be the case. Its orbit at aphelion and perihelion approaches near to Mars and Jupiter, and must have experienced considerable perturbations from the latter.

M. Leverrier calculated an ephemeris for its return in 1850, and, by means of this, the comet was observed on November 28th, of extreme faintness, at the Cambridge Observatory in England. With the exception of two other observations obtained in the Cambridge Observatory (U. S.), by Mr. Bond, it was not seen at any other observatory. Mr. Bond described it as a very faint object in the twenty-three feet refractor.

The last four periodic comets are known as such by the frequent observations of their returns to the perihelion. But others exist whose ellipticity is equally decided, although their returns have not been observed. Of these the first is the comet of De Vico, comprised, as in the other case, within the planetary system, and discovered by him at Rome in 1844. Its period was found to be 1980 days, but although its orbit has been ably investigated by Dr. Brünnow, it has not been seen either in 1850 or in 1855.

Brorsen's Comet.—This comet was discovered at Kiel, in Denmark, by Brorsen, on the 26th of February, 1846, and found to be elliptic, its period being upwards of five years and six months; Drs. Brünnow and Petersen consider that it was identical with the comets of 1532 and 1661. Dr. Halley likewise inferred the similarity of these comets; but, although expected, it was not visible in 1790. It appears probable that the influence of the planet Jupiter has a great effect on the elements of this body. Mr. Hind is of opinion that it experienced considerable perturbations from this planet in 1842.

D'Arrest's Comet.—This comet, whose period is six years and five months, was discovered by D'Arrest in 1851, June 27. It suffers great perturbation from the planet Jupiter in its path, and the time of its next return is not yet fixed.

Another remarkable comet of this class is that discovered by Messier in 1770, the orbit of which had been calculated by Levell, indicating a period of about five or six years. The planetary perturbations have, however, altered the path of this comet so considerably, that it has never been since observed.

Fig. 110 shews the *relative orbits of* four periodic comets, and the comet of Halley.

It must be remarked, however, that the orbits are not in the same plane, and that the cometary inclinations are sometimes very considerable. In Fig. 110, A is an orbit of comets of short period; B B, the comet of seven and a half years; C C, Gambart's comet; H H, Halley's comet; J J, orbit of Jupiter; M, the orbit of Mars.

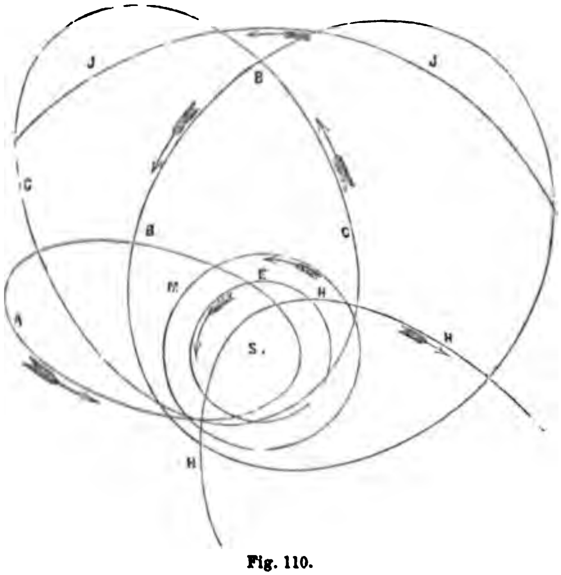


Fig. 110.

Nature of Comets. — Some attempts have been made to define with precision the difference between a star and a comet, and determine by what characteristics a newly discovered star is to be distinguished from one of these wanderers of the firmament. The planets always move in the same direction; the plane of their orbits being slightly inclined one to the other, and the eccentricity of these orbits are trifling. Comets, on the contrary, for the most part, describe orbits so much elongated that they seem to move in a sort of parabola; and, in the case of the few with whose movements we are best acquainted, the eccentricity of their orbits distinguishes them in a remarkable degree from the planets. Each newly discovered object that is observed in an ellipse, only slightly eccentric, has been classed among the stars, while those which do not satisfy these conditions are regarded as comets. The comets we have described, while differing in many respects, seem to agree in the great degree of eccentricity of their orbits, and would seem to satisfy the inquirer that they cannot have the same origin.

Comets generally present the appearance of a brilliant nucleus surrounded with a nebulosity which extends on one side more or less distant from the centre. This nebulous matter is so transparent, that stars, even of small magnitude, can be observed through the tail, and even through its thicker parts. The nebulosity of a comet, therefore, may be regarded simply as a vapour, extremely light, which accompanies the central nucleus. The rapid changes which have been occasionally observed in the form of comets would seem to confirm this impression. For example, Halley's comet was observed with much care by Herschel at the Cape of Good Hope at the end of 1835, and we have already noted the changes he observed; but the most striking instance of change on record occurred in January, 1846, when Gambart's comet was observed to be separated into two parts, each having formed an independent nucleus, with the usual nebulous accompaniment. When the comet again appeared in 1852, the two objects also appeared, but at an increased distance.

We here subjoin a table of calculated orbits of comets from the best authorities:—

No.	Greenwich Mean Time of Perihelion Passage.	Longitude of Perihelion	Longitude of Node.	Inclination.	Logarithm of per Dist.	Eccentricity.	Direction of Motion	Calculator.
	OLD STYLE.	° ′ ″	° ′ ″	° ′ ″				
1	371 Winter	150 to 210	270 to 330	Ab. 30°	very small	1	R	Pingré
2	137 Apr. 29	230 0 0	220 0 0	20 0 0	0·0043	1	R	Peirce
3	69 July	315 0 0	165 0 0	70 0 0	9·90	1	D	Peirce
4	12 Sep. 15	0 0 0	35 0 0	67 0 0	9·949	1	R	Peirce
5	66 Jan. 14·2	325 0 0	32 42 0	40 30 0	9·6480	1	R	Hind
6	240 Nov. 10	271 0 0	189 0 0	44 0 0	9·570	1	D	Burckhardt
7	451 July 3·5	Halley's Comet.						Langier
8	539 Oct. 20·62	313 30 0	58°or238°	10 0 0	9·53307	1	D	Burckhardt
9	565 July 14·5	80 0 0	159 30 0	59 0 0	9·92000	1	R	Burckhardt
10	568 Aug. 29·32	318 35 0	294 15 0	4 8 0	9·95771	1	D	Langier
11	574 Apr. 7·28	143 39 0	128 17 0	46 31 0	9·9836	1	D	Hind
(7)	760 June 11	Halley's Comet.						
12	770 June 6·588	257 7 0	90 59 0	61 49 0	9·80766	1	R	Langier
13	837 March 1	289 3 0	206 33 0	10° to 12°	9·763428	1	R	Pingré
14	961 Dec. 30·16	268 3 0	350 35 0	79 33 0	9·7418	1	R	Hind
15	989 Sep. 12	264 0 0	84 0 0	17 0 0	9·7546	1	R	Burckhardt
16	1066 May 30 or 31	120 0 0	230 0 0	70° to 80°	9·53	1	R	Pingré
17	1092 Feb. 15	156 20 0	125 40 0	28 55 0	9·9676	1	D	Hind
18	1097 Sep. 21·9	332 30 0	207 30 0	73 30 0	9·86832	1	D	Burckhardt
19	1231 Jan. 30·3	134 48 0	13 30 0	6 5 0	9·976698	1	D	Pingré
20	1264 July 17·25	275 45 0	178 45 0	30 25 0	9·61364	1	D	Pingré
21	1299 March 31·312	3 20 0	107 8 0	68 57 0	9·50233	1	R	Pingré
22	1301 Oct. 24	312 0 0	138 0 0	13 0 0	9·806	1	R	Langier
23	1337 June 15·074	2 20 0	93 1 0	40 28 0	9·91815	1	R	Langier
24	1351 Nov. 26·5	69 0 0	Indeterminate.		0·	1	D	Burckhardt
25	1362 Mar. 2·33	227 0 0	237 0 0	32 0 0	9·67214	1	R	Burckhardt
26	1366 Oct. 13	66 0 0	212 0 0	6 0 0	9·9814	1		Peirce
(7)	1378 Nov. 8·764	299 31 0	47 17 0	17 56 0	9·76604	1	R	Langier
27	1385 Oct. 16·26	101 47 0	268 31 0	52 15 0	9·8886	1	R	Hind
28	1433 Nov. 4·42	281 2 0	133 49 0	79 1 0	9·53079	1	R	Langier
(7)	1456 June 8·917	301 0 0	48 30 0	17 56 0	9·76754	1	R	Pingré
29	1457 Sep. 3·7	92 50 0	256 5 0	20 20 0	0·3229	1	D	Hind
30	1468 Oct. 7·4265	1 22 0	71 5 0	38 1 0	9·91893	1	R	Valz
31	1472 Feb. 28·218	48 3 0	207 32 0	1 55 0	9·751718	1	R	Langier
32	1490 Dec. 35·9	113 0 0	268 0 0	75 0 0	9·878	1	R	Peirce
33	1506 Sep. 3·662	250 37 0	132 50 0	45 1 0	9·58656	1	R	Langier
(7)	1531 Aug. 25·792	301 12 0	45 30 0	17 0 0	9·76338	0·967391	R	Halley
34	1532 Oct. 18·3324	111 48 0	87 23 0	32 36 0	9·715351	1	D	Olbers
35	1533 June 14·883	217 40 0	299 19 0	28 14 0	9·514362	1	D	Olbers
(20)	1556 Apr. 22·0233	274 14 54	175 25 48	30 12 42	9·70323	1	D	Hind
36	1558 Aug. 10·52	329 49 0	332 36 0	73 29 0	9·76140	1	R	Olbers
37	1577 Oct. 26·9476	129 42 0	25 20 24	75 9 42	9·24920	1	R	Wolstedt
38	1580 Nov. 28·5727	109 11 55	19 7 37	64 51 50	9·774903	1	D	Pingré
39	1582 May 7·348	281 26 45	214 42 35	59 29 5	8·602754	1	R	Pingré
	NEW STYLE.							
40	1585 Oct. 8·0262	9 8 21	37 44 10	6 5 52	0·0393530	1	D	Peters & Sawitsch
41	1590 Feb. 8·0271	217 57 21	165 37 5	29 29 44	9·7541386	1	R	Hind
42	1593 July 18·5685	176 19 0	164 15 0	87 58 0	8·94940	1	D	Lacaille
43	1596 July 23·612	274 24 0	335 39 0	52 48 0	9·75258	1	R	Valz
(7)	1607 Oct. 26·71594	301 38 10	43 40 28	17 12 17	9·769358	0·9670887	R	Bessel

No.	Greenwich Mean Time of Perihelion Passage.	Longitude of Perihelion	Longitude of Node.	Inclination.	Logarithm of per Dist.	Eccentricity.	Direc. of Motion.	Calculato
		° ′ ″	° ′ ″	° ′ ″				
44	1618 (1) Aug. 17·1268	318 20 0	293 25 0	21 28 0	9·71010	1	D	Pingré
45	1618 (2) Nov. 83·507	3 5 21	75 44 10	37 11 31	9·590556	1	D	Bessel
46	1652 Nov. 12·653	28 18 40	88 10 0	79 28 0	9·928140	1	D	Halley
47	1661 Jan. 26·881	115 16 8	81 54 0	33 0 55	9·646131	1	D	Méchain
48	1664 Dec. 4·494	130 41 25	81 14 0	21 18 30	0·011044	1	R	Halley
49	1665 Apr. 24·219	71 54 30	228 2 0	76 5 0	9·027309	1	R	Halley
50	1668 Feb. 28·8	277 2 0	357 17 0	35 58 0	7·680000	1	R	Henderso
51	1672 Mar. 1·359	46 59 30	297 30 30	83 22 10	9·843476	1	D	Halley
52	1677 May 6·026	137 37 5	236 49 10	78 3 15	9·448072	1	R	Halley
53	1678 Aug. 26·586	327 46 0	161 40 0	3 4 20	0·092727	1	D	Derowes
54	1680 Dec. 17·99338	262 49 5	272 9 29	60 40 16	7·793551	0·999985417	D	Encke
(7)	1682 Sep. 14·79505	301 55 37	51 11 18	17 44 45	9·7655898	0·96792019	R	Rosenberg
55	1683 July 12·72586	86 31 42	173 18 15	83 47 46	9·7430148	0·9832470	R	Clausen
56	1684 June 8·428	238 52 0	268 15 0	65 48 40	9·982339	1	D	Halley
57	1686 Sep. 16·606	77 0 30	350 34 40	31 21 40	9·511883	1	D	Halley
58	1689 Dec. 2·1403	270 16 0	344 18 0	30 25 0	8·01284	1	R	Peirce
59	1695 Nov. 9·7018	60 0 0	216 0 0	22 0 0	9·9261	1	D	Burckhar
60	1698 Oct. 18·706	270 51 15	267 44 15	11 46 0	9·839660	1	R	Halley
61	1699 Jan. 13·349	212 31 6	321 45 35	69 20 0	9·871570	1	R	Lacaille
62	1701 Oct. 17·410	133 41 0	298 41 0	41 39 0	9·772784	1	R	Burckhar
63	1702 Mar. 13·6065	138 46 34	188 59 10	4 24 44	9·810790	1	D	Burckhard
64	1706 Jan. 30·2060	72 36 25	13 11 23	55 14 5	9·630290	1	D	Struyck
65	1707 Dec. 11·9885	79 59 8	52 50 29	88 37 40	9·934013	1	D	Struyck
66	1718 Jan. 14·90573	121 39 57	127 55 31	31 8 6	0·010908	1	R	Argelande
67	1723 Sep. 27·62788	42 52 35	14 14 17	50 0 18	9·9994743	1	R	Spörer
68	1729 June 13·26350	320 31 22	310 38 0	77 5 18	0·6067570	1·0050334	D	Burckhard
69	1737 (1) Jan. 30·34767	325 55 0	226 22 0	18 20 45	9·347960	1	D	Bradley
70	1737 (2) June 8·3185	262 36 39	123 53 43	39 14 5	9·93802	1	D	Dansay
71	1739 June 17·4164	102 38 40	207 25 14	55 42 44	9·828389	1	R	Lacaille
72	1742 Feb. 8·6192	216 39 20	185 9 30	67 31 40	9·886523	1	R	Barker
73	1743 (1) Jan. 8·19403	93 19 37	86 54 30	1 53 43	9·9352858	0·7213086	D	Clausen
74	1743 (2) Sep. 20·8866	246 33 52	5 16 25	45 48 21	9·717310	1	R	Klinkenbe
75	1744 March 1·3282	197 11 58	45 46 6	47 10 53	9·346783	0·999100	D	Euler
76	1747 March 3·2991	277 2 0	147 18 50	79 6 20	0·342146	1	R	Lacaille
77	1748 (1) Apr. 28·78065	215 23 29	232 51 50	85 28 23	9·94486	1	R	Lemonnier
78	1748 (2) June 18·88751	278 47 10	33 8 29	67 3 28	9·796128	1	D	Bessel
79	1757 Oct. 21·38450	122 36 29	214 7 11	12 41 17	0·530610	1	D	De Ratte
80	1758 June 11·1373	267 38 0	230 50 0	68 19 0	9·333148	1	D	Pingré
(7)	1759 (1) Mar. 12·5517	303 10 28	53 50 27	17 36 52	9·7667990	0·9676844	R	Rosenberg
81	1759 (2) Nov. 27·02358	53 38 4	139 40 15	79 3 19	9·904218	1	D	Chappe
82	1759 (3) Dec. 16·53392	139 3 52	79 20 24	4 42 10	9·983064	1	R	Chappe
83	1762 May 28·33451	104 2 0	348 33 5	85 38 13	0·003912	1	D	Burckhard
84	1763 Nov. 1·86141	84 58 58	356 24 4	72 31 52	9·6974784	0·998680	D	Burckhard
85	1764 Feb. 12·57100	15 14 52	120 4 33	52 53 31	9·744462	1	R	Pingré
86	1766 (1) Feb. 17·3616	143 15 25	244 10 50	40 50 20	9·703570	1	R	Pingré
87	1766 (2) Apr. 26·98882	251 13 0	74 11 0	8 1 45	9·6009521	0·864000	D	Burckhard
88	1769 Oct. 7·62039	144 11 29	175 3 59	40 45 55	9·0890392	0·9992490	D	Bessel
89	1770 (1) Aug. 13·54085	356 16 51	131 58 56	1 34 28	9·8289491	0·786119	D	Leverrier
90	1770 (2) Nov. 22·2352	208 22 44	108 42 10	31 25 55	9·722833	1	R	Pingré
91	1771 Apr. 19·21271	104 3 10	27 51 49	11 15 19	9·9559104	1·0093698	D	Encke
92	1772 Feb. 19·09933	110 14 54	254 0 1	18 17 38	0·0058652	0·9031481	D	Bessel
93	1773 Sep. 5·37685	74 57 41	121 4 49	61 13 19	0·0512720	0·9935023	D	Pingré

No.	Greenwich Mean Time of Perihelion Passage.	Longitude of Perihelion	Longitude of Node.	Inclination.	Logarithm of per Dist.	Eccentricity.	Direc. of Motion.	Calculator.
		° ′ ″	° ′ ″	° ′ ″				
94	1774 Aug. 15·83010	317 27 40	180 44 34	83 20 6	0·1562066	1·0282955	D	Burckhardt
95	1779 Jan. 4·10445	86 53 0	23 40 0	32 43 0	9·853090	1	D	Olbers
96	1780 (1) Sep. 30·92630	246 35 56	123 41 15	54 23 12	8·9836418	9·9099640	R	Clüver
97	1780 (2) Nov. 28·84790	246 52 0	141 1 0	72 3 30	9·7120410	1	R	Olbers
98	1781 (1) July 7·18887	239 11 25	83 0 38	81 43 26	9·8897840	1	D	Méchain
99	1781 (2) Nov. 29·52320	16 3 7	77 22 55	27 12 4	9·9827230	1	R	Legendre
100	1783 Nov. 19·56218	49 31 55	55 12 0	47 42 0	0·1747341	0·6784	D	Burckhardt
101	1784 Jan. 21·19960	80 44 24	56 49 21	51 9 12	9·8499460	1	R	Méchain
?								
102	1785 (1) Jan. 27·32549	109 51 56	264 12 15	70 14 12	0·0581980	1	D	Méchain
103	1785 (2) Apr. 8·47200	297 34 30	64 44 40	87 7 0	9·6310240	1	R	Laron
104	1786 (1) Jan. 30·87350	156 38 0	334 8 0	13 36 0	9·5248100	0·84836	D	Encke
105	1786 (2) July 8·56747	158 38 30	195 23 32	50 58 33	9·5957630	1	D	Reggio
106	1787 May 10·82545	7 44 9	106 51 35	48 15 51	9·5427140	1	R	Laron
107	1788 (1) Nov. 10·30932	99 8 7	156 56 43	12 27 40	0·0265380	1	R	Méchain
108	1788 (2) Nov. 20·30254	22 49 54	352 24 26	64 30 24	9·8792760	1	D	Méchain
109	1790 (1) Jan. 16·79038	58 24 45	172 50 2	29 44 7	9·8735160	1	R	Laron
110	1790 (2) Jan. 28·31677	111 44 37	267 8 37	56 58 13	0·0266503	1	D	Méchain
111	1790 (3) May 21·24090	273 43 27	33 11 2	63 52 27	9·9019814	1	R	Méchain
112	1792 Jan. 13·53489	36 20 32	190 42 9	39 45 47	0·1114563	1	R	Zach
113	1792 (2) Dec. 27·25360	135 59 24	283 15 17	49 1 45	9·9851060	1	R	Prosperin
114	1793 (1) Nov. 4·84140	228 42 0	108 29 0	60 21 0	9·6057360	1	R	Laron
115	1793 (2) Nov. 28·59981	75 58 58	359 4 48	47 35 5	0·1461360	0·7347635	D	Burckhardt
(104)	1795 Dec. 21·44098	156 41 24	334 39 26	13 42 30	9·5243046	0·8488828	D	Encke
116	1796 April 2·82478	192 44 13	17 2 16	64 54 33	0·1981510	1	R	Olbers
117	1797 July 9·11424	49 34 42	329 16 30	50 35 50	9·7205310	1	R	Bouvard
118	1798 (1) April 4·49879	105 6 57	122 12 21	43 44 42	9·6853710	1	D	Olbers
119	1798 (2) Dec. 31·55350	34 27 27	249 30 30	42 26 4	9·8918290	1	R	Burckhardt
120	1799 (1) Sep. 7·22626	3 38 9	99 21 11	51 1 29	9·9244710	1	R	Olbers
121	1799 (2) Dec. 25·89640	190 20 12	326 49 11	77 1 38	9·7964370	1	R	Méchain
122	1801 Aug. 8·5574	183 49 0	44 28 0	21 20 0	9·417804	1	R	Burckhardt
123	1802 Sep. 9·89102	332 9 4	310 15 39	57 0 47	0·039061	1	D	Olbers
124	1804 Feb. 13·58813	148 44 51	176 47 58	56 28 40	0·029858	1	D	Gauss
(104)	1805 (1) Nov. 21·49987	156 47 19	334 20 5	13 33 30	9·5320168	0·84617529	D	Encke
(92)	1805 (2) Dec. 32·97420	109 32 23	251 15 15	13 38 45	9·9575120	0·745784	D	Gambart
125	1806 Dec. 28·93150	97 2 3	322 19 15	35 2 50	0·0340550	1	R	Burckhardt
126	1807 Sep. 18·738870	270 54 42	266 47 11	63 10 28	9·81031575	0·9954878	D	Bessel
127	1808 (1) May 12·953	69 12 57	322 58 36	45 43 7	9·59091	1	R	Encke
128	1808 (2) July 12·16768	252 38 50	24 11 15	39 18 59	9·783870	1	R	Bessel
129	1810 Sep. 29·0997	52 44 42	310 21 2	61 11 15	9·9893549	1	D	Iriesnecker
130	1811 (1) Sep. 12·25731	75 0 34	140 24 44	73 2 21	0·0151178	0·9950933	R	Argelander
131	1811 (2) Nov. 10·99048	47 27 20	93 1 45	31 17 11	0·1992359	0·9827109	D	Nicolai
132	1812 Sep. 15·31356	92 18 46	253 1 3	73 57 3	9·8904995	0·9545412	D	Encke
133	1813 (1) Mar. 4·52650	69 56 8	60 48 24	21 13 33	9·8445579	1	R	Nicollet
134	1813 (2) May 19·62378	197 28 37	42 39 36	80 55 5	0·084364	1	R	Olbers
135	1815 Apr. 25·99217	149 2 12	83 28 50	44 29 55	0·0838109	0·93921968	D	Bessel
(92)?	1818 (1) Feb. 7·397	95 7 0	254 0 0	20 2 24	9·86526	1	D	Pogson
136	1818 (2) Feb. 25·95890	182 45 22	70 26 11	89 43 48	0·0783711	1	D	Encke
(104)	1819 (1) Jan. 27·25616	156 59 12	334 33 19	13 36 54	9·5253771	0·8485841	D	Encke
138	1819 (2) June 27·73998	287 13 4	273 42 28	80 43 56	9·5340268	1	D	Nicolai
139	1819 (3) July 18·90020	274 40 54	113 10 48	10 42 48	9·8885382	0·75519035	D	Encke
140	1819 (4) Nov. 20·24553	67 18 42	77 13 51	9 1 16	9·9506368	0·6867458	D	Encke

No.	Greenwich Mean Time of Perihelion Passage.	Longitude of Perihelion	Longitude of Node.	Inclination.	Logarithm of per Dist.	Eccentricity.	Direc. of Motion.	Calculator
		° ′ ″	° ′ ″	° ′ ″				
141	1821 Mar. 21·53656	239 29 25	48 40 56	73 33 7	8·9629523	1	R	Rosenberger
142	1822 (1) May 5·57938	192 45 48	177 27 22	53 34 48	9·70280	1	R	Encke
(104)	1822 (2) May 23·96296	157 11 44	334 25 9	13 20 17	9·5390382	0·8444643	D	Encke
143	1822 (3) July 16·02433	219 53 48	97 51 23	37 43 4	9·92743	1	R	Heilligenstei
144	1822 (4) Oct. 23·76978	271 40 17	92 44 42	52 39 10	0·0588305	0·9963021	R	Encke
145	1823 Dec. 9·44408	274 34 30	303 3 1	76 11 57	9·3550726	1	R	Encke
146	1824 (1) July 11·51296	260 16 32	234 19 9	54 34 19	9·7717807	1	R	Rümker
147	1824 (2) Sep. 19·2440	4 44 24	279 5 49	54 22 3	0·0200454	1	D	Bouvard
148	1825 (1) May 30·55880	273 55 48	20 7 53	56 41 30	9·948964	1	R	Hansen
149	1825 (2) Aug. 18·71105	10 14 57	192 56 41	89 41 47	9·9461924	1	D	Claussen
(104)	1825 (3) Sep. 16·27312	157 14 31	334 27 30	13 21 24	9·5376348	0·3448885	D	Encke
150	1825 (4) Dec. 10·68186	318 46 41	215 43 14	33 32 39	0·0937189	0·9953690	R	Hansen
(92)	1826 (1) Mar. 18·41218	109 46 0	251 28 23	13 33 51	9·9554571	0·7465727	D	Santini
151	1826 (2) Apr. 21·97761	117 11 30	197 30 34	39 57 24	0·3016581	1·0089597	D	Nicolai
152	1826 (3) April 29·03904	35 48 1	40 29 1	5 17 2	9·2744275	1	R	Clüver
153	1826 (4) Oct. 8·95224	57 48 24	44 6 28	25 57 18	9·9308520	1	D	Argelander
154	1826 (5) Nov. 18·41206	315 31 38	235 7 48	89 22 10	8·4296128	1	R	Clüver
155	1827 (1) Feb. 4·92156	33 30 16	184 27 49	77 35 35	6·746000	1	R	Heilligenstein
156	1827 (2) June 8·41120	297 32 42	318 10 28	43 38 45	9·9074940	1	R	Heilligenstein
157	1827 (3) Sep. 11·69286	250 57 16	149 39 14	54 4 42	9·1393857	0·9992731	R	Clüver
(104)	1829 Jan. 9·74591	157 17 53	334 29 32	13 20 34	9·5385038	0·8446245	D	Encke
158	1830 (1) April 9·60506	212 23 19	206 22 43	21 11 9	9·9650486	1	D	Santini
159	1830 (2) Dec. 27·66040	310 59 19	337 53 7	44 45 30	9·0999822	1	R	Wolfers
(104)	1832 (1) May 3·97551	157 21 1	334 32 9	13 22 9	9·5358905	0·8454141	D	Encke
160	1832 (2) Sep. 25·52156	227 55 36	72 26 42	63 18 3	0·0732061	1	R	E. Bouvard
161	1833 Sep. 10·186	222 51 14	323 0 48	7 21 2	9·6612600	1	D	Peters
162	1834 April 2·821	276 27 3	226 1 31	5 59 48	9·7096600	1	D	Peters
163	1835 (1) Mar. 27·57651	207 42 55	58 19 46	9 7 39	0·3099084	1	R	W. Bessel
(104)	1835 (2) Aug. 26·36078	157 23 29	334 34 59	13 21 15	9·5371089	0·8450356	D	Encke
(7)	1835 (3) Nov. 15·93889	304 31 32	55 9 59	17 45 5	9·7683194	0·9673909	R	H.Westphalen
(104)	1838 Dec. 19·01224	157 27 4	334 36 41	13 21 28	9·5366085	0·8451775	D	Encke
164	1840 (1) Jan. 4·47111	192 12 17	119 57 54	53 5 41	9·7913112	1	D	Lundhal
165	1840 (2) Mar. 12·99064	80 18 20	236 49 16	59 13 20	0·0868563	0·9978836	R	Plantamour
166	1840 (3) April 2·49544	324 12 40	186 2 57	79 51 52	9·8740948	1	D	Rümker
167	1840 (4) Nov. 13·64438	22 31 34	248 56 15	57 57 23	0·1705070	0·9698527	D	Götze
(104)	1842 (1) April 12·01815	157 29 27	334 39 10	13 20 26	9·5378361	0·8447904	D	Encke
168	1842 (2) Dec. 15·95670	327 16 13	207 49 1	73 33 37	9·7026605	1	R	Langier
(50)?	1843 (1) Feb. 27·414015	278 43 12	1 26 59	35 38 34	7·7392979	0·9999365	R	Hubbard
169	1843 (2) May 6·05593	281 29 35	157 14 46	52 44 46	0·2085315	1·0001798	D	Göbtz
171	1843 (3) Oct. 17·14786	49 34 9	209 29 9	11 22 31	0·2285488	0·5559623	D	Leverrier
(53)?	1844 (1) Sep. 2·47402	342 30 50	63 49 0	2 54 50	0·0742309	0·6176539	D	Brünnow
172	1844 (2) Oct. 17·33613	180 23 59	31 38 55	48 36 22	9·9321180	1	R	Hind
173	1844 (3) Dec. 13·68294	296 0 30	118 22 23	45 36 34	9·4001230	1	D	Hind
174	1845 (1) Jan. 8·15698	91 19 42	336 44 31	46 50 36	9·9567272	1	D	Gougon
175	1845 (2) April 21·03098	192 33 34	347 7 0	56 23 36	0·0985330	1	D	Faye
176	1845 (3) June 5·67342	262 3 17	337 49 18	48 41 59	9·6038230	0·9898743	R	D'Arrest
(104)	1845 (4) Aug. 9·62627	157 44 21	334 19 33	13 7 34	9·5291008	0·8474362	D	Encke
177	1846 (1) Jan. 22·09387	89 6 25	111 8 29	47 26 6	0·1704680	0·9924026	D	Jelinck
(92)	1846 (2) Feb. 10·98766	109 2 26	245 54 45	12 34 53	9·9327011	0·7570030	D	Plantamour
(92)	1846 (2) Feb. 11·05401	109 2 45	245 56 7	12 34 13	9·9326965	0·7558991	D	Plantamour
178	1846 (3) Feb. 25·37406	116 28 22	102 41 6	30 55 53	9·8129827	0·7933879	D	Brünnow
179	1846 (4) Mar. 5·35599	89 16 50	76 49 35	85 34 58	9·8245330	1	D	Van Denise

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		° ′ ″	° ′ ″					
180	1846 (5) May 25·94071	84 48 14	161 31 51	57 43 40	0·1314317	1	R	Brorsen
181	1846 (6) June 1·21242	240 7 35	260 28 59	30 24 24	0·1842997	0·7213385	D	C. H. A. Peters
182	1846 (7) June 5·47921	162 6 2	261 53 12	29 18 47	9·8018857	0·9899389	R	Oudemans
183	1846 (8) Oct. 29·91663	98 47 19	4 38 22	49 39 3	9·9187601	0·9933127	D	Quirling
184	1847 (1) Mar. 30·32230	275 46 42	21 37 44	48 32 23	8·6457238	0·9993425	D	Villarceau
185	1847 (2) June 4·69052	141 37 13	173 57 43	79 33 43	0·3253373	1	R	Gauttier
186	1847 (3) Aug. 9·25914	21 20 33	76 42 2	32 38 24	0·1715239	0·9974348	R	Schoveizer
187	1847 (4) Aug. 9·44226	246 42 4	338 18 1	83 27 1	0·2472789	1	R	V. Littrow
188	1847 (5) Sep. 9·54272	79 12 41	309 49 24	19 8 25	9·6882986	0·9725603	D	D'Arrest
189	1847 (6) Nov. 14·41856	274 23 35	190 51 37	71 57 46	9·5187597	1	R	Rümker
190	1848 (1) Sep. 8·04532	310 34 41	211 32 31	84 24 50	9·5050568	1	R	Sometag & Quirling
(104)	1848 (2) Nov. 26·11567	157 47 8	334 22 12	13 8 36	9·5276717	0·8478281	D	Encke
191	1849 (1) Jan. 19·34781	63 14 18	215 12 56	85 2 54	9·9821497	1	D	Peterson & Loremtag
192	1849 (2) May 26·49803	235 43 48	202 33 15	67 9 19	0·0642078	1	D	Gougon
193	1849 (3) June 8·20364	267 6 30	30 32 22	66 55 19	9·9515250	0·9978312	D	D'Arrest
194	1850 (1) July 23·51966	273 24 27	92 53 25	68 12 7	0·0340307	0·9999868	D	Sometag
195	1850 (2) Oct. 19·34851	89 20 31	206 1 0	40 6 53	9·7518260	1	D	Roslhüher
(171)	1851 (1) Apr. 1·80560	49 41 38	209 31 19	11 21 39	0·2303547	0·5549226	D	Leverrier
196	1851 (2) July 8·69157	322 59 49	148 27 41	13 56 4	0·0696476	0·6607426	D	Villarceau
197	1851 (3) Aug. 26·23463	310 59 22	223 41 6	38 9 2	9·9931273	0·9968516	D	Brorsen
198	1851 (4) Sep. 30·79993	338 45 56	44 26 14	73 59 44	9·1503928	1	D	Götze and Sometag

This table will be found useful, inasmuch as it affords facilities to the computer to compare the elements of any recently-discovered comet with those given here.

Notes on Comets.—371 B.C. According to Diodorus of Sicily, about the end of the year 371, a comet appeared of prodigious magnitude, which was accompanied at the same time by inundations and an earthquake. Its apparent motion was from west to east, but its real motion was probably retrograde. From all recorded circumstances, it appears when observed to have been near its perihelion, and the inclination of its orbit to have been very great.

Seneca relates that the comet was divided into two portions near the end of its appearance. Struyck is of opinion that the comet was identical with that of 1664, but there are no grounds for this supposition.

344 B.C. At this period, Diodorus relates that Fabius and Sulpicius, being consuls, on the occasion of Timoleus of Corinth undertaking an expedition to Sicily, a burning torch appeared in the heavens the whole night, and preceded the train of Timoleus, even till its arrival in Sicily. We may then infer that it had a considerable north declination, and that it appeared in the western heavens.

203 B.C. M. Cornelius Cethegus and P. Sempronius Inditanus being consuls, a comet was seen at Sethia, extending from east to west. The Chinese annals confirm this by its appearance, about the month of August, near Arcturus.

172 B.C. P. De Maille relates that a great comet appeared in China, at the end of the summer of this year, which had a tail.

156 B.C. In China, near the end of October, and visible for a period of twenty-one

days, a comet in the west, which traversed the constellations of Aquarius, Equuleus, and Pegasus.

156 B.C. In this year, Seneca relates that, after the death of Demetrius, king of Syria, a comet appeared as large as the sun. At first its appearance was red and fiery, emitting sufficient light to dissipate the darkness of night. Gradually, however, its magnitude diminished, till it finally vanished.

136 B.C. In this year three comets were seen, which all may belong probably to the same body.

I. S. M. Emilius and C. Hostelius Mancinus being consuls, a burning torch was seen in the heavens at Prenéste.

II. Under the reign of Attatuss king of Pergamus, who reigned from 138 or 137 to 132 B.C., a comet was seen, which is supposed to be that of 136 B.C., and which, small at first, gradually increased till it reached the equator. Its tail equalled in length the parts of the heavens whose extent is the "Milky Way."

III. At the birth of Mithridates, who lived from this year to about 65 B.C., a comet, whose brightness was greater than that of the sun, lasted for about seventy days; the heavens appeared on fire, and the comet appeared to occupy one-fourth part of the sky, but it is doubtful whether this occurred in this year or 134 B.C.

133 B.C. In this year at Anisterne, the sun was seen during the night, and this lasted for some time; but it is likely that these appearances were due to the meteors, which we so frequently observe.

48 B.C. Lucian mentions that in this year a terrible comet appeared; the darkness of the night was illuminated by it. In 49 B.C., or probably 48, a comet was seen in China whose paths were extended from β Cassiopeiæ through ι Cassiopeiæ, till it was lost finally among those stars which never set.

43 B.C. In the latter part of September of this year, during the feasts in honour of Venus, a comet was seen, which was of great brightness, and was visible in all parts of the earth. It was visible about 5 P.M., when the brightness of the sun permitted it to be observed. It is very probable that it was in the sign Leo, with a north latitude of 35° to 40°, and that it was identical with the comet of 1680. In China, in this year also, a comet was seen in the months of May or June, which was in the constellation Orion; but if this is the same comet as the preceding, it will not agree with the elements of the comet of 1680.

Since the Christian Era.—14. According to Dio Cassius, S. Pompeius Magnus and S. Apuleius being consuls, many comets of the colour of blood were seen to shine.

39. In the Chinese annals mention is made of a comet in this year, which was visible from the 13th of March to the 30th of April, or for forty-nine days. Its path was from the Pleiades, through Pegasus, and finally to the head of Andromeda.

60. In China a comet was seen in the constellation of Capricornus, whose tail was eight degrees in length; its appearance lasted for fifty days, and it was last observed in the south of the head of the constellation Scorpio.

76. Pliny relates of a comet being observed this year, which was described by Titus Cæsar as having occurred in his fifth consulate. In China it was observed from the seventh of September, and was visible for forty days. Its tail was 3° in length, and the path of the comet was from the head of the constellation Hercules to 3° of the east of β Capricorn.

117. In the 9th of January of this year a comet was seen towards the west. On

January 14th it was seen near β Libræ and α Equulei; it afterwards passed to the constellation Musca.

141. According to the Chinese annals, on March 27th of this year, a comet, whose tail was 6° or 7° in length, was seen near α and β Pegasi. On the following 16th of April it was near ξ Andromeda. On the evening of the 22nd of April it had passed over the Pleiades, or about 11° of right ascension. On the 23rd of April it was seen in the feet and legs of Gemini, and finally disappeared in Leo.

240. Seen in China, on the 10th of November, a comet with a tail 30° in length in Scorpio; it passed through Capricornus, and on the 5th of December entered into Libra. Its latitude was 2° south on the 10th of November, but at the beginning of December it was not an entire degree. On December 19th the comet was south of the ecliptic, and was in the constellations of Aquarius and Cetus.

252. In this year two comets were seen in China. The first was seen in the wing of Pegasus on January 10. It was to the west, and appeared seventy days.

On the 25th of March a comet was visible in the constellation Musca, it was seen for twenty days, and the length of its tail was 50° or 60°.

336. In China, on the 16th of February, in the evening, a comet was seen in the southern arm and the girdle of Andromeda, and in the northern part of the constellation Pisces. It is probable that this is the comet which is mentioned as having been seen at the death of the Emperor Constantine, who, however, died on May 22nd in the following year.

363. At the end of August of this year, according to the Chinese annals, a comet was seen near the stars, α, ξ, κ, λ, ι, υ Virginis. Its brightness was so great that it was visible in full day.

389. In this year a brilliant comet was visible, in the month of August, in the morning, which Mascellin, Philosostorgus, and Nicephorus mention. It equalled Venus in brightness, and was erroneously placed near this planet. Payne has calculated that the planet Venus was at this time in inferior conjunction, and that it did not become visible in the morning till November; but he thinks that it was very likely that it was near Jupiter. Jupiter was, at this time, in August, to the west of the sun, and would rise a short time after midnight. The comet lasted for forty days, and finally disappeared in the constellation of Ursa Major.

390. In this year a comet appeared similar to a hanging column, which lasted for thirty days. There appears to be no doubt that this was a different body from that of the preceding year.

400. The historians of this year, Socrates and Sozerames, make mention of a terrible comet which occurred, and was seen in this year. Its form was that of a sword. It was also observed in China on the 19th of March, near the northern part of Pisces, and the girdle and southern arm of Andromeda; its tail was 30° in length. It successively passed through the constellations of Cassiopeiæ, and to the square of the great bear, where it was near χ Ursæ Majoris, then to α Hercules, and finally between γ and β Virginis. But this does not appear probable, for it would have been more so, if, instead of the head of Hercules, it had been written the tail of the Lion.

418. During an eclipse of the sun of this year, on the 19th of July, a comet was seen which had a conical form, and was visible for a period of four months. This is confirmed by an actual calculation of this eclipse, which occurred at this time. It was also seen in China, and its path was as follows:—At its first appearance, it was near

the star δ Cygni, from which it went to the square of the Great Bear; it afterwards proceeded to that part of the heavens which always remains above the horizon, to the constellations Bootes, Virgo, Leo, and Cerana. Struyck was of opinion, at first, that this comet was identical with that of 1596, but afterwards renounced this opinion.

467. Perseus and John being consuls, a great prodigy was seen for some days in the heavens. It was observed for 40 days, but in some places it was only seen for 10 days.

504. In this year a star or comet of great brilliancy was seen, but the Saxon annalist describes it in exaggerated terms.

530 or 531. The Chinese annals make mention of the first, which appeared in the month of October, 530. Its course was from Arcturus to λ and Ursæ Majoris. This would agree with the motion of the comet of 1680, which period Halley had assigned to it; and we may, therefore, infer its probability. The second, which appears to be a different body, was seen at Constantinople in 531. All the Byzantine authors speak of the last, which is described as having been a large and terrific body, and was visible with them for 20 days. Its rays extended to the zenith, and it was compared to a lamp.

539. This comet, which was visible for 40 days, occurred in the 13th year of Justinian. Its head was in the east, and its tail extended to the west. Its appearance is described as being equal in magnitude to a great man, which afterwards increased. It was afterwards observed in China, in the constellation Sagittarius, near μ, τ, φ, σ, τ and ζ. Its tail was 10 feet long. On the 1st of January, 540, the comet was within 3° of Venus; but this appears improbable.

565. On the 4th of August of this year, a comet was visible in China, and remained so for 100 days. It was first seen in *e*, *f*, θ, φ, *o*, and *h* of the Great Bear. Its tail was not a whole degree in length. It then approached γ Aquarii, and ϵ and θ Pegasi, at which time its tail was 10° long. Its last appearance was in Equuleus.

566. In this year a comet was seen which, according to one chronicler, lasted for a whole year; but, according to another, which is more probable, only remained for 66 days. It is described as throwing out a long flame, and appeared in the Arctic circle.

568 (first comet). On the 10th of July of this year, a comet was seen in China in the feet and thighs of Gemini.

568 (second comet). Another comet was observed by the Chinese this year, on the 3rd of September, in the face and heart of Scorpio, which proceeded eventually to the east. On the 8th of September, it had a tail of 40°, near the northern star of Delphinus. It then passed through the constellations near β Aquarius and α Equuli, and α Aquarius, and ϵ and θ Pegasi, and entered into the constellation the Eye of Pegasus. On the 8th of October it was in the head of Aries. It was observed altogether for 69 days.

582. At Easter day, which occurred on the 29th of March, a great comet is mentioned by many historians as having been seen at Soissons.

607. In China, two comets are mentioned as having been seen, which some astronomers consider identical. The first appeared on the 4th of April, in the west. It passed through the constellations, the northern Fish, the girdle and the southern arm of Andromeda, the head of Aries, α and ζ Virginis, and the feet of the Virgin, where it disappeared.

The second was seen on the 21st of October, in the same constellations. It passed over the northern part of Leo, of Virgo, and of other stars more to the north. The comet did not attain the constellation α, β, γ, δ, ϵ, ζ, η, θ, κ, Orionus.

615. In July a comet was seen in China, in the Great Bear. Its tail was 50° or

60° in length. During the night its head or nucleus had, as it were, a motion of libration.

676. In the beginning of this year a comet was discovered in China, whose tail was 5° in length, to the south of the constellations α and ζ Virginis, and the feet of the Virgo.

729. Bede and other chroniclers mention a comet as having appeared in the month of January of this year, which was near the sun for 14 days. But some mention two having been seen, which is improbable. It is likely that it had a right ascension little different from the sun, with a northern declination, which would account for its setting after the sun, and rising before it.

837. Father de Maille, according to Pingré, relates that on the 22nd of March, in China, a comet was seen near α Aquarii, ε and θ Pegasi. The tail of the comet was 7° long.

On March 29, it was seen in β Aquarii and α Equulei. On April 6, it had a tail 10° long, and its motion was towards the west. On April 10, the tail was 50° long, and separated into two portions. On April 6, its tail was undivided, and its length was 60°. On April 14, the length of the tail was 80°, at which time the comet was in the constellation Hydra. After this time the length of the tail decreased considerably, and on April 28 it was seen for the last time with a tail only 3° long. Pingré, after having carefully investigated all the preceding observations, deduces an orbit which appears in our catalogue; but there appear strong reasons for inferring that it was probably an apparition of Halley's comet, which, from some errors in the published accounts, will not agree with the observations. There also appears to be a doubt as to the year of this appearance—the Latin accounts agreeing consistently in placing it in 838, whilst the Chinese give 837. Were two comets seen, or is there an error in the Chinese account? The account of an anonymous Latin historian differs both in date and position. It is nearly as follows:—It was first seen in Virgo, and pursuing a retrograde course; it traversed successively Leo, Cancer, and Gemini, all in the space of twenty-five days, and finally disappeared in Taurus, under the feet of Auriga. Pingré's orbit, which is based on the Chinese observations, will not agree with the latter path; and we may infer, with some reason, that the year was 838, and the comet of which the Latin historian speaks was that of Halley.

855. In this year, an ancient chronicler mentions two stars being seen in the month of August, on ten successive occasions. They were likely comets, the larger being always visible, but the smaller not so frequently. A comet was also seen in France during 20 days.

875. On June 6, an extraordinary blazing comet was seen, with a fine tail. It appeared first in Aries, and was visible during the whole month of June. This comet was supposed, at the period, to be the announcement of the death of the Emperor Louis II.

891. In China, on May 12, a fine comet was seen in Ursa Major, with a tail 100° in length. This comet is also mentioned in the Saxon annals. Pingré was of opinion that this comet was identical with that of 1532 and 1661.

895. A fine comet, whose tail was 100° long, was visible in China on June 25. It was first seen near ι and κ of the Great Bear, and in the course of its appearance it passed over Cerene, part of Hercules, and Serpentarius. The length of the tail also increased to 200°, which, however, Pingré asserts is difficult to believe.

905. In this year the European and Chinese annals mention the appearance of a great comet in the months of May and June.

912. In this year the Latin authors relate that a comet appeared in the month of March, for fourteen days, in the north-west. A Greek author mentions its duration as being forty days.

931. A comet was visible in Cancer in the months of May, June, and July of this year.

939. On the 19th of July of this year a longer eclipse of the sun was seen; and in Italy, during eight successive nights, a comet of great splendour was beheld.

942. A comet was seen in October of this year for upwards of twenty days in the western heavens. Its motion was towards the east. Its head was faint, but the tail projected and resembled smoke.

975. In the autumn of this year a comet is mentioned by the Latin and Chinese historians. In China it was seen on August 3rd in Hydra; its tail was 40° in length. The comet, during the time of its appearance, which was for eighty-three days, passed through the constellation Cancer to the space between γ Pegasi and α Andromedæ. Pingré remarks that there is a considerable similarity in the above path to that of 1556, which is supposed, with great probability, to be identical with that of 1264. Taking its perihelion passage a few days before the end of 975, July, Pingré finds that it might have been seen about the 11th of July, which Father Ganbil states to be the case in China. On August 3rd it would have been in conjunction with the sun, but its northerly latitude being considerable, it would rise some time before it, and might leave, as mentioned, a tail 40° long. Its course would then be retrograde, apparently through the constellations of Cancer, Gemini, Taurus, and Aries, in which latter sign it would probably be visible about the month of October. There is sufficient accordance in the above to give this opinion of Pingré's considerable weight.

989. An appearance of Halley's comet is supposed to have occurred in the autumn of this year. On August 5th it was seen in the constellation of Gemini, and pursuing a retrograde direction, it passed through the constellations of Leo and Cancer. Burckhardt has computed an orbit from the rough account of the Chinese annalist, which appears in our table.

1000. A comet of extraordinary brilliancy was seen this year for nine days; but there appears to be some doubt as to the exact period.

1066. In the April of this year the Chinese and European annalists mention that a comet of great brilliancy was seen. It was supposed to be the forerunner of the conquest of England by William duke of Normandy. The orbit has been computed by Pingré, from the Chinese annalists, and there appears a strong probability of its identity with that of 1677.

1097. Although the comet of this year was only seen for a very short time (in Europe only fifteen days), it is remarkable as projecting two rays or tails, which were directed to the east and south-east respectively. In China, on October 6th and 9th, its tail was noted as being respectively 30° and 50° in length. On October 6th it was seen in China near α and μ Libræ. On October 16th it was seen near the head of Hercules. It ceased to be visible in China on October 25.

1106. A great and fine comet was seen in Palestine on February 7, and in China on February 10. It was seen on February 7 in Pisces. Its tail was similar in colour to the whiteness of snow. Its appearance lasted for fifty days. In China, on February 10, its tail was 60° in length, and extended from Gemini to Orion. During its apparition, it traversed the path from the end of Pisces to the end of Taurus. There is some similarity supposed between this comet and that of 1686.

1222. In the months of August and September a comet of extraordinary magnitude, very red, and accompanied with a great tail, was seen. In China it was observed on September 10, beneath the feet of the Virgin, Arcturus, and Berenices. It disappeared on October 8.

1231. A comet was seen in China, on February 6, in Cygnus; its magnitude was equal to Saturn. From the recorded path, Pingré has calculated an orbit, which appears in our table.

1264. A great and celebrated comet, of which all the historians make mention. It is supposed to be identical with the comets of 975 and 1556. It was visible for some months—at least three, probably four. Its disappearance occurred the same day as the death of Pope Urban IV., or on October 3. Pingré and Dunthorne have calculated its orbit.

1265. An historian mentions that, at the commencement of autumn of this year, a comet was seen with a long tail, which commenced to shine after midnight.

1266. In the month of August of this year a comet was visible in France. It was also seen at Constantinople, near the sign of Taurus; there appears some doubt, however, with respect to this last account. It is questionable whether the comets of 1265 and 1266 be not identical.

1299. On January 24 of this year a comet was seen near Columba. It was visible for sixty-three days. Pingré has deduced its orbit from all the observations.

1301. A great comet, which is mentioned by the Latin historians as having occurred at the time of the autumnal equinox. It was also seen in China for forty-six days, commencing with September 16. The elements deduced by Pingré are very uncertain, but Mr. Hind is of opinion that this is identical with the comet of Halley.

1337. At the summer solstice a comet is noted as having occurred about the time of the death of Frederic, king of Sicily. It was also seen in China. The European historians give its duration for three or four months.

1337. A second comet was seen in this year, but it was not observed in China. The European historians mention it as having lasted two months.

1351. In the month of December a comet was seen in Cancer in the east. The account, however, is obscure, and gives its position with uncertainty. From the Chinese positions of November 24, 26, 29, and 30, Burckhardt has determined the time of the perihelion passage on November, 26 days 12 hours.

1362. On March 5 a comet was seen in China, near the constellation of α Aquarius and ϵ and θ Pegasi. It disappeared on the 7th of April. It was visible in Europe during Lent as a very great and brilliant star.

1362. On June 29 this comet was seen in China for forty days. Its first appearance was in Capricornus, and it had an extensive tail.

1366. On August 26 a comet was seen in China in Ursa Major. On August 27 it was visible in the tail of Scorpio. On August 29 it was near ϵ and μ Aquarius. On August 30 it was near β Aquarius and α Equulei.

1378. On the 22nd of September of this year a comet was seen in China to pass over the stars in the western foot of Antinous. It was also seen in Europe. On September 29 it was near the constellation Ursa Major, between Aries and Taurus. (This is certainly an error—the Great Bear is not near these constellations). It was visible for five days, and its motion was in an opposite direction to the apparent diurnal motion. This is an appearance of Halley's comet.

1385. On October 23 a comet was seen in the constellations of Leo and Virgo, to the south of the constellation Ursa Major. On the morning of October 30, it was seen near χ Hydræ. It afterwards passed to the south of the constellations ϕ, μ, λ, υ, and κ Hydræ. The tail was 10° long.

1402. A very large and very brilliant comet, which commenced to appear on the first day of Lent, on the 8th day of February, and which remained to the beginning of March.

1433. At Bologna a remarkable and brilliant comet was seen, which appeared from evening to morning. It lasted more than a month, or rather less than three months. Some dated its first appearance on October 12.

1456. A return of the celebrated comet of Halley. It is represented by all the historians as being grand, terrible, and of extraordinary magnitude, with a tail equivalent in extent to 66°. But the extent of this appendage was fleeting and uncertain, varying to only 7°. The period of its durability was about one month.

1457. In the month of June of this year a comet was seen in Pisces. Its body was small. Its tail, at first very long, was equal to 15° of a great circle. The colour of the comet was of the appearance of lead.

1468. The second comet of this year, which appeared in September, October, and November. It was seen at first in Leo, near the tail of Leo. Its colour was blue, with some mixture of paleness. It was a very small body.

1472. In the month of December, 1471, a fine comet was discovered in China. At Japan it was observed on the 9th of January, 1472, where it is described as being very great, its tail being equal in length to a street. In Europe it appears to have been first seen about Christmas—at first small, but afterwards very large. The greater number of historians represent it as very fine, and as altogether fearful.

1490. At the commencement of this year a great comet, with a very white and very long tail, was seen. It was visible at Bologna about the middle of February. Its head was small, and its tail long, but of little brightness. Pingré makes the year of this appearance to be 1491, but another was seen in China in January, 1491, in Cygnus, which cannot be reconciled with the European observations.

1506. In the month of August of this year a comet was visible with a long and bright tail, which extended between the stars of Ursa Major. It was observed in China and Japan, and appeared for a period of eighteen or twenty days.

1531. On July 31, the comet of Halley again made its appearance in Europe. In China and Japan it was seen on the 13th of July. Apian, astronomer at Ingoldstadt, observed it from August 13 to August 23, from which observations, however rude and imperfect, Halley has computed elements. Apian also observed that the tail was always directed from the sun. It appears, from the Chinese observations, that the train was 7° in extent, and that the comet was visible for thirty-four days.

1532. A comet was visible this year, which is supposed to be identical with that of 1661. The orbit has been deduced from Apian's observations, which, as in the preceding case, are not much worthy of confidence. Its head appeared constantly in the morning before the sun, and was three times larger than Jupiter. Its duration was about seventy days. According to the Chinese accounts, it was visible altogether for 115 days, and its tail varied from 1° to 10° in extent.

1533. A comet appeared this year about the middle of June. It was seen in the summer solstice in Taurus, with a very long tail. Its appearance is generally dated at the end of June, and it was seen to the first days of September.

1556. This is the celebrated comet whose return is so anxiously expected. It began to appear about the end of February, at which time it equalled in magnitude one-half of the full moon. Its tail, however, was short and variable. It was not more at the greatest than 4° in length, and its flickering nature resembled much a flame agitated by the wind. The comet disappeared altogether on the 23d of April, near the chain of Cassiopeæ, after having been seen for many days; but its brightness being effaced by the rays of the sun, which it was very near. It created great terror in the mind of Charles V.

1558. This comet appeared on July 14 in Leo. It disappeared on August 24 and 25, apparently behind the clouds. It was at first not brilliant, but its brightness increased in the latter part of its appearance.

1569. In the November of this year a comet was seen in Serpentarius, and in the signs of Sagittarius and Capricornus. Its motion in longitude equalled the extent of these two signs. The comet was discovered in November, and its tail was directed towards the east.

1577. A great comet, which was discovered at Peru on November 1. It was seen in Europe by Tycho on November 13, at Uraniberg, in the island of Hoène; on this day he estimated the diameter of the head to be 7′, and the length of its tail 22°; the head was white, but less bright than the fixed stars. It was observed to January 26 of the following year.

1580. This comet was discovered at Tubingen, by Mestlim, on October 2. Tycho discovered it on the 10th of the same month, and his observations have been used by Pingré in calculating the orbit. Tycho notes that the diameter of the head was 8′, its light faint, its colour livid, and its tail difficult to be distinguished. Dr. Halley's orbit was based on the uncertain observations of Mestlim. Tycho observed the comet to December 12.

1582. The second comet of this year, which was seen for only fifteen days at the longest. Tycho discovered the comet, and observed it at Uraniburg from May 12 to May 18. On May 17, the magnitude of the head was scarcely equal to stars of the fourth magnitude. Its tail was more than 3° long, and very faint. Pingré computed two orbits from Tycho's observations, which are both necessarily uncertain.

1585. This comet was discovered by the Landgrave of Hesse, and Rothmaun, his astronomer, on October 18, and afterwards observed by Tycho to November 22. At its first appearance it equalled Jupiter in magnitude, but of less brightness. This comet had neither coma nor tail.

1590. This comet was discovered on March 5 by Tycho, and observed to March 16. It is noted as being of a medium magnitude; but it had a great tail, which extended to the zenith. On the day of its discovery the comet appeared as a star of the second magnitude, and shortly afterwards, on the same night, of the first magnitude. Its brightness, however, was not so great as stars of this description.

1593. This comet, which escaped entirely Tycho's scrutiny and observations, was discovered by his pupil, De Ripen, at Zerbst, in Anhault. On August 4, it was noted as being of a livid and reddish colour; its head equalled in magnitude stars of the third class, with a tail 4½° in length. On August 9, it was equal in brightness to stars of the fourth magnitude, and a very slight vestige of a tail could be perceived.

1596. This comet appears to have been discovered on July 11, and observed to the 12th of August. Its colour was feeble and pale, with a tail small, fleeting, and undefined. Valz considered that this comet was identical with that of 1845.

1607. The third apparition of Halley's comet, as ascertained by the elements, which, however, are not considered so accurate as could be wished. It was observed by Kepler in Prague, by Langemontanus in Copenhagen, and Malmoe in Scania. Bessel's orbit is deduced from the observations of the English astronomer Harriott.

1618. The first comet of this year was discovered at Caschau, in Hungary, on August 25, and observed by Kepler from September 1 to September 25.

1618. The second comet of this year was discovered at Silesia, on November 10, and at Rome on the same day. At Ispahan, in Persia, the Spanish ambassador saw this comet for the space of fifteen or sixteen days, commencing with November 10, two hours before the rising of the sun. The length of its tail equalled in extent one-sixth part of the zodiac. On November 18, the Jesuits at the Roman College noted the extent of the tail to be 40°.

1647. Seen in Prussia on September 29, soon after the setting of the sun. This comet was small, and remained visible for a very short time.

1652. This comet, which was of a pale and livid colour, almost equalled the moon in magnitude, according to Hevelius and De Caniers. It was observed by many astronomers, but, with the exception of those by Hevelius at Dantzic, the observations are very rough. Dr. Halley has computed his orbit from the observations of Hevelius.

1661. This comet, which was considered at first to be a return of the comet of 1532, was discovered by Hevelius from February 3 to March 10. On these observations Dr. Halley has calculated its orbit; but no body, which bears any similarity to these elements, was observed in 1789 or 1790.

1664. This comet appears to have been discovered on November 17, in Spain. It was observed by Huygens on December 2, and by Hevelius on December 14. The observations were continued by Hevelius to February 18 of 1666. At its first appearance it was noted as large as a star of the first magnitude, but not so bright. The length of the tail was from 5° or 6° to 10°.

1665. This comet was seen at Aix on March 27. From the observations of Hevelius, from April 6 to April 20, Dr. Halley has calculated its orbit.

1668. This comet, of which the tail appears to have been only seen at Bologna, was observed principally in the southern hemisphere. There appears to be some similarity of elements with the comet of 1843.

1672. This comet was observed by Hevelius at Dantzic on March 2, and seen by him till April 21. On these data Dr. Halley has computed his orbit. The comet was small, having a train only 1° or 1½° in length.

1676. Father Fontenay, of the Society of the Jesuits, observed a comet at Nantes on the 14th of February. It was visible to March 9, and was equal to stars of the third magnitude. This comet had no tail.

1677. This comet was discovered at Dantzic, on April 27, by Hevelius. It was observed to the 8th of May, and it is on these observations Dr. Halley has calculated his orbit. The magnitude of this body was equal to Jupiter, and it had a tail about 2° in length.

1678. La Hire discovered this comet on September 11, and observed it to October 7. The orbit, however, deduced by Dawes is very rough, principally in consequence of the uncertainty of the observations.

1680. This great and celebrated comet was discovered by a person whose name is unknown, at Coburg in Saxony, on November 4. It was independently discovered by Godfrey Kirch, on the 14th of November, while about to observe the Moon and Mars. *Flamsteed*, at Greenwich, saw the comet on the 20th of December, and observed it to the 15th of February, 1681. The length of the tail on December 16 was 70°. Several elliptic orbits have been calculated of this comet.

1682. Halley's comet. It was first observed at Paris by Picard and La Hire, on August 26, and last observed by Flamsteed on September 19. Its head was 2′ in diameter, and the length of the tail varied from 12° to 15°.

1683. Hevelius observed this comet at Dantzic from the 30th of July to the 4th of September. Clausen's Elliptic Elements, from Flamsteed's observations, give its period as 190 years. Its nucleus was equal to a star of the fourth magnitude, with a tail varying from 2° to 4°.

1684. This comet was discovered by Bianchini, at Rome. It was observed from July 1 to July 17; on which observations Halley has calculated its orbit.

1686. In the August of this year a comet was visible at Para, in Brazil, during the whole month. Its head equalled stars of the first magnitude, and its tail was 18° in length.

1689. This comet was discovered at Pekin on the 11th of December. It was not seen at all in Europe. On December 11, the part visible of the tail was from 10° to 12°. In the southern hemisphere it was regularly observed, and its tail at the greatest was 60° long.

1695. This comet was almost obscured by the atmosphere or coma by which it was surrounded, so that the nucleus was scarcely distinguishable. It was seen in the southern hemisphere in the Brazils by Father Jacob, a French Jesuit, on October 28. The length of the tail was then 18°.

1698. The orbit of this comet is very uncertain, in consequence of the roughness of the observations. Cassini discovered it at Paris, in the beginning of September, in the constellation Cassiopeia. La Hire observed it to September 28. This comet was only seen at Paris, and was not larger than a star of the third magnitude.

1699. Discovered at Pekin, by Father de Fontenay, on February 17, and observed to February 26. It was seen at Paris from February 20 to March 2.

1701. This comet was discovered at Pau, by Father Palla, from October 28 to November 1. It was a small body without tail, which diminished sensibly in magnitude.

1702. This comet, which was the second discovered this year, was observed from April 20 to May 5, at Paris, Rome, and Berlin. The orbit is uncertain, in consequence of the roughness of the observations. This comet was compared to a nebulous star on April 20.

1706. Observed at Paris by Cassini and Maraldi, from March 18 to April 16. This comet was also similar in appearance to a nebulous star.

1707. This comet appears to have been discovered at Bologna on November 25, and observed to January 23 of the following year. Of all known comets, this body has the greatest inclination. Viewed with a telescope, it appeared to be nebulous, and of the second magnitude.

1718. This comet was observed at Berlin by Kirch, from January 18 to February 5. It appeared to be equal to a star of the fourth or fifth magnitude, with a nebulous diameter of 5′ or 7′.

1723. Discovered at Bombay on October 12, and observed at Lisbon and other European stations to the middle of December. It appeared of about the third magnitude, with a very faint tail, not more than 1° in length.

1729. This comet, which is remarkable in the length of its visibility, and the greatness of its distance from the sun and earth, was discovered at Nimes, by P. Sarabat, on July 31. It was a small nebulous body, scarcely visible to the naked eye.

1737. The observations of this comet were made at Paris, Rome, Bologna, Oxford, Lisbon, and also in Jamaica and Madras.

1739. This comet was observed at Bologna, from May 28 to August 18.

1742. First comet. Seen at the Cape of Good Hope on February 5. It was also seen in Europe in March, and was visible to the naked eye. The length of the tail was 5° to 8°.

1743. First comet. This comet was observed at Bologna, Paris, Vienna and Berlin. In Berlin, it appears to have been discovered on February 10, by M. Grischon. It was a small body.

1743. Second comet. Observed at Harlem from the 18th of August to the 13th of September. This comet was small, but was seen, notwithstanding, with the naked eye.

1744. This comet appears to have been discovered at Harlem on December 9, 1743, and was observed to March, 1744, at several observatories. It was one of the finest bodies which had occurred since the comet of 1680. In February, M. Cassini noticed that the head was divided into two portions. The tail was also divided shortly after into two branches.

1747. Discovered at Laussaune, by Cheseaux, on the 13th of August, 1746, and observed there to September 22. His observations were then interrupted by illness, but it was last seen on the 23rd of November near a star in Capricornus.

1748. The first comet of this year was discovered at Paris about the latter end of April. It was also observed in South America, and in Pekin in China, as well as the Royal Observatory, Greenwich. It appears to have been a fine object, easily seen with the naked eye, and having a tail 20° long. This comet was observed at Paris by Maraldi to the 30th of June.

1748. The second comet of this year was seen at the same time as the first, but in a different part of the heavens. Its nucleus was brighter than the preceding, but there was no appearance of tail. The orbit depends on three approximate observations made at Harlem, and is consequently very uncertain.

1757. Observed at Greenwich from September 13 to October 18. The orbit of this comet is very approximate.

1758. Discovered in the island of Bourbon on the 26th of May, and seen in London on June 18. Messier first observed the comet on August 15, and continued his observations to the 2nd of November. It appears to have been a diffused body, equal in diameter to Jupiter.

1759. Halley's comet, first seen on Christmas day, 1758, by Pazlitch, near Dresden, and on the 28th by Dr. Hoffmann. This return is very celebrated in the history of astronomy, as being the first predicted appearance of the apparition of a comet. Dr. Halley roughly estimated that the time of arriving at its perihelion would be at least one year longer than the interval between the two preceding returns. Clairaut, by a laborious calculation of the planetary perturbations, fixed its return on the 13th of April, the true time having been determined by Rosenberger, from a minute discussion of all the observations, to have taken place on March 12. From the beginning of 1759 to the middle of February, the comet was regularly observed, till it became plunged in the solar rays about the latter period. On its reappearance at the end of March, it was again observed to the 17th of April, till its declination became too great to permit its observation at European stations. It was observed, however, from the 20th of April to the middle of May in southern stations. La Nux, at the Isle of Bourbon, found the length of its tail on March 21 to be 8°; on the 28th of March, 25°; on May 1, 33° to 34°; and on the 5th of May, 47°. It afterwards diminished, till on the 14th of May it was not more than 19°. It afterwards reappeared above the European horizon, and was observed in France and Portugal to the 3rd of June.

1759. The second comet of this year is reckoned in the order of its time of perihelion passage. It was discovered on the 25th of January, 1760, in the constellation Leo, by the celebrated astronomer Messier, and regularly observed to the 16th of March.

1759. The third comet of this year (reckoning, as before, in the order of perihelion passage), was discovered on the 1st January 1760, by all the astronomers of the French Academy, and observed till the 8th of February. Its motion was exceedingly rapid, being, on the day of its discovery, 2° 25′ of a great circle in two hours, or at the rate of 29° daily. The diameter of the nebulosity was found to vary from 20″ to 30″. Its tail was about 4° in length.

1762. This comet was discovered by Klinkenberg on the 17th of May, and observed by Messier and Maraldi to the 2nd of July. Its appearance at the time of its discovery was similar to a star of the fourth or fifth magnitude, with a slight tail. The nucleus was readily visible to the naked eye, but in a telescope was bright and ill-defined.

1763. Discovered by Messier on September 28, and observed to the 25th of November. Its appearance was that of a nebulous star. On the 4th of October, its diameter was 7′ or 8′. The orbit of this comet gave Pingré considerable trouble. That of Burckhardt is founded on unpublished observations.

1764. This comet was also discovered by Messier, and observed by him from January 3 to February 11. It was a bright nebulous object, visible to the naked eye, with a tail 2½° in length.

1766. Discovered by Messier on the 8th of March, when looking for the supposed satellite of Venus, which had created great sensation about this period. It appeared as a small nebulosity, with a luminous centre.

1766. Second comet. This comet appears to have been discovered by Helfenzriella, of Dillingen, in Suabia, on the 1st of April. On April 9, it presented a tail from 3° to 4°, its nucleus being similar to a star of the fourth magnitude. Pingré supposed that it would be visible after the perihelion passage, which proved to be the case; but not, however, in European latitudes. La Nux observed it in the Isle of Bourbon, from April 29 to May 13.

1769. This remarkable comet was discovered by Messier, on the 8th of August, and observed by nearly all the astronomers. La Nux observed it at the Isle of Bourbon, from August 26 to September 26. On September 11, he found its tail by measurement 97° long. Pingré, the same day, measured its length to be 90°. On August 28, Dr. Maskelyne noticed its tail as 7°; Messier as 15°; La Nux and Pingré as 19° to 20°. On September 9, Dr. Maskelyne estimated its length 43°; Messier as 55°; La Nux upwards of 60; and Pingré as 75°. This comet was also observed after its perihelion passage, from October 24 to the 1st of December.

1770. First comet. Messier discovered this comet on the 14th of June, and observed it regularly to the 2nd of October. This comet is celebrated for the trouble it has successively given astronomers in the attempts to investigate its orbit. Leverrier has recently determined its period to 5·6 years nearly, but it has never been observed since the time of its discovery.

1770. Second comet. This comet appeared in 1771, on the 10th of January of which year it was seen at Paris by Messier, and at Milan by Boscovich. It appears, however, to have been discovered by La Nux, at the Isle of Bourbon, on January 9.

1771. Messier discovered this comet on the 1st of April, and observed it to June 19. It was also observed by Dr. Maskelyne, at Greenwich, from April 14 to May 30. Its appearance was similar in brightness to a star of the third magnitude, with a train varying in length from 1° to 3°. The orbit is supposed, with some probability, to be hyperbolic.

1772. *Biela's Comet.*—This body, of which there is no doubt as to its identity with the comet of Gambart, or Biéla, was discovered by Montague, at Lemoges, on the 8th of March. Messier observed it on three occasions, on March 27, March 30, and April 3.

1773. Discovered by Messier on the 12th of October, and observed to the 14th of April, 1774. It was with difficulty seen by the naked eye.

1774. Montague discovered this comet on the 11th of August, at Lemoges. This was a small body, not visible to the naked eye.

1779. The small comet of this year was discovered by Bode, on January 6, at Berlin, and independently by Messier on January 18, who observed it to the 17th of May.

1780. First comet. This comet was discovered by Messier, on October 26.

1780. Second Comet. This was discovered by Montague at Lemoges, on October 18. It was also seen by Olbers, on the same day.

1781. First comet. This small comet, which was not visible to the naked eye, was discovered by Mèchain on the 28th of June. It had no tail. The diameter of the nebulosity did not exceed 3′ or 4′.

1781. Second comet, discovered by Mèchain on the 9th of October, and observed to December 25. On November 9 it was visible to the naked eye, the nebulosity being 4′ or 5′ in diameter, and its tail 3° or 4° in length.

1783. Discovered by Pigott at York on November 19, as a small faint body, with difficulty bearing any illumination of the telescope. There is little doubt as to the ellipticity of its orbit, the period varying from five years and ten years, in Burckhardt's two orbits.

1784. La Nux first observed this comet on the 15th of December, 1783, at the Isle of Bourbon. At Paris, in the month of January, 1784, it was visible to the naked eye, with a tail from 2° to 3° in length.

1785. First comet discovered by Messier and Méchain on the 7th of January. It was invisible to the naked eye.

1785. Second comet discovered by Méchain on the 11th of March, and observed to the 7th of April. On April 4, the nucleus was visible to the naked eye, with a tail 5° in length.

1786. First comet. This is a well-ascertained appearance of Encke's comet, which was discovered by Méchain on January 17, and only observed on two occasions.

1786. Second comet. This body was discovered at Slough on the 1st of August.

1787. This comet was discovered by Méchain on the 10th of April. It was a small luminous body, only visible in a telescope.

1788. First comet. This comet, which at its first appearance was not visible to the unaided vision, was discovered by Messier on November 25. Its brightness increased so much that it was visible to the naked eye on November 30. It appeared with a train 2° or 3° in length. The last observation was on December 29, when the comet had diminished considerably in brightness.

1788. Second comet. This was discovered by Miss Caroline Herschel on December 21. It was a small telescopic body, presenting, at its first appearance, a nebulosity of five or six minutes in diameter.

1790. First comet. Discovered by Miss Herschel on January 7. This was also a nebulous body, only visible by the help of a telescope, and of a diameter equal to about five or six minutes of arc.

1790. Second comet. Discovered by Méchain on January 9. It appears to have been nearly similar to the preceding, and visible at the same time.

1790. Third comet. This was discovered by Miss Herschel on the 18th of April. It was observed by Messier from the 1st of May to the 29th of June. Its brightness on the 17th of May was so great that it was visible to the naked eye; the length of the tail varied from 2° to 4°.

1792. First comet. Discovered by Miss Herschel on the 15th of December, 1791. It was a faint nebulous body.

1792. Second comet. Discovered by the Rev. E. Gregory on the evening of January 8, 1793. It was a dull nebulous body, with a faint appearance of tail.

1795. First comet. Discovered by Messier or Méchain on the 27th of September.

1795. Second comet. Discovered by Peray on September 24. Miss Herschel detected it on the 7th of October. Burckhardt estimated its period to be twelve years, which, from a revision of the elements by D'Arrest, is improbable. The latter astronomer makes its period to be 422 years.

1795. Encke's comet. Discovered by Miss Herschel on November 7, and observed by Dr. Maskelyne, and other astronomers, to the end of the month. It was a round, ill-defined body, without a nucleus.

1796. This comet was discovered by Olbers on March 31. It was a very faint body, and would bear no illumination of the field.

1797. This comet was discovered by Bouvard on August 14. It was a nebulous body from 3′ to 5′ in diameter.

1798. First comet. Discovered by Messier on April 12.

1798. Second comet. Discovered by Bouvard on December 6. It was a faint body, with a slight increase of light in the centre.

1799. First comet. This body was discovered by Méchain on August 7, and visible to October. It was very faint when first seen; but at the end of August it was visible to the naked eye, with a tail about 1° in length.

1799. Second comet. Discovered by Méchain on December 26; it was visible to the naked eye as a star of the fourth or fifth magnitude. It had a tail from 1° to 3° in length.

1801. Discovered by Reissig, jun., of Cassel, on June 30, and observed by Messier to July 23. It was a small body.

1802. Discovered by Pons, at Marseilles, on August 26; it was not visible to the naked eye.

1804. This comet, which had neither nucleus nor tail, was discovered by Pons on March 7. It was seen the last time on the 1st of April.

1805. (1) Encke's comet. It was visible to the naked eye, and much resembled the nebula in Andromeda. It was discovered by Thulis, of Marseilles, on October 19. One observer mentions its tail as being 3° in length.

1805. (2) Biela's comet. Discovered by Pons, on November 10, as a very faint body, the nebulosity scarcely bearing the slightest illumination of the field.

1806. Discovered by Pons on the 10th of November. The coma was from 5′ to 7′ in diameter.

1807. Discovered by Pons on September 20; but it appears to have been first seen eight days previously by an Augustine monk in Italy. This was the finest comet that had appeared since 1769. On the 30th of September, the nucleus was equal to a star of the first magnitude. Olbers, on November 7, noted the division of the tail into two branches.

1808. (II.) Discovered by Pons, in Camelopardalus, on March 25, and observed to the 2nd of April.

1810. Discovered by Pons on the 22nd of August, and observed to October 8. It was a faint and small round nebulous body.

1811. The great comet of this year, which was first discovered by Flangergues on March 25, and visible to the end of October. This is a most remarkable body.

1811. The great comet of this year, remarkable in several particulars, was discovered by Flangergues at Vivieres, on the 26th of March, and observed to August 17th, 1812. From the laborious investigations of Argelander, its orbit was found to be elliptical, the period being 3065 years. This comet, in addition to its magnitude and the remarkable duration of its visibility, presented, on September 7, a tail bent off in two branches; but these branches did not proceed from the comet itself, but were hung together at a slight distance from it, and separated from it by a dark interval, so that they enclosed the comet as a parabola does its focus. At this period its tail was 5° in length. On September 20, Bode found the length of the tail to be 10°. On October 11, the tail was about 13° long; the diameter of the nebulosity being 1′ 20″. Its maximum length appears to have occurred in the first week of October, when the tail was found to be 25° long, and about 6° broad. Sir W. Herschel paid considerable attention to the physical appearance of this comet, and the reader is referred to the Philosophical Transactions, 1812, for his remarks, which are too extensive to be given here.

1811. The second comet of this year was discovered by Pons on November 16, and observed to the end of January, 1812. It had a well-defined nucleus, with a faint surrounding coma.

1812. This comet was discovered by Pons, on July 20, and observed to the end of September. At first it was only visible in a telescope; but in September it increased in brightness, and had a tail 2° in length. Professor Encke found its orbit elliptical, the period being 70·7 years.

1813. First comet. This comet was discovered on the 4th of February, by the preceding observer; and its appearance was that of a small confused nebula.

1813. Second comet. This comet, the eighteenth discovered by Pons, was first seen on March 28. It appears to have been visible to the naked eye on April 24th and 25th, as a small round nebulosity without tail.

1815. This comet was telescopic, and was discovered by Olbers, on March 6th. It is remarkable as being one whose elliptical orbit is decided; the periods of revolution, by several computers, varying from 72 years to 77 years.

1818. First comet. Discovered by Pons, in Cetus, on the 23d of February. It was very faint, and has recently been supposed to have been an appearance of the comet *of Biela.*

1818. Second comet. Discovered by Pons, on December 26th, 1817. It was a faint telescopic body.

1818. (Third comet.) Discovered by Pons, on November 28th, as a small, round, and well-defined body. Bessel, independently, discovered a comet on December 22nd, which proved to be the same body.

1819. The first comet of this year was remarkable in affording to Encke the discovery of its periodic nature, and the identity of it with the comets of 1786, 1795, and 1805. It was discovered by Pons, on November 26th, 1818, as a small, ill-defined nebulosity. The history of this body has been given in the preceding part of this section.

1819. Second comet. This was discovered by Professor Tralles, at Berlin, on July 1st. It appeared with a well-defined planetary nucleus, and a tail from 7° to 8° in length.

1819. Third comet. Discovered by Pons, on June 12th. Encke computed its orbit, which he found to be elliptical, with a period of 2052 days.

1819. (Fourth comet.) Discovered by Pons, on December 4; and previously at Marseilles, by Blanpain, on November 28th. Encke found its orbit elliptical, and its period 4·8 years. Clausen was of opinion that it was identical with the comet of 1743.

From 1821 to 1851 we can do little more than record the various comets which have appeared in our hemisphere, with their dates and the names of their discoverers:—
1821. Discovered by Pons on January 28. Santini saw it with the naked eye on February 19, and estimated its tail to be 2½° long. It was observed to May 3.—1822 (1). Gambart discovered this comet on May 12.—1822 (2). Encke's comet. Discovered at Paramatta by Rumker on June 2.—1822 (3). Discovered by Pons on May 31, but not observed much in Europe in consequence of its southerly declination.—1822 (4). Discovered by Pons on July 13. On August 21 it was visible to the naked eye, and had a tail 1½° long.—1823. Discovered by Pons on December 29. Santini found it on January 3. De Zach, on January 23, noticed that in addition to the usual tail, directed from the sun, it had another in a contrary direction, varying from 4° to 7° in length.—1824 (1). Discovered by Rumker on July 15.—1824 (2). Discovered by Scheithauer, at Chemitz, on July 23.—1825 (1). Discovered by Gambart on May 19, in Cassiopeia. This comet has some resemblance in its elements with the third comet of 1790.—1825 (2). This small comet was discovered by Pons on the 9th of August.—1825 (3). Encke's comet. Discovered by Valz on July 13.—1825 (4). Discovered by Pons on July 15. It was visible to the naked eye, and in October presented a remarkable appearance in the heavens.—1826 (1). Gambart or Biela's comet. Discovered by Biela on February 27.—1826 (2). Discovered by Pons on November 6, 1825.—1826 (3). Discovered by Flangergues on March 29.—1826 (4). Discovered by Pons on August 7, and observed to October.—1826 (5). Discovered by Pons on October 22. It was visible to the naked eye in December, with a beautiful train.—1827 (1). Discovered by Pons on December 26, 1826.—1827 (2). Discovered by Pons on June 20.—1827 (3). Discovered by Pons on August 2.—1829. Encke's comet. Discovered by Struve in 1828, October 13.—1830 (1). Discovered by Professor D'Abbadie in the Mauritius on March 16. It was also seen on March 17, during a voyage from Calcutta to Boston, as a bright object, with a tail 8° in extent.—1830 (2). Discovered by Herapath on January 7, 1831. It exhibited a tail 2½° long; the diameter of the nucleus was 3′ or 4′.

1832 (1). Encke's comet. Discovered by Mossotti, on June 1, at Buenos Ayres, and at the Royal Observatory, Cape of Good Hope, on the next day. It was only seen once in Europe, by Professor Harding, on August 21.—1832 (2). Discovered by Gambart, on July 19.—1832 (3). Gambart's or Biela's comet. Discovered at Rome, on August 25.—1833. Discovered at Paramatta, by Dumlop, in September.—1834. Discovered by Gambart, on March 8, in Sagittarius.—1835 (1). Discovered by Boguslawski, on April 20.—1835 (2). Encke's comet. Discovered by Boguslawski, on July 30.—1835 (3). Halley's comet. Discovered by Dumonchel, at Rome, on August 6. (See the history of this comet, at the commencement of the section.)—1838. Encke's comet. Discovered by Boguslawski, on August 14.—1840 (1). Discovered by Galle on December 3, 1839. 1840 (2). Discovered by Galle on January 25. It was very faint, without any appearance of tail.—1840 (3). Discovered by Galle, on March 6. It had a tail several degrees in length.—1840 (4). Discovered by Bremicker, on October 26.

1842. Encke's comet. Discovered on March 9, by Valz.—1842 (2). Discovered by Langier on October 28. It was very faint, and without tail.—1843. The great comet of this year is within the recollection of many readers of this work. It was seen in South America, on February 27, and observed afterwards in Europe to the middle of April. This was the finest body since the great comet of 1811.—1843 (2). Discovered by Mauvais, on May 2.—1843 (3). Discovered by Faye, on November 22. Its orbit is elliptical, and it was again observed at the Cambridge Observatory in 1850.—1844 (1). Discovered by De Vico, on August 22.—1844 (2). Discovered by Mauvais, on July 7. It had a small brilliant nucleus of 3′ in diameter.—1844 (3). Detected by Captain Wilmot, at the Cape of Good Hope, on December 19.—1845 (1). Discovered by D'Arrest, on December 28, 1844.—1845 (2). Discovered by De Vico, on February 25. —1845 (3). Discovered by Coller, on June 2. It was a beautiful object, visible to the naked eye, with a tail 2½° in length.—1845 (4). Encke's comet. Discovered by De Vico, at Rome, on July 10.—1846 (1). Discovered by De Vico, on January 24.—1846 (2). Gambart's or Biela's comet. Discovered by De Vico, on November 26, 1845. At this appearance, the singular phenomenon of a double comet was seen.—1846 (3). Discovered by Brorsen, on February 26. Its elements have some resemblance to the comets of 1532 and 1661.—1846 (4). Discovered by De Vico, on February 20. There is no doubt as to the elliptical orbit of this comet, the period being 73 years.—1846 (5). Disvered by Hind and De Vico, on July 29.—1846 (6). Discovered by Peters, on June 26. —1846 (7). Discovered by Brorsen, on April 30.—1846 (8). Discovered by De Vico, on December 23.—1847 (1). Discovered by Hind, on February 6.—1847 (2). Discovered by Coller, on May 7.—1847 (3). Discovered by Schweizer, on August 31.—1847 (4). Discovered by Mauvais, on July 4.—1847 (5). Discovered by Brorsen, on July 20.—1847 (6). Discovered by Miss Mitchell, on October 1.—1848 (1). Discovered by Petersen, on August 7.—1848 (2). Encke's comet. Discovered by Hind, on September 13. 1849 (1). Discovered by Petersen, on October 26, 1848.—1849 (2). Discovered by Goujon, on April 15.—1849 (3). Discovered by Schweizer, on April 11.—1849 (4). Discovered by Mr. Jenkins, on November 28, at sea.—1850 (1). Discovered by Petersen, on May 1.—1850 (2). Discovered by Bond, on August 29.

1851 (1) Faye's comet. Discovered at the Cambridge Observatory, on November 28, 1850.—1851 (2). Discovered by D'Arrest, on June 28.—1851 (3). Discovered by Brorsen, on August 1.—1851 (4). Discovered by Brorsen, on October 22.

◆

ECLIPSES AND OCCULTATIONS.

The striking phenomena attending eclipses of the sun and moon, in which, apparently, the course of nature is interrupted, and for a time the discs of the sun and full moon lose their wonted lustre, and their light is almost extinguished in the mid-heavens, have ever been a source of curiosity, and occasionally of terror, to mankind. Whilst they have furnished many examples of credulity which have been taken advantage of by superior minds, they have sometimes served a nobler purpose; and the biography of astronomers is filled with instances in which the accidental occurrence of a solar or lunar eclipse has been the cause of their first directing their attention to the science subsequently enriched by their investigations. *Eclipses* may be defined as a short interruption in the passage of light to the earth, which is either *real*, as in the case of the interposition of an opaque body between the earth and sun; or *apparent*, as takes place when the earth itself passes between the sun and the object previously illumined. The first are termed solar eclipses, and take place at the time of the new moon; the latter lunar eclipses, and occur at the time of full moon, or when the earth is interposed between the sun and her satellite; the earth being a body of such dimensions that its shadow is sufficient to reach to the moon.

Lunar Eclipses.—Taking the latter case, or that of a lunar eclipse, we readily see by Fig. 111 the form of the shadow cast by the earth. The sun, S, scatters its rays in all directions, and such as are directed towards the earth, T, will be interrupted in their onward passage by the interposition of this body; and we perceive that if the rays of light, A B, A′ B′, preserve their straight path they cannot penetrate within the portion B O B′, or between the summit of the cone A O A′ and the earth, and that this is the form and dimensions of the shadow cast by the globe T. In order that the moon may be eclipsed, it must be placed between the earth T and the summit of the cone O; and the distance of those two points is easily determined in the following manner:—Draw the line T C parallel to O A, and in the two similar triangles, O T B and T S C, we have the proportion S C : T S :: T B : O T. The radius of the earth, T B, being taken for unity, the line S C will be equal to the radius of the solar sphere diminished by the former quantity or to 111 radii of the earth. As the mean distance, T S, of the earth and sun are equal to 24,000 radii of the earth, we conclude that the distance O T is equal to about 216 terrestrial radii; or, more exactly, that the length of the axis of the shadow is 216,531 radii of the earth. At the time when the sun is in perigee this will decrease to 212,896 radii of the earth, and at the time of apogee to 220,238 radii. As the distance of the moon from the earth never exceeds 64 terrestrial radii, and the least length of the shadow is 212 terrestrial radii, it is obvious that the moon must be obscured when the earth is situated between it and the sun. The breadth of the shadow of the earth at the distance of 108 terrestrial radii will be equal to half the diameter of the earth, and considerably greater at the distance of only 60 radii; but the exact dimensions of the latter point may be obtained as follows:—

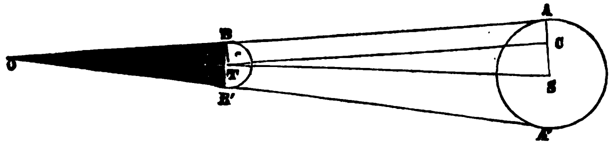

Fig. 111.

Dimensions of the Shadow.—Let M N (Fig. 112) be the surface of the celestial sphere, which is here supposed to pass through the centre of the moon; this surface will cut the cone of the earth's shadow at M M′. and the angle M T M′ is the apparent angle of the shadow which it is required to determine. The half of this angle, or M T O, is equal to the angle B M T, diminished by the angle M O T; but the former is the parallax of the moon (since M T is the distance of the moon from the earth), and the latter is equal to the angle A T S (the semi-diameter of the sun) diminished by the angle B A T (the parallax of the sun). In order, therefore, to obtain the apparent semi-diameter of the shadow at the distance of the centre of the moon, we must add the parallax of the sun to that of the moon, and subtract the apparent semi-diameter of the sun.

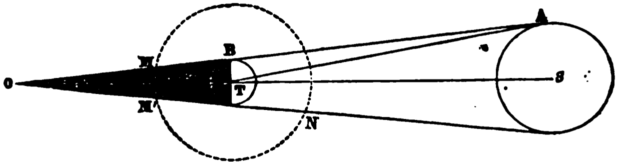

Fig. 112.

As the moon moves over a space about equal to its diameter in an hour, it follows that it may be entirely within the shadow for two hours. The greatest value of the dimensions of the earth's shadow at the moon may be readily obtained by taking the greatest parallax of the moon and the least of the sun; and as the former is 61′ 29″ at its maximum, and the sun's least semi-diameter and corresponding parallax, respectively, are 15′ 46″ and 8″·6, it would follow that the greatest semi-diameter of the earth's shadow at the distance of the moon was 45′ 52″.

The apparent semi-diameters of the shadow for other distances of the sun and moon may be readily calculated, and are here annexed:—

		′	″
Sun in perigee	Moon in apogee	37	42
	„ at mean distance	41	31
	„ in perigee	45	20
Sun at mean distance	Moon in apogee	37	58
	„ at mean distance	41	48
	„ in perigee	45	37
Sun in apogee	Moon in apogee	38	14
	„ at mean distance	42	3
	„ in perigee	45	52

Respective Positions of Moon and Shadow.—Let A C (Fig. 113) be the great circle of the ecliptic, and B D the orbit of the moon, N will be the nodes of this orbit. The shadow O moves along the first circle with a velocity equal to that of the sun, and the moon passes along the second circle with a velocity about thirteen times greater. In order that the moon may meet the shadow, it must happen that the centre of the shadow is sufficiently near the nodes N at the moment of opposition. By taking into

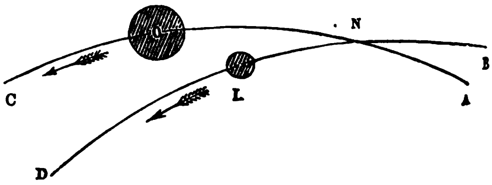

Fig. 113.

consideration the above facts—that the apparent diameters of the moon and the earth's shadow vary from one epoch to another—and remarking that the distance of the centre of the shadow from the node N is precisely equal to the distance of the centre of the sun from the opposite node of the moon, we find that if the distance of the centre of the sun from the node at the time of full moon is greater than 12° 3′, there *cannot* be an eclipse; and if the distance is less than 9° 31′, there *must* certainly be an eclipse. Between these two extremes, the case is doubtful; but a more exact calculation will show whether an eclipse really does occur.

Partial Eclipses of the Moon.—When the moon is wholly obscured, as we see it can be, the eclipse is termed total; but when only a portion of the lunar disc enters into the shadow, the eclipse is partial. In the latter case, the outline of the earth's shadow projected upon the disc of the full moon clearly shows its round and globular figure, although the diameter of the conical shadow is large in proportion to that of the moon. The accompanying diagram (Fig. 114) gives an idea of the relative proportions of the shadow of the earth and the disc of the moon at those times, and of the curvature of the circumference of the shadow *a b c*. But the definition of the earth's shadow is far from being so sharp as is here represented, and, like that of the shadow of any other opaque body, is edged with a cloudy and imperfectly-defined penumbra, the dimensions of which may be estimated by the following considerations:—Let the lines A O′ B′ C′ and A′ O′ B C be drawn (Fig. 115), another cone, A O′ A′, having its summit O′ between the sun and the earth, is formed, and enveloping the sun and the earth in its opposite parts A O′ A′, B O′ B′. It is clearly seen that all the part situated within the space C B B′ C′, and without the real shadow B O B′, would only receive a portion of the rays of the sun coming from the part of the hemisphere turned towards it, the other part being hid by the earth; and that the portion of the sun which is visible is greater according as this is nearer to the exterior surface of the space C B B′ C′, and, on the contrary, smaller as it is nearer to the real shadow of the earth, or B O B′. The consequence would be, that as the moon passed into the part C B B′ C′ before meeting with the real shadow, it would insensibly lose a portion of its light, and its lustre would become gradually less intense as it approached the real shadow. As, however, the breadth of the penumbra is equal to the angle O B C = A B A′, or the diameter of the sun as seen from the earth; and as the apparent diameters of the moon and sun are nearly equal, it follows that the whole disc of the moon may be within the penumbra C B O. When *any part* of the disc of the moon is obscured by the real shadow, it follows that the *whole* of its surface is more or less hidden by the penumbra. The insensible melting away of the real shadow into the penumbra is noticed at every lunar eclipse, but it is impossible to tell precisely where the penumbra ends and the shadow begins.

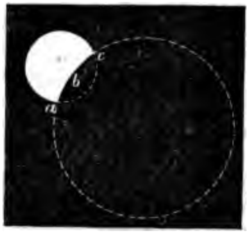

Fig. 114.

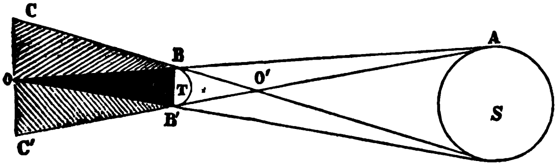

Fig. 115.

Effect of the Atmosphere of the Earth.—Hitherto we have supposed the rays proceeding from the sun to pass in straight lines until they meet the lunar disc, but this is not the case; for those which pass through the earth's atmosphere are subject to refraction, and change their direction in the manner already explained in the case of a star. This will be more apparent when we consider the direction of the ray S A (Fig. 116), which traverses the atmosphere of the earth, and passes beyond it. The direction which it takes previous to entering the atmosphere of the earth makes an angle of 33' with its direction when it arrives at that point; and after passing through it, it is still further deflected from its original course, and by the same amount as before. The consequence of this will be, that it is altogether deflected more than a degree, and that the solar rays will meet before arriving at the point O; and, instead of the rays A B and A' B' taking a straight direction, they will be deflected to the point D (Fig. 117), which is considerably nearer the earth than the point O. Thus it will only be the inner cone B D B, which is the real cone of the shadow; the remaining parts of the *mathematical* cone B O B' will be traversed by the solar rays, which are bent from their primitive direction by the action of the terrestrial atmosphere. The distance of the point D from the centre of the earth can be calculated as in the former case, and it has been found that its average distance is forty-two terrestrial radii; and we conclude from this, that as the moon's mean distance is equal to sixty-four radii of the earth, it can never be inclosed within the real shadow of the earth B D B'; but at the time of total eclipse it falls within that part of the shadow where the rays refracted by the atmosphere penetrate. This is the cause that when the moon is wholly eclipsed it still shines with a reddish light, being illumined by the faint rays refracted by the earth's atmosphere. This, however, is not the sole effect of the atmosphere, for the apparent diameter of the earth's shadow has been found by observation to be *greater* than might be expected from the preceding investigation; and it has been accounted for, by supposing that the solar rays do not really touch the surface of the earth, but that the lower strata of the air absorbs those which approach its margin; and if such be the case, the diameter of the globe of the earth, and its shadow would accordingly be greater.

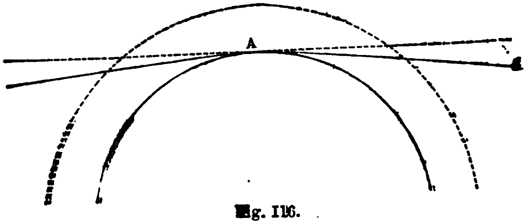

Fig. 116.

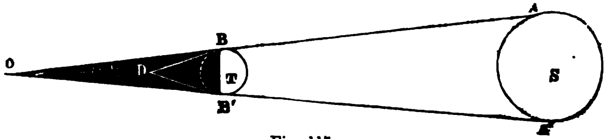

Fig. 117.

The *condition* of the atmosphere through which the rays of the sun pass, produces considerable changes in the appearance of the lunar disc at the times of total eclipse. In ordinary cases, as before mentioned, the colour of the moon is of a red or coppery tint, similar to that frequently assumed by the setting sun, and produced by the same causes, viz., an absorption of the blue rays of light when passing through great depths of the atmosphere. If that portion of the atmosphere is in addition charged with cloud *and dense vapours*, the whole of the elementary rays will be stopped in their passage to the lunar surface, and several instances are on record in which the disc of the moon was completely invisible at the time of total eclipse. In the eclipses of 1601 and of June, 1620, this was the case, although the air was sufficiently clear to allow the light of stars of the fifth magnitude to be distinguishable; and, in a similar manner, the moon could not be perceived (even with the aid of a telescope) during the eclipse of April, 1642, although the sky was equally as clear as in the former case. When, however, the sky is clear at those portions of the earth's surface through which the solar rays pass, the red rays are transmitted in great number, and the lustre of the lunar surface is but slightly dimmed in consequence. Such was the case with the eclipse of the moon which happened in March, 1848, when the lunar disc was almost as apparent as on ordinary dull nights, and when many persons who were observing it could scarcely be persuaded that it was really eclipsed. The dark spots and bright places were as well seen on its surface as if viewed through thin cloud or vapour; and one observer, who had witnessed lunar eclipses for more than sixty years, never remembered one in which the illumination was so strong—appearing like the glowing heat of fire from the furnace, and tinged with a deep red. Sometimes the moon becomes invisible during the progress of its immersion in the shadow, as was the case with that observed by Wargentin, in 1761, when the moon was very bright for ten minutes after its total immersion, but for an hour afterwards became so completely invisible that not the slightest trace of it could be detected, either with the naked eye or telescope, although the sky was clear and stars in the immediate vicinity of the moon appeared bright and distinct. In other cases, the region of the atmosphere, through which the rays have passed, have been clear at some parts and obscured at others; and the consequence has been that whilst the lunar disc appeared clear and bright in some portions, it was very dark at others. Kepler states that, during the total eclipse of August, 1598, one half of the moon appeared so bright that it seemed doubtful if it was eclipsed at all, whilst the other was seen with the utmost difficulty. A similar variegated appearance was presented by the lunar disc in October, 1837. Sir W. Herschel perceived many bright spots on the occasion of a total eclipse, and was induced to believe that they were volcanoes in action. Such an appearance, however, it will plainly be seen, may be supposed due to the various reflective qualities of the surface of the moon, and that the bright spots noticed by Herschel were only the bright mountainous districts illuminated by the red light of the sun.

Prediction, Duration, and Magnitude of a Lunar Eclipse.—If the moon did not depart from the plane of the ecliptic, there would be a lunar eclipse at each full moon and a solar one at new moon; but as it is sometimes above and sometimes below this plane, an eclipse can only take place when it approaches the nodes of its orbit. It was known to the ancients that at certain intervals the new and full moon returned again on the same day of the month, and they had observed that at the end of eighteen years eleven days, or a period of 223 lunations, there was a return of the same eclipses, and were thus enabled to foretell them with considerable, but not with perfect accuracy; for the exact recurrence, if it took place, would depend upon the return of the sun's place, the moon's place, the position of the moon's apogee, and that of the ascending node of the moon to exactly the same situation. This exact return cannot happen, but is sufficiently approximate to foretell eclipses with a remarkable degree of accuracy, and Dr. Halley found that if the period of 18 years 10 days 7 hours 43¾ minutes were added to the middle of the time of any eclipse, the return of the corresponding one might be predicted within 1h. 30m. If only four leap years occur in the interval, the period would be enlarged to 18y. 11d. 7h. 43¾m. It may, however, happen that when the eclipse is very slight, or an *appulse* occurs, that at the succeeding period of 18y. 11d. it will not be observed. Since the position of the heavenly bodies at any epoch, as well as the laws which govern their motion, are at present equally well known, the eclipses of the sun and moon are now rigorously calculated, although, to save time and trouble, the above period of 18y. 11d. (the period of 521y. 3h. 3m. would be still more exact) is still made use of. The "Nautical Almanac" contains the exact positions of the sun and moon in relation to the earth at any given time, and it is from these data that we shall determine the beginning and end of an eclipse at any given time.

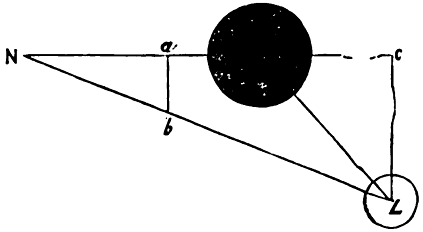

Fig. 118.

In order to *calculate* an eclipse of the moon, the following data are necessary, and the manner in which they are used will be seen by the diagram (Fig. 118) and explanation. Suppose the moon's place at opposition to be at *b*, the centre of the earth's shadow being at *a* at the same time, the latter passing through the space T *a*, the path of the ecliptic, whilst the moon's centre describes the line L *b*, the path of the moon's motion (which may be regarded as straight lines, as well as those of L *c*, *a b*):

Let m = moon's horary motion in longitude.
n = moon's horary motion in latitude.
s = sun's horary motion in longitude.
λ = moon's latitude when in opposition at b.
t = time taken by moon to pass from L to b.
d = distance from T to L.

The moon's motion in longitude will be $a\,c$ in the interval it takes to pass from L to $b = m\,t$. Its motion in latitude, during the same time, or $\mathrm{L}\,c - a\,b = n\,t$. The sun's motion in longitude, or the distance $\mathrm{T}\,a = s\,t$ in the same manner. Then

$$\mathrm{L}\,c = a\,b + n\,t = \lambda + n\,t \text{ and } \mathrm{T}\,c = a\,c - \mathrm{T}\,a = m\,t - s\,t$$
$$d^2 = \mathrm{L\,T}^2 = \mathrm{L}\,c^2 + \mathrm{T}\,c^2 = (\lambda + n\,t)^2 + (m\,t - s\,t)^2.$$

Expanding this expression, we obtain a quadratic equation of which t is the unknown quantity (the others being derived from the above data), and will depend on the value given to d. Such values may be given to d as correspond to the phases of the eclipse, and the interval between the time of opposition and the occurrence of the phases will thus readily be obtained, the time of *opposition* being known from the position of the sun and moon as given by the tables. Arranging them we obtain

$$d^2 - \lambda^2 = t^2\,[(m - s)^2 + n^2] + 2\,t\,\lambda\,n.$$

Substituting $\tan^2\theta = \dfrac{\sin^2\theta}{1 - \sin^2\theta}$ for $\dfrac{n^2}{(m - s)^2}$, we obtain, instead of the above,

$$n^2\,t^2 + 2\,\lambda\,n\sin^2\theta\,t = (d^2 - \lambda^2)\sin^2\theta; \text{ or, completing the quadratic,}$$
$$n^2\,t^2 + 2\,\lambda\,n\sin^2\theta\,t + \lambda^2\sin^4\theta = (d^2 - \lambda^2)\sin^2\theta + \lambda^2\sin^4\theta$$
$$= \sin^2\theta\,(d^2 - \lambda^2)\,(1 - \sin^2\theta)$$
$$= \sin^2\theta\,(d^2 - \lambda^2\cos^2\theta).$$
$$\therefore t = \frac{1}{n}\left(-\lambda\sin^2\theta \pm \sin\theta\sqrt{d^2 - \lambda^2\cos^2\theta}\right).$$

And from this expression the values of the times for any values of d may be obtained. For the determination of the time, t, at which the moon immerges into the earth's penumbra, $d =$ moon's horizontal parallax + sun's horizontal parallax + the sun's semi-diameter + moon's semi-diameter $= P + p + \frac{D}{2} + \frac{\delta}{2}$; the first value of t giving the instant of immersion, the second that of emersion. At the time of immersion in the umbra $d = P + p + \frac{\delta}{2} - \frac{D}{2}$. At the period when the whole disc has entered the shadow $d =$ sum of the parallaxes — sum of the semi-diameters, or $P + p - \frac{\delta}{2} - \frac{D}{2}$. The time at which the middle of the eclipse occurs (or the *greatest phase*) is when the two values of t are equal, or when $d^2 - \lambda^2 \cos^2 \theta = 0$, and $t = \frac{\lambda \sin^2 \theta}{n}$. In this case the distance $d = \lambda \cos \theta$. When the latter term is known, the *magnitude* of the eclipse may be determined. Some part must be eclipsed when $\lambda \cos \theta$ is less than the distance $P + p + \frac{\delta}{2} - \frac{D}{2}$, as the latter quantity shows when the moon's limb just touches the shadow, and the portion of diameter eclipsed is $\left(P + p + \frac{\delta}{2} - \frac{D}{2}\right) - \lambda \cos \theta$. The portion of the diameter of the moon which is not eclipsed will, therefore, be $\lambda \cos \theta + \frac{\delta}{2} + \frac{D}{2} - P - p$. The eclipse will be *exactly* total when this is nothing, and will be *more* than total when it is negative. The part eclipsed may be expressed either in digits or twelfths of the lunar diameter (δ), or in decimal parts, the moon being taken for unity.

Example.—Partial eclipse of the moon, April 20, 1856.

By the tables of the sun and moon, the time of opposition is at 9h. 13m.·6 morning, Greenwich mean time.

′ ″
Moon's latitude at time of opposition $\lambda = 33\ 18$
„ horary motion in latitude . $n = 2\ 46$ lat. increasing.
„ horary motion in longitude $m = 30\ 8$
Sun's horary motion in longitude . $s = 2\ 27$
Moon's apparent diameter . : . $\delta = 29\ 44$
„ horizontal parallax . . . $P = 54\ 33$
Sun's apparent diameter . . . . $D = 31\ 52$
„ horizontal parallax . . . $p = 9$

$$\text{Tan } \theta = \frac{n}{m - s} = + \frac{166''}{1661} \therefore \theta = + 5° 43' 10''.$$

The middle of the eclipse, or

$$-\frac{\lambda \sin^2 \theta}{n} = \frac{1998}{166} \times \sin^2 (5° 43' 10'') = - 7\text{m.}\cdot 16.$$

Therefore the middle of the eclipse will occur at 9h. 6m.·4 morning, reckoning from opposition. The times at which it entered the shadow and emerged from it are calculated by finding the value of d at those times:—

$d = \frac{\delta}{2} - \frac{D}{2} + p + P = 53' 38''$, and adding the one-sixtieth part of $\left(P + p - \frac{D}{2}\right)$

for the effect of the earth's atmosphere, $d = 54' \, 17''$. Introducing this value into the expression $-\frac{\lambda \sin^2 \theta}{n} \pm \frac{\sin \theta \sqrt{(d^2 - \lambda^2 \cos^2 \theta)}}{n}$ we obtain for the two values of t:

$$\begin{array}{llll} & \text{m.} & \text{h. m.} & \text{h. m.} \\ \text{End of eclipse} & = 7{\cdot}16 & + 1\ 32{\cdot}68 & = 1\ 39{\cdot}84 \\ \text{Beginning} & = 7{\cdot}16 & - 1\ 32{\cdot}68 & = 1\ 25{\cdot}52 \end{array}$$

The commencement of the eclipse accordingly took place at 7h. 33m.·8 morning, and the end at 10h. 39m. morning. The distance of the centres corresponding to the middle of the eclipse, or $\lambda \cos \theta = 33' \, 8''$; and the eclipsed part, or $\frac{\delta}{2} - \frac{D}{2} + p + P - \lambda \cos \theta = 21' \, 9''$, (allowing for the effect of atmosphere), or the magnitude of the eclipse, reckoning the moon's diameter at 1 = 0·71.

In order to determine the time at which the moon enters the *penumbra*, we assume, $d = P + p + \frac{D}{2} + \frac{\delta}{2} = 86' \, 9''{\cdot}8$, whence the two values of t are 2h. 44m.·32 and 2h. 58m.·64, which applied to the time of opposition, 9h. 13m.·6, gives the time of commencement at 6h. 15m.·0 a.m., and the time of ending at 11h. 57m.·9 a.m.

Graphical Construction of a Lunar Eclipse.—Instead of calculating the various phases of a lunar eclipse, they may be obtained by the following graphical process, applied to the eclipse of November 13 and 14, 1845.

At Paris mean noon of November 13, the longitude of the sun exceeded that of the moon by 186° 20′ 7″·4. On the 14th, at the same hour, it exceeded it by 174° 45′ 8″·6. Finding, by interpolation, the instant at which the difference of longitudes was exactly 180°, or the moment at which the moon was in opposition, we obtain, November 14, at 1h. 4m. 20s.·9 morning. At this time the parallax of the moon was 55′ 39″·6; that of the sun, 8″·7; the semi-diameter of the moon, 15′ 10″·1; and that of the sun 16′ 12″·3; whence, as before, we conclude that the semi-diameter of the shadow of the earth was 39′ 36″, or increasing it by $\frac{1}{60}$ = 2415″·6. In addition to this, we find that at 0h. 30m. on the morning of the 14th, the longitude of the sun exceeded that of the moon by 180° 16′ 33″·7; and that the latitude of the moon was 0° 25′ 57″·6 S. At 1h. 30m. of the same morning, the excess of the longitude of the sun was 179° 47′ 37″·7, and the latitude of the moon 0° 28′ 51″·5 S. Let the circle A B C D (Fig. 119) represent the dimensions of the shadow of the earth at the distance of the moon (semid. = O A = 2415″·6), let E E′ be the ecliptic.

At 0h. 30m. of the morning of the 14th, the longitude of the sun surpassed that of the moon by 180° 16′ 33″·7, the longitude of the centre, O, of the shadow, therefore, only exceeded that of the moon by 993″·7. According to the scale adopted, set down O F = 993″·7. Make F G perpendicular to E E′, and mark off F G = 25′ 57″·6, which is the latitude of the moon at the time, and G is the position of the centre of the moon at 0h. 30m. morning. In the same manner, take O H = 12′ 22″·3, or 742″·3, and H K = 28′ 51″·5. or 1731″·5; and K will be the position of the centre of the moon at 1h. 30m. morning. Draw a right line, M M′, through the points G and K, this line will represent the path of the moon during the eclipse in reference to the shadow of the earth. The point N is the position of the moon at the moment of opposition, or at 1h. 4m. 20s.·9 morn.

With a radius equal to the sum of the radii of the shadow and of the moon, or, 3325″·7 describe a circle with the centre O. The circumference will cut the line M M′ at two points, L L′; and it is plain that, if with those two points as centres, we describe two lesser circles with the radius of the moon, or 910″·1, these two circles will touch the circumference of the shadow A B C D, and represent the position of the moon at the beginning and end of the eclipse. If, moreover, the perpendicular, P O, is raised on M M′, the point P represents the position of the centre of the moon at the middle of the eclipse.

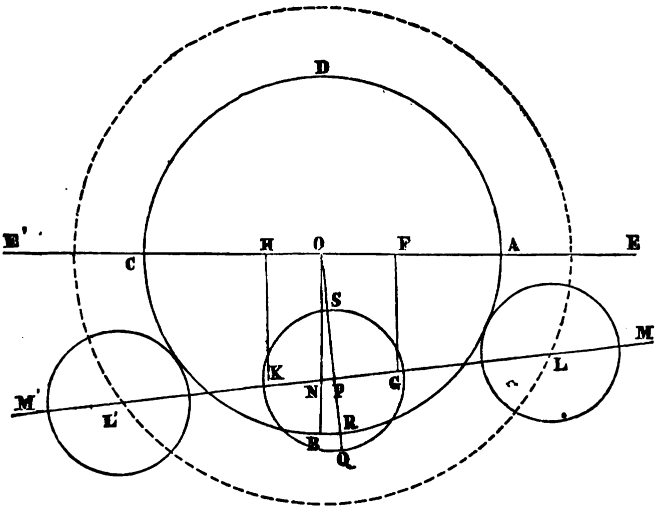

Fig. 119.

As the moon employs an hour in passing from G to K, we may, by proportion, determine the time it takes in passing from N to P (if we make the figure large enough); and we thus find that it takes 5m. 41s. in going through this distance, or 5m. 41s. before the opposition, or it is at 0h. 58m. 40s. that the middle of the eclipse happens. In the same manner, we find that the moon should be 1h. 39m. 19s. in passing from P to L′, and it is therefore at 11h. 29m. 21s. of the evening of November 13 that the eclipse commences, and at 2h. 38m. of the morning of the 14th that it ends. The *magnitude* of the eclipse is found, likewise, by this graphical construction, and in this case we perceive that it is partial, since at the moment when the centre of the moon is nearest the centre of the shadow, a portion of its disc lies *without* the latter circle. If we draw the diameter Q S, directed towards the point O, and we take the proportion of this diameter S R, which is in the shadow, and the diameter itself, we find the magnitude. In the present case this amounts to 0·92, the diameter being 1·00. Should the diameter Q S be within the circle of the shadow, or the eclipse be total, the various phases, it will be seen, can be measured off with equal facility, as in the instance here given.

Instead of making use of latitudes and longitudes, the right ascensions and declinations of the sun and moon may be employed, as in the following example of the total eclipse of October 24, 1855 (Loomis Ast.). The Greenwich time of opposition in right ascension is 19h. 17m. 55s.·6, or the Washington time = 14h. 10m. 29s.·6, for which time the "Nautical Almanac" furnishes the following data:—

		° ′ ″	
Declination of moon	=	11 42 24·3	north.
Declination of sun	=	11 56 48·0	south
Moon's horizontal parallax (lat. 45°)	=	59 39·0	
Sun's horizontal parallax . . .	=	8·6	
Moon's semi-diameter	=	16 18·8	
Sun's " "	=	16 7·9	
Moon's hourly motion in R. A. .	=	33 22·1	
Sun's " " " . .	=	2 23·5	
Moon's hourly motion in declination	=	15 40·1	north.
Sun's " " " "	=	0 52·0	south.

The semi-diameter of the earth's shadow, or C B = 59′ 39″·0 + 8″·6 — 16′ 7″·9 = 43′ 39″·7, to which adding one-sixtieth for refraction, C B = 44′ 23″·4, and the distance of the moon's centre, or C E = 60′ 42″·2. The hourly motion of the moon from the sun in right ascension is 30′ 58″·6, and this reduced to the arc of a great circle by multiplying it by the cosine of the moon's declination = 1819″·9.

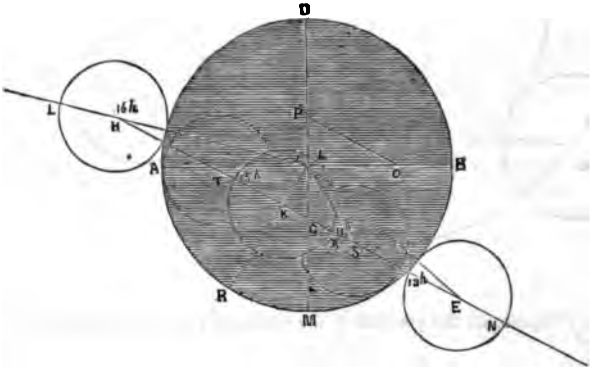

Fig. 120.

From the centre C with the radius C B describe the circle A R B, or the shadow of the earth. Draw the line A C B to represent a parallel to the equator, and make C G perpendicular to it equal to 14′ 23″·7, or the sun's declination minus the moon's declination, the point G being taken below C as the centre of the moon is south of the centre of the earth's shadow. Make C O = 1819″·9 and C P to the hourly motion of the moon in declination minus hourly motion of the sun in declination; or 14′ 48″·1, the point P being placed above C as the moon was moving northward. Join O P and draw parallel to it through G the line N G H, which represents the path of the moon, and on N H let fall the perpendicular C K. At 14h. 10m. 29s.·6 the moon was at G. To find its position at 14h. we make the proportion:—As 60m. : 10m. 29s.·6 :: O P : G X. The position of the moon's centre at 14h. is set at X, and its position at 16h., 15h., &c., may be laid dawn in the same manner by adding the distance O P. Subdividing those into sixty parts, the beginning, end, and middle of the eclipse will be ascertained, answering to the times when the moon's centre is at E, H, and K.

The calculation of the eclipse may be performed accurately as follows:—In the right-angled triangle O C P we have C O = 1819″·9 and C P = 888″·1, then $CP : R :: CO : \tan CPO$.

C O = 1819″·9 = 3·2600475
888″·1 = 2·9484619

C P O = 63° 59′ 16″ = 0·3115856 log. tan.
Sin C P O : R : : C O : O P.
C O = 3·2600475
Sin C P O = 9·9536150

O P = 2025″·0 = 3·3064325

The angle C P O = angle C G K, G K and O P being parallel. The angle C G E = 116° 0′ 44″; the side C G = 863·7, and the line C E = 3642″·2. The angles C E G and the side E G of the triangle C E G are found thus:—

C E : sin C G E : : C G : sin C E G.
Comp. C E = 6·4386362
Sin C G E = 9·9536150
C G = 2·9363629

C E G = 12° 18′ 18″ Sin 9·3286141

Therefore the angle E C G = 51° 40′ 58″.

Sin C G E : C E : : sin E C G : E G.
Comp. sin C G E = 0·0463850
C E = 3·5613638
Sin E C G = 9·8946428

E G = 3179″·7 = 3·5023916

To Determine the Time of Describing E G.

As O P : 1h. : : E G (= 3179s.·7) : 1h. 34m. 12s.·8, which, subtracted from the time of apposition in right ascension, or 19h. 17m. 55s.·6, (Greenwich time) gives the time of the beginning of the eclipse, or first contact with shadow, at 17h. 43m. 42s.·8. The middle of the eclipse is found by the triangle C G K similar to C P O, in which the angles and hypothenuse are given to find C K and K G.

R : C G : : sin C G K : C K : : cos C G K : G K.

Sin C G K = 9·9536150
C G = 2·9363629

C K = 776″·2 = 2·8899779

Cos C G K = 9·6420319
C G = 2·9363629

G K = 378″·8 = 2·5783948

To Determine the Time of Describing G K.

2025″ : 1h. : : 378″·8 : 673s.·4 = 11m. 13s.·4, which, added to the Greenwich mean time of opposition, gives 19h. 29m. 9s. for the middle of the eclipse. The duration of the eclipse will consequently be 3h. 30m. 52s.·4, and the end 21h. 14m. 35s.·2 Greenwich mean time. The magnitude of the eclipse is found by subtracting C K = 12′ 56″·2 from C R = 44′ 23″·4, whence K R = 31′ 27″·2; to which adding the moon's semi-diameter we obtain R I = 47′ 46″·0. Dividing this by the moon's diameter, or 32′ 37″·6 the magnitude of the eclipse = 1·464 on the northern limb.

The beginning and end of total darkness is found as follows:—With the radius of the earth's shadow, or C B diminished by the moon's semi-diameter, or 44′ 23″·4 — 16′ 18″·8 = 28′ 4″·6, or 1684″·6, describe a circle about the centre C cutting L N in the points S and T, which will represent the points of beginning and end of total darkness.

In the triangle C G S, C G = 863·7, C S = 1684·6, and the angle C G S = 116° 0′ 44″. Hence C S : sin C G S : : C G : sin C S G.

Comp. C S = 6·7735032
Sin C G S = 9·9536150
C G = 2·9363629

C S G = 27° 26′ 12″. Sin = 9·6634811

And the angle S C G = 36° 33′ 4″.

Sin C G S : C S : : sin S C G : S G.
Comp. sin C G S = 0·0463850
C S = 3·2264968
Sin S C G = 9·7749107

G S = 1116·3 = 3·0477925

To Determine the Time of Describing G S.

2025·0 : 1h. : : 1116·3 : 1984·6 = 33m. 4s.·6, which being subtracted from 19h. 17m. 55s.·6, gives 18h. 44m. 51s. for the time of disappearance or beginning of total darkness. The duration of total darkness is therefore 1h. 28m. 36s., and the time of the end of total darkness or re-appearance = 20h. 13m. 27m.,the *middle* of the eclipse occurring at 19h. 29m. 9s.

The semi-diameter of the penumbra is equal to the semi-diameter of the shadow + the sun's diameter, or 44′ 23″·4 + 32 15·8 = 76′ 39″·2. Let the circle A R B represent, in this case, the limits of the penumbra, then C E = 76′ 39″·2 + 16′ 18″·8 = 92′ 58″·0. In the triangle C G E, the angle C G E = 116° 0′ 44″. C G = 863·7 and C E = 5578″·0.

C E : Sin C G E : : C G : sin C E G.
Comp. log C E = 6·2535215
Sin C G E = 9·9536150
C G = 2·9363629

C E G = 7° 59′ 56″ Sin = 9·1434994

And angle E G G = 55° 59′ 20″.

Sin C G E : C E : : sin E C G : E G
Comp. sin C G E = 0·0463850
C E = 3·7464785
Sin E C G = 9·9185174

E G = 5144″·9 = 3·7113809

As 2025 : 1h. : : 5144·9 : 2h. 32m. 26s.·5, which gives the time of describing E G. This subtracted from 19h. 17m. 55s.·6, gives 16h. 45m. 29s.·1 for the time of first contact with the penumbra, and the whole duration being 5h. 27m. 19s.·8, we get the time of ending 22h. 12m. 48s.·9 for the last contact with the penumbra.

An eclipse of the moon can only be seen at those parts of the earth where the moon is above the horizon, as well as the shadow of the earth, or at least a portion of this shadow. As this can only take place when the sun is below the horizon, it is only during the night that eclipses of the moon are visible. It may happen, however, that the eclipse may be seen for a few moments before the setting of the sun, or after its rising. This is due to the refraction of the atmosphere, which, when the sun is *altogether* below the horizon, as well as that part of the moon which is eclipsed, eletes both objects above the horizon, and an observer at A (Fig. 121) will be able to ı the sun at one side, and the eclipsed moon on the other.

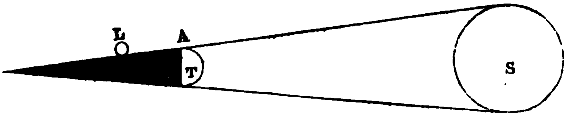

Fig. 121.

Eclipses of the Sun.—Whilst eclipses of the moon follow from the interposition the earth, and prevent the light of the sun from reaching to and illuminating the :face of our satellite in the usual manner, eclipses of the sun are due to the con- ıry cause, being the shadow of the moon falling on certain parts of the earth (passing er it like the shadow of a cloud), and hiding the disc of the sun at those districts. The nensions of the shadow at the surface of the earth may be found in the same manner in the former case, and an idea of the different kinds of solar eclipses may be gathered ım the following diagrams. Let L be the position of the moon (Fig. 122) situated ıctly in a line with the sun S and the earth T, and at a distance L T, which corresponds the distance of the sun and moon at the time. The smallest distance of the centres of ı earth and moon being equal to 55,947 semi-diameters of the former body, and the ıatest value of the shadow O L (calculated in the manner already mentioned) being ·73 semi-diameters of the earth, it would follow that, under those circumstances, the ıdow of the moon would extend beyond the centre of the earth, and consequently ıt at that region of the earth where the shadow would fall (as at *a b* Fig. 123) the ipse would be total—the moon completely obscuring the sun's disc. On the con- ıry, if we take the greatest distance of the earth and moon, which is equal to 63,802

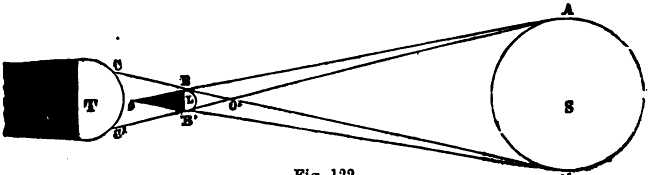

Fig. 122.

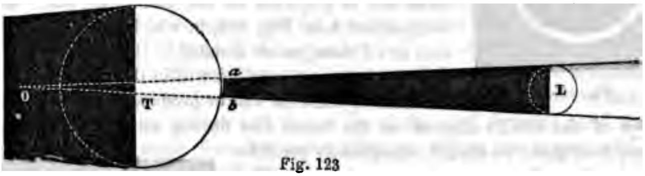

Fig. 123

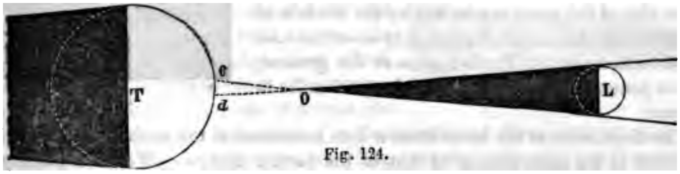

Fig. 124.

radius of the former body, and the smallest value of the length of the shadow, which is found to be 57·76 radii, the summit of the cone of shadow does not reach to the earth (Fig. 124), and, when this is the case, the eclipse cannot be total for any part of the earth. On all that hemisphere turned towards the sun, we perceive a portion, though not the whole of its disc. There is, however, a peculiarity to be noted, which is, that if we prolong the cone of the shadow of the moon beyond the summit O, the base of the second cone formed will be situated on the portion of the surface of the earth *c d*, and for all this part the eclipse will appear *annular*—*i. e.*, the moon will appear projected upon the disc of the sun at all those parts of the earth, and the exterior portion of the disc of the sun will consequently appear under the form of a luminous ring. When the moon is placed between the sun and the earth, the eclipse will be either total or annular for certain parts of the earth, according as the distances of the sun and moon are more or less great. This is also apparent from considering the respective dimensions of the discs of the sun and moon at different periods. The apparent diameter of the moon, as seen from the surface of the earth, may increase to 34' 6", whilst the diameter of the sun may decrease to 31' 31", and if an eclipse takes place under those circumstances, it must be total. As the diameter of the moon on other occasions may decrease to 29' 22", whilst that of the sun can attain its maximum value of 32' 35"·6, it will follow that when an eclipse happens under such circumstances, that it must be annular, and that for a few moments, at such parts of the earth as are exactly in a line with the centres of the sun and moon, the moon will be seen projected upon the sun, the exterior parts of the sun will continue visible and shine with their usual brilliancy (Fig. 125). At the parts of the earth adjoining those which are exactly in a line with the centres of the sun and moon, the eclipse cannot be total or annular, as only a portion of the moon can be projected on the disc of the sun. This will be apparent from Fig. 122, in which it is easy to perceive that at all those points situated to the interior of the cone C' O' C', and not comprised within the real shadow B O B', a similar appearance will be produced as in the case of the penumbra of the earth's shadow on the lunar disc during an eclipse of the moon. From any such point we should consequently perceive the circular disc of the moon intercepting a portion of the surface of the sun (as in Fig. 126), and the nearer the observer is situated to the real cone of shadow B O B', or the farther he is distant from the surface of the cone C' O' C', the greater will be the portion of the solar surface covered by the moon. We have seen, in the case of lunar eclipses, that the whole disc of the moon can be hid by the shadow of the earth; but the moon's shadow only covers an area whose diameter is 180 English miles at the greatest, but the penumbra may reach to nearly 4,900 miles in diameter.

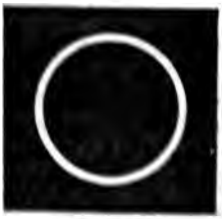

Fig. 125.

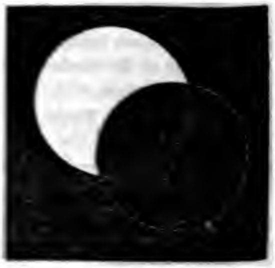

Fig. 126.

The dimensions of the lunar shadow here mentioned at the surface of the earth are calculated in the same manner as that of the earth's shadow. We have already seen that the semi-diameter of the latter was equal to the parallax of the sun added to that of the moon, and from which the apparent semi-diameter of the sun was subtracted; in the same way the semi-diameter of the moon's shadow at the earth is equal to the parallax of the earth added to the parallax of the sun at the distance of the moon, from which sum we subtract the apparent semi-diameter of the sun as seen from the moon. The latter quantity is found by increasing the semi-diameter as seen from the earth, in the proportion of the distances of the moon and earth from the sun at those times, or the apparent semi-diameter of the sun seen from the moon, = apparent semi-diameter of the sun $\left(\frac{D}{2}\right)$ as seen from the earth $\times \frac{K}{k}$, K being the distance of the earth from the sun, and k the distance of the moon from the sun. The sun's horizontal parallax at the moon is equal to the sun's horizontal parallax at the earth, increased by the ratio of the distances and diminished by the ratio of the diameters of the sun and earth; or the sun's horizontal parallax at the moon = sun's horizontal parallax on the earth $(p) \times \frac{r}{R} \times \frac{K}{k}$, r being the moon's *true* semi-diameter, and R the earth's *true* semi-diameter. Substituting those values in the preceding expression, the semi-diameter of the moon's shadow will be $\frac{\delta}{2} - \frac{D}{2} \times \frac{K}{k} + p \cdot \frac{r}{R} \cdot \frac{K}{k}$. The parallaxes of the sun and moon may be used instead of the ratio of their distances in this expression, and as $p = \frac{R}{K}$ and $P = \frac{R}{K - k}$, consequently $\frac{K}{k} = \frac{P}{P - p}$. As $\frac{r}{R} = \frac{\delta}{2P}$, we also find $\frac{\delta}{2} + p \frac{r}{R} \cdot \frac{K}{k} = \frac{\delta P}{2(P - p)}$; and the apparent semi-diameter of the moon's shadow $= \frac{\delta - D}{2} \times \frac{P}{P - p}$. We perceive from this that when the moon's diameter is equal to that of the sun, the apparent diameter of the moon's shadow is nothing, or the apex of the cone just reaches to the earth. When the moon's diameter is less than that of the sun, the expression is negative, and the shadow cannot reach the earth's surface. The formulæ for the dimensions of the penumbra may be adapted to the moon's shadow in a similar manner. If the apparent semi-diameter of the earth, or the moon's horizontal parallax, be added to the apparent semi-diameter of the shadow, the distance of the centres of the moon's shadow and the centre of the earth is found; and the limits of the distance of the moon from her nodes when an eclipse can happen will readily be obtained, and is found to be 17° 27′.

It is thus seen that the same formulæ apply to the calculation of both lunar and solar eclipses, in so far as the times at which the earth in general would become enveloped in the shadow of the moon is concerned; but this would only have reference to an observer situated on the moon, who would thus be enabled to foretell and note the duration, times, and magnitude of an eclipse, the instant at which the penumbra entered upon the enlightened disc of the earth and the period of total eclipse. In Figure 127 we perceive the phenomena which would follow from the interposition of the moon between the sun and earth; the fictitious observer situated at L, as the moon moved in the direction of the arrow, carrying with it the real shadow and the penumbra, would first see the penumbra touch the disc of the earth T; then (if the eclipse were total) the contact of the real shadow and the

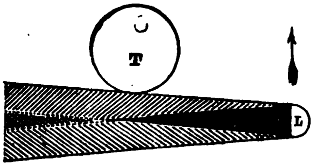

Fig. 127.

above formulæ would give the instant at which those appearances would happen. The two cones, that of the real shadow and the penumbra, would continue to traverse the portion of the hemisphere turned towards the moon, covering successively the various parts of its disc, advancing on new regions, and leaving those which it had previously shadowed. Finally, the cone of the shadow, then that of the penumbra, would again become tangents to its surface; and the instant at which this would take place would mark the end of the eclipse. But in the case of a solar eclipse on the earth's surface, the time at which it became generally visible on the earth, or the period of first contact, would not be of such interest to any observer, as the time at which it would become visible, or total, at his own particular locality; and the position of the observer at different parts of the moon was not taken into consideration in the above calculations, the latitude and longitude of the several places on the moon being a matter of indifference to an observer on the earth. But to an observer on the earth it is a matter of great consequence to know the exact time at which the eclipse would commence at his own station (for it will not begin for him until such period), and also the extent and duration of the shadow of the moon when it passes over a known station of any assigned latitude and longitude. The calculation of an eclipse of the sun is, therefore, more difficult than that of the moon, because more is required to be determined. The foregoing formulæ may be applied to the case of an eclipse of the sun viewed from the centre of the earth, to which they will exactly apply. But to the position of an observer on the surface of the earth, various corrections of parallax have to be applied on account of the angle which he makes with the centre of the earth, whether in latitude or longitude, as the various phases of the eclipse at the different parts of the earth, by which it is visible at some parts and not at others, is due altogether to parallax. In calculating a solar eclipse for any assigned position, it is necessary to correct the angular distances, or the longitudes and latitudes, for the effects of parallax. This requires a long and intricate calculation; but the principal appearances and phenomena, as likewise the times of contact, greatest eclipse, duration, and magnitude, may be determined in the following simple manner:—

Geometrical or Graphical Construction of a Solar Eclipse.—If an observer be supposed to be situated at the centre of the sun, and the moon is interposed between the sun and earth, he will observe it passing across its disc in the same manner as an observer on the earth sometimes perceives the satellites of Jupiter traversing the bright disc of the latter planet. In the same manner, likewise, as an observer on the earth perceives the equator and axis of the sun inclined at different angles to the north and south points and to the ecliptic, at the various seasons of the year (see page 231, Figs. 23 and 24), the observer at the sun would perceive the poles and equator of the earth change their situations in respect to the ecliptic, but in a greater degree, the solar equator being only inclined at an angle of 7° to the ecliptic, whilst that of the earth is inclined 23½°. At the time of the vernal equinox, the plane of the equator passes through the sun, and the north and south poles of the earth will consequently appear to an observer at the sun situated at the margin of the disc, the parallels of latitude describing straight lines, as in Fig. 128. At the summer solstice the north pole of the earth will be turned towards the sun, the south pole will be invisible, and the parallels of latitude will to an observer on the sun be projected into ellipses, as at Fig. 129. At the autumnal equinox the parallels of latitude will again appear as straight lines, but the poles of the earth will be on the opposite side of the poles of the ecliptic, as at Fig. 130. Finally, at the winter solstice the south pole will be turned towards an observer on the sun, and the parallels of latitude projected into ellipses, as in the former case, but in the contrary direction (Fig. 131).

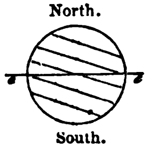

Fig. 128.

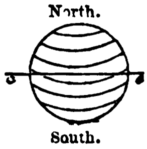

Fig. 129.

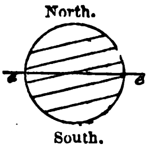

Fig. 130.

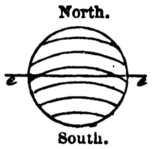

Fig. 131.

By projecting in this manner the surface of the earth at the various seasons of the year, as it appears to a spectator on the sun, we can represent any parallels of latitude. By doing so for the day on which the eclipse is expected to occur, we can mark along this projection the position of the place at the different hours of the day; and the moon's apparent path across the earth's disc may be laid down in a similar way, and its position from hour to hour determined. In order to determine the time at which the eclipse appears greatest at this locality, we must find that point in the moon's path and the path of the observer marked with the same times, and which are at the *least* distance from each other. The beginning or end of the eclipse will be determined by finding those points in the moon's path and in the path of the spectator which are marked with the same times, and whose distance is equal to the sum of the semi-diameters of the sun and moon. As an illustration of this method we will choose the next solar eclipse visible in the British Islands, viz., that of March 15, 1858 (Figs. 132 and 133), which will be generally seen throughout the north of Europe, and which is the greates which has occurred or will happen in this country for many years. The times and phases of the eclipse are for the latitude of Greenwich, or 51° 28′ 38″.

By the "Nautical Almanac" the time of new moon is March 15d. 0h. 12m. 0s., mean time corresponding to March 15d. 0h. 2·9m. apparent time, the equation of time being — 9·1m. The following elements are calculated for that epoch:—

	°	′	″
Sun's longitude	354	38	38
Sun's declination	2	7	15 S
Moon's latitude	0	37	43 N
Moon's hourly motion in longitude . . .	0	34	18
Sun's hourly motion in longitude . . .	0	2	29
Moon's hourly motion in latitude . . .	0	3	9
Moon's equatorial horizontal parallax . . .	0	58	15·2
Sun's equatorial horizontal parallax . . .	0	0	8·6
Moon's true semi-diameter	0	15	54·6
Sun's true semi-diameter	0	16	6·5

The *geocentric* latitude, which is found by applying the angle of the vertical (p. 214) to the *geographical* latitude (and which in this case is 11′ 13″), = 51° 17′·4. The moon's *equatorial* horizontal parallax being that which is given by the tables, the horizontal parallax for any other latitude will be found by applying the following correction:—

Latitude.	Moon's Horizontal Equatorial Parallax.		
	53′	57′	61″
°	″	″	″
15	0·7	0·8	0·8
20	1·2	1·3	1·4
25	1·9	2·0	2·2
30	2·6	2·8	3·0
35	3·5	3·7	4·0
40	4·4	4·7	5·0
45	5·3	5·7	6·1
50	6·2	6·7	7·2
55	7·1	7·7	8·2
60	8·0	8·6	9·2
65	8·7	9·4	10·0
70	9·4	10·0	10·8
75	9·9	10·7	11·4
80	10·3	11·0	11·9
85	10·6	11·4	12·1
90	10·6	11·4	12·2

The moon's horizontal parallax for Greenwich will therefore be 58′ 8″·0; and as the relative positions of the sun and moon will remain the same if the sun be supposed to retain a fixed position, and the moon be supposed to be affected with the difference of both their parallaxes, the relative parallax, or that which affects the relative positions of the two bodies, will be 57′ 59″·4, which is the semi-diameter of the earth as seen from the moon, that of the moon seen from the earth being 15′ 54″·6. The relative sizes of their discs will be in the same proportion if viewed from any distance. With

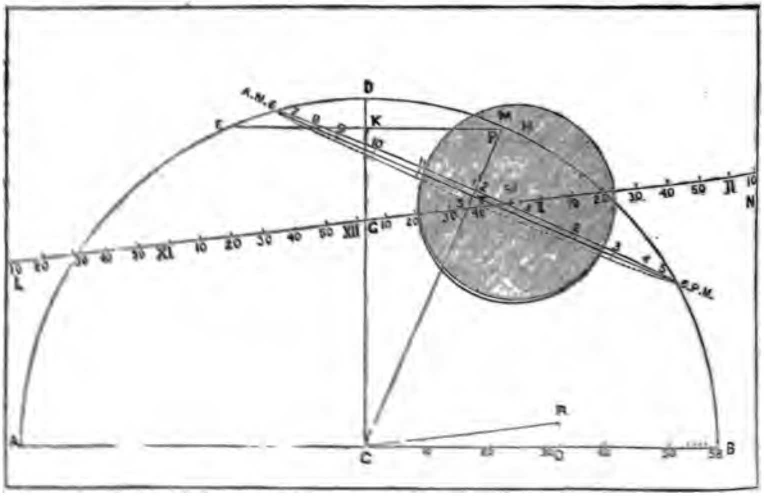

Fig. 132.

a radius A C equal to 57′ 59″·4, or if practicable with one of 3,479 parts from any scale of equal parts, describe the semi-circle A D B with the centre C, which represents the northern half of the enlightened part of the globe of the earth viewed from the sun. If the place were in south latitude, the lower half of the earth's disc should be that represented. The line A C B represents the ecliptic, and the perpendicular C D the axis of the ecliptic. Take from a sector, or any other convenient scale, the chord of 23° 28′ (which is the obliquity of the ecliptic) to the radius C B, and set it off from D to H and E in the periphery of the circle, or the angle D H = 23° 28′ may be at once set off. Draw the straight line E H, cutting C D in K. By considering Figs. 128, 129, 130, 131, it will be seen that the north pole of the earth, as viewed from the sun, will constantly appear on this line E K H, E being its position at the autumnal, H at the vernal equinox, and K at the solstices. Its distance from H at any time will be equal to the versed sine of the sun's longitude, or its distance from K will be the sine of the difference between the sun's longitude and 90° or 270°. To the radius E K take the sine of 354° 38′·6 — 270° = 84° 38′·6, and set it off from K to P, the north pole of the earth; for when the sun's longitude is between 270° and 90°, the north pole lies to the right of the pole of the ecliptic, and to the left when it is in the other six signs. Draw C P, which is the northern half of the earth's axis, and produce it until it cuts the circle A D B in M.

In order to draw the parallel of latitude of Greenwich on the earth's disc as seen from the sun, from sunrise to sunset, the following method may be taken:—If the latitude of the place were exactly equal to the sun's declination, it would be vertical at noon, and the place of observation would be at the centre C. But as the latitude of the place exceeds the sun's declination by 53° 24′, it must be seen that distance north of the place where the sun is vertical, or when projected on the disc; this will be the sine of that angle which is set off from C to 12, which is the apparent position of Greenwich at noon of March 15. The position of Greenwich at midnight is determined in the same manner by taking C S equal to the sine of 49° 10′ to the radius A C, and the point S will be its position at midnight. It is easily seen that the line 12 S will be the shortest diameter of the ellipse into which the circle of latitude is projected. To determine the length of the longest diameter, as it is not shortened by being viewed obliquely, it will be equal to the diameter of the parallel of latitude, or to the cosine of the co-latitude = 38° 43′; and setting of this distance from the point T (which is situated exactly between S and 12) on the line perpendicular to C M, we obtain the extremities of the longest axis of the ellipse, or the points at which Greenwich will appear at 6 A.M. and 6 P.M. At the times of the equinoxes these hours will be projected at the edges of the disc (as they are almost so in the present instance), but at all the other times of the year they will be more or less distant.

In order to obtain the position of Greenwich at any other hour of the day, we take, with a radius equal to T 6, the sine of 15°, or one hour of time, which is set off on each side of the point T. The sines of 30°, 45°, 60°, 75° may be obtained and set off in the same way. Through those points draw lines parallel to C M. With a radius equal to S T take the sines of 75, 60, 45, 30, 15, which are marked off on each side of the line C 6 respectively at the points 15°, 30°, 45°, &c., as before. By drawing an elliptic curve through those points, the several positions of Greenwich at each hour of the day from sunrise to sunset are readily obtained; and by continuing it on the opposite side, its position (here represented by the dotted line) is represented during the night. The points at which the ellipse touches the circle A D B show the time of sunrising and sunsetting. It will be seen that the ellipse will be more open near the time of the solstices, and more eccentric near the equinoxes (as is the case in the present instance). The points 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5, 6, represent the position of Greenwich at those hours, which are marked from noon towards the right. If the sun's declination had been north, the diurnal path of Greenwich would be on the lower or dotted portion of the ellipse. If possible, the figure should be constructed on such a scale that the hours might be divided into minutes with sufficient certainty.

Having represented the position of Greenwich at the various hours of the day, as they appear to a spectator on the sun, the next thing to be done is to draw the moon's apparent path across the earth's disc. From the same scale of equal parts used in the radius C B, measure off an interval equal to the moon's latitude, or 37′ 43″; and, as the moon's latitude is north, make it equal to C G, above the ecliptic A B. Take in the same scale, C O, equal to 31′ 49″, and apply it on the line C B, from C to O. This quantity represents the hourly motion of the moon from the sun in longitude. Draw O R perpendicular to C B, and make it equal to 3′ 9″, the hourly motion of the moon in latitude; and as the moon is moving northwards, set it above the line C B. The line C R, joining the points C and R, represents the relative hourly motion of the sun and moon; and parallel to this line draw the orbit of the moon, or L G N, on which, by means of the hourly motion C R we can mark the position of the moon from hour to hour; and as the moment of new moon at Greenwich is at three minutes past noon of apparent time, the position of the moon at apparent noon, is found by taking the proportion of 60m. : 3m. :: the distance C R, to the distance G XII., which is measured to the left of the point G, and shows the position of the moon at twelve o'clock, or noon. By taking the line C R in the compasses, the other hours may be marked off which represent the places of the moon at those times. If the scale is made sufficiently large, those spaces may be subdivided into sixty parts or minutes.

In order to find the times at which the eclipse is greatest, those times must be found on the path of the spectator and the path of the moon, which are marked with the same times, and which are at the *least* distance from each other. The times which best correspond to those conditions in the present case are those of 12h. 52m., or 52 minutes past noon of apparent time. The appearance of the moon at any hour may be shown by taking its semi-diameter to the adopted scale of equal parts—and which in this instance = 15′ 54″·6—and with this radius to describe a circle whose centre is the point where the moon's centre is at the given time. The position of the disc of the sun at the same moment may be represented in the same manner, by taking a radius equal to the semi-diameter of the sun = 16′ 6″·5, and describing a circle taking the *same* time on the path of the spectator for a centre. The relative positions given in the diagram are those at which the eclipse are greatest, when the magnitude of the eclipse = 0·976, on the northern limb, the sun's diameter being 1·000; and as the moon's semi-diameter is less than that of the sun, the eclipse will be annular and partial. The sun's centre must be invisible over all that portion of the earth represented as being covered by the moon's disc, and throughout a much larger area some portion of the sun's disc will be obscured. The extent of this area will be found by describing a circle with the same centre, and whose radius is equal to the sum of the semi-diameters of the sun and moon measured on the adopted scale.

In order to determine the times of the beginning and end of the eclipse, we take from the scale of equal parts a distance equal to the sum of the semi-diameters of the sun and moon = 32′ 1″·1, and beginning near L, place one foot of the compasses on the moon's path, and the other on the path of the spectator, and shift them backwards and forwards until the *same times* are found on the two paths, which are this distance apart. This will give the time of the beginning of the eclipse, which in the present case is 11h. 32m. A.M., *apparent* time. In the same manner find the corresponding times on the other side of the moon's path, and the end of the eclipse will be determined, which in the present case is 2h. 9m. P.M., of apparent time. We thus determine:—

	Apparent Time.	Mean Time.
Beginning of Eclipse . . .	11h. 32m. A.M.	11h. 41m. A.M.
Greatest Obscuration . . .	0h. 52m. P.M.	1h. 1m. P.M.
End of Eclipse . . .	2h. 9m. P.M.	2h. 18m. P.M.

When this projection is made on a sufficient scale, and carefully finished, it will give the above times accurately to one or two minutes, and furnish an useful chart of the various phases of the eclipse. By drawing different parallels of latitude, we can determine the phases of the eclipse for any place. The results are obtained in apparent time, because both the moon's path and the path of the spectator correspond to apparent time.

Projection by Right Ascensions and Declinations.—Instead of employing longitudes and latitudes in the projection of a solar eclipse, we may make use of right ascensions and declinations; and as those are the elements given in the "Nautical Almanac" for these phenomena, it may be convenient to append an example for this eclipse, but for a different latitude and longitude. The radius of projection, C B, is the same in this case as in the last, being the difference of the horizontal parallaxes of the sun and moon; C D is the meridian or circle of declination; the ellipse represents the projection of the parallel of latitude; L G N the moon's apparent path; C G the difference of *declination* of the sun and moon at the conjunction in right ascension, and

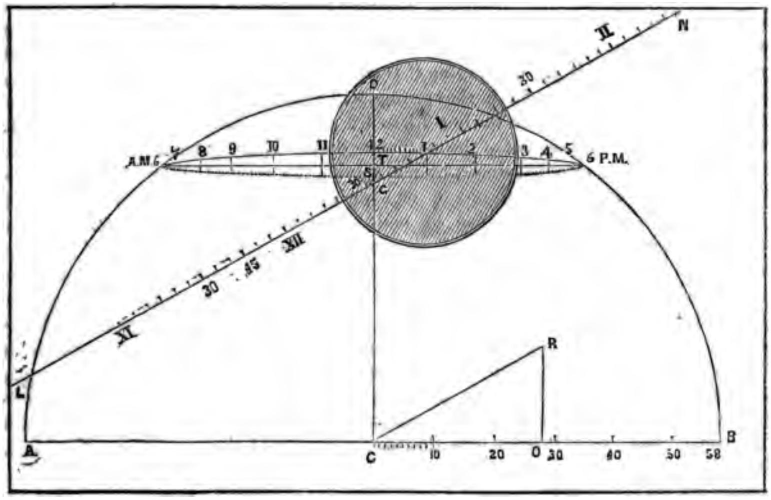

Fig. 133.

C 12 the sine of the sun's zenith distance at noon. The elements for the above eclipse given in the "Nautical Almanac" are as follows, and the place may be considered as situated 0° 6′ west of Greenwich, and at 52° 41′ of north latitude :—

Greenwich mean time of conjunction in right ascension = March 15, 0h. 44m. 7·6 or March 15, 0h. 43m. 44s.·2 of the mean time at the given place.

The apparent time will be March 15, 0h. 34m. 37·8s., the equation of time being — 9m. 6s·4. The moon's hourly motion in right ascension = 30′ 16″·4; and the sun's, 2′ 17″·1; and the relative hourly motion of the moon from the sun on right ascension 27′ 59″·3, = which must be reduced to the arc of a great circle by being multiplied by the cosine of the declination, or 1° 24′ 21″·8 = 27′·98.

The radius A C = 58′·0 as in the last case.

Hourly motion of moon from sun	= 27′·98
Moon's hourly motion from sun in declination . .	15′ 31″
Moon north of sun	42′ 53″
Sum of semi-diameter of sun and moon	= 32′ 1″
Difference	= 11″·9

The semicircle A D B represents the northern half of the earth's disc drawn with a radius, A C, equal to 58′·0, as in the last case C D is the axis of the earth. The position of a point of 52° 41′ of north latitude, or 52° 30′ of geocentric latitude, will be obtained in a similar manner to the last, and its place on the axis of the earth at noon and midnight will be 54° 37′ and 50° 23′ to the north of the point where the sun is vertical. Obtain, by means of a sector or graphically, the sines of those angles to the radius C B, and set it off on the line C D from C to S. The position of the locality whose latitude is 52° 30′ at 6h. in the evening, will be obtained by taking the cosine of the latitude, and marking it off on the line T 6 — T 6, perpendicular to C D on each side of T, that point being the bisection of the two points S 12. These distances may likewise be obtained, and with more exactness, by multiplying the radius C B = 58′ by the sines of 54° 37′, and 50° 23′, the former of which gives the distance C 12, and the latter C S, the distance T 6 being obtained by multiplying 59′ by the cosine of 52° 30′; and those quantities may be marked off by means of the same scale of equal parts, as adopted in the radius C B. The other times in path of the observer, are determined as in the former instance. The moon's path, according to right ascension and declination, is laid down as before. Take the difference of the declinations of the sun and moon = 42′ 53″, and set it off, from C to G above the line A C B, because the moon is north of the sun. Take C O equal to the hourly motion of the moon from the sun in right ascension, reduced to a great circle, and set it off from C to O. Make O R, which is perpendicular to C B, equal to 15′ 31″, the moon's hourly motion in declination from the sun. The line C R represents the hourly motion of the moon from the sun in regard to right ascension and declination, and the line L G N, drawn parallel to this, represents the moon's path across the earth, the point G being the position of the moon's centre at the moment of conjunction in right ascension. The even hours may therefore be marked off on this path, by taking the point G to be 34m. 38s. past noon on March 15; and the line C R to be the hourly motion of the moon, and the position of the moon at noon will be found by the proportion as 60m. : 34m. 38s. : : C R : G XII. : : 31·6 : 18·2. The hours on the moon's path should if possible be sub-divided to minutes, and the same on the path of the spectator. The sum of the semi-diameters of the sun and moon being taken on the adopted scale = 32′ 1″, that distance is taken in the compasses, and those are shifted backwards and forwards, as in the former case, until the *same* times are found on the moon's path and the path of the spectator, which are exactly at this distance apart. When the left leg of the compass is on the moon's path, and the right leg on that of the spectator, this will give the commencement of the eclipse; and when the *right* leg of the instrument is on the moon's path, the left on that of the spectator, and the corresponding times are again found at the same distance apart, we obtain the end of the eclipse in the same manner. By applying the side of a small square to the moon's path, and moving it along until the other side cuts the same hour and minute on both lines, we obtain the nearest approach the centre to of the sun and moon. If the scale of the projection be made of sufficient dimensions, the distance of those two points will show whether the eclipse be total or annular; if less than the difference of the semi-diameters of the sun and moon, it will be annular at that locality. By describing a circle whose radius is equal to that of the moon, and whose centre is situated on the moon's path at the time of the nearest approach of the centres, we have a circle representing the moon's disc. Taking the radius of the sun in the same manner, and the corresponding time on the path of the spectator for a centre, we describe the position of the solar disc, and their intersection will represent the phase of the eclipse at the moment of greatest darkness. With the points of the compasses at a distance apart, equal to the differences of the semi-diameters of the sun and moon, the times of the formation and rupture of the annulus may be determined in a similar manner, as the beginning and end of the eclipse. In the present case we obtain for the beginning and end of the eclipse at the given position 11h. 32m. A.M., and 2h. 8m. P.M. of apparent time, which is equal to 11h. 41m. and 2h. 16m. of mean time. The time of greatest obscuration is 12h. 52m., found by taking the shortest distance between the corresponding times on the paths of the moon and spectator, answering to 1h. 1m. P.M. of mean time as before.

From the relative dimensions and positions of the sun and moon, it will be seen that the eclipse is central and annular at this locality, viz., at 0° 6′ (of arc) of west longitude and at 52° 41′ of north latitude, or at a point which is a little to the east of Peterborough. The line of central eclipse will pass across England from Bridport in Dorsetshire (at 2° 51′ of west longitude, and 50° 43′ of north latitude) to the Wash, and a little to the north of the towns of Sherborne, Marlborough, Oxford, Buckingham, and Wisbeach. The diameter of the moon so closely approaches to that of the sun, that this eclipse may be *total* in the vicinity of the island of Madeira by the augmentation of the former value. At the station for which the above calculation is made, the duration of the annulus will only be 12s.·3. Should the position of the sun and moon be exact, and likewise their diameters, we are thus furnished with the remarkable phenomenon of the eclipse being both an annular and total one in different parts of the globe.

Annular eclipses are more common than total eclipses, and the prediction of their times and duration show more palpably than anything else the great accuracy to which the tables of the sun and moon have been carried; but astronomers still rely upon the careful observation of those phenomena, in order to correct the places of those bodies as given by theory; and the prediction of the approaching eclipse of March 15, 1858, will, it is supposed, be slightly affected by small errors in the tabular places of the sun and moon, whilst errors which are known to exist in the received diameters of those bodies will affect the duration of the annulus. These errors will probably, however, only amount to a fractional part of a second at most. How different is this from the prediction of the eclipse which was to have happened at Rome in 1684, which the tables at that time in use predicted would be *total*, whilst in reality only three-fourths of the sun disappeared. In the prediction of the eclipse of 1706, the tables of Lahire were four or five minutes in error.

Annual Number of Eclipses of the Sun.—Eclipses of the sun on the earth generally are more frequent than eclipses of the moon, in the proportion of three to two, as it will be seen should follow from the following considerations. In order that there be an eclipse of the moon, it must happen that the moon penetrates the cone of shadow B O B′ at C (*Fig.* 134); and it is easily seen that in order that there be an eclipse of the sun at *some portion* of the earth, it must penetrate into the cone A O A′ at the point D. As the diameter of the cone is larger at D than at C, it consequently happens that the moon passes more frequently through the former than the latter, and that eclipses of the sun will be more frequent than those of the moon. Theory and observation alike prove that in the period of 18 years 11 days, in which, as before mentioned, the moon passes to the same position in respect to the sun and its nodes, that there are in general seventy eclipses, of which forty-one are of the sun, and twenty-nine of the moon. There cannot be more than seven eclipses in one year, nor less than two; and if the latter, they must both be eclipses of the sun. In any *given place*, however, the eclipses of the moon will appear to be more frequent than those of the sun, as will be apparent when we come to consider that the former are due to the total or partial extinction of light on the surface of our satellite, and are seen on every occasion on that hemisphere of the earth where the moon is above the horizon. Eclipses of the sun, on the contrary, can only be seen at one zone of the earth, and that a very narrow one; for, in addition, it will be seen that the dimensions of the shadow of the moon are small as compared with that of the earth. Out of the forty-one solar eclipses, which may occur in eighteen years, there are in general twenty-eight of those which may become central according to circumstances—that is, either total or annular. It results from Du Sejour's calculations that the greatest possible duration of an eclipse of the sun cannot be more than 4h. 29m. 44s. for a place situated at the equator, or 3h. 26m. 32s. for the latitude of Paris. In total eclipses, the greatest possible duration of total obscuration may only be 7m. 58s. at the equator, and 6m. 10s. at the latitude of Paris. In the case of annular eclipses the greatest possible duration of the phase is 12m. 24s. at the equator, and 9m. 56s. at the latitude of Paris. Such combinations are of course very rare. The total eclipse of 1706 lasted for 4m. 10s.; that of 1715, at London, 3m. 57s.; the total eclipse of 1806, at Kinderhook, in America, continued for 4m. 37s.; and that of 1724, at Paris, for 2m. 16s. In the eclipse of 1778, the darkness continued for four minutes.

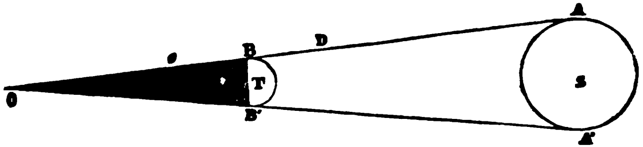

Fig. 134.

Phenomena observed during a Total Eclipse.—The great and sudden darkness which takes place at the moment of central eclipse is, of course, the great phenomenon to be witnessed in solar eclipses, but many other curious circumstances have been unexpectedly noticed by the telescopic observers during the eclipses of 1842 and 1851, which serve to throw some light on the constitution of the sun; so true is it, as Arago observes, that, in the intellectual world as in the terrestrial, we cannot advance a step without discovering a new horizon. It is not, however, to be strictly understood that those appearances were first perceived on these occasions, but rather that they were observed with greater care; and from the number of observers engaged, and the different instruments they made use of, the results are placed beyond doubt, and confirm the desultory notes of former astronomers.

The darkness which follows the total obscuration of the sun's disc, though momentary, is generally very great, though not complete. The total eclipse which occurred in England on June 17, 1433, was long remembered under the name of the "Black Hour." That of 1598 was equally imprinted on the memories of the peasantry, and remembered by the name of the "Black Saturday;" whilst the total eclipse of 1652 is recorded in Scotland by the name of "Mirk Monday." Nor, if we are to allow the testimony of Dr. Halley, are those titles misapplied in such cases. "I forbear," he says, in his communication to the Royal Society on the eclipses of 1715, "to mention the chill and damp which attended the darkness of this eclipse, of which most spectators were sensible and equally judges. Nor shall I trouble you with the concern that appeared in all sorts of animals, birds, beasts, and fishes, upon the extinction of the sun, since ourselves could not behold it without some sense of horror." In the eclipse of 1706, during the period of total darkness at Geneva, bats flew about as at dusk, swallows were seen flying about, and cage-birds put their heads under their wings. The stars appeared as thickly strewed as at the time of full moon. In the eclipse of 1842, several stars of the first magnitude, as well as the planet Mars, were distinctly visible. The colour of the sky at those times does not appear, however, either to belong to the darkness of night nor the hue of twilight; in some places it was even noticed to be of a violet tint; and *red* stars, such as those of Alpha Orionis and Aldebaran, were noticed to be *white*. At Montpellier, in the eclipse of 1842, the light had acquired a livid tint, imparting to the human countenance an aspect which it was painful to contemplate; and the spectacle was generally allowed to be of an extraordinary and appalling character. An owl was seen to leave the tower of St. Peter, the bats left their retreats, the swallows disappeared, the fowls went to roost, and the cattle stood still in the field. A heavy dew fell at Perpignan, Turin, and Vienna during the obscuration. Similar appearances were noticed during the total eclipse of 1851, by the numerous observers stationed in the path of central eclipse in Sweden.

Even during the greatest obscuration, and when the disc of the sun is completely hidden by that of the moon, its place is still made apparent by the brightness in the part of the heavens in which it is situated. This light appears in the remarkable form of a corona or lustrous ring, and it has sometimes been so bright as to be observed and mistaken for an *annular* eclipse. This luminous ring was seen during the eclipse of 1567, and was mistaken for the margin of the solar disc; it was equally visible in that of 1598. Its *radiating* appearance seems to have been first noticed in the eclipse of 1652, and appeared to be endowed with a sort of rotatory motion. In every subsequent eclipse observed under any favourable circumstances, the corona has been observed with equal plainness, generally of about one-tenth or one-twelfth of the diameter of the moon, and of a pearl-white colour, or, in the words of an observer, "of that bluish tint which distinguishes the colour of quicksilver from that of a dead white." In the eclipses of 1842 and 1851, it was the principal circumstance to which the attention of the observer was directed; and M. Arago was able to perceive what might be termed *two* rings—the inner one, or that which bordered on the moon's limb, being of an uniform brightness, whence it faded imperceptibly outwards, and terminated irregularly. The inner ring was 3′ or 4′ in breadth; but the whole breadth of the corona was differently estimated by various observers as 8′, 16′, and 25′ in breadth. Several luminous jets of light were noticed in the corona by M. Struve at Lipesk, as in the accompanying diagram (Fig. 135), and it had a decidedly radiating appearance; but the observers in France only perceived one or two radiations, and in some cases only the brighter or inner ring of light was observed. In the eclipse of 1851 the same discordance in the estimation of the breadth and in the radiating aspect of the corona existed between the several observers. The direction of the diverging rays did not seem to be perpendicular to the circumference of the moon, but were greatly inclined; and M. Arago noticed that at one part of the corona the rays were entwined within each other like a skein of thread which was entangled.

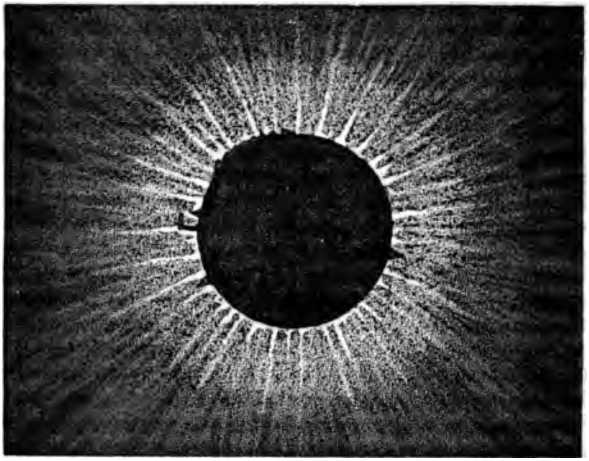

Fig. 135.

The appearance of this ring of light is now generally attributed to the effect of the atmosphere of the sun, rather than to anything of the same nature in the case of the moon. The latter was the supposition of Kepler, and before the want of an atmosphere to our satellite was established by numerous proofs, it was the most natural one which could be imagined; although the great breadth of the ring would go to prove that its height must be three or four hundred miles, or fifty times the extent of the earth's atmosphere. No such difficulties exist in the case of its being supposed to be the solar atmosphere, while its concentric form, in regard to the sun at the different periods of observation, tend still further to confirm this conjecture. By what has already been stated of the photosphere of the sun, and the different strata which surround its surface, as well the appearance of the zodiacal light, this may be deemed to be, in every respect, the most probable conjecture. The corona, however, cannot be identified as forming the zodiacal light, for it has mostly been observed of a circular form, whilst the latter is elliptical; even where the corona has been noticed to be protuberant on opposite sides, the greater axis does not take the direction of the ecliptic.

Red Flames.—In the eclipses of 1715 and 1733, when the sun was wholly eclipsed, and only the corona was visible; the margins of the moon were noticed to be marked with some red spots, which remained visible for some seconds. Whilst observing the corona in the eclipse of 1842, these appearances again became unexpectedly conspicuous to the different observers. They were first observed a few seconds after the time of total obscuration, and were of a fine crimson tint—being compared by one observer to the peaks of the Alps illumined by the setting sun; and, by another, to beautiful sheaves of flames, which remained visible even after the sun had emerged. The length of the most considerable was, by actual measurement, found to be equal to 1¾ minutes of arc. Only two or three isolated prominences were perceived by the observers stationed in the south of France; but to MM. Struve and Schidlowsky, at Lipesk, these rose-coloured flames burst out at several parts of the lunar disc—a *very large portion* of the periphery being garnished with this reddish bordering. M. Littrow noticed that they changed their colour as the eclipse advanced, being at first white, then rose colour, and finally violet, passing afterwards in a reverse order through the same tints. In the eclipse of 1851, these crimson projections were beautifully seen, and by some observers in great number. Some seen were, in this instance, crooked, and resembled a flame bent aside by the wind, but their colour was of the same remarkable tint as in the former case. In the eclipse of 1706, observed by Captain Stannyan, at Berne, "a blood-red streak of light" was noticed; and Halley, in 1715, saw a similar "long and very narrow streak of a dusky but strong red light." Equally with the corona, these appearances have been conjectured to be in the atmosphere of the sun, and to be in no way connected with the moon. Their reddish light would seem to show that they were of a cloudy nature, resembling our terrestrial clouds, and absorbing all the rays of the spectrum, with the exception of the red ones.

Baily's Beads.—In some cases when the margin of the moon comes in contact with that of the sun, instead of the faint and regular thread of light which would be supposed to ensue under those circumstances, the appearance presented is a broken glimmer of light, which was first noticed by the late Mr. Baily, who compared it to *beads* of light. These are noticed in total as well as annular eclipses, and were seen by different observers in the last eclipse of 1851. They are generally regarded as being due to the rough and mountainous edges of the moon coming in contact with the margin of the sun, and the light proceeding from the latter shining through the *chinks* or valleys of the moon, the effect being greatly increased by irradiation.

Method of Observation.—We can follow the different phases of a solar eclipse without the help of a telescope, and by merely making use of a piece of coloured glass, that colour which is technically termed by opticians *London Smoke* being the most convenient for this purpose. If a pin-hole be made in a piece of card, and the image received on a screen, the disc of the sun and the various phases it assumes from the interposition of the moon likewise become very apparent, and we have thus a still more simple method of following the phases of an eclipse. The *form* of the aperture, or the pin-hole, is of little consequence in this instance, provided it is small. This is apparent from the shadows cast by a tree or hedge, through the narrow interstices of which, under ordinary circumstances, the sun will throw a circular image, which arriving obliquely on the ground, will appear of an elliptic form. But at the time of an eclipse of the sun, instead of those regular ellipses, we perceive the horned or crescent appearance of the moon at the first or last quarter, changing according to the phase of the sun at the time, as in the following Figure. The peculiarity which the shadow of a tree presents during an eclipse is very apparent, and even if we were not aware of the phenomenon which has happened, we could scarcely fail to remark the curious and regular shapes of the patches of light which are cast upon the ground on those occasions.

Application of Photography.—In the eclipse of May 26, 1854, the American astronomers, assisted by M. Victor Prevost, took several photographic pictures of the sun at its different periods of eclipse, and it may probably happen that during the approaching annular one of March 15, 1858, the *annulus* (which in this case lasts for fifteen seconds) may continue for a sufficient length of time to imprint its image on a plate of the requisite degree of sensibility.

Occultations of Stars by the Moon.—The occultations of stars by the moon are analogous to eclipses of the sun, and their graphical projection may be made with some slight modifications as in that case, and the same formulæ may be applied to their prediction. All the planets may be occulted by the moon, but only such of the fixed stars as are situated near the ecliptic, and within the limits of the moon's latitude. Among the brighter stars which can suffer eclipse—those of Regulus, Spica, Aldebaran, Antares, and the group of the Pleiades, may be mentioned. An occultation can be seen at many different parts of the earth's surface, and their accurate observation forms one of the best means of determining the longitude, being much more exact than the occultations of Jupiter's satellites, as the moments of immersion and emersion are instantaneous. The portion of the earth's surface at which the star appears to be occulted at the same instant, will have a diameter equal to that of the moon, and will pass, like the shadow of the moon, over a certain zone of the earth. As the star has no perceptible diameter, there can be no penumbra in such a case, even if the light of the star were sufficient to cast a shadow. This circumstance renders the calculation more simple, which likewise follows from the star having neither motion nor parallax.

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ON UNIVERSAL GRAVITATION.

ALL experience shows that every heavy body requires force, or effort, or support, in order to prevent its falling to the earth. What are called light bodies also require a similar counteracting force when their tendency to fall is not resisted by the presence of the air, as numerous experiments with the air-pump have sufficiently proved.

This influence of the earth upon all bodies suspended over it, and which renders force necessary to *keep* them suspended, is itself called force—the *force of gravitation.* Such is the name which philosophers give to the hidden cause, whatever it be, which forces bodies to fall to the earth when left to themselves, instead of remaining suspended wherever they may be placed, or moving in any other direction.

That this force of gravitation or attraction is not exclusively expended on terrestrial bodies, was a doctrine entertained by several astronomers before Newton's time. Kepler had a steady conviction that the moon gravitated towards the earth; but he conceived that some additional force, animal or spiritual, was in operation to check the gravitating influence, and prevent the two bodies from rushing into collision.

It was reserved for Newton to prove that the one single principle of gravitation, combined with one single impulse given to each planet, was sufficient to account for all the celestial movements, and to render clear and intelligible the entire mechanism of the heavens.

In addition to the first distinct enunciation of the *principle*, Newton also discovered the *law* of gravitation. The principle is, that all particles of matter mutually attract one another; the law is, that the intensity of the attraction varies inversely as the square of the distance at which it acts.

To verify these important propositions, observations of a peculiar kind were indispensable. These, fortunately, had already been supplied by Kepler; they are embodied in the three following statements, which have, however, a greater degree of generality than Kepler's observations strictly warrant. His *laws*, as they are called, thus generalized, have been mentioned at page 261: it is convenient to repeat them here.

1. If a line be supposed to connect any planet with the sun, this line will describe equal areas in equal times.

2. The planets revolve about the sun in elliptic orbits, the sun occupying one focus of the ellipse.

3. The squares of the times in which any two planets perform their revolutions about the sun, are as the cubes of their mean distances from it.

The first of these three laws is not peculiarly connected with astronomy, and is of no special importance in establishing the physical theory of the planetary motions; for, as shown by Newton, it is a universal truth that a body, moving round a point of attraction in any orbit and attracted by any force, *must* describe equal sectorial areas in equal times. This truth, therefore, is quite independent of every specified law of attraction, and is irrespective of all considerations as to the distance of the revolving body, or the form of its orbit. If only the direction of the force be towards a fixed point, the equable description of areas necessarily has place; and, conversely, if this latter have place, the direction of the attractive force is always towards the same fixed point. This is proved as follows, after the manner of Newton:—

Suppose that a planet, moving at any distance from the sun, be subjected to the influence of an attractive force tending to draw it directly to that body; and imagine this force, instead of acting continuously, to act by successive impulses, after certain equal intervals of time. Let A B (Fig. *a*) be the path described by the planet during one of these intervals, which path will, of course, be a straight line, since in describing it the planet is not acted upon by any force.

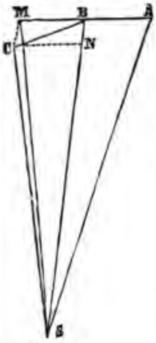

Fig. *a*.

Arrived at the point B, the attractive force at S, acting merely as an instantaneous impulse, changes both the direction and velocity of the planet, which takes another rectilinear path, B C, described in an interval of time equal to the former interval; at the end of which another impulse, in the direction C S, is given to the planet; and so on.

Now, confining our attention for the present to the two sectorial areas (in this case the two triangles S A B, S B C) described in the first two intervals, we readily see that they must be *equal*. For if no fresh impulse had been given to the planet when at B it would have proceeded onwards from B to M, in the continuation of A B, and in the second interval of time would have described B M equal to A B; the impulse from S, however, if acting alone, would have brought the planet to some point N in B S, and this impulse, combined with that which acting alone would have brought it to M, causes it to describe the diagonal B C of the parallelogram M N, as the first principles of mechanics show.

Now M C being parallel to B S, the two triangles B M S, B C S are equal (Euc. Prop. 37, B. i.); but the triangle A B S is also equal to the triangle B M S; hence the triangle A B S is equal to the triangle B C S; so that in the equal times the equal triangles A B S, B C S are described, however intense or however feeble the impulse from S upon the planet at B may be.

By increasing the number of intervals of time, and the corresponding impulses from S, the path of the planet would be represented by the sides of an irregular plane polygon, as in the diagram (Fig. *b*); and however these sides may differ in length and direction, it is plain, from what is shown above, that they are the bases of *equal triangles* of which the point S is the common vertex. As each triangle is described in the same time, by the line joining the planet and the sun, it follows that equal areas are always described by this line in equal times.

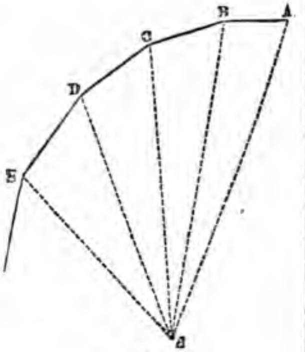

Fig. *b*.

It is obvious that this conclusion has nothing to do with the length of time which has been supposed to intervene between the impulses upon the planet, only upon the equality of the intervals. We may therefore imagine these equal intervals to be as small as we please, and therefore the impulses to succeed one another with the utmost rapidity, and, in fact, to unite in one continuous force; in which case the sides of the polygon, becoming shorter and shorter, must unite in one continuous plane curve, the centre of attraction being itself in that plane. We infer, therefore, that a planet, moving in virtue of a primitive impulse, and diverted from its rectilinear path by an attractive force residing in the sun, describes a curvilinear orbit round the source of attraction, such that equal sectorial areas are generated by the *radius vector* of the planet in equal times.

However accordant this description of equal areas in equal times may be with actual observation, we could not fairly infer that the centre of force is necessarily in the sun, unless it were further proved that such equal areas could not be described if the centre of attraction were situated elsewhere than at the common vertex of the equal sectors. And this may be done thus:—If the force acting on the planet when at B (see Fig. *a*, page 360) had a direction out of the line B S, B N would make some angle with B S, and C M, which is parallel to B N, would not then be parallel to B S; and the two triangles B C S, B M S, on the same base B S, would have their vertices at unequal distances from that base; so that the surfaces of those triangles would be unequal. The triangle A B S, always equal to B M S, would no longer be equal to the triangle B C S; nor, consequently, would the triangles A B S, B C S, C D S, &c. (Fig. *b*), described by the radius vector in equal times, be equal. We conclude, therefore, that, 1st, if the force acting on a planet is constantly directed towards the sun, the areas described by the straight line joining the planet and sun are proportional to the times of describing them; and, 2nd, if the force which acts on the planet be not directed towards the sun, the proportionality of the sectorial areas to the times does not exist. The first law of Kepler, therefore, warrants the conclusion that the attractive force acting on a planet, is constantly directed towards the sun; and this is the only conclusion derivable from that law. As respects the intensity of the attractive force—how it varies with the distance of the planet, or whether it varies at all, are particulars of which nothing can be learnt from the observed fact of the equal description of areas; for as the preceding reasoning shows, equal areas are always described about a point, provided only the force acting on the moving body be in the direction of the line from the body to the point, however the intensity of the force may vary. But if the body move in a circle round the centre of attraction, then we may infer, from the law of equal areas, in equal times, that its motion must be uniform; for only equal arcs of circles can belong to equal sectors, and from the constancy of the distance, or radius vector, we cannot but infer the constancy of the attractive force.

Kepler's other two observed laws of planetary motion evidently imply a certain law of attractive force. If the attraction be such as to cause the planet to describe an elliptic orbit, any alteration in the force acting upon the planet at any particular point in that orbit, would necessarily alter its path. And if two planets revolve round a centre of force, and it be observed that their mean distances from that centre and their times of revolution bear the same relation to one another, a uniform law of force is again implied. Let us endeavour to discover what this law is from Kepler's third proposition, namely, that the squares of the times are as the cubes of the mean distances.

It will be observed that the enunciation of this truth involves no condition in reference to the *eccentricities* of the planets; the proposition should hold, therefore, for all eccentricities, and even for *circles*, in which the eccentricities are zero. For simplicity, then, we shall consider the case of planets revolving in circular orbits, and consequently, as shown above, moving uniformly; and shall thence endeavour to discover the law of force necessary to justify the proportion that the squares of the times of revolution are as the cubes of the radii of the circular orbits.

The intensity with which a force acts upon a body is measured by the addition it imparts to the body's velocity in the direction of that force in a second of time. If no force act, no addition can be made to velocity; if a body be at rest, or move uniformly in a straight line, we infer at once that it is not acted upon by any force. If a force act continuously, and in the same direction, it must generate equal increments of velocity in equal times; and the motion of the body is thus said to be uniformly accelerated. It must be borne in mind, however, that the force must act with the same intensity upon the body throughout. The attraction of the earth, or terrestrial gravity, may be regarded as such a constant force in all our experiments and observations respecting bodies falling to its surface, because these observations and experiments can never be made in a region so remote from the surface as to render the variation in the force of the earth's attraction sensible.

If a body fall from rest to the surface of the earth, it will acquire a velocity at the end of the first second, which, as stated above, is the measure of the earth's attraction; throughout this second, the velocity increases uniformly from nothing up to the final velocity which measures the force. The velocity midway is, therefore, half this final velocity; and if the body were to move *uniformly* with this midway velocity, it is pretty obvious that in one second it would describe the same length of path as in falling from rest as above supposed.

The length of path in uniform motion is evidently found by multiplying the uniform velocity (the length passed through in a second) by the number of seconds; that is, calling the length of space s, the velocity v, and the number of seconds t, we have $s = v\,t$, or if $t = 1$, $s = v$. Now, if v be the final velocity at the end of the second of accelerated motion, adverted to above, and S the space actually passed through in that second; then, putting $\frac{1}{2}$ V for v, and S for s, we have $S = \frac{1}{2} V$; therefore $V = 2S$; that is, the final velocity, or the measure of the force of gravity, is twice the space described by a body falling from rest in the first second of time.

The same conclusion applies, however remote from the surface of the earth the falling body be conceived to be placed; the intensity of gravity *there* would still be measured by double the space through which the body would fall from rest in one second, for during so short a time the force may, as before, be considered as constant. And a like conclusion applies whatever centre of force be considered; the measure of its intensity on a body anywhere subjected to its action, will always be accurately expressed by twice the space through which the body falls towards that centre in one second.

Let the circle in Fig. *c* represent the path of a planet revolving about the sun at S. When at A, the planet is actuated by a velocity in the direction of the tangent A M, and in this direction it would, of course, continue to move uniformly if no force diverted it from its path. But it is acted upon by a force which is constantly directed towards the sun, S, and which tends to draw it towards that body, or to deflect it more and more from the tangent. It may thus be said to fall towards the sun, just as a body near the earth projected forwards in a straight line, A M (Fig. *a*), falls towards the earth's surface, the actual motion in either case being in a curved line, A B. Let A B, on the orbit of the planet, be the portion of that orbit described in one second of time; the distance D B will be that which the planet has fallen, from its wonted path, towards the sun, in this second of time, and the double of D B will measure the *attractive force of* the sun at that distance.

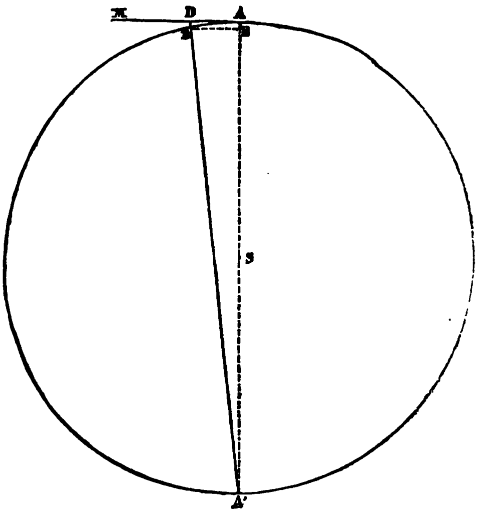

Fig. c.

Now, to find the length of D B we proceed as follows:—From the point B draw B E perpendicular to the radius S A; draw also from B a straight line to the point A' of the orbit diametrically opposite to the point A, and make A E equal to D B. It is plain that the arc A B differs insensibly from its chord, and may be replaced by it; so that the angle A B A', being in a semicircle, is a right angle. In the right-angled triangle A B A', we have the proportion

$$AE : AB :: AB : AA'$$

(Euc. 8 of vi.)

for B E cannot differ sensibly from parallelism with D A,

$$\text{therefore, } \frac{AE}{AB} = \frac{AB}{AA'}$$

$$\therefore AE = \frac{AB^2}{AA'} = \frac{AB^2}{2AS}$$

As the arc A B is the portion of the orbit described by the planet in a second of time, it measures the uniform velocity with which it moves; and we see that the distance A E, or B D, which the planet falls in that time towards the sun, is found by dividing the square of the velocity by twice the radius of the orbit. We have seen that the attractive force of the sun, acting at that distance, is measured by double the distance fallen in a second. The intensity of the force, therefore, at the planet, is expressed by the quotient of the square of its velocity by the radius of the planet's orbit.

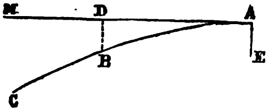

Fig. d.

We may now compare together the intensities of the forces which act at different distances, or on different planets, by means of the third law of Kepler. In order to this, let us suppose that planets, moving uniformly in circles round the sun, be situated at distances from that central body proportional to the numbers

1, 2, 3, 4, 5, &c. . . . . (A).

To obtain the velocity of one of these planets in its circular orbit, we must divide the length of the circumference by the number of seconds occupied in describing it; and to get the square of the velocity, we must divide the square of the circumference by the square of the number of seconds. But the squares of circumferences are as the squares of their radii, that is, in the present case, as the numbers

1, 4, 9, 16, 25, &c.

Also, from the third law of Kepler, the squares of the times of revolution are as the cubes of the radii of the orbits, that is, in the present case, as the numbers

1, 8, 27, 64, 125, &c.

The squares of the velocities of the several planets will, therefore, be related to one another as the numbers

$$1,\ \frac{4}{8},\ \frac{9}{27},\ \frac{16}{64},\ \frac{25}{125},\ \&c.$$

Or as the numbers

$$1,\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \frac{1}{5},\ \&c.\ .\ .\ .\ .\ .\quad (B)$$

And as the force acting on each planet is measured by the square of its velocity divided by its distance, we shall evidently discover the law by which the intensity varies, from one planet to another, by dividing the several numbers (B) by the corresponding numbers (A); that is to say, the forces at the several distances (A) will be as the numbers

$$1,\ \frac{1}{4},\ \frac{1}{9},\ \frac{1}{16},\ \frac{1}{25};$$

or inversely as the *squares* of the distances (A). And in this way is the law of solar attraction established; and not only does this follow from the third law of Kepler, but this law itself may be shown to be a necessary consequence of the law of force just established. For, since the length of orbit, divided by the uniform velocity, gives the periodic time in seconds, and that circumferences vary as their radii, the periodic time varies as the radius divided by the velocity. Hence, calling the periodic time P, the radius R, the velocity v, and the force F, we have in symbols,

$$P \propto \frac{R}{v} \therefore P^2 \propto \frac{R^2}{v^2}. \quad \text{Now } F = \frac{v^2}{R}.$$

If, then, $F \propto \frac{1}{R^2} \therefore \frac{v^2}{R} \propto \frac{1}{R^2} \therefore \frac{R^2}{v^2} \propto R^3 \therefore P^2 \propto R^3$; so that if the force vary inversely as the square of the distance, and bodies move round the common centre of attraction in circles of different radii, the squares of the periods of revolution will be to one another as the cubes of the distances.

Having proved that the relation observed by Kepler between the periodic times and the distances was a necessary consequence of the above-named law of gravitation, Newton sought to determine whether the forms of the planetary orbits were not also a necessary consequence of the same law; and he accordingly found that, under the influence of that law of attraction, it was impossible for a body to move in any other curve than a *conic section;* that is to say, the orbit must be either an ellipse (of which a circle is a particular case), a parabola, or an hyperbola. This may be proved as follows:—

Let v be the velocity in a circle of radius R; v' the velocity in a parabola, ellipse, or hyperbola, whose radius of curvature at perihelion is R′; then

$$F = \frac{v^2}{R} = \frac{v'^2}{R'}; \therefore R' = R\frac{v'^2}{v^2}$$

Now R′ is what, in the doctrine of the conic sections, is called the *semi-parameter* of the curve; and R being the distance of the focus from the vertex of the curve, it is also proved in that doctrine that the curve will be a parabola, an ellipse, or an hyperbola, according as R′ is $= 2$ R, $\angle$ 2 R, or $>$ 2 R. Hence, if $\frac{v'^2}{v^2} = 2$, the orbit is a parabola; if $\frac{v'^2}{v^2} \angle 2$, the orbit is an ellipse; and if $\frac{v'^2}{v^2} > 2$, the orbit is an hyperbola. And as one or other of these conditions must necessarily have place, whatever be the velocity v', it follows that the planets and comets must all move in one or other of these three curves.

The above established law of force is thus competent to account for the revolutions of all the planets and comets round the sun, as also for the motions of those comets in the system, if any there be, which describe parabolic or hyperbolic orbits, and which consequently proceed onward in space continually and never return.

The same law of force accounts, in like manner, for the revolutions of the satellites round the planets to which they belong, for these all describe elliptic orbits.

This general statement, however, is, in strictness, only an approximation to the truth, although an approximation so close as to be but very little at fault in explaining the great phenomena of the planetary motions, and therefore regarded by Kepler as strictly accurate. In the foregoing reasonings the attracting body has been supposed to be absolutely fixed in space. This is not entirely consistent with Newton's law of *universal gravitation*, according to which all bodies in the universe mutually attract one another with a force directly proportional to their masses, as well as inversely proportional to the squares of their distances. On account of the masses of the planets being very small in comparison with the mass of the sun, the rejection of the planet's attraction on the latter body involves an error comparatively insignificant.

"It is true, that when observations are carried to a high degree of precision, and when each planet is traced through many successive revolutions, and its history carried back, by the aid of calculations founded on these data, we learn to regard the laws of Kepler as only *first approximations* to the much more complicated ones which actually prevail; and that to bring remote observations into rigorous mathematical accordance with each other, and at the same time to retain the extremely convenient nomenclature and relations of the ELLIPTIC SYSTEM, it becomes necessary to modify, to a certain extent, our verbal expression of the laws, and to regard the numerical data or *elliptic elements* of the planetary orbits as not absolutely permanent, but subject to a series of extremely slow and almost imperceptible changes. These changes may be neglected when we consider only a few revolutions; but going on from century to century, and continually accumulating, they at length produce considerable departures in the orbits from their original state."—*Herschel.*

From the principle of universal gravitation, the third law of Kepler, from which the law of attractive force has been deduced above, requires to be modified as follows:—The cubes of the mean distances of the planets from the sun are as the products of the squares of the periodic times, by the sum of the masses of the attracting and attracted bodies. In the original enunciation of the law, the mass of the attracted body was neglected; the trifling amount of error thus introduced—seeing that even Jupiter, the largest of the planets, has less than a thousandth part of the matter contained in the sun—is evidently such as to render the departure from strict accuracy too minute to be detected, except by delicate observations extended over a long period of time.

Perturbations of the Planetary Motions.—In consequence of the law of universal gravitation, each planet exercises an influence over every other planet; these mutual disturbances necessarily modify, in a slight degree, as just noticed, the orbits of all, and occasion what are called *perturbations* and *inequalities*. But all these are fully accounted for and satisfactorily explained by referring them to Newton's great principle. Calculations founded upon this principle enable us to predict the position of a planet at any future time with a degree of accuracy that appears little short of marvellous, when we consider the complication involved in the mutual actions of the sun and all the planets on one another. "The motion of Jupiter, for instance, is so perfectly calculated, that astronomers have computed, ten years before hand, the time at which it will pass the meridian of any specified place; and we find the prediction correct within half a second."—*Airy.*

No physical law has ever been enunciated which long and careful observation more completely verifies than Newton's law of universal gravitation. The distances and masses of the planets being known, the effect of the attractions of all, in modifying the path of each, can be correctly ascertained, though only by aid of investigations of the most difficult and recondite character; but one of the most remarkable attestations to the truth of this universal principle is that which has been furnished in our own day by Mr. Adams of Cambridge. Knowing that there were certain small perturbations of the orbit of Uranus that the combined attractions of all the other planets of the system were insufficient to account *for*, and fully confiding in the competency of Newton's law to explain every movement of the heavenly bodies, he had the boldness to pronounce that a yet unseen planet existed beyond the bounds of what had till then been regarded as the remotest planet in our system; and, taking the unaccounted-for perturbations of Uranus as data, he had science enough to assign the place where, at a specified time, the *new* planet would be found. An eminent French mathematician, Leverrier, had independently, and but a short time afterwards, arrived at a similar conclusion; and upon transmitting the necessary instructions to a German astronomer, Galle, the predicted planet, Neptune, was revealed to his telescope on the evening of the day that the communication reached him.

One of the most important results to which the various researches into the planetary perturbations have conducted is, that the major axes of the variable elliptic orbits always preserve the same values. The disturbances to which each planet is subjected, by the action of the others, affect all the elements of its elliptic path, except its major axis, which throughout every other change continues itself invariable; this invariability insures at the same time the invariability of the period of revolution, agreeably to the third law of Kepler. And thus the stability of the system is secured. We are indebted for this great truth to the genius and researches of Lagrange.

Masses of the Planets as compared with the Sun.—The mass of a planet is ascertained by observing the disturbances which its attraction produces in the orbits of other planets, or, if it have a satellite, by determining the force exerted upon that satellite. The mass of the sun being represented by unit, the masses of the planets have been estimated as in the following table:—

Names of the Planets.	Masses.	Names of the Planets.	Masses.
Mercury . . .	$\frac{1}{2025810}$	Jupiter . .	$\frac{1}{1050}$
Venus . . .	$\frac{1}{401841}$	Saturn . . .	$\frac{1}{3500}$
Earth . . .	$\frac{1}{354936}$	Uranus . .	$\frac{1}{24000}$
Mars	$\frac{1}{2680337}$	Neptune . .	$\frac{1}{14446}$

It must be understood that the *mass* of a body is not the same thing as its bulk or volume. The mass is estimated by volume and density; if one body have only half the volume of another, but be twice as dense, their masses are equal. If a planetary body be so minute in volume, or so feeble in density, as to render its attractive energy too small to perturb sensibly the motions of the other planets, the mass of that body cannot be determined. On these accounts there is still some uncertainty as to the mass of Mercury; and respecting the masses of the small planets between Mars and Jupiter—asteroids, as they have been called—we know nothing. In like manner, as respects the comets, all we know is, that their masses must be exceedingly small; that is, that they contain but a very small quantity of matter, as they produce no appreciable disturbance of the planetary movements; indeed, it has happened that a comet has crossed among the satellites of Jupiter without occasioning any observable disturbance even in the motions of those comparatively small bodies.

Before concluding this section on the principle of universal gravitation, we must devote a short article to one of the most noticeable effects of that principle on our own planet, the phenomena of the tides.

The Tides.—The principal physical cause of those periodical oscillations of the surface of the ocean called *tides* is the attraction of the moon; the sun contributes to the general results, but in a far less degree.

Suppose the earth were entirely covered with water: from the mutual attraction of all its parts, its surface would assume an exactly spherical form. Its rotation on its axis, however, as at present, would cause the equatorial parts to recede from the centre, and to bulge out.

This receding from the centre, of the equatorial regions of the earth, is caused by a new force, opposed to that of gravitation, being brought into operation by the planet's rotation. It is called the *centrifugal force*, from its imparting a tendency in the outer particles of the rotating mass to fly off or recede from the centre. Just such a tendency would be excited if the earth were at rest, and an attractive external influence were uniformly diffused round its equatorial regions, and penetrating only to a small depth below the surface. If, however, the external attractive influence, instead of being thus diffused as a ring round the earth, were confined to a limited place, the bulging out of the waters would of course be confined to the place immediately under the attracting body; and if the force of attraction extended to the centre of the earth, the elevation of the waters would evidently be due not to the force acting on their surface, but only to the difference of the forces acting on the surface and on the centre. The waters would, as it were, be drawn away from the earth by the influence of this difference.

If the attractive force of the external body extended to the opposite part of the earth's surface, then the force on the centre being greater than that on the remote surface, the earth would, as it were, be drawn away from the waters at the further surface, and there would in consequence be also a bulging out there; and as the difference of the forces on the centre and near surface must be very nearly the same as the difference of the forces on the centre and remote surface, the waters will be heaped up to nearly the same extent at both places.

The application of these general considerations to the moon and the tides will sufficiently show that the bulging out of the waters by the attraction of the moon M (Fig. *e*) is nearly equal at the two places under the moon marked *m* on the earth's surface, diametrically opposite to each other, causing the waters of the entire globe to assume a spheroidal form, the elevations at *m* necessitating a subsidence or depression at C and D. As different portions of the earth's aqueous surface are brought under the direct influence of the moon, those portions in like manner bulge out, producing a continuous succession of tides.

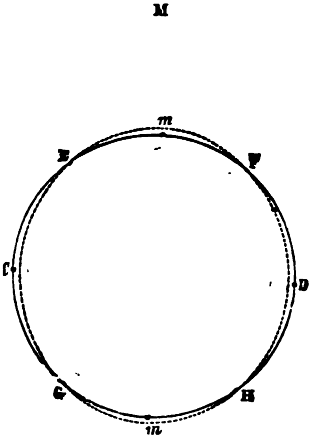

Fig. *e*.

If the moon alone acted, and always moved directly over the equator, the interval between two consecutive high tides would be just half a lunar day. In like manner did the sun alone act on the waters, its motion being supposed to be directly over the equator, the interval would be just half a solar day; but the combined action of both these bodies causes, of course, the intervals to vary.

In the actual state of the earth, whose surface presents both land and water, the phenomena of the tides cannot be expected to be in strict agreement with what they would be if no solid matter existed on it. Headlands, coasts, the shallowness and contraction of channels, &c., continually obstruct the free motion of the tide-wave; these obstructions often greatly delay the time of high water, and cause a much greater rise than would take place if the advance were unimpeded. Such local influences, indeed, greatly modify the results of pure theory, both as to the time and height of high water at different ports. Special observations, therefore, at each place are necessary to supply the proper data for predicting the time of high water, and to determine what is called the "establishment of the port." The height of the tide, too, often depends upon the set and force of the wind; but, "of all the causes of difference in the height of the tides, local situation is the most influential. In some places, the tide-wave, rushing up a narrow channel, is suddenly raised to an extraordinary height. At Annapolis, for instance, in the Bay of Fundy, in Nova Scotia, it is said to rise 120 feet. Even at Bristol, the difference of high and low water occasionally amounts to 50 feet."—*Herschel.*

The same influences which produce the tides of the ocean, operate also on the surrounding atmosphere of the earth; and produce tides in *it*. It is popularly supposed, too, that the weather is subjected to the moon's influence; but careful and long-continued observation, by competent persons, has shown this supposition to be fallacious. A change in the moon is imagined to be attended with an immediate change in the weather; it being overlooked that the moon changes her position, and passes through her several phases, by imperceptible gradations; which is incompatible with a sudden change in the weather.

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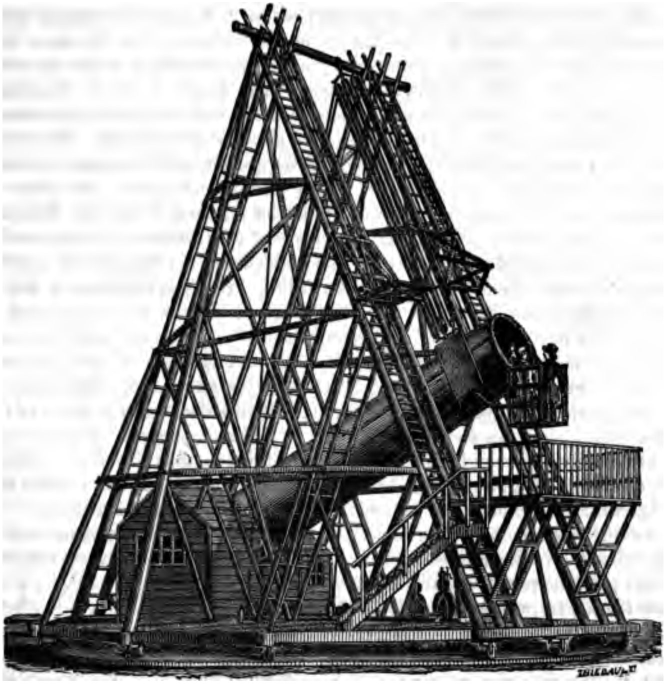


SIR JOHN HERSCHEL'S TELESCOPE.

PRACTICAL ASTRONOMY.

E have now reached—perhaps we are somewhat beyond—the point where the tronomer's labours must have ceased but for the invention of instruments, by whose d he has been enabled to sweep the heavens, and reveal some of its hidden mysteries. e propose, at this stage of our labours, to give some description of these instruments, the principles on which they are constructed, of the manner in which they tend to e augmentation of our power of sight, and the most convenient methods of mounting em, with directions for their practical application.

The apparent size of an object, which, properly speaking, is the size of its image the retina, depends on the distance between the eye and the object. Thus, for stance, if the object be M (Fig. 6), and the eye be placed at *o*, the ht line which joins the extreme ints, A and B, will be seen under angle A *o* B. If the eye be placed the point *o′*, or at half the distance at which it was previously situated, the line

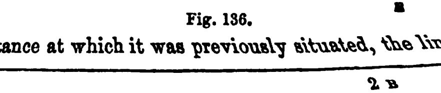


Fig. 136.

A B will, in the same manner, be viewed under the angle A o' B, which will be the double of the preceding, supposing that the line A B is small in comparison with the distance of the object M from the eye of the observer. In the same manner, the angle A B will be three or four times greater, as the distance of the object is one-third or one-fourth of the primitive distance. The angle A o B is, therefore, the measure of its *apparent* size (which is different from its *real*, the latter being invariable), and is called the *visual* angle, or the apparent *diameter* of the object. The surface of the body becomes four, nine, and sixteen times greater, as the distance is decreased to one-half, one-third, and one-fourth of the primitive distance. In other words, the diameter of an object varies inversely as the distance of the eye from it, and its apparent area varies inversely as the square of this distance. The power of the eyesight is limited, and it is generally held that when the diameter of an object is less than a minute, the object ceases to be visible. But, the *brightness* of the image performs as important a part in this respect as its *size*, as is easily proved by the visibility of the fixed stars and of the planets, none of the former of which can subtend an angle as great as a second. Where the colour or brightness of an object, however differs less from the ground on which it is placed, the above rule holds true. The *distinctness of vision*, as every one is aware, depends likewise on the distance of the eye from the object; and this distance is variable for different persons. Even the one eye of the same individual may have a longer range of vision than the other. Every one knows, also, that if an object be held at a greater or less distance from the eye than that which is customary or natural, in either case it becomes indistinct.

In regard to the *brightness* of any object, according as it is more or less removed from the eye, the following considerations are to be taken into account. From any bright object as M, rays of light are sent in all directions, and from the point m, as from any other point on its surface, these rays are sent before it, as represented in Fig. 137. Of all these rays of light, the eye only receives those which penetrate through the pupil $a\,b$. If the eye now approaches the object M, so that its distance is one-half of what it was in the previous case, and its position $a'\,b'$, the diameter of the cone of light will be only one-half what it was in the previous case, and consequently the area of the section in that part is only one-quarter of that at the base $a\,b$, where it was equal in area to the surface of the pupil of the eye. Hence it follows that the opening of the pupil at $a'\,b'$ will take in a greater number by four times of rays than when it was situated at $a\,b$, so that the light received from the point m will be four times as much in the latter case as in the former. At the same time, however, as the eye approaches the object M, a greater portion of its surface is taken in, and the position m becomes four times as great. The bright light is thus spread over a greater surface, and in a like proportion; and it follows that the brightness of the surface is unchangeable. If it happens that this is not the case in nature, it must be remembered that the interposition of the air causes a loss of light, and even a change in the colour of objects; for distant mountains appear blue, as likewise does the sky—being the effect when viewed through great depths of the atmosphere. Without this, a white object would appear of equal brightness whether it were seen close at hand or at a distance.

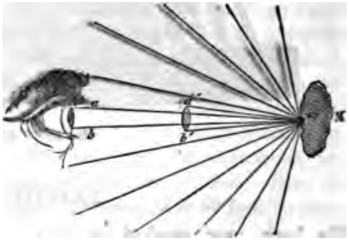


Fig. 137.

Lenses.—Many solid bodies, and a greater portion of liquid ones, are transparent; but, in order that the former become perfectly so, it is necessary that their surfaces should be highly polished, which condition is fulfilled, in liquids, by the natural effect of gravity. In the grinding and polishing of glass and crystal, or even reflecting metals, into various shapes, it was early discovered that various effects were produced; but none more extraordinary than those which took place in the apparent magnitude of objects when viewed through these artificial media. The fundamental law on which all those changes are based is, that when a luminous ray passes obliquely from one transparent medium into another, it departs from its primitive direction, and undergoes a *refraction.* On this law the theory of lenses is based.

There are two different species of lenses, the surfaces being, in both cases, spherical, or one of the faces may be plane. The first are those in which the surface is convex or converging, and the other in which it is concave and diverging. The converging glasses are those which are doubly convex (Fig. 138), the form being *lenticular* (whence they derive their general name of lenses)—2° those which are plano-convex (Fig. 139), and 3° concave convex (Fig. 143), the convexity being in this case more considerable than the concavity—and which are likewise known by the name of *menisci,* in consequence of the form of their section. The converging glasses are those which are doubly concave, as in Fig. 141; plano-concave, or convex-concave, the concavity being more considerable than the convexity, and likewise known as menisci. The *radii* of lenses are the radii of the spheres of which their surfaces are the segments.

Fig. 138.

Fig. 139.

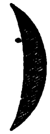


Fig. 140.

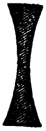


Fig. 141.

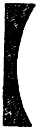


Fig. 142.

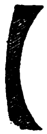


Fig. 143.

If we expose convex glass to the rays proceeding from the sun, they will be converged to a point, A (Fig. 144); and if we remove the lens more or less away from the surface on which the converging rays are received, this bright spot (A) will be of greater or smaller extent. To that point where it is of the smallest dimensions, the name of *focus* has been given, and its distance from the nearest surface of the lens the *focal distance.* If we turn the lens round, the same effect is produced; and if both the surfaces have the same radius, the distance will be alike in both cases, though it will vary but very slightly should they be different—even in the case of a *meniscus* lens.

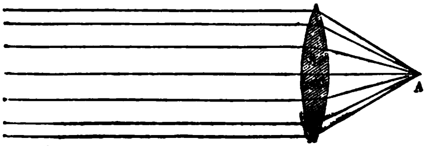


Fig. 144.

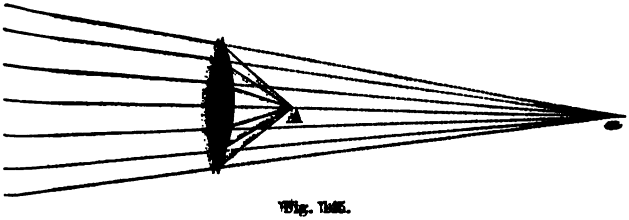


Fig. 145.

The line *a* A (Fig. 145) is termed the *axis* of the lens, and the point o its optical centre. In the above case the rays are supposed to pass parallel to the optical axis; but if they proceed from a luminous point, A, a short distance from the axis (Fig. 146), or rather if the parallel rays make a small angle with the optical axis, the manner in which they converge to a focus, *a*, is the same. In both cases they pass through the optical centre *o* of the lens. In concave lenses, the parallel rays of light which pass from an object are rendered divergent; or if already divergent, are rendered more so by their interposition, as in Fig. 147.

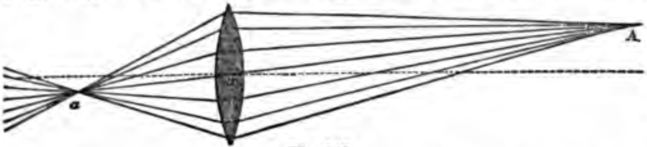


Fig. 146.

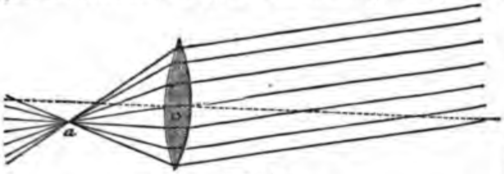


Fig. 147.

Supposing that, instead of proceeding from a point, A, the rays of light from a luminous object, A B (Fig. 148), pass through a convex lens: In this case the rays of light coming from the point A will, as in Fig. 148, converge to the point *a*; and those from the point B to the focus *b* (Fig. 148); and in the same manner, all the other points of the object A B will be represented by opposite points in the image formed by the lens. It thus follows that the image is reversed, and if the eye be placed at the focus *a b*, the object A B will appear turned *upside down*; but in all other respects, a perfect picture of the object A B will be perceived. This

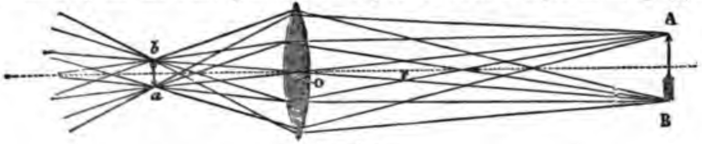


Fig. 148.

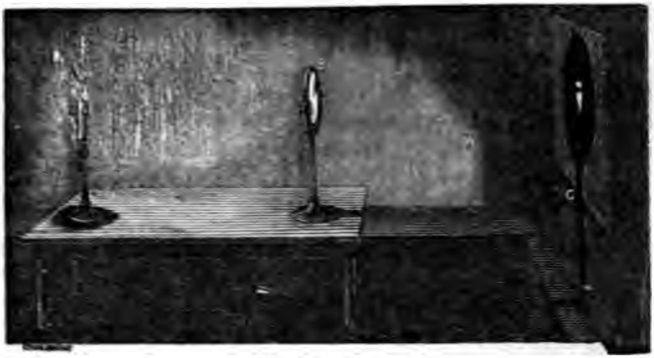


Fig. 149.

effect may likewise be seen by placing a candle, A (Fig. 149), in a dark chamber, at a certain distance from the mounted lens B, the light from the candle passing through the optical axis of the lens, to the opposite wall, C, which is at the proper distance from the lens to receive a well-defined image of the flame of the candle: A reversed image of the candle may here be perceived, as well as of that portion of the candle which is illumined by the flame.

The well-known effect of convex glasses in magnifying small objects will be seen from the following diagram (Fig. 150). The object A B being placed between the lens and its focus E, the rays of light proceeding from the point A do not lose all their divergence, but appear to come from a more distant point *a*, formed by the prolongation of the line O A; and in the same manner the point B seems to be situated at *b*, formed by the prolongation of the line B O. To the eye situated at the other side of the lens, therefore, it will appear as if the object A B was replaced by the image *a b*, and the latter object will appear more or less distant from the eye, according as the object A B is nearer or more removed from the focus of the lens. The lens can be so shifted in respect to the object, that the image *a b* will appear distinct and well defined as well as magnified. We can readily perceive that the image *a b* is greater and more distant when the focal distance is smaller, and, in consequence, that a lens magnifies so much the more as its focal distance is less. The magnifying power of lenses is somewhat illusory, and nearly every one judges differently in estimating the magnified size of the object under examination. This may arise from two causes—in the first place, the distance of distinct vision is different for almost every person; and, in the second place, when we look through magnifying glasses, almost every guide which serves to regulate the judgment on the distance and size of the object looked at is removed.

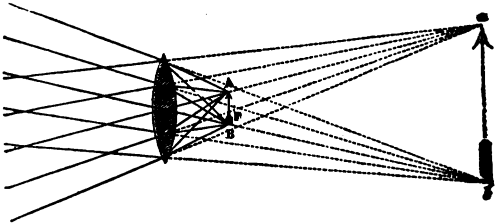


Fig. 150.

Telescopes.—Every convex glass, as we have seen, produces an image of the object from which it receives rays of light; and by a combination of such glasses, viz., by examining the image formed by one lens (as if it were a real object) by means of another, and thereby magnifying the image as explained in the last paragraph, distant objects can be seen with a distinctness unapproachable with the naked eye. The most simple and most common description of the telescope, used by the astronomer, is that formed by two convex lenses, L and L′ (Fig. 151); the first of which, from being turned in the direction of the object, is termed the *object-glass*, the second the *eye piece*. In this the object A B, passing through the convex lens L, forms an image *a b* in the focus of the lens. The second lens serves to magnify the reversed image *b a* in the same manner as if it were as a

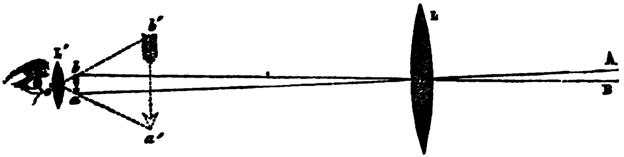


Fig. 151.

tangible object. The image $b\ a$ is not always at the same distance from the object-glass, but varies more or less according to the distance of the object; and when the latter is so far removed that the rays which fall on the object-glass may be considered as parallel—which is the case with all celestial objects—the image $b\ a$ falls in the focus of the lens. As the object should be seen with the requisite distinctness at $b'\ a'$, which is the distance of distinct vision, and as the latter varies almost with every person, it is necessary that the eye-piece should be drawn in or out from the image $b\ a$ in order to give perfect definition. It will be seen from the diagram that the telescope does not, like the microscope or magnifying-glass, increase the *natural* size of the objects viewed; for the image $b'\ a'$ is much smaller than the object A B, which is at a great distance from the object-glass: it only tends to increase the *apparent* size of an object when viewed from a distance.

To compare the size of a distant object when viewed with natural and telescopic sight, we have only to compare the angles $A\ o\ B$ (Fig. 151), or $a\ o\ b$, which is the angle subtended by the object to the unassisted vision, and the angle $a'\ o'\ b'$, which is the angle subtended by the image to the eye of the observer. The proportion between the relative sizes of the object A B, and the image $a'\ b'$, is consequently the same as that between the angles $a\ o\ b$ and $a\ o'\ b$, and this is what is termed the magnifying power of the telescope. The angles $a\ o'\ b$ and $a\ o\ b$ being always small, they may be regarded as in the inverse proportion of the two points o and o' from the image $a\ b$; or, in other words, that as these distances may be regarded as the focal lengths of the object and eye-glasses, that the magnifying power of the telescope is in the same proportion. This instrument allows a large *field of view* (or the circular space which the eye would take in without the assistance of any telescope), for this depends on the dimensions of the instrument at $a\ b$, or the common focus of both lenses. Equally important with the perfect definition of the object, is, as already stated, the *brightness* with which it appears when under examination. If a ray of light passes from any point, M (Fig. 152), of an object, it will fall on the whole surface of the object-glass, and thence through the same point, m, of the focus, $a\ b$, after which it will form a pyramid of rays, the divergence of which will be lessened by passing through the eye-piece in such a manner that they will appear to come from the point m' in the image $a'\ b'$. If the eye viewed the image, $a\ b$, without the interposition of the eye-piece, it would only receive a portion of the light and rays emitted from the point M, many of the outer diverging rays having been lost, the opening of the eye being too small to take the whole in. But by employing a second convex glass, the divergence is diminished, and the eye receives the whole of the light emitted from the point M, which falls on the object-glass. Comparing, therefore, the brightness of an object, as seen with the naked eye and seen through a telescope, and supposing, in the latter case, that all the light passes through the lenses, it would follow that the light received by the eye, in using a telescope, would be as many times greater than that of the eye, as the surface of the object-glass exceeds that of the pupil of the eye. This, however, is only theoretically true, for there is a considerable absorption of light by passing through the lenses, as well as by reflection

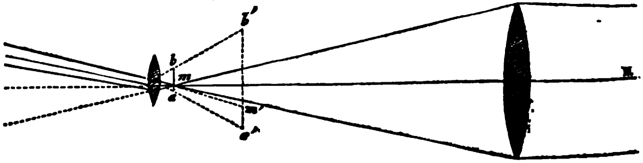


Fig. 152.

from their surfaces. If the size of the objects was increased by the telescope in the same proportion as the brightness of their surfaces, it would follow that the brightness would always remain the same. The magnifying power, however, is quite distinct from the illuminative power of the telescope, the first depending on the proportion between the focal distance of the two lenses, the latter upon the area of the object-glass. It thus happens that the brightness of the image will vary greatly with the power applied; and, with high powers applied to the same telescope, the objects become dim and indefinite, and the two excellencies of brightness and good definition are very difficult to be obtained.

This description of telescope was not, however, the first invented. The optical principles on which its construction is founded were not started by Kepler until some time after the discovery of Galileo. In this latter form, the object-glass, as in all other instruments of this sort, is a convex glass, but the eye-piece is a concave one, the focus of which is very short. The disposition of the lenses is seen by the diagram (Fig. 153.) The reversed image of distant objects produced by the object-glass L is formed in *a b*, but the concave eye-piece is placed between the two, in such a manner that the eye placed behind the lens L′, instead of the image *b a*, will see *a′ b′*; and the eye-piece can be so adjusted that the image *a′ b′* will appear well defined. This telescope magnifies the apparent diameter of objects as many times as the focal distance of the eye-piece is contained in the focal distance of the object-glass. It will not admit of any considerable magnifying powers, the field of view being very small, on which account it is never used at the present time as an astronomical telescope. On the other hand, it is very convenient as a pocket telescope, as it does not reverse objects; and as the eye-piece is placed between the object-glass and its focus, it is shorter than the common telescope. In the common opera-glasses, which are of this construction, the magnifying power does not exceed three; but in some of those made by Galileo the magnifying power reached to thirty-two. Another form of the erecting or terrestrial telescope is that imagined by Rheita, in which three convex glasses, of equal but short focus, are fixed in a tube, and form the eye-piece. The first of those lenses, or that most distant from the eye, would give a reversed image, which, passing through the second one, produces an erect image, and finally, as it is before the focus of the third lens, it is viewed and much magnified by this one. The magnifying power of this is measured as in the preceding ones, being in the proportion of the focal distances of the object-glasses and one of the lenses of the eye-piece. The magnifying power of a telescope may be roughly determined by comparing the sizes of an object, such as the sun or moon, as seen with it and with the naked eye, and the extent of the field of view may be found by taking the diameter of the sun at about half a degree. The former, however, is determined more exactly by an instrument called the *dyanometer*, which serves, when a telescope is directed towards the sky, and a sheet of paper is held behind the eye-piece, to measure the diameter of the luminous circle which falls upon the white surface, and the best definition of which is found by trial. By dividing the diameter of

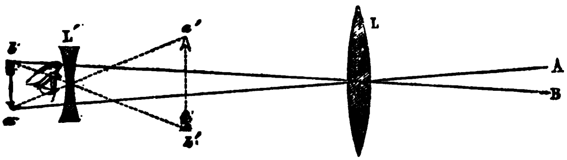


Fig. 153.

the object-glass by the diameter of this circle, the magnifying power of the telescope is obtained. The luminous circle is itself an image of the object-glass, and is contained as many times in the latter as the telescope magnifies distant objects.

Hitherto the object-glass has been supposed to be formed of one piece and of one sort of glass, but in all modern instruments this is not the case. The image formed by a convex glass is situated at a greater or a less distance from it, according to the refracting power of the glass, and is so much shorter as the refracting power is more considerable. In addition to this, the different colours of which white light is composed are not equally refracted and cannot have the same focus. If the eye of an observer is situated at the focus of the lens, he will not see the image perfectly defined and colourless, but it will appear surrounded with the prismatic colours; the violet rays of the object is more strongly refracted than the red ones, and will be thrown nearer the lens. In addition to this source of indistinctness of the image, there is another less important, known by the name of *spherical aberration*, which depends on the figure of the lens, there being no curvature in which all the rays of light coming from any object are *exactly* united in a common focus. In order to get rid of the first source of indistinctness, in using a single lens for the object-glass, it was with high magnifying powers found necessary to have the lens of very long focus (some of those used by the earliest observers were 300 feet in length), for by this means the causes which led to the formation of colour, viz., the considerable curvature of the lens, were diminished and the larger images were formed. By an examination of different species of glass, Dollond found that some sorts refracted light and dispersed the colours much more than ordinary glass; and by passing a ray of light through two prisms made of different glass, he had the satisfaction of finding that a white light was transmitted. After this, he employed the two sorts of glass in forming an object-glass, which would transmit light without decomposing it, which he effected by uniting a convex lens of crown-glass, of a greenish colour, with a concave lens of flint-glass, of a white tint. (Fig. 154.) When a ray of light falls upon this object-glass, it is acted upon by both—that of the crown-glass renders it convergent and decomposes it, the concave flint lens, on the contrary, destroys the effect of the first, and an uncoloured image is formed at the focus. By this beautiful discovery, the refracting telescope has been made a convenient and easily managed instrument, to which higher powers can now be applied, and with better effect, to one of 10 feet in length, than to those formerly made, and which were some hundreds of feet in focal distance.

Fig. 154.

It is a matter of considerable moment to have the telescope mounted as steadily as possible, and to be able to direct it easily to any particular part of the heavens, so as to follow any object with the requisite facility. In the following figure (Fig. 155) a telescope-stand is represented, which serves these useful purposes. It consists of a firm support, A, resting on the ground by means of the feet B′ B′ B′ and the rollers B B B, and two moveable supports, C and D D, the former of which directly supports the telescope E, and is connected with the stand A by means of hinges at M M, by which it can be inclined more or less to the horizon. The second support, D, is connected with the first in a similar manner by the hinges *n n*, by which means the angle between the two supports C and D may be varied at will. The inferior beam, *o o*, of the branch D, should slide along the inclined side of the stand A A, and this motion produces the requisite elevation or depression of the extremity *n n*, which turns on the hinge *m m*. Two endless chains, *q q*, pass round the stand A, and are attached to the ends *o o* of the branch D.

An axis, *r*, terminated by two handles, *s s*, carries a pinion which works in a wheel mounted on a second axis, *t*; this second axis is furnished with two pinions at both ends, of which the teeth catch in the links of the endless chain *q q*. In turning round the handle *s s*, the chain is thus worked up and down; the portion *o o* of the support D slides on the inclined plane *p p*, and the projecting branch is raised more or less. The telescope, E, rests in *u u*, the latter part being connected with the support C by a pin

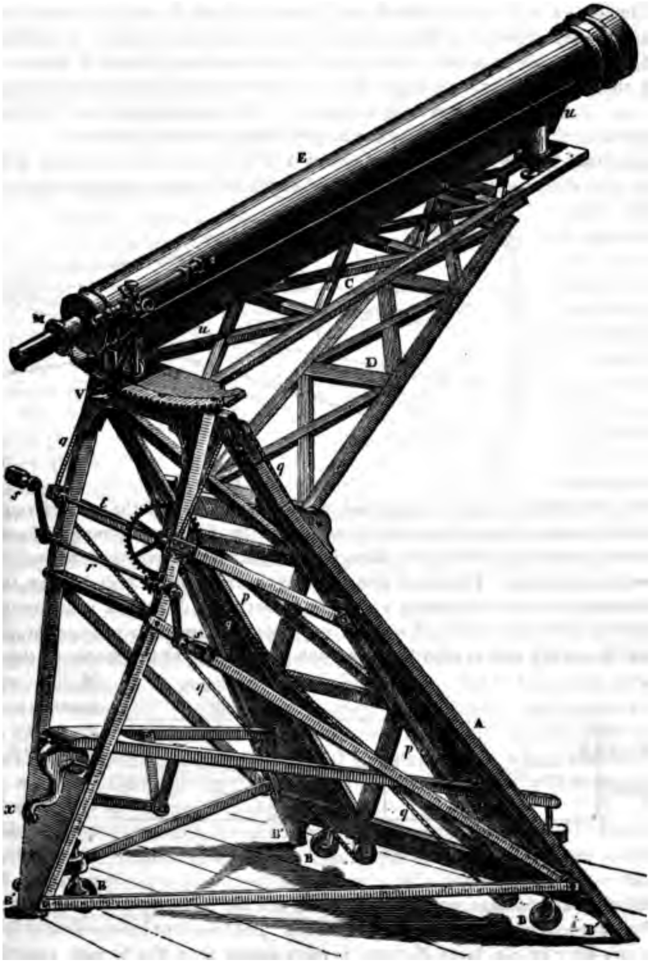


Fig. 155.

at the object end, round which it can easily be turned as on a pivot. At the eye end of the telescope, the support *u u* rests upon two wheels, and a milled head, V, attached to a pinion, works in the toothed edge of the support C in such a manner that in turning the milled head it turns the telescope to the right or left. The observer, by turning one of the handles, *s s*, with the one hand and the milled head, V, with the other, is thus able to change the direction of the instrument according to his wish. A smaller telescope, Z, is commonly attached to the tube of the larger one, which is known by the name of the *finder*. Having a larger field of view than the principal telescope, the observer can discover the object he is in search of with great facility, and by placing it in the centre of the finder, he guides the larger telescope directly upon it.

Reflecting Telescopes.—If an object A B is placed before a concave reflecting mirror M (Fig. 156), a similar effect is produced as when the rays of light proceeding from such an object passes through a convex lens. The ray of light passing from the point A will be reflected to the point *a*, while those passing from B will take the direction *b*, and the inverted image *a b* will be produced. The image *a b* can be magnified in the same manner as the image formed by a convex lens, though evidently not with the same facility; for here the image is formed between the object and the mirror, and there is a difficulty in knowing in what manner the eye-piece is to be placed. To obviate this difficulty the inventor of this telescope, the celebrated Gregory, thought of placing a smaller concave lens N (which would only intercept a portion of the rays proceeding from any object) after their passage through the image *a b* (Fig. 157), and to send them through the opening at the centre of the mirror M, in such a manner as to produce a second image, *a′ b′*, which could be examined with the requisite distinctness by the eye-piece *o*. By this combination an erect image would be produced, and the telescope would be used in precisely the same manner as the Galilean telescope. The telescope independently invented a few years later by Sir Isaac Newton, is more simple than the former, receiving the rays which pass from the great mirror on a plane reflector N (Fig. 158), inclined at an angle of forty-five degrees, before they arrive at the focus *a b*, and produces the image *a′ b′*, which can be examined by means of the eye-piece O. In this form of the reflecting telescope, the observer looks in a direction perpendicular to that of the object observed. In order to obtain as much light as possible, a considerable portion of which was lost

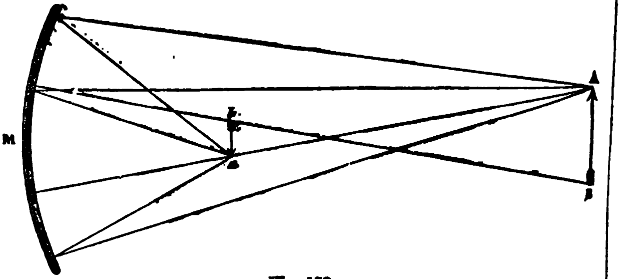


Fig. 156.

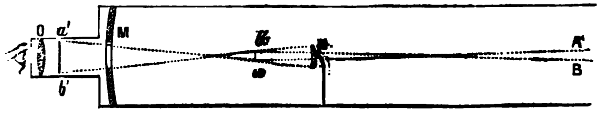


Fig. 157.

by being reflected from successive surfaces, Herschel conceived the plan of inclining the great concave mirror at a small angle to the object observed, by which means the rays of light proceeding from the object A B (Fig. 159) were reflected to *b a*, and the image thus formed could be examined in the ordinary manner by the eye-piece, the back of the observer being turned to the object. This disposition (termed by Herschel *front view*) is, however, only suited for instruments with large apertures, in which the loss of light is less than when reflected from a second mirror.

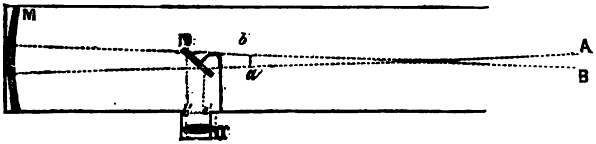


Fig. 158.

The great forty feet telescope of Herschel, with a mirror of four feet in diameter, was mostly used in this manner, the observer being placed in a gallery at the end. The engraving at the head of this section of our work gives a view of this magnificent instrument, which was moved upwards and downwards by means of numerous ropes and pulleys, whilst the motion to the right and left was facilitated by rollers. With this mighty tube Herschel was able to apply a magnifying power of six thousand! Unfortunately it was too much exposed to the weather; the polished mirror was dimmed in one night by the damp atmosphere and the instrument remained in use for only a very few years, when it was found unfit for service.

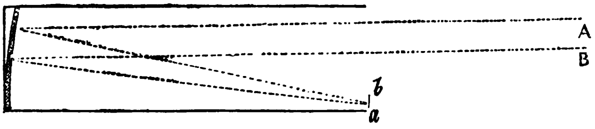


Fig. 159.

Instruments for Measuring the Angle.—Hitherto our attention has been directed to instruments of great magnifying power, intended to bring distant objects nearer to the observer. We have now to speak of graduated instruments of more delicate construction, by whose aid the astronomer is enabled to measure, with wonderful accuracy, the size and distance of celestial objects; and by which also is obtained the true solar time, by comparing their angular distances from other well-known objects; in other words, by measuring their angle.

In order to accomplish this, let us imagine the circumference of a circle described on its plane, having its summit as a centre. The length of the arc of a circle is described between the two sides of an angle, valued by means of a particular arc taken as unity.

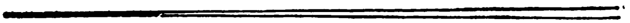


Fig. 160.

This arc, by common consent, is taken as the three hundred and sixtieth part of the entire circumference, which is called a degree, marked thus, 1°; and the angle which corresponds with it is the angle of a degree (Fig. 160).

It rarely happens, however, that the angle we seek to determine comes out exactly in degrees. It becomes necessary, therefore, to divide the degrees into fractional parts. For this purpose the degree is divided into sixty parts, called minutes, written thus, 1′. further accuracy the minute is again subdivided into sixty parts, called seconds, wri thus, 1″. When the instruments which we are about to describe are properly arranged, and the two telescopes are presented to their proper side of the object whose position or magnitude is to be measured, the angle is read off the graduated scale which surrounds the circumference of the circle by means of an index, which extends its limb over the whole length of the arc.

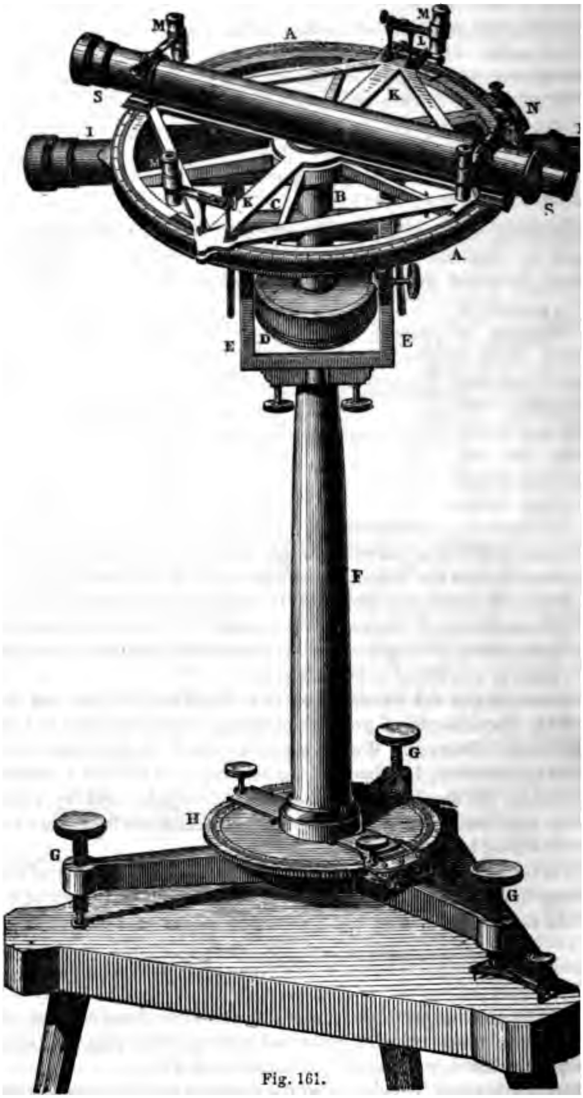


Fig. 161.

In measuring the angle by the visual rays which abut on each side of an object, two operations are required. The two rays of the graduated circle must coincide with the two sides of the angle, which is effected by viewing it successively in the direction of each of its sides; the object being to value the degrees, minutes, and seconds contained in the arc of a circle comprised between the two

The Repeating Circle consists of a graduated circle A A (Fig. 161) divided into 360° with their respective fractional parts, and fitted with two telescopes furnished with micrometers, the telescopes varying in power according to the size and value of the instrument, and the whole mounted on a pedestal, as represented in the figure. It is also so arranged as to permit of its being turned in any required direction. The circle, A A, turns on its own plane round an axis implanted in it perpendicularly and in its centre. This axis traverses a socket, B, which is fixed to the horizontal axis, C, and is terminated by the weighted drum of the pedestal D. This weighted drum is intended as a counterpoise to the circle and its telescopes, and to prevent it from swinging while turning on the axis C; the extremities of the axis C are supported by the mounted frame E, which is arranged so as to turn freely in the openings left by the mounting. In short, the pedestal, F, can itself turn, with all it carries, round an axis which penetrates its interior, and which is fixed in the foot of the instrument. By this disposition of its parts in turning the circle round the axis C, and at the same time turning all the instrument round the axis of the pedestal E, the plane of the circle can be trained in any desired direction.

A telescope, S S, is fixed on the upper face of the circle in the direction of one of its diameters, which turns freely round its centre without effort. A second glass, I I, is adapted, in the same manner, to the under-face of the circle at an angle with the other, but so adapted as to turn freely and independent of the circle. The eccentricity of the under telescope, and the great distance of the objects usually under observation, render the errors scarcely appreciable.

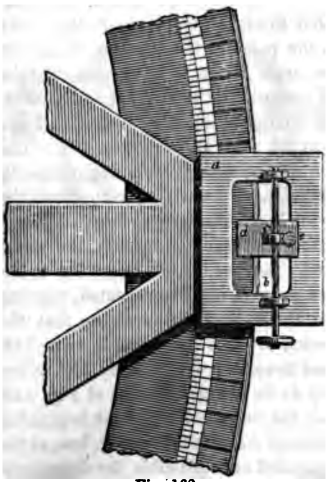


Fig. 162.

In Fig. 162 is represented an enlarged portion of the circle, showing the graduated scale and the extremity of the piece, *k*. The screw *a* is pierced with a rectangular opening, traversed in its middle by the screw *b*, which is formed to turn upon itself in the collars fixed at the two extremities of the opening, *a*. Screw *c* acts upon *b*, and is besides attached to the clamp *d*, which clasps the edge of the limb above and below. The screw *e* is to regulate the place of the clamp on the opening, pressing the edge of the limb between its jaws, and thus fixing the screw, *c*, to the limb. When it is required to displace the telescope rapidly, it is necessary to press the screw *c*, which throws it loose, so as to move round the circle under the impulse of the hand. When the glass has nearly attained its proper position, it is trained more exactly by turning the screw *e*. The vice *d* and the screw *c* are acted on and fixed again to the limb, and by turning the screw *b*, a slow movement is communicated to the screw *a*, which trains the telescope to the exact spot required.

Another disposition has been adopted for producing, in an analogous manner, the movement of the circle round its centre. The axis, after having traversed the socket BB (Fig. 161), and the cylindrical counterpoise D slightly inclined from below, and carries a toothed wheel of the same diameter as the counter weights. An endless screw is connected with this wheel, and is carried by a collar fixed to the counterpoise D, as is in Fig. 163. If the endless screw *a b* is moved by turni of the two milled heads, the toothed wheel with whic connected will receive a rotatory motion, in which the a which it is fixed by the same axis will necessarily participate. But the screw *a b* may be withdrawn from the toothed wheel in such a manner as to stop all commun between them. To do this, it is only necessary to tu finger *a* of the screw when the two are disengaged, as in Fi Having thus explained the mechanical details of the rep circle, its application to measuring the angle is as follows

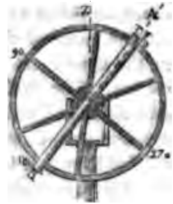


Fig. 169.

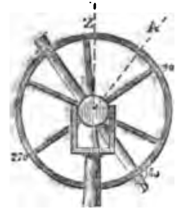


Fig. 168.

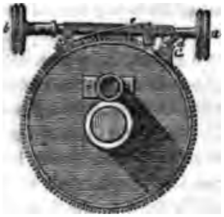


Fig. 163.

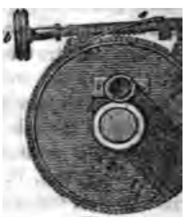


Fig. 164.

Let A and B (Fig. 165) be the two distant points, are the sides of the angle to be measured. Having plac upper glass at zero of the graduated scale, and fixed circle in that position, the circle is now placed in the of the angle and made to turn in this plane, so that th scope of the face is directed towards the point H, the telescope being directed to the point B, the circle remair the position Fig. 165. The circle is now to be turned ur lower instrument is directed towards the point A, as in Fi The circle being again fixed, the upper telescope is detach trained so as to be directed to the point B, as in Fig. 167. upper instrument is thus made to describe an angle e double that sought for, and the index travels over the lir exact measure of this double angle, which it represents in d minutes, and seconds; dividing this number by two, the value of the angle is obtained. Supposing the operation quire confirmation, the same process may be repeated, tı the whole instrument round the axis of the circle, so th upper telescope is again directed towards the point A, as i the other detached and turned towards point B, as in 169, the instrument stands exactly in its first position, as at Fi the only difference being that the index, which at the beg of the first operation was at zero of the graduated scale, has, end of the first operation, travelled over it double the dist the angle sought; and, at the end of the second operation, this distance from zero, drupled; in order to get the exact value of the angle, the portion of the circl travelled over has to be divided by four. In order to diminish the chances of e reading off the scale at the end of these operations, four different verniers, re subdivided, are fixed upon the circle; one of the indexes only which accompan

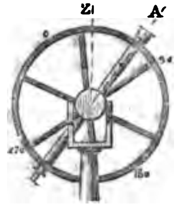


Fig. 167.

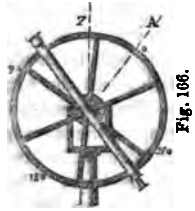


Fig. 166.

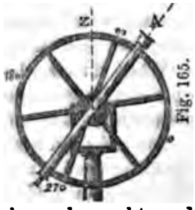


Fig. 165.

verniers is employed to determine the entire number of divisions of the limb which the telescope has turned; but the four verniers give each besides a value of the fraction of division which is to be added to the entire number, and it is the mean of their indications which is given as the value of this fraction of division. The microscopes M M (Fig. 161) are so disposed that the divisions of the vernier can be easily observed, as well as the coincidence of one of them with the scale of the limb itself. It is hardly necessary to add that the repetition of the operation secures immunity from error, and that it also gives its name to the instrument.

To Measure the Zenith Distance.—In measuring the angle we have real objects to deal with; it is otherwise in the present operation. We name the zenith that point of the heavens to which the vertical line is directed, and this direction is very neatly indicated by the plumb-line (Fig. 170). The zenithal distance of a point is the angular distance of this point and of the zenith—that is to say, the angle that the ray directs towards the point made in the vertical at the place of observation. The zenith is not, as we have said, a point that we can observe with an instrument; but the zenith distance is, nevertheless, to be found by the following operation:—In order to render the repeating circle available for measuring the zenith distance, the axis of the column F (Fig. 161) is rendered perfectly vertical. For this purpose three powerful screws, G, are required, on which the instrument is to rest, turning these screws until it is perfectly level, which is ascertained by means of a spirit level (Fig. 171) fixed to the tube of the lower telescope, in a horizontal position, while the circle itself is nearly vertical, as in Fig. 172; the screws, M *n q* (Fig. 174), being used to regulate the position and render the whole perfectly square.

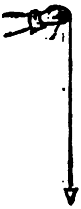


Fig. 170.

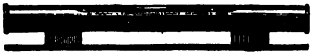


Fig. 171.

Having ascertained that this position is attained, turn the whole instrument round the axis, F (Fig. 172), until the spirit level is parallel to the line *p q* and perpendicular to the line *m n* (Fig. 173).

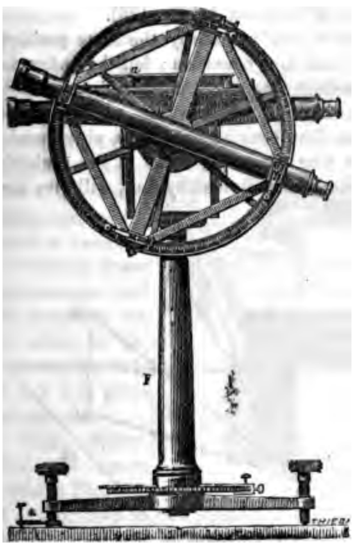


Fig. 172.

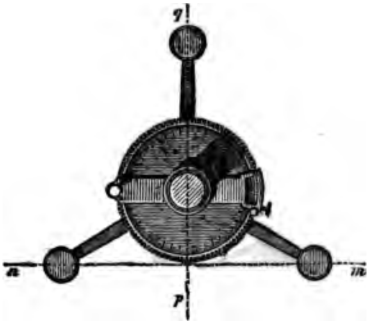


Fig. 173.

The axis of the column of the circle and the plane of the circle being thus rendered perfectly vertical, the upper telescope is to be turned upon the circle so that its index stands at zero of the graduated scale, and the circle is fixed in this position. The circle is then turned with the telescope, first around the axis of the column so as to lead the vertical plane of the circle by the point A (Fig. 174), and afterwards round the axis of the circle so as to train the optical axis of the glass exactly towards the same point. The circle being fixed in this position by means of the tangent screw acting on the extremity of its axis, a half turn is given to all the instrument, which carries it round the axis of the column, placing it in the position indicated (Fig. 175), detaching the instrument, and training it along round the axis of the circle in such a manner as to point towards A (Fig. 176). It is clear that in this movement the telescope has made an angle double the zenithal distance A O Z (Fig. 179), which is to be determined; and that by reading the number of degrees, minutes, and seconds of the graduated scale to which the index, which accompanies the telescope, corresponds, we have only to take the half of this number to have the value of zenithal distance.

The operation may here terminate, if the operator is satisfied with the measure obtained; but should he wish to verify the calculations by increasing the multiple of the angle, he may continue the operation by making a half turn round the axis of the column, crossing the circle in its plane, so that the telescope, which remains fixed, is again directed towards the point A (Fig. 178).

The instrument is now in exactly the same position as in Fig. 174, and the first operation; but the index, in place of being at zero, is now at an angular distance from zero just double the data we seek to value. A new operation, exactly like the first, consequently gives, at its termination, a value of four times the value of the zenithal distance, and the sum found, divided by four, will give the precise value.

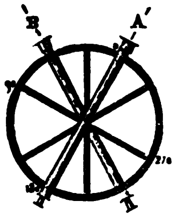


Fig. 178.

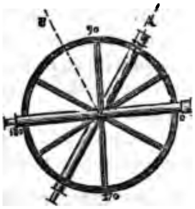


Fig. 177.

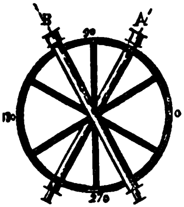


Fig. 176.

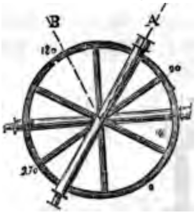


Fig. 175.

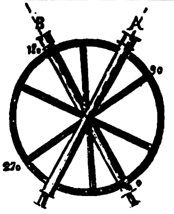


Fig. 174.

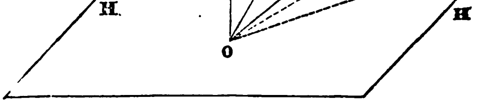


Fig. 179.

The Theodolite.—The repeating circle measures with great exactness the angle

H O B (Fig. 179), formed by the right lines joining the two points A and B at the point O. But it frequently happens that the angle comprised between the vertical planes Z O A, Z O B, which passes between these two points, is required—that is, the angle *a* O *b*, formed by the intersections, O *a* O *b*, of the two planes, vertical with the horizontal plane, H H. Our knowledge of the angle A O B, measured by the repeating circle, added to that of the angles, Z O A, Z O B, which are the zenith distance of the points A and B, enables us, by deducting the angle *a* O *b*, either by geometrical construction, or by trigonometrical calculation, to determine the angle. But it is sometimes more convenient to ascertain its value by direct measurement, and the theodolite we are about to describe enables us to do this with great exactness.

This instrument is represented in Fig. 180. It is composed of two graduated circles, of which the one is vertical, the other horizontal. The first of these two circles, A, is adapted to the horizontal axis B, round which it turns. The axis B is carried by the upper extremity of the vertical axis C, round which the circle A and the axis B can be turned by one common movement.

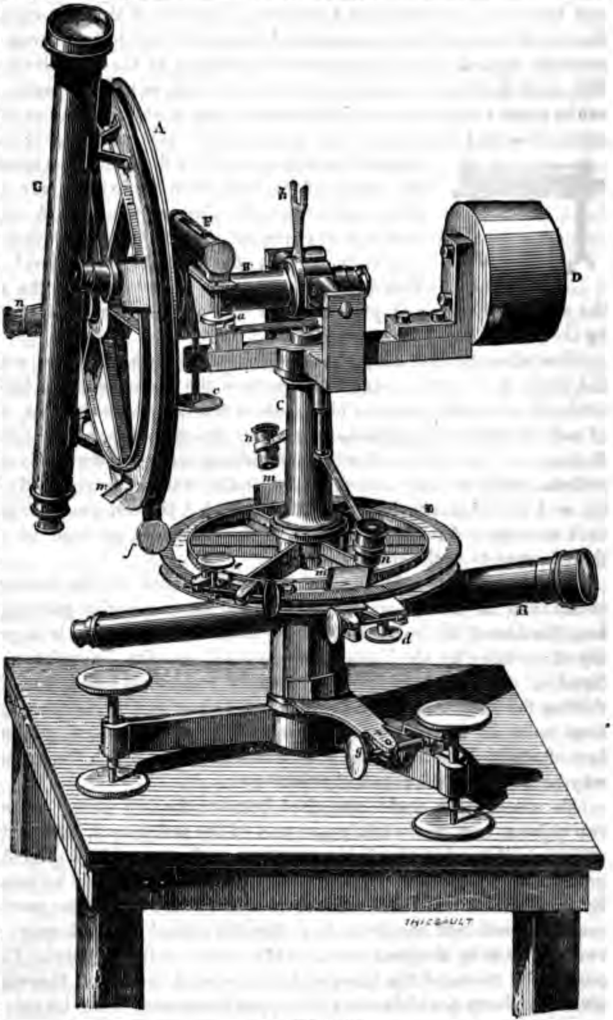


Fig. 180.

The weight D acts as a counterpoise to balance the circle A, preserving it steadily on the vertical axis C. The second circle, E, has it centre upon the same vertical axis C, and turns on its plane round the axis. The foot of the instrument is furnished with three double screws like the repeater, by means of which the instrument can be rendered perfectly vertical, which is ascertained by means of the spirit level, F, on the axis B. This spirit level is not carried round with the circle, as in the repeater, but a slight motion can be given to it by means of the screw *a*, thus slightly raising or depressing one of its extremities, and making it turn round a small axis at its other extremity. In this manner it is so managed that the bead of the level will be exactly in the centre of its tube when the axis C is in a vertical position; and, consequently, the plane of the circle A will be vertical also. As a test of circle and the axis C being perfectly vertical, and the axis B horizontal, the moveable spirit level· Fig. 181, is used. This instrument is made so as to rest on the axis B; the feet of the spirit level being adapted to it for that purpose, while it is supported in its place by the fork *h* (Fig. 180).

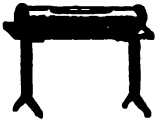


Fig. 181.

The telescope G is adapted to an inner circle, which moves round the interior of the circle A. In like manner the parts of the instrument attached to the horizontal circle, E, are fixed to an inner circle, which moves round its interior, the outer extremity of each circle carrying a graduated scale, to which the inner circle acts as an index guide. A clamp, *a*, having an adjusting and guiding screw, serves to fix the circle E to the pedestal, while another clamp, *e*, in a similar manner, serves to fix the upper parts to it; and a third clamp, *f*, serves to fix the limb A in such a manner as to be in opposition to it when turned round its centre. A fourth clamp, not seen in the figure, attaches the telescope G to the circle A.

A second telescope, H, is adapted to the pedestal of the instrument, whose movements from its position are very limited; it serves no other purpose, however, than to keep the foot of the instrument steady while the observation is in progress. With this object, profiting by the limited movements it can take, its optical axis is trained in the direction of a point which can be easily recognized; and from time to time, while shifting the instrument, serves to assure the observer that the telescope is directed to the same point as at the commencement of the operation. A guiding-screw, *g*, in the foot of the pedestal serves as a slow movement by which the optical axis of this glass may be trained to the required point.

To measure the angle comprised between the two vertical planes passing between two objects, turn the whole upper part of the instrument, independently of the graduated limb, E, so that the index traced upon the inner circle coincides with zero of the graduation, and fix this circle to the limb E in this position by means of the clamp *e*. By turning the limb E and all it carries, and at the same time moving the telescope G round the centre of the circle A, so that the optical axis is directed to the first of the two objects to be observed upon, the first operation is completed. Fix the limb E in this position by means of the clamp *d*, for the second operation, throwing loose at the same time the clamp *e*, and turning the upper instrument round its axis so as to lead the telescope G direct to the second object. The index of the circle moving in the interior of the limb E has described upon this limb an arc representing the value of the angle sought, and of which we can read the value on the graduated scale.

If it is intended to apply the principle of repeating the angle, fix the higher instrument to the limb E in the position it has now attained; loosen the clamp *d* and turn the limb E, with all above it, so that the telescope G is again directed to the first object. Fixing the circle E in this position by means of the clamp *d*, and turning the upper part of the instrument until the telescope G is directed to the second object, and it is clear that the index of the inner circle has described another arc equal to that already described in the first operation; and so on, continuing the same series of operations until the multiple is sufficiently enlarged to secure a result free from error when it is reduced to its real value, which is done by dividing the gross result by the number of multiples. The readings on this instrument is accomplished by means of verniers, whose divisions are enlightened by small plates of polished glass *m m*, and by the microscopes, *n n*, trained on the scale; by these means indications are easily read off.

The Altitude and Azimuth Instrument.—This is the most useful of all the portable instruments, and to the scientific traveller an invaluable one, measuring with great accuracy both vertical and horizontal angles.

In applying the instrument to astronomical purposes, it was formerly the custom to clamp it in the direction of the meridian, and after taking an observation, or series of observations, with the face of the instrument one way, to wait till the next night or till opportunity permitted, and then take a corresponding series of observations of the same object with the face reversed, by way of verification. This is now seldom practised, being obviously imperfect. The instrument consists of a central tripod, to which is fixed the azimuth circle, having a horizontal motion of about three degrees, so that its zero can be brought exactly in the meridian by means of a slow moving screw beneath the circle. The tripod rests upon foot screws, which are described by Mr. Troughton in the memoirs of the Astronomical Society, as "being double, that is, a screw within a screw, the exterior one having its female in the end of the tripod, and the female of the interior screw being within that of the exterior," by which ingenious contrivance three distinct motions are gained for regulating the azimuth. Brass cups are placed under the spherical ends of the foot screws; this screw, invented by Mr. Troughton, is intended to give a very slow motion to one of the feet, and the foot of the tripod is designed to be placed either north or south. Above the azimuth circle, and concentric with it, is a strong circular plate, which carries the whole of the upper works. This plate rests on the axis of the azimuth, and moves concentrically with it. Rising from this plate are two strong conical pillars, on which the transit instrument is supported. Upon the axis, as a centre, is fixed a double vertical circle with the telescope between them, the circles being fastened together by small brass pillars, while the graduated scale is made on a narrow silver ring, inlaid on the outer face of one of the circles. Two reading microscopes are placed at each extremity of the circle, supported by two attached near the top of the pillars. The adjustments required are as follows:—The horizontal circle is first to be leveled, which is to be effected in the same manner as with a theodolite. The axis of the telescope must also be leveled, as in the transit instrument, and the spider lines adjusted for collimation and verticality. The meridional point on the azimuth circle is its reading when the telescope is pointed north or south, and may be determined by observing a star at equal altitudes east and west of the meridian, and finding the point midway between the two observed azimuths; or the instrument may be adjusted to the meridian, in the same manner as a transit. The horizontal point of the altitude circle is its reading when the axis of the telescope is horizontal, and may be found, as with the mural circle, by alternate observations of a star directly and reflected from the surface of mercury. The telescope usually carries in its principal focus a spider line micrometer, as in the transit instrument to be described presently.

The Micrometer.—We have had occasion to mention this instrument, and a short account of it may not be out of place here. It consists of a system of very fine wires (Fig. 182), by which the apparent magnitude of heavenly bodies may be measured, and the exact moment of their transit across the wires calculated with great exactness. In the case of refracting telescopes, the micrometer is fitted in the instrument itself. In circular instruments, and the larger instruments of observatories, it forms a part of the microscope attached to the outer edge limb for reading off the graduated scale.

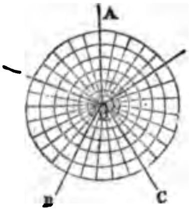


Fig. 182.

When a telescope is fitted with the micrometer, the right line of the optical axis *o* (Fig. 183), exactly coincides with the centre of the crossed lines B. When it receives the ray A, it is very obvious that by a proper arrangement of wires, the utmost exactness of observation may be attained. The micrometer is usually formed of the finest platinized wire, having three horizontal and four transverse wires. When the telescope is furnished with this instrument, the reticulated frame A (Fig. 184) is placed at the end of the tube B C, near the eye-piece, which again shuts into the principal tube D of the telescope. On the other hand, the object-glass E, F, formed of two lenses, also shuts into the same tube, each requiring to have their focal distance from the object-glass adjusted either by the hand or by means of a guiding screw.

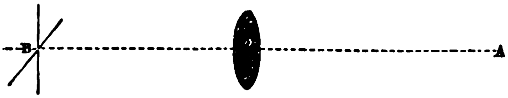


Fig. 183.

Fig. 184.

In the case of circular instruments, the micrometer is fitted in a microscope, as represented in Fig. 185. It is firmly fixed in its position, so as to enable the observer to read off the angle from the graduated scale on the outer side of the limb C D. In small instruments, the screws which serve to fix the microscope in its position are usually arranged so as to regulate its distance from the limb of the circle. In the body of the microscope at *a* are placed two wires, crossing each other diagonally, which may be made to traverse the field of view, either horizontally or perpendicularly, by turning the micrometer screw, *a*, working in the box underneath it.

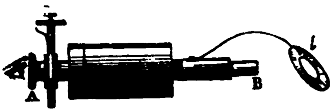


Fig. 185.

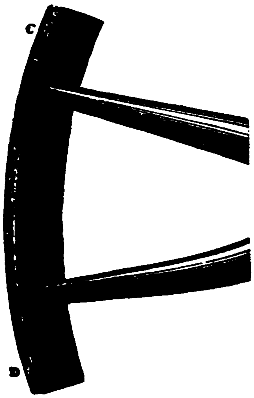


Fig. 186.

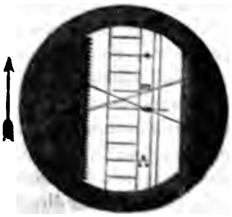


Fig. 187.

Figure 186 represents the field of view, with the magnified bars of the graduated scale as seen through the microscope. A small mirror, *l* (Fig. 185), is sometimes fixed to the micrometer, arranged in such a manner as to throw the light of a lamp or a jet of gas upon the part of the limb which is opposite to the microscope; it is sometimes pierced with an opening in the centre, which receives the rays and serves to illumine the microscope itself. In the larger instruments of observatories, they are fixed in the wall which supports the circle, the graduated scale being illuminated by a lamp or jet of gas. The shaded part represents a diaphragm with the cross wires, the angle of which may, by turning the micrometer screw, *a*, be bisected by any line on the circle in the field of view. On the left hand of the diaphragm, is a comb, or scale, each tooth of which represents one minute, and one revolution of the screw moving the wires over one tooth of the comb is equal to one minute of space.

The adjustment of the microscope consists in making the cross wires in its focus, and the divisions on the circle both appear at the same instant of time, and free from parallax, the adjustment is such that five revolutions of the micrometer screw shall measure a five minute space on the graduated circle. For the former of these adjustments in the telescope draw out the eye-piece until the distinct vision of the wires is obtained, and the divisions or bars of the instrument are well defined.

The motion of the comb, or scale of minutes, is regulated by a screw, and the micrometer head by friction can be made to read either zero, or any required second when the cross wires bisect any particular bar, by holding the milled head of the micrometer screw.

The Equatorial.—The instruments we now come to describe belong exclusively to observatories, and in these establishments the equatorial is specially adapted to mark the diurnal movements of the heavens. The axis A A (Fig. 188), round which the whole of the instrument can turn, is so arranged as to follow the axis of our planet the earth, and it carries, laterally, the graduated scale B B, which can turn both in its plane and round its centre. The telescope C C, fixed to this circle follows its movements, and its optical axis thus makes an angle variable with the earth's axis. A second graduated circle, D, D, whose plane is parallel to the celestial equator, and having the axis A A as its centre. It is fixed in such a manner that it follows all the movements of the instrument in its rotations round this axis; the position of this second circle determines the attributes and gives name to the instrument. The clamp E E, with guiding and reversing screws, intended to fix the circle and the telescope to the axis A A, when the circle turns round its centre, is carried by the pieces F F attached to the axis. Two micrometers, G G, are adapted to the extremities of the cross beam, firmly attached to the axis in such a manner as to permit of the divisions of the graduated scale carried on the side of the limb of the circle being read off. Other micrometers are attached to the mason-work which carries the extremities of the axis, and are intended to read the graduated scale of the second circle D D, which in this instance is on the upper face, and not on the outside of the limb. Such an instrument affords great advantages in measuring the relative position of two contiguous bodies, in measuring the diameters of the planets, &c. The circle which is connected with the polar axis is graduated into hours, minutes, and seconds of time, to indicate the right ascension of the object under examination; while the circle connected with the declination axis is graduated into degrees, minutes, and seconds of arc, to indicate declination or polar distance.

From this disposition of the instrument it will be seen that the optical axis or line of collimation of the telescope turns towards all points of the heavens, and in making it turn with the circle B B round its centre, it can be made to form any angle with the axis of the earth. If the circle B B is fixed in one particular position, by means of the clamp

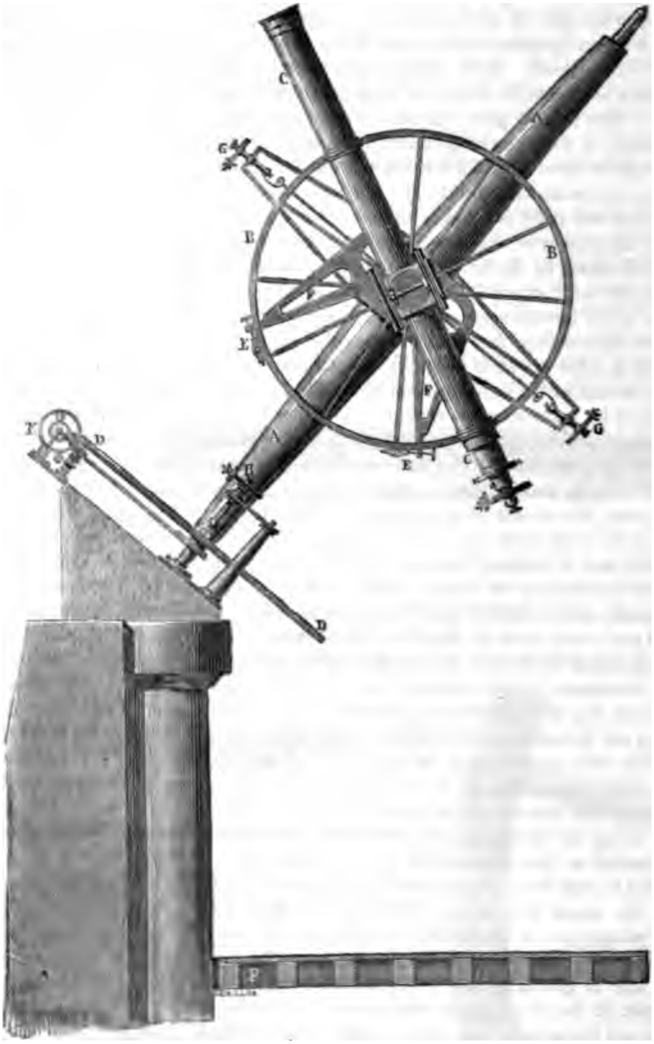


Fig. 188.

E E, and an entire turn is made all round the axis A A, it is evident that the optical axis of the telescope meets the celestial sphere at various points of the same parallel.

A particular piece of mechanism, K, permits the equatorial circle, D D, to be put in communication with a clock movement, so disposed that the circle, D D, performs an entire turn on its axis in a sidereal day. If the optical axis of the telescope is directed to a star after the circle B B is fixed to the axis A A, and the circle D D put in communication with the clock-work, the whole instrument will be carried along with it, and the telescope will continue to follow the movements of the star; thus affording a means of verifying the uniform rotatory movement of the celestial sphere, after having made due allowance for the effects of atmospheric refraction.

The telescope of the equatorial requires to be in such a position that it can be directed to every part of the heavens above the horizon. This renders it necessary that the instrument should be so placed as to be clear of interruption from neighbouring objects. It is usual, therefore, to place it in the upper part of the observatory. The engraving at page 273 represents a section of the equatorial room of the Paris observatory, Fig. 187 representing a part of the instrument, of which we shall here give a brief description.

The axis is supported at its lower extremity by a massive piece of masonry, L, its upper extremity being supported by the iron beam M, made as light as the safety of the instrument and steadiness required will permit. The instrument is protected from the weather by a roof in the form of a hemisphere, having a long opening following its vertical plane, and formed with doors sliding laterally so as to leave a free opening in every required direction. By this arrangement the glass can sweep the heavens on its vertical plane from the zenith to the horizon; and by sliding the other parts of the roof, the whole of which is placed on friction rollers, and furnished with a crank, R, for the purpose of moving it, every portion of the heavens can be swept with the instrument. The equatorial, consisting, as we have seen, of one circle, parallel to the plane of the equator, and of another circle which follows the axis of the earth, is arranged so as to coincide with any circle of declination, and is well adapted for measuring the differences of right ascension and declinations of two neighbouring stars.

The Transit Instrument.—The place of an object in the heavens is mostly defined by two elements, viz., by its right ascension and north polar distance. In the first approaches to accurate results of observation, these elements were generally determined by a mural arch or quadrant, firmly fixed in the direction of the meridian, the position of which was frequently checked by equal altitudes of stars. It was by means nearly similar that Flamsteed observed his catalogue of three thousand stars, which has been so ably reduced and edited by the late Mr. Baily, and is generally known as the "British Catalogue."

Lacaille, at the Cape of Good Hope, observed his zones in like manner, using certain well-determined stars as "zero points," whose positions had been obtained by independent methods. The first astronomer was content with noting his observations to seconds of time—an accuracy which, from the imperfection of his instruments, he considered to be quite sufficient. The invention of the transit instrument by Roemer, in the year 1690, enabled astronomers to make great advances in the accurate determinations of right ascensions. This element, the most delicate and important, essentially depends upon time, and is, therefore, more difficult of direct estimation than the other, which consists of angular measurement, *read off* on graduated instruments, which modern artists have been enabled to bring to an astonishing degree of perfection. The transit instrument has been successively improved since the first idea of Roemer. Dr. Halley, when Astronomer Royal, introduced its use at Greenwich, the construction being similar to that given by Lerrebow in his "Praxis Astronomiæ." In this arrangement t telescope was placed twenty-six inches nearer one end of the axis, instead of being c tral—a construction which evidently rendered the determination of error of collimati very troublesome.

The form of the transit instrument used by Dr. Bradley (the next Astronom Royal) differs little from that adopted in modern times, viz., a telescope fixed at rig

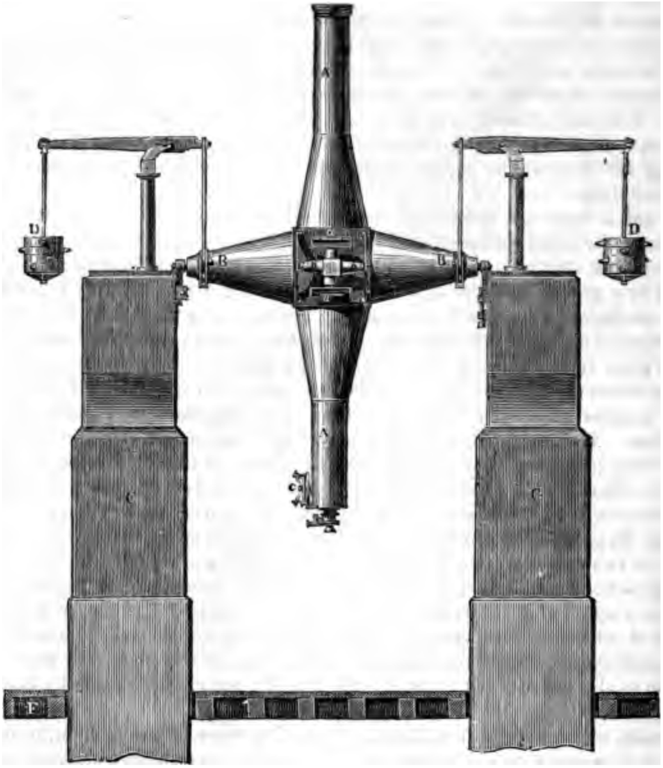


Fig. 189.

angles to a cross axis, and capable of taking all positions in the meridian of the plac This axis terminates in two cylindrical pivots, resting in Y's, so that they only touc in two points for the purpose of diminishing the friction. One of these Y's is moveabl in such a manner as to correct either an error of horizontality of the axis, or an err of deviation from the meridian. The instrument is furnished with a graduated settin circle, sometimes placed at one extremity of the axis concentric to this axis (as was th case formerly with Troughton's Greenwich transit instrument), or two small divide circles placed near the eye end of the telescope. In the common focus of the object glass and eye-glass, a system of wires is placed for the purpose of defining the positio

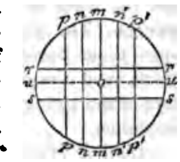


Fig. 190.

of an obect as it passes through the telescope. These wires are of extreme tenuity, and generally consist of five or seven vertical wires and two horizontal (Fig. 190), the central vertical wire closely approaching to the meridian. The object of the lateral wires is to diminish the error of estimation of a star's transit, by taking the mean of all, since the time of transit over a single wire would be liable to some uncertainty. To prevent any error arising from parallax in the observation of the wires, the eye-piece is moveable by a screw, so that the transit over each wire may be observed in the centre of the field. The adjustments of a transit instrument are, first, that its optical axis, or the line joining the centre of the object-glass and the middle wire, be perpendicular to the axis of rotation; secondly, that the axis be horizontal; and, lastly, that when corrected for the two preceding adjustments, the line of collimation describes the meridian.

The other instrument, equally important in connection with a transit instrument, is a sidereal clock, beating seconds, and so regulated that the hour-hand will describe a complete revolution, or 86,400 seconds, from the time of a star's transit over the meridian fibre to its return to the same point. The difference of the clock's gain or loss from this number is termed *its rate;* the excellency of a clock, therefore, depends on the steadiness of its rate, and every means are taken by mechanical refinements to preserve its uniformity. Astronomical clocks are generally funished with mercurial pendulums for the purpose of compensating for extremes of temperature.

The transit clock requires to be mounted on a pier of solid masonry for the purpose of insuring steadiness.

The method of observing transits is as follows :—The observer, being comfortably seated (the telescope having been previously directed to the colimating star), notes carefully the minute, second, and fractional part of a second at which a star transits each wire. The hour is afterwards noted, and the counting of the clock verified when the transit is completed. This is done for several objects, the mean of the wires is taken, and thus, the differences of their right ascensions, subject to the rate of the clock, can be found. The fractional part is estimated by carefully noting the position of an object before its passage over the wire, and after its passage over the same wire. By a little practice, the instant when it was exactly behind the wire can be very accurately proportioned. In good observations the mean of wires is seldom liable to an error greater than one-tenth part of a second.

Recently a new method of recording transit observations, by the agency of galvanism, has been brought into use at the Royal Observatory, Greenwich, which promises to supersede the old systems of observing. Its first introduction appears to have been in the coast survey of the United States, and in the hands of Mr. Walker, Dr. Locke, and Professor Mitchell, it has been considerably improved. The form of the invention, as described by the Astronomer Royal, is as follows :—From the principal instruments of the observatory, galvanic wires are connected with a recording surface placed in another part of the building. The transit clock at each beat completes a galvanic circuit, which is registered by proper apparatus on this recording surface, consisting of a cylindrical barrel, covered with prepared paper. Above the barrel (which is driven by clock-work regulated by a conical pendulum), a system of prickers is placed, carried by a travelling frame, moving slowly in the direction of the barrel axis. Thus, the pricker, which is connected with the transit clock, will register seconds on this barrel, and will form a series of spiral lines as it revolves. The method of arrangement is as follows:—A wheel of 60 teeth is fixed on the escape-wheel axis, and the teeth of this wheel in succession make momentary contacts of the galvanic springs. The position of the springs is so adjusted that the effect of the wheel-tooth upon them occurs only when an escape-tooth has passed the sloping surface of the pallet, and the other escape-tooth is dropping upon its bearing; and thus the resistance of the springs does in no way affect the legitimate action of the train upon the pendulum.

Another pricker, carried by the same travelling frame, is connected, by arbitrary touch, with an index at the eye end of the transit circle. At the instant of a star's passage behind the wire, the observer touches this index, which will, therefore, register on the barrel a series of punctures equivalent to a transit observation. An advantage is gained by this method, insomuch that the equatorial intervals of the wires may be reduced to three seconds of time—the duty of the observer merely consisting in writing down, in addition to the preceding signals, the name of the object observed. For the purpose of registering on the barrel the beginnings of some minutes, and hours and minutes, from which the rest may be inferred, a provision is made by certain prearranged signals at known instants of the transit clock. In this manner the paper, when taken off the barrel, may be easily translated. This system has so far succeeded admirably at Greenwich, requiring only the use of the nerves of sight and sensations of touch, of which there is a more intimate connection than those of the eye and the ear. It will, therefore, most probably reduce the amount of "personal equation."

It frequently happens that two observers will not give the same time of the occurrence of the same phenomenon; as, for instance, the passage of a star over the wires of a transit instrument. The amount of this difference, which is termed "personal equation," varies from two-tenths to half a second, though instances have been known of this quantity exceeding one second of time. Dr. Maskelyne, and Kinnebrook his assistant, differed in this manner seven-tenths of a second. In modern times, Bessel and Argelander have differed upwards of a second. The method of determining it is to compare the "clock error" of one observer with that of another independently obtained (at a short interval of time). Or it can be deduced by taking transits of the same star over some wires of a transit instrument, by one observer, compared with the transits over other wires by another observer. Reducing each separately to the middle wire, the difference of their methods of observing may be at once deduced. We refer to the annual volumes of the Greenwich observations for examples of the first of these methods.

It can also be obtained by a binocular eye-piece, an instrument which permits two observers to see the star in its passage over the wires at the same time, and thus to observe the whole transit. This instrument is the invention of the late Mr. J. Jones.

In order to determine the relative accuracy of transit observations at the separate wires, the following method, first introduced by Bessel, and recently used by Struve and Dr. Oudemans, may be applied. Let x be the mean error of one observed wire, arising from a defect of hearing, y the mean error produced by an imperfection in the sight, which may be caused by a wrong estimation of the time at which the star passes behind the wire (varying with the thickness of the wires); then the mean error of an observed transit at one wire will be

$$M = \sqrt{(x^2 + y^2 \sec^2 \delta)}.$$

The error of hearing, or x, will arise from the estimation of the precise instant of the origin of second, and will, therefore, be the same for all stars. The other will vary as the secant of the declination.

Then if the transits of several stars are reduced to the middle wire by the known intervals of the wires, and the difference of each result from the mean is taken, the squares of these differences will be $= (n - m)\,M^2$, where $n =$ the number of wires, $m =$ the number of observed culminations. The probable error of M, determined by this equation, will be $M\frac{0{\cdot}4769}{\sqrt{n}}$, the probable error of the value M^2 will be $M^2\frac{0{\cdot}9538}{\sqrt{n}}$.

Forming for each culmination $\Sigma\,(\epsilon^2)$, and their sums for all the culminations of the same star, the equation will become

$$(n - m)\,x^2 + (n - m)\,y^2 \sec^2\delta = \Sigma\,\Sigma\,(\epsilon^2).$$

Proceeding in this manner, Dr. Oudemans has found, from 228 culminations observed from October 1847 to October 1848, the mean error of a single wire =

$$\text{or } M = \sqrt{(0\overset{s.}{\cdot}1242^2 + 0\overset{s.}{\cdot}0603^2 \sec^2\delta)}.$$

But the probable error

$$w = \sqrt{(0\overset{s.}{\cdot}0838^2 + 0\overset{s.}{\cdot}0407^2 \sec^2\delta)}.$$

A similar investigation from October 1848 to October 1849, gave a mean error, or,

$$M = \sqrt{(0\overset{s.}{\cdot}0962^2 + 0\overset{s.}{\cdot}0602^2 \sec^2\delta)};$$

and the probable error

$$w = \sqrt{(0\overset{s.}{\cdot}0649^2 + 0\overset{s.}{\cdot}0406 \sec^2\delta)}.$$

It would, therefore, appear that his hearing had improved, whilst that his sight had remained the same as before. This determination rests on 360 culminations.

Dr. Robinson has recently investigated this subject, and has introduced another error arising from atmospheric tremor, which he considers to vary as the secant of the zenith distance. From his observations in 1830, he deduces the following probable errors for a star whose declination is δ:—

$$x = \pm\, 0\overset{s.}{\cdot}0445 \qquad y = \pm\, 0\overset{s.}{\cdot}0619, \qquad \text{Z, or the atmospheric tremor} = \pm\, 0\overset{s.}{\cdot}0381.$$

Several years afterwards, he found the following values :—

$$x = \pm\, 0\overset{s.}{\cdot}0732 \qquad y = \pm\, 0\overset{s.}{\cdot}0554 \qquad z = \pm\, 0\overset{s.}{\cdot}0049.$$

But between the periods of the two observations, new and more numerous wires had been inserted in the transit instrument.

Astronomers are very careful that the three adjustments of the transit instrument, before mentioned, are satisfied. And, firstly, with regard to the line of collimation: This adjustment is generally made by the observation of a well-defined terrestrial mark in one position of the axis, measuring its distance from the central vertical wire (in terms of the micrometer) with which the telescope is furnished. If, on reversal of the axis, the mark is again seen at the same distance from the central wire, the collimation of the instrument is correct; but, if not, the wire frame must be moved to one-half the difference of the two bisections. In the practice of modern observatories, the complete correction of this error is seldom attempted, this adjustment, though probably in large instruments the most permanent, being nearly corrected, and then left for occasional verification.

The following extract from the Greenwich Observations, 1850 (Introduction, page xvi.), will show the method of determining the error of collimation :—

"The value of this error, which is given in seconds of space, is supposed positive when it implies an additive correction to the transits of stars above the pole. For its measurement the following method has been used:—

"A small transit instrument is fixed on temporary wooden piers in the north opening of the transit room, and its object-glass is turned towards the principal transit instrument. In this position, when the principal transit instrument is turned towards the small one, the wires of the latter are seen as well-defined marks at an infinite distance, and the error of collimation of the principal transit may be determined by repeatedly reversing it, and viewing these wires.

"In November, 1846, a new determination of the value of the micrometer-screw (of the transit telescope) was made by means of six transits of Polaris over the two wires moved by the micrometer. The mean of the intervals of the times of transit was found to be 3m. 1s.·17; and the north polar distance of Polaris being 1° 30′ 9″·3, this corresponds to an interval of space = 71″·188. But, by bringing each micrometer wire several times into contact with the fixed central wire, this interval was found to correspond also to 4r.·368 of the micrometer. Hence, one revolution = 16″·297.

"April, 29d. 22h. The St. Helena transit instrument was used in the manner stated above. The wire appeared as a broad white line, there being no reflector; and the edges of this line were observed alternately by placing upon them the micrometer wire of the transit instrument.

"OBSERVER M.

	s.
Illuminated End East.	
Micrometer reading on coincidence with collimator (12 measures) .	10·494
Illuminated End West.	
Micrometer reading on coincidence with collimator (12 measures) .	8·895
Illuminated End East.	
Micrometer reading on coincidence with collimator (12 measures) .	10·500
Illuminated End West.	
Micrometer reading on coincidence with collimator (12 measures) .	8·873
Hence reading for true line of collimation by 1st and 2nd sets .	9·695
„ „ „ „ 3rd and 4th sets .	9·687
Reading for true line of collimation	9·691
Micrometer reading on coincidence with D or central vertical wire .	9·674
Hence apparent error of collimation for D	0·017

"The illuminated end of the axis was left east, which was also the position of the micrometer head; and as the readings of the micrometer increase as the wire moves from the head, D (or central vertical wire) was therefore east of the line of collimation, and stars pass it too late, or the error of collimation of D is — 0r.·017, or — 0″·277. Also, with the illuminated end of axis east, Polaris passes the mean of wires earlier than it passes D by 0s.·07, which in arc is equivalent to 0″·03; and the correction for diurnal observation is — 0″·19. Hence the corrected error of collimation is — 0″·44."

The second method of determining the error of collimation is by observations of the transit of Polaris, or any other close circumpolar star, in one position of the axis, compared with its transit in a reversed position of the axis. By proper apparatus, this reversal of the instrument can be effected during the time of transit. If, therefore, three wires of Polaris, or λ Ursæ Minoris, be observed in one position of the axis, and the remainder of the transit in a reversed position, and if each set of wires be separately reduced to the central wire, half the difference of the two results will be the error of collimation for the object observed, which can be reduced to the error of collimation (in arc) for an equatorial star (dividing by the proper factor). It is necessary, however, in the use of this method, that the reversal be very carefully made, and that the mean of several results of separate stars be taken.

The following example of this method is extracted from the Georgetown Observations:—

Transit of λ Ursæ Minoris on August 7, 1845.

End C of the axis to the east.

		h.	m.	s.	
	At 3rd wire . .	20	4	24	
Moveable wire—	x . .	20	11	3	
	4th wire . .	20	17	48	Here the instrument was reversed.
	x . .	20	24	31	
	3rd wire . .	20	31	10	

	m.	s.	
Interval from 3rd to 4th wire . . .	13	24	$= t$
„ 4th to 3rd wire . . .	13	22	$= t\,o$

$$\text{Difference} = 2 = t - t\,o$$

$$1 = \frac{t - t\,o}{2}$$

	m.	s.	
Interval from x to 4th wire	6	45	$= t'$
„ 4th wire to x	6	43	$= t'\,o$

$$t' - t'\,o = 2$$

$$\frac{t' - t'\,o}{2} = 1$$

$$\text{Error of collimation or C} = \frac{(t - t\,o)\ 15 \sin p}{2} = 0''{\cdot}303$$

The third method was originally introduced by Professor Struve, at the Pulkova Observatory. It consists in mounting firmly two telescopes in the direction of the meridian, the wires of which can be adjusted on each other. It is evident that by comparing the principal instrument (transit or meridian circle) with each successively, the true line of collimation may be obtained without reversal.

For the purpose of determining the line of collimation of the Greenwich transit circle (as the instrument is not capable of reversion), two collimating telescopes, the axes of which are as nearly as possible in the same line with the principal instrument, are firmly mounted on brick piers a few feet north and south of it. These collimating telescopes are furnished with proper wires, which can be accurately adjusted to coincidence, and will therefore represent (when viewed with the telescope of the transit circle) two objects at an infinite distance, and exactly opposed to each other. When a series of bisections is taken of each collimator in terms of the micrometer of the transit circle telescope, the mean of these will give the geometrical line of collimation. The annexed form will show the work:—

ROYAL OBSERVATORY, GREENWICH.

Readings of the Transit Micrometer at coincidence of central wire with the wires of the Collimators, for determining the position of the Line of Collimation of the Transit Circle, in the year 1851.

Approx. Solar Time.	Oct. 11, 7^{h}		12, 23^{h}		13, 8^{h}		13, 22^{h}	
Observer.	H.		R.		R.		H. B.	
Collimator observed.	South.	North.	South.	North.	South.	North.	South.	North.
	r	r	r	r	r	r	r	r
Readings of Transit Micrometer.	28·358	34·572	28·296	34·602	28·312	34·580	28·302	34·563
	349	586	294	600	310	596	318	587
	347	578	293	595	307	590	306	594
	346	580	290	592	314	592	301	556
	345	581	300	590	315	585	303	575
	340	582	294	598	310	600	307	584
Sum of Readings.	45	479	567	577	68	543	37	459
Mean.	28·348	34·580	28·295	34·596	28·311	34·591	28·306	34·577
Mean of South Readings.		28·348		28·295		28·311		28·306
Sum.		62·928		62·891		62·902		62·883
Concluded Reading for line of Collimation.	r 31·464		r 31·446		r 31·451		r 31·442	

Approx. Solar Time.	Oct. 14, 8^{h}		14, 22^{h}		15, 6^{h}		15, 22^{h}	
Observer.	H. B.		J. H.				E.	
Collimator observed.	South.	North.	South.	North.	South.	North.	South.	North.
	r	r	r	r	r	r	r	r
Readings of Transit Micrometer.	28·327	34·582	28·355	34·592	28·332	34·626	28·284	34·692
	291	614	345	600	323	626	278	683
	330	600	330	578	329	618	290	695
	312	593	337	598	340	592	284	695
	337	612	348	585	350	590	285	685
	331	604	342	602	342	605	280	686
Sum of Readings.	128	05	257	555	216	57	501	536
Mean.	28·321	34·601	28·343	34·593	28·336	34·610	28·284	34·689
Mean of South Readings.		28·321		28·343		28·336		28·284
Sum.		62·922		936		62·946		62·973
Concluded Reading for line of Collimation.	r 31·461		r 31·468		r 31·473		r 31·487	

The error of horizontality of the axis is generally determined by the application of a spirit level to the pivots. But it must be previously known whether the pivots are perfectly circular, and of equal diameter. For this purpose, careful series of levellings are taken in reversed positions of the axis of the transit instrument. If we assume the Y's to be perfectly horizontal, but that on application of the level the same pivot shows a quantity too high in reversed positions of the axis, it is evident that this pivot is the larger of the two; and it is equally evident that if the Y's are not perfectly horizontal, we can determine the difference of the pivots from level readings in reversed positions of the axis. The following example, from the Oxford Observations, 1840, page v., will illustrate the whole process. The sign + is used when the west end of the axis is highest, — when the east end is highest :—

Day, 1840.	Position of Lamp.	Level Reading.	E.—W.
		″	″
Jan. 16	East	+ 2·31	+9·14
	West	— 6·83	
17	East	+ 1·90	+ 5·60
	West	— 3·70	
20	West	— 3·86	+ 4·91
	East	+ 1·05	
March 14	West	— 2·60	+ 5·61
	East	+ 3·01	
April 14	East	+ 2·37	+ 4·94
	West	— 2·57	
15	East	+ 4·50	+ 5·10
16	West	— 0·60	
	East	+ 4·00	+ 4·60
July 7	East	+ 2·50	+ 3·60
	West	— 1·10	
July 10	West	— 0·43	+ 3·54
	East	+ 3·11	
11	West	— 1·16	+ 5·20
	West	— 0·22	
	East	+ 4·35	+ 4·86
	East	+ 4·12	
	West	— 0·62	
	West	— 1·02	
Aug. 10	West	— 1·09	4·89
	East	+ 3·80	
Nov. 26	West	— 5·43	+5·76
27	East	+ 0·33	+6·16
28	West	— 5·83	+6·31
29	East	+ 0·48	

"From these it results, that when the lamp is east the west end of the axis is highest, and when the lamp is west the east end is highest; thus they show that the solid pivot is larger than the perforated one.

"The angle of the level's Y's is the same as that of the pivot's Y's, both being rectangular, or very nearly so: therefore the difference of the radii of the pivots

$$= \frac{\text{Difference of the Level Readings in Opposite Positions of the Axis.}}{4\sqrt{2}}$$

"The column E — W shows the difference of the level readings, the arithmetical mean of which $= 5''\cdot445$; therefore $\frac{5''\cdot445}{5\cdot660} = 0''\cdot962$ is the mass of the radius of the solid above that of the perforated pivot, and $0''\cdot962 \times \sqrt{2} = 1''\cdot361$ is to the correction to be applied to the level readings to obtain the inclination of the pivot centres. This correction is to be added *algebraically* to the level readings when the lamp is west, and subtracted when the lamp is *east*."

A careful series of levellings at different elevations of the object-glass showed that there was no appreciable difference in the circularity of the pivots, as the reader will observe by reference to the descriptions of the Altazimuth and Transit Circle in the Greenwich Observations, 1847 and 1852, for the method adopted by the Astronomer Royal to insure perfect circularity of the pivots.

Dr. Oudemans, in his essay on the Transit Instrument, gives the following formulæ for the correction of the level indication caused by unequal diameters of the pivots. The error of level being determined after frequent reversals of the transit instrument, this correction may be easily deduced.

Let $2\,g$ be the angles between the planes of the Y's, on which the pivots are placed; $2\,f$, the angles between the feet of the level, by which it is placed on the pivots; u, the difference of inclinations which are observed in both positions of the axis; r and r', the two radii of the pivots; L, the distance between the points, where the two pivots rest on the planes of the Y's. Now g, f, u, and L being known, the difference of the radii r and r' are made known by the following formulæ:—

$$r - r' = \tfrac{1}{2}\, u\, \mathrm{L} \sin 1''\cdot\frac{\sin g \sin}{\sin g + \sin f}.$$

He then gives a determination of the values before and after reversal, and finds an arithmetical mean of the differences, or of $u = +\,4''{\cdot}15$. Since this value of u found will depend on one unequal breadth of the extremities of the axis, the correction of any inclination can be obtained by the level.

$$z = \pm\, \tfrac{1}{2}\, u\, \frac{\sin g}{\sin g + \sin f}$$

In his transit circle, $2\,f = 98^{\circ}\ 40'$, $2\,g = 91\ 25'$; therefore

$$z = \pm\, \tfrac{1}{2} \times 0{\cdot}4854 \times 4''{\cdot}15 = \pm\, 1''{\cdot}01 = \pm\, \overset{s.}{0{\cdot}067}.$$

In order to eliminate any errors in the level itself, it is the practice to reverse it several times, or to take a series of readings in one position, and then to turn it, end for end, and to take a set of readings in an opposite position. The following is an example:—

ROYAL OBSERVATORY, GREENWICH.

1848.	April $\overset{d.}{14}$, $\overset{h.}{21}$.	
Position of Illum. End.	East.	
	East Scale.	West Scale.
Cross Level.	div.	div.
E	115·2	110·3
W	107·0	119·0
E	115·3	110·4
W	107·0	119·0
E	115·4	110·5
W	106·6	119·0
Sums	666·5	688·2
Subtract Sum of East Scale		666·5
		12)21·7
		1·81
Add for inequality of Pivots	·	
West End high in Scale Divisions . .		
Multiply by 1″·2 . .		
West End high in seconds		+ 2″·17

The value of one division of the level scale may be determined by the application of the level to a vertical circle, moving it so as to produce changes in the circle reading, and in the position of the bubble. Thus, in the Greenwich Observations, 1836, introduction, pages xvi. and xvii., " on January 27, the level of the transit instrument was lashed to Troughton's circle, and the circle being moved till the bubble was alternately near the end of one and the other scale, the microscopes A and B were read. This operation was repeated sixteen times. The mean of the results gave 2′ 0″·03 = 112·18 parts at one end, = 108·93 parts at the other end of the scale; the mean value of one part being, therefore, 1″·086."

It can also be determined by the comparison of the results of the transit of a star observed by reflection and direct vision. A slowly moving star is generally selected for this purpose, four or more wires being observed in each position. The difference of the two results (separately reduced) is equal to double the level error of the object observed, which can be converted into " error of inclination" for an equatorial star by the proper factor. By comparing this result with the level indication, the value of one division of the scale may be found.

Lastly, it can be obtained by elevating or depressing the Y's, by the proper screw, and finding the effect of a change of level indicative of the transits of a close circumpolar star. The level error may also be obtained by a Bohneberger's eye-piece, an instrument consisting of an ordinary eye-piece, perforated at the side, and so arranged that, by means of a glass reflector, placed near the eye lens, inside the tube, the light of a lamp may be thrown into the instrument. In this manner the system of vertical and horizontal wires may be seen by directing the telescope nearly to its nadir position, and placing underneath the object-glass a trough of mercury. By causing the two images to overlap, by means of the right ascension micrometer (noting its readings), and comparing these with the corresponding readings for the line of collimation of the instrument previously obtained, the level error may be easily deduced. This method is now practised in the use of the Greenwich transit circle. The following form shows the readings of the transit micrometer for coincidence of the central wire for determination of error of level of the axis of the transit circle in 1851 :—

Approx. Solar Time	Oct. 11, 7^h	12, 22^h	13, 8^h	13, 22^h	14, 8^h	14, 22^h
Observer	H.	R.	R.	H. B.		J. H.
	r	r	r	r	r	r
Readings of Transit Micrometer	31·634	31·650	31·650	31·625	31·620	31·589
	·635	·642	·658	·660	·675	·580
	·640	·650	·660	·625	·642	·583
	·643	·640	·650	·669	·662	·578
	·635	·639	·643	·655	·621	·582
	·650	·650	·648	·667	·661	·580
Sum of Readings	·237	·271	·309	·301	·281	·492
Mean	31·639	31·645	31·652	31·650	31·647	31·582
Subtract from Reading for Line of Collimation	31·449	31·455	31·455	31·455	31·455	31·455
Remainder	0·190	0·190	0·197	0·195	0·192	0·127
Reduction for Value in Arc	....	....	....	....	....	....
	″	″	″	″	″	″
Error of Level	2·81	2·81	2·91	2·88	2·84	1·88

The azimuthal error may be determined by the transits above and below the pole of any close circumpolar star. In observatories, where the adjustments of an instrument are sufficiently steady to be relied on, the successive passages of Polaris or δ Ursæ Minoris for several days are used for this purpose. But in small instruments, whose foundations are not so firm, it is usual to obtain an azimuth error by corresponding observations of high and low stars, or an observation of Polaris or δ Ursæ Minoris, compared with the transit of a star south of the zenith. Thus Polaris and θ Ceti can be used advantageously for this purpose, the difference of their right ascensions being only thirteen minutes, and their tabular right ascensions being known with sufficient precision.

Mr. Johnson, in his Oxford Observations, uses certain pairs of stars for azimuthal errors, whose positions have been determined with considerable accuracy. These stars are distributed in such a manner, that the observer can have little trouble in finding one transiting above the pole, and another passing below the pole within a short interval of time of each other. Thus, the result is entirely free from any doubt as to the rate of the clock, or other causes, and the principal error can only arise from the unavoidable uncertainty of observation.

It is frequently necessary, in the use of a small transit instrument, to have a fixed mark in the direction of the meridian, which can be easily referred to for the purpose of determining the error of collimation, and also its deviation from the meridian, in cases when, from cloudy weather, observations of circumpolar stars cannot be obtained.

At the distance of 286 feet, a circle of two-tenths of an inch in diameter will subtend an angle of 12″ of azimuth. By noting the positions of the central wire of the transit in reversed positions of the axis, an approximation may be made at once to the error of collimation from its relative positions in this circle. This meridian mark may be frequently verified by observations of circumpolar stars.

If the instrument is furnished with a micrometer, this method of determining the collimation will not be necessary.

We will now proceed with the investigation of the formulæ for the three preceding errors, and will shew the method of application to the results of observations.

Error of Collimation.—The error of collimation is the distance of the middle line from the true line of collimation, east or west. When the middle wire is east of the true line of collimation, the transit is observed too soon, and consequently the correction for collimation is additive for stars above the pole, and subtractive for those below the pole. The contrary takes place when the middle wire is west of the line of collimation. According to this, the transit telescope may be supposed to describe a circle parallel to the meridian. Thus, let C S A (Fig. 191) be the circle described by the telescope, or the circle in whose plane the middle wire is. Then A N, or S N, is the error of collimation, which is generally *expressed* in seconds of arc. Let S be the place of a star, P *s* B a great circle passing

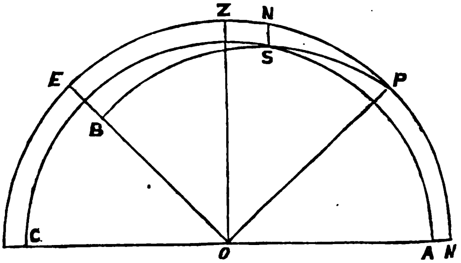


Fig. 191.

through it. We have now to find B E, the corresponding part of the equator; but $\frac{s\,n}{\sin P S}$ = B E in arc, or $\frac{s.n}{15 \sin P S}$ = B E in time, which is the correction to be applied to the observed transit. In that investigation, stars, however, near the pole are supposed to transit the middle wire, that is, their distance from the pole is supposed to be greater than the error of collimation. If this were not the case, such stars could not pass the middle wire.

The correction for the error of collimation is written thus,

$$\text{Correction} = \text{error} \times \frac{1}{15 \times \sin \text{stars N P D}}.$$

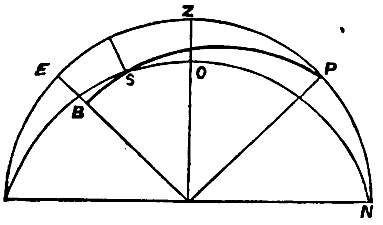


Fig. 192.

Correction for Level.—The error of level is considered positive when the western pivot is too high. In that position the great circle, passing through the north and south points of the horizon, and which is the circle on which observations are made, lies to the east of the meridian, and the observed time of passage is less than the true time for stars above the pole.

Thus (Fig. 185), O Z = the error of level.

$$\text{Z O} \times \cos \text{S O} = \text{S } x\text{, and}$$

$$\text{B E} = \frac{\text{S } x}{15 \sin \text{N P D}} = \frac{\text{S } x}{15 \sin \text{P S}}.$$

$$\therefore \text{B E}' = \frac{\text{Z O} \times \cos \text{zenith distance south}}{15 \sin \text{N P D}}.$$

Azimuthal Error.—This error is considered positive when the eastern pivot is too far north. In this position stars to the south of the zenith are observed before they come to the true meridian, and those north of the zenith after they pass the true meridian.

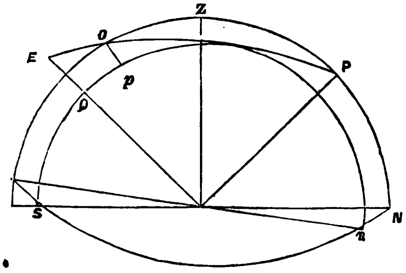


Fig. 193.

Let N *n* or S *s* (Fig. 193) be the azimuthal error, O the observed place of the star, the S *s* × sin Z *o* = *o p*, draw the great circle P O E from the pole to the equator, passing through the star O, then E Q is the correction in R A in arc, which, being divided by 15, will give the correction in time; but E Q = O *p* × $\frac{1}{\sin \text{P O}}$; therefore, E Q' = S *s* × $\frac{\sin \text{Z } o}{15 \sin \text{P } o}$ = azimuthal error × $\frac{\sin \text{Z D south}}{15 \sin \text{N P D}}$. Since the sign of the correction to the north of the zenith is contrary to that to the south, if zenith distances *south* have the sign *plus*, and those to the north the sign *minus*, the above expression will give the correction in either case.

The preceding corrections for errors of level, azimuth, and collimation being all supposed positive, may be represented by

$$\frac{l \cos \mathrm{Z\,D}}{\sin \mathrm{N\,P\,D}} + \frac{a \sin \mathrm{Z\,D}}{\sin \mathrm{N\,P\,D}} + \frac{c}{\sin \mathrm{N\,P\,D}}$$

where l represents the error of level, a that of azimuth, and c the error of collimation. By introducing the latitude L, the expression for the sum of these corrections may be written thus (δ being the declination),

$$\left.\frac{l \cos (\mathrm{L} - \delta) + a \sin (\mathrm{L} \mp \delta) + c}{\cos \delta}\right\} \text{or} = \begin{array}{l} \dfrac{l (\cos \mathrm{L} \cos \delta \mp \sin \mathrm{L} \sin \delta)}{\cos \delta} \\ + \dfrac{a (\sin \mathrm{L} \cos \delta \pm \cos \mathrm{L} \sin \delta)}{\cos \delta} \end{array}$$

$$= l \cos \mathrm{L} + l \sin \mathrm{L} \tan \delta + a \sin \mathrm{L} - a \cos \mathrm{L} \tan \delta + c \sec \delta.$$

$$= (l \cos \mathrm{L} + a \sin \mathrm{L}) + (- a \cos \mathrm{L} + a \cos \mathrm{L}) \tan \delta + c \sec \delta.$$

$= m + n \sin \delta + c \sec \delta$, which is the form of these corrections Bessel gives in the "Tabulæ Regiomontanæ."

To Find the Azimuthal Errors by Transits of a Circumpolar Star.

Let T = the time of superior transit, supposed too late, t = the time of an inferior transit, which will be too early; then, since the difference between these transits corrected for motion in R. A. should be 12h., all the other corrections being made, we shall have—

$$\mathrm{T} - \frac{a \sin \mathrm{Z\,D}}{15 \sin \mathrm{N\,P\,D}} = 12'' + t + \frac{a \sin \mathrm{Z\,D'}}{15 \sin \mathrm{N\,P\,D}}$$

Let $\mathrm{T} - t = 12\text{h.} + b$, then—

$$b = \frac{a (\sin \mathrm{Z\,D} + \sin \mathrm{Z\,D'})}{15 \sin \mathrm{N\,P\,D}} = \frac{a (\sin (\text{colat} - \mathrm{N\,P\,D}) + \sin (\text{colat} + \mathrm{N\,P\,D}))}{15 \sin \mathrm{N\,P\,D}}$$

$$\text{or } b = \frac{2\, a \sin \text{colat} \cos \mathrm{N\,P\,D}}{15 \sin \mathrm{N\,P\,D}} \quad \therefore a = \frac{15\, b \times \tan \mathrm{N\,P\,D}}{2 \sin \text{colatitude}}$$

At the end of this section, we have given numerical examples of the determination of azimuthal errors.

The preceding factors for collimation, level, and azimuth, viz.—

$$\text{Error of collimation} \times \frac{1}{15 \sin \text{stars N P D}}$$

$$\text{Error of level} \times \frac{\cos \text{Z D south}}{15 \sin \mathrm{N\,P\,D}}$$

$$\text{Error of azimuth} \times \frac{\sin \text{Z D south}}{15 \sin \mathrm{N\,P\,D}}$$

are tabulated for different north polar distances at the principal observatories, the multiplication being easily made by means of a sliding ruler. Several other contrivances have been invented for forming these corrections by mechanical methods, by the

Astronomer Royal, Professor Challis, and Mr. Carrington, for which we beg to refer to the memoirs of the Royal Astronomical Society.

Having thus explained the formulæ and methods of determining the three preceding errors, we shall now proceed with the investigation for a determination of the equatorial intervals of the wires, which are immediately necessary for the reduction of imperfect transits.

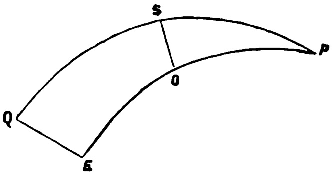


Fig. 194.

When a heavenly body is observed with a transit instrument at a distance from the meridian, if this distance be known, and also the distance of the object from the equator, the time to be added to or subtracted from the sidereal time of observation can be computed thus:—Let P be the pole, Q E a portion of the equator, O the object at the distance O S measured on a great circle. (It may be well to remark that to whatever part of the heavens a transit instrument is pointed, the distance from any wire to the middle wire is an arc of a great circle; it is the same as if the equator had been moved into that position). Now in the triangle we have O S and O P given to find S P O, which expresses the time; therefore $\sin \text{S P O} = \frac{\sin \text{S O}}{\sin \text{O P}}$, or if $h =$ the distance $\delta =$ the declination, $c =$ angle S P O $\qquad \sin c = \sin h \sec \delta$

Now it frequently happens, especially in cloudy weather, that an object has not been, or cannot be, observed at all the wires of the transit instrument; but if the intervals of the wires be known—that is, the times employed by an equatorial star in passing from the first to the second, from the second to the third, &c.—the true time at which an object, which has been observed on any wire or wires, would transit the meridian, can be found. The manner in which the equatorial intervals are given is the distance of the first wire from the mean of wires, which is called the first interval; the distances of the second and third from the same point are the second and third intervals—that is, the distance of the mean of all the wires from each wire is the interval: the intervals before the meridian point having the sign *plus*, and those after it the sign *minus*, for stars above the pole—the signs to be changed for stars below the pole. If the instrument be reversed, the signs are to be changed. For the determination of these intervals, Polaris, or some other star near the pole is observed; then for each transit the difference between each wire and the mean of all the wires is taken, and the mean of all these differences for the same wire for all the observations; then, if we call this mean for any wire $= c$, we have

$$\sin h = \sin c \cos \delta = \sin c \sin \text{stars N P D}$$

$$\text{therefore } h = \frac{\sin c \times \sin \text{stars N P D}}{15 \sin 1''}$$

This gives the equatorial distance in seconds of time from the wire under consideration to the mean of all the wires.

Now let $+a$, $+b$, $+c$, $+d$, $-e$, $-f$, $-g$, be the equatorial intervals for a telescope having seven wires, and suppose the times of transit of a star to have been observed on two of these wires; if the mean of these two observed times of transit be taken, it will be the time at which a star passed a point equally distant from both wires. We shall suppose the two wires to be the first and second; to find the distance of this point from the mean of wires, or from what we consider to be the meridian point. The distance between the first and second wires is $a - b$; hence the distance of the first wire from the required point is $\frac{a-b}{2}$, the distance of the first wire from the meridian point is a, therefore $a - \frac{a-b}{2} = \frac{+a+b}{2}$ is the distance of the point, for which we have the time of transit to the meridian point. This shows that we have only to take the sum of the equatorial intervals for the wires observed; this sum, divided by the number of wires, will be the distance of the point for which we have the time of transit from the meridional point. This distance being called h, and c representing the correction required, we have—

$$(1)\ \sin c = \sin h \sec \delta, \text{ or}$$
$$(2)\ c^{\sec} = h^{s} \sec \delta.$$

For the pole star and others near it (1) must be used.

Determination of Intervals of the Greenwich Transit Instrument by the following Transits of Polaris:—

Distance of each Wire from the Mean of the 7th Wires (Polaris), 1848, May 2 S. P. to May 12 S. P.

1848.	A	B	C	D	E	F	G	Decl. of Polaris.
	m. s.	m. s.	m. s.	s.	m. s.	m. s.	m. s.	° ′ 88 29 ″
May 2 S. P.	—24 29·86	—16 20·86	—8 9·86	— 0·86	+ 8 10·14	+16 17·14	+24 34·14	50·3
3	32·00	21·00	9·00	+ 0·50	9·50	17·50	34·50	50·0
4 S. P.	32·14	22·14	10·14	+ 1·36	10·36	17·86	34·81	49·8
4	32·57	20·57	8·57	+ 0·43	10·43	18·43	32·43	49·7
5 S. P.	33·29	21·29	11·29	— 0·29	9·71	20·71	35·71	49·5
5	30·14	19·14	10·14	— 1·14	9·36	17·36	33·86	49·4
6 S. P.	33·14	21·64	8·14	+ 0·36	8·86	18·36	35·36	49·2
7	28·57	22·57	8·57	— 0·57	6·43	18·43	35·43	48·9
8	30·86	20·86	11·86	+ 1·14	12·14	19·14	31·14	48·7
9 S. P.	31·57	21·57	9·57	+ 0·43	8·43	19·43	34·43	48·6
9	29·57	19·07	8·57	+ 0·43	8·93	15·43	32·43	48·5
May 12 S. P.	—24 29·00	22·00	10·00	— 1·00	8·00	16 19·00	35·00	48·0
								° ′ 88 29
Means	—24 31·06	—16 21·06	— 8 9·64	+ 0·07	+ 8 9·36	+16 18·23	m. s. +24 34·11	″ 49·22
Or in Arc	° ′ ″ 6 7 45·9	° ′ ″ 4 5 15·90	° ′ ″ 2 2 24·60	+ 1·05	+2 2 20·40	+4 4 33·45	+6 8 31·65	
Log Sines Constant	9·0284678 2·5571186	8·8529932 2·5571186	8·5514513 2·5571186	4·6855740 2·5571186	8·5512029 2·5571186	8·8517407 2·5571186	9·0293639 2·5571186	
Log. Equat. Interval	1·5855864	1·4101118	1·1085699	7·2426935	1·1083216	1·4088593	1·5864825	
Equatorial Interval	s. + 38·511	25·711	12.840	0·002	12·633	25·636	38·591	
In Arc	′ ″ 9 37·67	′ ″ 6 25·67	′ ″ 3 12·60	″ + 0·03	′ ″ 3 12·50	′ ″ 6 24·54	′ ″ 9 38·87	

Log cos 88 29 49·22	8·4187848
Ar. co. log sin 1″	5·3144251
Ar. co. log 15	8·8239087
Sum = constant . . .	2·5571186

Computation of Intervals from Middle Wire for Polaris for Declinations 88° 29′, 88° 30′, and 88° 31′.

	A	B	C	D	E	F	G
Log sin equatorial interv.	7·4472539	7·2717901	6·9702297		6·9700041	7·2705157	7·4481553
Ar. co. log cos 88° 29′	1·5772832	1·5772832	1·5772832		1·5772832	1·5772832	1·5772832
Log sin interval	9·0245371	8·8690733	8·5475129		8·5472873	8·8477989	9·0254385
Interval	° ′ ″ 6 4 26·38	° ′ ″ 4 3 3·45	° ′ ″ 2 1 18·27		° ′ ″ 2 1 14·50	° ′ ″ 4 2 20·65	° ′ ″ 6 5 11·94
In. time	m. s. 24 17·76	m. s. 16 12·23	m. s. 8 5·22	s. 0·07	m. s. 8 4·97	m. s. 16 9·37	m. s. 24 20·80
Log sin equatorial interval	7·4472539	7·2717901	6·9702297		6·9700041	7·2705157	7·4481553
Ar. co. log cos 88° 30′	1·5820810	1·5820810	1·5820810		1·5820810	1·5820810	1·5820810
Log sin interval	9·0293349	8·8538711	8·5523107		8·5520851	8·8525967	9·0302363
Interval	° ′ ″ 6 8 30·17	° ′ ″ 4 5 45·73	° ′ ″ 2 2 39·16		° ′ ″ 2 2 35·33	° ′ ″ 4 5 2·45	° ′ ″ 6 9 16·29
In. time	m. s. 24 34·01	m. s. 16 23·05	m. s. 8 10·61	s. 0·08	m. s. 8 10·36	m. s. 16 20·16	m. s. 24 37·09
Log sin equatorial interval	7·4472539	7·2717901	6·9702297		6·9700041	7·2705157	7·4481553
Ar. co. log cos 88° 31′	1·5869324	1·5869324	1·5869324		1·5869324	1·5869324	1·5869324
Log sin interval	9·0341863	8·8587225	8·5571621		8·5569365	8·8574481	9·0350877
Interval	° ′ ″ 6 12 39·52	° ′ ″ 4 8 31·66	° ′ ″ 2 4 1·86		° ′ ″ 2 3 57·99	° ′ ″ 4 7 47·89	° ′ ″ 6 13 26·15
In. time	m. s. 24 50·63	m. s. 16 34·11	m. s. 8 16·12		m. s. 8 15·87	m. s. 16 31·19	m. s. 24 53·74

Comparing the intervals for 88° 29′ and 88° 30′ we have—

s. + 16·25	s. + 10·82	s. + 5·39		s. — 5·39	s. — 10·79	s. — 16·29

And dividing these by 60, we have the change for one second of arc—

+ 0·271	+ 0·180	+ 0·090		— 0·090	— 0·180	— 0·271

Therefore for Polaris, declination = 88° 29′ + *n*″.

	m.	s.
A	+ 24	17·76 + *n*″ × 0·271
B	+ 16	12·23 + *n* × 0·180
C	+ 8	5·22 + *n* × 0·090
D	—	0·07
E	— 8	4·97 — *n* × 0·090
F	— 16	9·37 — *n* × 0·180
G	— 24	20·80 — *n* × 0·271

The following examples will illustrate the whole method of completing imperfect transits:—

SPECIMEN PAGE OF THE COMPLETION OF IMPERFECT TRANSITS.

Day.	1848, July 20	July 20.	July 20.	July 20.	July 20.
Object.	B. A. C. 5509	α Herculis.	β Draconis.	α Ophinchi.	B. A. C. 6125
N. P. D.	27° 57′	75° 26′	37° 35′	77° 20′	111° 27′
1st wire.		51·5	50·0		
2 "		9·6	10·8		
3 "		22·8	32 0	32·7	
4 "	21 39·5	36·4	52·7	46·0	
5 "	22 6·7			59 5	11·6
6 "	22 34·0			12·5	25 7
7 "	16 23 1·6	17 7	17 26	17 28 25·6	17 58 39·6
	4) 89 21·8	4) 65·3	85·5	29 6 3	3) 76·9
Mean of wires.	16 22 20·45	17 7 16·33	17 26 21·38	17 27 59·26	17 58 25 63
Correction to imperfect transit *	— 41·11	+ 19·91	+ 31·59	— 13·16	— 27·6
Concluded transit over mean of the seven wires.	16 21 39·34	17 7 36·24	17 26 52·97	17 27 46·10	17 57 58·39

* See the following Form.

COMPLETION OF IMPERFECT TRANSITS OF POLARIS.

Day.	1848, July 21	July 22.	Sept. 5.	Sept. 21.	1848, Oct. 2.
Object.	Polaris.	Polaris, S. P.	Polaris S. P.	Polaris, S. P.	Polaris, S. P
1st wire.	40 26·0				40 29·0
2 "	48 34·0	48 40·0			48 47·0
3 "	56 46·5	56 47·0	56 24·0		57 0·0
4 "	4 54·0	57·0	4 35·0	5 19·0	5 10·0
5 "	4·0		12 40·0	13 28·0	13 21·0
6 "			52·0	21 40·0	
7 "	1 13	13 4	13 20	13 29 50·5	13
	5)28 3 44·5	3) 144·0	4) 151·0	137·5	28 4 47·0
Mean of observed wires.	56 44·90	12 56 48·00	13 8 37·75	13 17 34·38	12 56 57·40
Reduction*	+ 8 10·09	+ 8 8·85	— 4 5·64	— 12 17·14	+ 8 11·94
Concluded transit over mean of the seven wires.	1 4 54·99	13 4 56·85	13 4 32·11	13 5 17·24	13 5 9·34

* See the following Form.

FOR POLARIS AND δ URSÆ MINORIS.

Day.	July 21.	July 22.	September 5.	September 21.	October 2.
Object.	Polaris.	Polaris S. P.	Polaris S. P.	Polaris S. P.	Polaris S. P.
For observation on A	+ 24 17 76 + *n* × 271			+ 24 17 76 + *n* × 271	
„ B	+ 16 12 23 + *n* × 181		+ 16 12 23 + *n* × 181	+ 16 12 23 + *n* × 181	
„ C	+ 8 5 22 + *n* × 090		+ 85 22 + *n* × 090	+ 8 5 22 + *n* × 090	+ 85 22 *n* × 090
„ D	— 0 07	— 0 07	— 0 07	— 0 07	0 07
	48 35 14 *n* × 542	— —*n* +	24 17 38 *n* × 271	48 35 14 *n* × 542	8 5 15 *n* × 090
„ E	— 8 4 97 *n* × 090	— 8 4 97 — *n* + 0 090	— 8 4 97 + *n* × 090	— —*n* +	— 8 4 97 — *n* × 090
	40 30 17 *n* × 452	— 16 9 37 — *n* × 180	16 12 41 *n* × 181	— —*n* ×	0 18 — *n* × 000
F	— —*n* ×	— —*n* ×	— —*n* ×	— —*n* ×	— 16 9 37 — *n* × 180
	— —*n* ×	— —*n* +	— —*n* ×	— —*n* ×	— 16 9 19 — *n* ×
G	— —*n* ×	— —*n* ×	— —*n* ×	— —*n* ×	— 24 20 80 — *n* × 272
Sum	+ 40 30 17 *n* × 452	— 24 14 41 — *n* × 270	16 12 41 *n* × 181	48 35 14 *n* × 542	— 40 29 99 — *n* × 452
Value of *n*	944	59 44	51 65	7 16	756
Decl. = 88° 29′ + *n*″	1808	1080	905	3252	2712
	181	108	109	54	226
	41	24	2	38	32
		1	1		
Product	20 30	— 12 13	10 17	+ 33 44	— 29 70
Add Sum independent of *n*	40 30 17	— 24 14 41	16 12 41	48 35 14	— 40 29 99
Sum	5) 40 50 47	3) — 24 26 54	4) + 16 22 58	4) 49 8 58	5) — 40 59 69
Mean	8 10 09	— 8 8 85	4 5 64	12 17 14	— 8 11 94

ROYAL OBSERVATORY, GREENWICH.

Calculations for Completing Imperfect Transits in the Year 1848. For Stars and Planets in general.

Day.	July 21.									
Object.	B.A.C. 5509.		α Hercules.		β Draconis.		α Ophinchi.		B.A.C. 6125.	
For observation on A .			+ 38·511							
,, B .			+ 25·711							
,, C .			+ 12·840				+ 12·840			
,, D .	— 0·002		— 0 002				— 0·002			
			77·060				12·838			
,, E .	— 12·833						— 12·833		— 12·833	
							0·005			
,, F .	— 25·636						— 25·636		— 25·636	
							— 25·631			
,, G .	— 38·591						— 38·591		— 38·591	
Sum & mean	— 77·062	— 19·266		19·265		.. 19·265	— 64·222	— 12·844	— 77·060	— 25·687
Declination and nat. sec.	62° 3′	553·312	14° 34′	23·301	52° 25′	6936·1	12° 40′	7420·1	21° 27′	4470·1
		38·532		1926·5		19265		12844	25687	
		1·927		578		11559		257	1798	
		578		58		578		51	103	
		58		4		173		9	10	
		10				12				
		1								
Product . .		41·106		19·905		31587		31·161	— 27598	

Another method of reducing imperfect transits is to form a table of intervals from the middle wire for different north polar distances, by multiplying the equatorial intervals by the seccant of the declination (see preceding formulæ). When the wires observed of any object have been separately reduced to the middle wire by this table, the mean of all the results will be the correct number. For a planet, the change of right ascension in the interval may be applied in the formation of the results for each wire.

The following is an example of Polaris by this method:—

1848, October 2. Wires observed—A, B, C, D, E.

Declination of Polaris 88° 29′ 65″·7.

	h.	m.	s.	Reduction to Mean of Wires.		m. s.
A...		40	29·0 ...	+ 24 17·76 + 17·80	= 24 35·56	= 5 4·56.
B...		48	47·0 ...	+ 16 12·23 + 11·83	=	5 11·06
C...		57	0·0 ...	+ 8 5·22 + 5·91	=	5 11·13
D...		5	10·0 ...	— 0·07	=	5 9·93
E...	13	13	21·0 ...	— 8 4·97 5 91	=	5 10·12
					5)	46·80
						13 5 9·36

The intervals of the transit wires may also be determined by directing the telescope of a theodolite to the object-glass of the transit instrument. In this manner the wires may be distinctly seen; but it is also necessary that the axes of the two instruments be exactly in the same parallel. Gauss first introduced this method at Gottingen. The following is an example. It is to be premised that the axis of the theodolite was higher than that of the centre of the circle, and therefore that the measured readings must be diminished by multiplying by the cosine of the inclination, or 23° 58′ 52″.

1823, October 23.

	Horizontal Distance.		True Distance.	
	′ ″	′ ″	′ ″	′ ″
Wire I.	11 25·050	11 23·000	10 25·93	10 24·06
„ II.	7 45·900	7 46·275	7 5·69	7 6·03
„ III.	3 58·225	3 57·750	3 37·66	3 37·23
„ V.	3 58·850	3 59·375	3 38·24	3 83·71
„ VI.	7 53·625	7 53·875	7 12·74	7 12·98
„ VII.	11 17·600	11 17·775	10 19·11	10 19·28

At the Royal Observatory, Greenwich, the Astronomer Royal has made use of the south collimator for the determination of the intervals of the wires of the transit circle. "The eye-piece of the transit circle was turned round 90°, so as to bring the wires into a sensibly horizontal position, and with the head of the micrometer upwards when the telescope was directed to the south. Each of the wires in succession was then brought upon a determinate point of the image of one of the nearly horizontal wires of the south collimator, partly by turning the instrument and partly by the right ascension micrometer, and the instrument being clamped, the six microscopes of the zenith distance circle were read as well as the right ascension micrometer."

In this manner, the following concluded circle readings for each wire were obtained:—

	′ ″
Wire I.	59 33·696
„ II.	56 6·746
„ III.	52 39·290
„ IV.	49 12·018
„ V.	45 44·880
„ VI.	42 17·242
„ VII.	38 50·482

The mean of these numbers is 49′ 12″·051, and, comparing this with each number separately, we finally get for the distance in arc of each wire from the mean of all:—

	′ ″	s.
For Wire I.	+ 10 21·645	or + 41·443
„ „ II.	+ 6 54·659	+ 27·646
„ „ III.	+ 3 27·239	+ 13·816
„ „ IV.	— 0·033	— 0·002
„ „ V.	— 3 27·171	— 13·811
„ „ VI.	— 6 54·809	— 27·654
„ „ VII.	— 10 21·569	— 41·438

Dr. Drew has usefully applied a wire micrometer, attached to a five-feet telescope, for the determination of the intervals of his transit circle—an instrument of forty-two inches focal distance. The value of one revolution of the micrometer being known, he was able to measure easily the distances of the wires, by directing the two object-glasses towards each other. The result agreed closely with that deduced from a transit of δ Ursæ Minoris.

To find the correction to an imperfect transit of the moon or a planet:—

Let h'' be the apparent distance from the middle wire.
,, $\mu =$ the right ascension of the zenith.
,, $\alpha' =$ the apparent right ascension of the interval.
,, $\delta' =$ the apparent declination.

Then we have—

$$\text{Sin } h = \cos \delta' \sin \overline{\mu - \alpha'} = \frac{d}{d'} \cos \delta \sin (\mu - \alpha).$$

Put $\sin h = h \sin 1''$, and $\sin \overline{\mu - \alpha} = \overline{\mu - \alpha} \sin 1''$.

Then we shall have—

$$h = \frac{d \cos \delta}{d'}(\mu - \alpha) \ldots \mu - \alpha = \frac{h \times d'}{d \cos \delta} \ldots (36).$$

This is the expression for all those bodies which do not change their right ascension. The fixed stars are the only ones which do not, and their distances are so immense that α' and d may be regarded as equal, which is also the case with the planets, on account of their distance. Let λ^s be the moon's or planet's increase of R. A. in one second of time, then $(1 - \lambda)^s$ is the equatorial space described by the moon or planet in one second of sidereal time; consequently, the sidereal time of describing $\mu - \alpha$, will be represented by—

$$\frac{1}{1-\lambda}\left(\frac{d' h}{d \cos \delta}\right) = (1 + \lambda)\frac{d' h}{d \cos \delta} = (1 + \lambda) \times \frac{\sin \text{Geo. Z. D.}}{\sin \text{app. Z. D.}} \times \frac{h}{\cos \delta}, \text{nearly} \,(37).$$

This is the formula given in the Greenwich Observations for the same purpose. In the case of the planets d' is very nearly equal to d, and the formula is—

$(1 + \lambda) \times \frac{h}{\cos \delta}$, where λ is the increase of the planet's right ascension in one second of sidereal time.

COMPLETION OF AN IMPERFECT TRANSIT OF THE MOON.

Day.	1848, August 9.
Object.	☽ 1 L.
1st wire.	47·0
2 ,,	0·9
3 ,,	14·7
4 ,,	
5 ,,	
6 ,,	
7 ,,	17 4
	3) 2·6
Observed mean of wires . .	17·4 0·87
Reduction . .	27·79
Concluded transit over mean of seven wires .	17 4 28·66

1848, August 9.	° '
☽'s geo. N.P.D. . . .	107 41
Co. lat. of Greenwich . . .	38 31
☽'s geo. Z.D.	69 10
Parallax in altitude . . .	49
☽'s app. Z.D.	69 59

I = 131″·62 = the increase (in seconds of time) of the Moon's right ascension for a transit over a meridian upon the earth, distant by 1h. of terrestia longitude (Nautical Almanac, 1848. Section—Moon Culminating Stars).

Log $\frac{3600+I}{3600}$	0·01559
Log sin. ☽'s geo. Z.D.	9·97064
Ar. co. log. sin. ☽'s app. Z.D.	0·02706
Log sec. ☽'s geo. decl.	0·02102
	0·03431
Log 25·687 s. (see below*) .	1·40972
	1·44403
Completion of imperfect transit, to reduce to mean of seven wires . . .	s. 27·79

1848, August 9.

Wires Observed.

For an Equatorial Star.	A . . .	+ 38·511
	B . .	25·711
	C . . .	12·850
	3)	77·062
		+ 25·687*

COMPLETION OF AN IMPERFECT TRANSIT OF A PLANET.

Day,	1848, August 9.	
Object,	Mercury 2 L.	
1st wire.		27·2
2 ”		40·2
3 ”		54·2
4 ”		8·1
5 ”		
6 ”		
7 ”	8 6	
	4)	189·7
Mean of observed Wires . .	8 3	47·42
Reduction * . .		+ 20·53
Concluded transit .	8 4	7·95
Object . . .	Mercury 2 L.	

Planets, N.P.D. = 70° 22′

I = variation in R.A. in 1h. = + 12·92 s.

Wires observed :—

A	+ 38·511
B	+ 25·711
C	+ 12·840
D	— ·002
4)	+ 77·060
	+ 19·265
Log	+ 1·28477
Log co. sec. planets, N.P.D.	0·02601
Log $\frac{3600+I}{3600}$.	0·00155
	1·31233
L. No. . .	s. 20·53*

Formula for Determining Collimation.—In the operations of the Brussels Observatory, the following formula for the determination of the collimation, independently of the other errors of the instrument, is used. The investigation is to be found in M. Lingré's memoir, "Sur les Corrections de la Lunette Meridienne"—one of the prize essays of the Brussels Academy, vol. xviii., 1844—45. The stars best suited for this method are, one near the pole, another near the zenith, and a third of south declination. The example and formula are as follows :—

Example—Greenwich Observations, 1842, May 10.

		h.	m.	s.		h.	m.	s.		°	′	″	
β Corvi . .	H =	12	25	39·53	RA =	12	26	8·86	p =	112	31	39	Observer Henry.
γ Ursæ Majoris	H° =	11	45	3·88	RA° =	11	45	33·11	$p°$ =	55	22	39	
Polaris S.P. .	H′ =	13	1	30·42	RA′ =	13	2	6·52	p' =	1	31	59	

Calculation of the Collimation.

$$C = D^{\circ}\frac{\sin p^{\circ}\cos\frac{1}{2}(p-p')}{2\sin\frac{1}{2}(p-p^{\circ})\sin\frac{1}{2}(p^{\circ}-p')} + D'\frac{\sin p'\cos\frac{1}{2}(p-p^{\circ})}{2\sin\frac{1}{2}(p-p')\sin\frac{1}{2}(p'-p^{\circ})}$$

$$D^{\circ} = 15\,[(H + RA^{\circ}) - (H^{\circ} + RA)] = -1''\cdot 50.$$

$$D' = 15\,[(H + RA') - (H' + RA)] = +101''\cdot 40.$$

$$\tfrac{1}{2}(p - p') = 57^{\circ}\ 1'\ 49'' : \tfrac{1}{2}(p - p^{\circ}) = 38^{\circ}\ 33'\ 0'' : \tfrac{1}{2}(p^{\circ} - p') = 18^{\circ}\ 28'\ 49''$$

Log D°	$-0\cdot 17609$	Log D'	$2\cdot 00604$
Log sin p° . . .	$9\cdot 76317$	Log sin p' . . .	$-8\cdot 42738$
Log cos $\tfrac{1}{2}(p - p')$ . .	$9\cdot 73516$	Log cos $\tfrac{1}{2}(p - p^{\circ})$ . .	$9\cdot 89324$
Ar. co. log 2 . .	$9\cdot 69897$	Ar. co. log 2 . . .	$9\cdot 69897$
Ar. co. log sin $\tfrac{1}{2}(p - p^{\circ})$.	$0\cdot 20537$	Ar. co. log sin $\tfrac{1}{2}(p - p')$.	$0\cdot 07626$
Ar. co. log sin $\tfrac{1}{2}(p^{\circ} - p')$	$0\cdot 49897$	Ar. co. log sin $\tfrac{1}{2}(p' - p^{\circ})$.	$-0\cdot 49897$
	$-0\cdot 07773$		$+0\cdot 60086$
1st Number =	$-1''\cdot 20$	2nd Number	$+\ 3\cdot 99$
			$-\ 1\cdot 20$
		C =	$+\ 2\cdot 79$

A similar calculation made on May 13, 1842, using β Corvi, γ Ursæ Majoris, and Polaris, gave $C = +2''\cdot 67$. This agrees very satisfactorily with the preceding result.

The error determined (by reversal of the instrument in the usual way practised at Greenwich) was found to be, on April 6, $= +1''\cdot 51$, which is a very good agreement.

The Astronomer Royal, in the Cambridge Observations, 1828, proposed a method nearly similar for determining the clock errors, and the errors of collimation and azimuth, but was obliged to discontinue it on account of the imperfect places of the star catalogues, and the difficulty of making observations sufficiently accurate. The following is the investigation:—"Let x, y, z, be the clock error and the errors of collimation and deviation (the clock being supposed slow). O, O′, O″, the observed times of transit corrected for the error of level and the clock rate; T, T′, T″, the true sidereal times at which they pass the meridian; Y, Y′, Y″, the respective multipliers of the error of collimation, which give results additive to the observed transits; Z, Z′, Z″, the same for the error of meridian. Then the following equations subsist:—

$$\begin{aligned} O + x + Yy + Zz &= T \\ O' + x + Y'y + Z'z &= T' \\ O'' + x + Y''y + Z''z &= T'' \end{aligned}$$

And subtracting the 2nd from the 1st, and the 3rd from the 2nd—

$$\begin{aligned} (Y - Y')\,y + (Z - Z')\,z &= (T - O) - (T' - O') \\ (Y' - Y'')\,y + (Z' - Z'')\,z &= (T' - O') - (T'' - O'') \end{aligned}$$

"The catalogued R. A. and the observation of the transit must be extremely accurate, in order that any result worthy of confidence may be obtained from these equations. The most favourable stars are Polaris, a star below the pole, and a star above the pole, whose N. P. D. is considerable."

Method of Finding the Time.—The method of finding the time given in the following pages is capable of so many useful applications to travelling astronomers, that, in a treatise on this subject, it would be unwise to neglect it.

Professor Hansteen, in "Schumacher's Astronomische Nachrichten," vol. vi., p. 189, gives a simple and expeditious means of determining the time by a small transit instrument, which requires only the observed transit of Polaris in a certain parallel, and the transit of a known fundamental star in the same vertical. The formulæ of *reduction* are as follow:—

$$A = \frac{\tan\phi - \tan\delta}{\tan\delta - \tan\delta'\cos P}$$

$$B = A^2\frac{\tan\phi + 2\tan\delta'}{\tan\delta - \tan\delta'\cos P}$$

$$P = 15\,(\alpha - \alpha' + T)$$

$$0 = \frac{A\sin P}{\sin 15''} + \frac{B\sin P^3}{\sin 90}$$

In which δ and α are respectively the declination and right ascension of the pole star δ' and α', are the same co-ordinates of another fundamental star; ϕ is the latitude of the place, and T is the sidereal interval of the observations.

Example.—At Christiana, on the 8th of November, 1823, the following transits of stars were observed with an universal instrument of Reichenbach :—

Star.		Eye-piece towards the East. h. m. s.	Star.		Eye-piece towards the West. h. m. s.
Polaris	τ . .	13 36 31·5	Polaris . .	τ' . . .	14 0 28·50
η Ursæ Majoris .	τ . .	13 42 4·96	Polaris . .	τ' . . .	14 10 47·46

The apparent places of the stars are—

		h. m. s.		° ′ ″
Polaris . .	α =	24 58 32·90 . .	δ =	88 22 26·68
η Ursæ Majoris .	α' =	13 40 34·47 . .	δ' =	50 11 34·79
α Bootes . .	α'' =	14 7 37·38 . .	δ'' =	20 6 8·66

Therefore we have—

	h. m. s.		h. m. s.
$\alpha - \alpha'$ =	11 17 58·43	$\alpha - \alpha''$ =	10 50 55·52
τ =	+ 5 33·46	τ' =	+ 10 18·96
$\alpha - \alpha' + \tau$ =	11 23 31·89	$\alpha - \alpha'' + \tau'$ =	11 1 14·48
P =	170° 52′ 58″	P′ =	165° 18′ 35″

The latitude ϕ is assumed to be 59° 54′ 8″·4

Log tan δ' . .	0·0791592	tan δ' . . =	35·22949
Log cos P . .	— 9·9944783		
	— 0·0736375		
			1·18478
		N . . .	36·41427
Log (tan ϕ — tan δ') .	9·7204185	Log tan ϕ . . =	1·7752525
Log N	1·5612716	Log tan δ' . . =	1·1999391
Log A	8·1591469	(Tan ϕ — tan δ')	0·5253134
Log sin 15″	4·1383339	Tan ϕ + 2 tan δ'	4·1251307
Log sin P	9·1999056		s.
	1·4973864	. . . . .	+ 31·433
Log A²	6·31829		
Log (tan ϕ + 2 tan δ')	0·61543		
Ar. co. log N	8·43873		
Ar. co. log sin 90″ . .	3·36018		
Log sin P³	7·59971		
	6·33234	. . . . . =	0·000
			31·433

In the same manner, if a calculation be made for α Bootes, we shall have—

$$\theta' = \overset{s.}{133{\cdot}205} + \overset{s.}{0{\cdot}004} = +\overset{m.\ \ s.}{2\ \ 13{\cdot}21}$$

From these values of the quantities θ and θ' are found the binary angles t an these two stars; and the zenith distance of the pole star by this formula—

$$\text{Cos}\ \xi = \cos\phi\cos\delta\cos(P - t) + \sin\phi\sin\delta.$$

Therefore, by calculation—

$$\begin{array}{rrr@{\qquad}rrr} & {}^\circ & {}' & & {}^\circ & {}' \\ P - t = & 170 & 45 & P' - t' = & 164 & 45 \\ \xi = & 31 & 43 & \xi' = & 31 & 40 \\ \kappa = & 0 & 23 & \kappa' = & 0 & 25 \\ \delta - \phi = Z = & -9 & 42 & \delta' - \phi'\ Z' = & -39 & 48 \\ C = & \multicolumn{2}{r}{\cdot 0{\cdot}13754} & C' = & \multicolumn{2}{r}{0{\cdot}15755} \end{array}$$

The daily gain of the clock was 2s·5; therefore for the time $\tau' - \tau$ or 28m., = 0s·047. Therefore, if κ and $\kappa' = 0$, $C + C'$ will be equal to 0·29509.

$$\begin{array}{rrr} & m. & s. \\ \tau' - \tau = & 28 & 42{\cdot}50 \\ -\alpha = & - & 0{\cdot}05 \\ -(\alpha'' - \alpha) = & -27 & 2{\cdot}91 \\ -\ \theta' - \theta) = & -\ 1 & 41{\cdot}78 \\ \hline S = & - & 2{\cdot}24 \end{array}$$

$$\frac{S}{C + C'} = \gamma = \frac{-2{\cdot}24}{0{\cdot}29509} = \overset{s.}{7}{\cdot}59.$$

By this value of γ itself is found.

$$\begin{array}{rl} & \quad\ \ s. \\ \text{for } \eta \text{ Ursæ Majoris } C\gamma = & -\ 1{\cdot}044 \\ \alpha \text{ Bootes } C'\gamma = & -\ 1{\cdot}196. \end{array}$$

Wherefore the state of the clock is as follows—

η Ursæ Majoris.

	h.	m.	s.
$\alpha =$	13	40	34·47
$\theta =$		+	31·43
$- C\gamma =$		+	1·04
Sidereal time =	13	42	6·94
Clock time =	13	42	4·96
Error of Clock		+	58·02

Arcturus.

	h.	m.	s.
$\alpha' =$	14	7	37·38
$\theta' =$		2	13·21
$+ C'\gamma =$			—1·20
Sidereal time =	14	9	49·49
Clock time =	14	10	47·46
Error of Clock		+	58·07

This method is likely to be of considerable value to the astronomers travelling place to place, and for this purpose we give it here.

On the Determination of Right Ascension by the Transit Instrun —The origin of right ascensions is the intersection of the ecliptic and equator vernal equinox. For the purpose of determining this origin, Dr. Maskelyne se α Aquilæ, the place of which he arbitrarily assumed, and with which he compa other principal stars, applying the clock error and rate; and, by these means, fo apparent right ascensions. These were afterwards converted into mean right ascen for the beginning of each year, by the application of precession, nutation, and aberr from certain assumed constants. The places of all the others were, therefore, af with the error of the assumed right ascension of α Aquilæ. To determine the amount of this error, he computed the sun's right ascension from the transit observations depending on the assumed place of α Aquilæ; and he also computed a right ascension of the sun, from observations of declination by the quadrant, by the formula.

Rad. × sin. R. A. = cotan. obliq. × tan. declination, and comparing the two results for certain periods of time, on each side of the equinoxes, so as to eliminate any errors arising from refraction, parallax, obliquity of the ecliptic, and latitude of the place, he obtained a correction to his assumed place of α Aquilæ, which was common to the remaining stars of the catalogue.

When a number of stars have had their right ascensions determined by referring them to some fundamental star, they will all be charged with the error which may happen to belong to this star; and it is an object of the utmost importance to ascertain the existence and quantity of such error. The difficulty lies in determining accurately the position of the first point of Aries, from which the right ascensions of all the stars are counted. The course pursued, therefore, by astronomers, is first to find the sun's right ascension, by comparing the transit of his centre with the transit of the fundamental star, or with the transits of several principal stars, related to it by known differences; and, secondly, to compute from his observed declination the right ascension belonging to the moment of the meridian passage. These operations should be performed on several days, near both the vernal and autumnal equinox. The right ascensions derived from a comparison with the stars should agree with those derived from the observed declinations of the sun. If there be a constant difference, this will be the correction to be applied to the assumed right ascension of the fundamental star. The sun's right ascension is deduced from his declination in the following manner:—Let A C (Fig. 195) represent a part of the equator, A D a part of the ecliptic, and A be the first point of Aries. Suppose the sun to be at S, and draw S B perpendicular to A C; then A B will be the right ascension of the sun, and S B his declination.

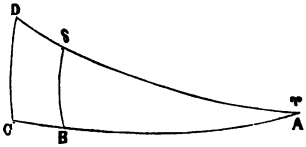


Fig. 195.

But, by Napier's rule, rad. × sin. A B = cotan. S A B × tan. S B; that is, sin. R. A. = cotan. obliquity × tan. dec.

Or, representing the sun's declination by δ, and the obliquity of the ecliptic by ω, we have

$$\sin \text{R. A.} = \frac{\tan \delta}{\tan \omega}$$

Example.—The following observations of the sun's centre were made at Greenwich in 1851:—

Date.	Sun's R. A. Observed.	Sun's Dec. Observed.
	h. m. s.	° ′ ″
Sept. 15	11 34 16·15	2 47 0·18 N.
16	37 51·22	2 23 50·70 N.
21	55 48·55	0 27 15·35 N.
22	59 24·19	0 3 51·53 N.
23	12 3 0·32	0 19 33·76 S.

The following example, from the appendix to the Greenwich Observations, 1851, will explain this process:—

Errors of the Assumed Position of the First Point of Aries, by Observations made near the Vernal and Autumnal Equinoxes of 1802.

Assumed latitude of Observatory . . . . 51° 28′ 38″·9
Assumed mean obliquity, 1802, January 1 . . . 23° 27′ 55″·58
Assumed mean R. A. of α Aquilæ, 1802, January 1, 19h. 41m. 7s.·00.

Month and Day. 1802.	Sun's R. A. from observation of Transits.	Sun's R. A. computed from observed Declination.	Excess of computed R. A.	Month and Day. 1802.	Sun's R. A. from observation of Transits.	Sun's R. A. computed from observed Declination.	Excess of computed R. A.
	° ′ ″	° ′ ″	″		° ′ ″	° ′ ″	″
Feb. 13	326 27 48·6	326 28 17·6	+29·0	Oct. 30	214 2 14·9	214 1 40·7	—34·2
14	327 26 28·5	327 26 52·5	+24·0	28	212 5 53·7	212 5 29·5	—24·2
26	338 56 40·4	338 57 7·2	+26·8	16	200 42 44·8	200 42 19·9	—24·9
27	339 53 17·3	339 53 38·5	+21·2	15	199 46 52·5	199 46 30·2	—22·3
Mar. 1	341 45 54·8	341 46 21·1	+26·3	14	198 51 5·5	198 50 45·0	—20·5
7	347 20 51·8	347 21 18·5	+26·7	7	192 24 31·6	192 24 12·0	—19·6
15	354 41 56·4	354 42 12·9	+16·5	Sept. 29	185 8 23·6	185 8 10·4	—13·2
18	357 26 4·0	357 26 10·4	+ 6·4	26	182 25 52·5	182 25 40·3	—12·2
20	359 15 18·0	359 15 33·5	+15·5	24	180 37 54·6	180 37 35·1	—19·5
23	1 58 53·7	1 59 7·8	+14·1	31	177 55 57·8	177 55 43·9	—13·9
25	3 47 46·4	3 47 57·5	+11·1	19	176 8 15·6	176 7 55·2	—20·4
26	4 42 14·7	4 42 26·8	+12·1	18	175 14 20·3	175 14 11·8	— 8·5
27	5 36 41·3	5 36 50·7	+ 9·4	17	174 20 33·3	174 20 10·4	—22·9
28	6 31 11·0	6 31 24·1	+13·1	16	173 26 38·7	173 26 22·6	—16·1
29	7 25 40·2	7 25 57·5	+17·3	15	172 32 44·3	172 32 33·3	—11·0
31	9 14 39·2	9 14 53·9	+14·7	13	170 45 1·6	170 44 41·6	—20·0
Apr. 1	10 9 9·7	10 9 22·0	+12·3	12	169 51 7·5	169 50 51·9	—15·6
4	12 52 57·6	12 53 11·5	+13·9	9	167 9 17·0	167 9 15·3	— 1·7
5	13 47 37·1	13 47 53·9	+16·8	8	166 15 12·8	166 14 56·4	—16·4
10	18 21 36·9	18 21 46·3	+ 9·4	3	161 44 9·0	161 44 16·5	+ 7·5
12	20 11 37·5	20 11 58·3	+20·8	1	159 55 15·9	159 55 9·0	— 6·9
14	22 1 58·9	22 2 9·4	+10·5	Aug. 30	158 6 5·0	158 5 58·7	— 6·3
		Mean....	+16·72			Mean....	—15·58

The correction applicable to the right ascensions of all stars observed in 1802 is

$$\frac{+16''\cdot72 - 15'\cdot58}{2} = +0''\cdot57 = +0^{s}\cdot04.$$

Errors of collimation may be determined by a micrometer attached to the eye-piece of the telescope, by which the angular distance of the object from the central wire is first measured. The instrument is then reversed, and the distance of the same object from the central wire again measured. The azimuthal direction may be found by comparing any two stars, differing in declination, whose places are known, Polaris being generally one of these stars. At the present time the places of certain fixed stars are used for these purposes, and a correction of the assumed equinox is obtained from all the right ascensions and north polar distances of the sun. (See the annual volumes of *the* Greenwich and Cambridge Observations.)

When the preceding errors of collimation, level, and azimuth have been applied to the results of observations, the transit instrument can be usefully employed in the determination of accurate time and the investigation of apparent right ascensions. The "Nautical Almanac" contains the apparent places, for every tenth day in the year, of certain fixed stars called clock stars, which can be immediately compared with the corrected instrumental results. A series of clock errors for certain epochs of time will thus be obtained (by taking the differences of the foregoing numbers); which, being grouped on successive days, will give at once a rate of the clock. In adopting the *rate* for use, due weight must be allowed for the interval of time on which each *deduced rate* depends, and also on its relative accuracy with regard to the number and distribution of the clock stars.

Apparent Errors of the Transit Clock, and Adopted Clock Rates and Clock Errors at 0h. Sidereal, in the Year 1848.

MONTH, DAY, and HOUR.	Observer.	Apparent R. A.	True Transit and Mean of each Group.	Apparent Error, and Mean, and Reduction to 0h. by adopted Rate.	Mean Error reduced to Standard Observer.	Interval of consecutive Means of True Transit.	Difference of consecutive Means of Apparent Errors.	Clock's loss in 24h.	Adapted Losing Rate.
		h. m. s.	h. m. s.	s.	s.	s.	s.	s.	s.
March 14, 7	H.	6 38 28·04	6 38 23·65	4·39					0·34
7	,,	7 11 3·66	7 10 59·10	4·56					
8	,,	7 24 54·90	7 24 50·40	4·50					
8	,,	7 31 21·74	7 31 17·86	4·38					
9	,,	8 38 44·87	8 38 40·32	4·55					
6) 39			37 21	2·38					
14·7			7 28	4·48 ·11	4·49				
		d. h. h. March 13 23 at 0 sidereal		4·37		74·32	1·17	0·37	
March 17, 10	H.	10 0 17·94	10 0 12·29	5·66					
	,,					22·21	0·59	0·64	
March 18, 7	R.	6 52 40·08	6 52 33·39	6·69					0·54
7	,,	7 11 3·58	7 10 56·77	6·81					
8	,,	7 24 54·83	7 24 48·20	6·63					
8	,,	7 31 21·68	7 31 15·01	6·67					
8	,,	7 36 1·68	7 35 55·03	6·65					
8	,,	8 1 5·66	8 0 58·96	6·70					
11	,,	11 6 2·77	11 5 56·16	6·61					
11	,,	11 11 46·68	11 11 40·22	6·46					
8) 68			66 48	5·22					
18·8			8 21	6·65 —·18	6·25				
		March 17d. 23h. at 0h. sidereal.		6·47		26·47	0·48	0·44	
March 19, 11	H.	11 6 2·77	11 5 55·87	6·90					0·47
11	,,	11 11 46·68	11 11 40·15	6·53					
19·11			11 8	6·72 ·21	6·73				
		March 19d. 0h. at 0h. sidereal.		6·51					

The following forms will show the method of reducing transit observations at the Royal Observatory, Greenwich, as well as the formation of clock errors at 0h. sidereal and adopted rates :—

Example I.—Observations of Transits in the Year 1851 with the **Transit Circle,** and Computations of Right Ascension.

Approx. Sol. Time.	d. h. m. August 15 8 27	d. h. m. 15 10 26	d. h. m. 15 10 32	d. h. m. 15 10 49
Name of Object.	μ Sagitarii.	θ Aquilæ.	α ²Capricorni.	ε Delphini.
Approx. N.P.D. of Object.		° ′ 91 16		° ′ 79 12
Observer.	H. B.			
Reading of Transit Micrometer.	r. 31·500			
	h. m. s.	h. m. s.	h. m. s.	h. m. s.
Trans. over wire I.	26·2	15·2	24·4	43·1
" " II.	41·2	29·1	38·3	57·3
" " III.	55·9	42·8	52·3	11·4
" " IV.	10·7	56·7	6·4	25·3
" " V.	25·6	10·4	20·9	39·6
" " VI.	40·7	24·4	35·2	53·6
" " VII.	18 4 54·7	20 3 38·0	20 9 49·4	20 26 7·8
	7) 25·0	7) 36·6	7) 36·9	7) 38·1
Add....	3·57	5·23	5·27	5·44
	7·14	1·43	1·43	
Observed Transit..	18 4 10·71	20 2 56·66	20 9 6·70	20 25 25·44
″				
Col. error — 0·76 ×	+ ·071 = — ·05	+ ·067 — ·05	+ ·068 — ·05	+ ·068 — ·05
Level err. — 4·34 ×	+ ·021 = — ·09	+ ·040 — ·17	+ ·029 — ·13	+ ·052 — ·23
Azim. err. — 0·36 ×	+ ·068 = — ·03	+ ·053 — ·02	+ .062 — ·02	+ ·044 — ·02
True Transit over Meridian	18 4 10·54	20 2 56·42	20 9 6·50	20 25 25·14
Clock slow at 0h. Siderl. preceding	41·83	41·83	41·83	41·83
Adop. los. rate 1·14	·86	·95	·96	·97
Ob. R. A. of Object	18 4 53·23	20 3 39·20	20 9 49·29	20 26 7·94
Star's Correc. with Sign changed...	— 2·09	— 2·24	— 2·33	— 2·25
Obser. Mean R. A. Jan. 1, for Stars.	18 4 51·14	20 3 36·96	20 9 46·96	20 26 5·69

Having obtained the error of the clock at any certain period, and also the rate, by a simple interpolation we can find its error at any other time. When this is applied to the observed transit (properly corrected), the apparent right ascension is at once obtained. This can be converted into the mean right ascension for the beginning of each year, by the day numbers, A, B, C, D, given in the "Nautical Almanac," combined with the star constants, a, b, c, d, of the British Association Catalogue. This method of determining the corrections for precession, nutation, and aberration, is so lucidly explained in the latter work, that we shall content ourselves by simply referring to it; but we may mention that, for the purpose of avoiding *negative* signs in their computations, the Astronomer Royal has ingeniously transformed these formulæ. (See the Introduction to the Greenwich Twelve Year Catalogue.) The annexed examples worked out will illustrate all the preceding formulæ.

Example II.—Computation of Star Corrections applicable to Mean R. A. by the New Constants, in the Year 1851.

Day.	August 15.							
Star's Name.	μ Sagittarii.		θ Aquilæ.		α^2 Capricorni.		ϵ Delphini.	
Star's R. A.								
Star's N. P. D.								
Reference for Star Constants.	1578 Gr. 12 yr. Cat.		1804 Gr. 12 yr. Cat.		1816 Gr. 12 yr. Cat.		1836 Gr. 12 yr. Cat.	
Log. E	1·60053		1·60053		1·60053		1·60053	
Log. e	0·07971		0·09141		0·09228		0·09355	
		s.		s.		s.		s.
Sum and Number	1·68024	47·890	1·69194	49·197	1·69281	49·296	1·69408	49·440
Log. F	1·09921		1·09921		1·09921		1·09921	
Log. f	0·05253		0·05794		0·05774		0·05897	
Sum and Number	1·15174	14·182	1·15715	14·360	1·15695	14·353	1·15818	14·394
Log. G	0·18447		0·18447		0·18447		0·18447	
Log. g	1·45615		1·44865		1·45226		1·44507	
Sum and Number	1·64062	43·714	1·63312	42·966	1·63673	43·324	1·62954	42·613
Log. H	1·45769		1·45769		1·45769		1·45769	
Log. h	0·07899		0·07891		0·07618		0·08191	
Sum and Number	1·53668	34·410	1·53660	34·403	1·53387	04·188	1·53960	34·642
L		74·433		74·433		74·433		74·433
l		87·459		86·878		86·736		86·727
Sum of Numbers		302·088		302·237		302·330		302·249
Subtract Constant		300·000		300·000		300·000		300·000
Star Correction = Remainder		2·088		2·237		2·330		2·249

SPECIMEN OF THE GREENWICH METHOD OF RECORDING TRANSIT OBSERVATIONS.

Transits observed with the Transit Circle, and Computations of Apparent Right Ascension, at the Royal Observatory, Greenwich, in the year 1851.

Month and Day.	No. for Reference.	Name of Object.	Observer.	Seconds of Transit over the Wire I.	II.	III.	IV.	V.	VI.	VII.	Concluded Transit over the Mean of the Wires.	Error of Collimation (Level) [Azimuth]	Seconds of Transit Corrected.	Tabular R. A. of known Stars.	Clock apparently Slow.	Adopted Clock slow at 0h. Sidereal and [Rate].	Correction for Semi-diameter.	Apparent R.A. of Centre from the Observation	Correction to Mean R.A. 1851 Jan. 1.
				s.	s.	s.	s.	s.	s.	s.	h. m. s.	"	s.	s.	s.	s.	s.	h. m. s.	s.
June 17	1	☉ 1 L.	H. B.	49·2	4·2	19·5	34·7	49·7			5 39 34·56	— 0·73	34·10			16·05	+ 68·95	5 40 59·17	
	2	Polaris S.P.		12·0	6·5	59·5	52·5				13 4 51·12	(— 4·97)	58·90	14·84	15·94	[0·32]		1 5 15·12	+ 3·41
	3	Spica.		23·7	38·0	51·9	6·2	20·4	34·4	48·3	13 17 6·13	[— 2·50]	5·77	22·01	16·24			13 17 22·00	— 1·06
	4	Arcturus.		53·2	7·9	22·7	37·5	52·2	6·6	21·4	14 8 37·36		36·92	53·12	16·20			14 8 53·16	— 1·13
	5	μ Bootis.		21·7	37·8	53·7	10·2	26·0	42·2	58·5	14 25 10·01		9·51					14 25 25·75	— 1·25
	6	ε Bootis.		27·5	43·0	58·7	14·4	29·7	45·6	1·2	14 38 14·30		13·81					14 38 30·06	— 1· 1
	7	ξ 2 Bootis.			47·2	1·8	16·8	31·7	45·9	0·7	14 44 16·68		16·21					14 44 32·49	— 1·34
	8	ζ Ursæ Minoris.				5·7	57·9	51·2	44·5		14 50 58·52		57·40					14 51 13·65	— 2·84
	9	ψ Bootis.					49·2	5·2	20·9	36·2	14 57 49·51		49·03					14 58 5·28	— 1·40
	10	ξ Libræ.		3·5	17·4	31·5	45·4	59·5	13·3	27·3	15 8 45·41		45·04	1·29	16·25			15 9 1·29	— 1·58
	11	ζ Libræ.			8·8	23·2	37·7	51·7	6·5	20·8	15 19 37·59		37·25					15 19 53·50	— 1·70
	12	α Coronæ.		21·8	37·7	53·1	8·6	24·2	39·7	55·4	15 28 8·65		8·16	24·35	16·19			15 28 24·42	— 1·53
	13	B.A.C. 5188.				35·8	50·2	4·2	18·7	32·8	15 34 50·07		49·72					15 35 5·98	— 1·73
	14	Irene.			15·7	30·2	44·5	58·8	13·2	27·5	15 38 44·52		44·17					15 39 0·43	
	15	γ Serpentis.				5·8	20·3	34·5	49·2	3·6	15 49 20·29		19·85					15 49 36·11	— 1·60
	16	ξ' Scorpii.			3·4	18·2	32·7	47·4	2·2	16·8	15 56 32·80		32·46	48·68	16·22			15 56 48·72	— 1·85
	17	Lalande 29,490.		4·0	18·2	32·4	46·4	0·7	15·2	29·3	16 3 46·60		46·25					16 4 2·51	— 1·80
	18	ε Ophiuchi.			44·7	58·6	12·4	26·5	40·2	53·9	16 10 12·46		12·08					16 10 28·35	— 1·73
	19	γ Herculis.			37·6	52·2	6·7	21·6	36·4	50·7	16 15 6·87		6·43					16 15 22·70	— 1·67
	20	Antares.			32·0	47·2	2·4	18·2	34·1	49·6	16 20 2·78		2·47	18·77	16·30			16 20 18·74	— 2·01
	21	ζ Ophiuchi.		1·2	15·5	29·4	43·5	57·6	11·7	25·8	16 28 43·53		43·17					16 28 59·44	— 1·80
	22	ζ Herculis.		37·5	53·6	9·8	26·2	42·5	58·8	15·2	16 35 26·23		25·73					16 35 42·00	— 1·80
	23	* N. P. D. 74° 29		58·1	12·2	26·5	41·0		9·3	24·2	16 44 40·94		40·51					16 44 56·78	— 1·74
	24	ε Herculis.			49·4	5·2	21·2	37·7	53·7	10·0	16 54 21·46		20·96					16 54 37·24	— 1·84
	25	Vesta.		30·2	44·9	59·4	13·8	28·5	43·0	57·4	17 1 13·88		13·54					17 1 29·82	
	26	θ Ophiuchi.		51·7	7·6	22·7	37·7	53·2	8·4	23·6	17 12 37·85		37·53					17 12 53·81	— 2·06
	27	Juno.		56·5	10·0	24·4	38·2			19·7	17 18 38·07		37·69					17 18 53·97	

ILLUMINATED END OF AXIS WEST. Order of Wires for Stars above the Pole, A, B, C, D, E, F, G.

Correction to Equatorial Transits:—For I. + 41s·443; II. + 27s·646; III. + 13s·816; IV. + 0s·000; V. — 13s·811; VI. — 27s·654; VII. — 41s·438.

20. The image very confused and coloured.

The following examples will show the methods of determining azimuthal errors of the transit instrument at the Royal Observatory, Greenwich:—

Approximate Solar Time. 1848.	Transits Corrected for Error of Collimation and Level Error, with the Corrections for Clock Rate and Change of R. A., and inferred Azimuthal Errors.		
d. h.		h. m. s.	
April 28, 9	Transit of β Leonis corrected for two errors	11 40 45·45	+ z × 0·041
	Tabular Right Ascension	11 41 19·74	
	Clock Slow	34·29	
	s. h. m.		
	Rate [1·25] for 1 20	+ ·07	
		34·36	— z × 0·041
		h. m. s.	
April 28, 10½	Transit of Polaris S. P.	13 3 25·85	+ z × 1·638
	Tabular Right Ascension	1 4 8·76	
		42·91	— z × 1·638
	$\therefore Z = \frac{+8{\cdot}55}{1{\cdot}597} = +5''{\cdot}354$		
		h. m. s.	
June 13, 8	Transit of Polaris S. P.	13 4 5·46	+ z × 1·638
20	Transit of Polaris	1 4 15·84	— z × 1·533
	s. h.		
	Clock Rate [1·06] for 12	+ 0·53	
	Subtract Increase of R. A. in 12h.	— 0·40	
		1 4 15·97	+ z × 1·533
	$\therefore Z = +\frac{10{\cdot}51}{3{\cdot}171} = +3''{\cdot}314$		

The practical directions for using them, as given in the Greenwich Observations are as follows (1848, page xx.):—

"If two consecutive transits of Polaris have been observed, the first transit (as corrected for error of collimation and level error) is altered by the change of the star's right ascension, and by the estimated clock error for 12h.; and the difference, in seconds of time, between the altered first transit and the second transit (rejecting 12h.) is divided by 3·171 to obtain the azimuthal error in seconds of space.

"If three transits have been observed, the difference between the first and second is taken (rejecting 12h.), and the difference between the second and third in like manner; and the mean between these is supposed to be independent of the change of right ascension and the clock's rate, and is then divided by 3·171.

"If several transits have been observed, the same process is used for every successive set of three, and the results are used separately, or the mean of the results is taken, according as there appears reason to think that the position of the instrument has or has not undergone a change."

Example of the Determination of Azimuthal Error by Three Consecutive Transits of Polaris :—

Transit corrected for two errors.

			h. m. s.	
1846.	Dec. 15.	Polaris . . .	1 4 49·39	
				+ 14·09
	Dec. 15.	Polaris, S. P. .	13 4 35·30	
				+ 11·78
	Dec. 16.	Polaris . . .	1 4 47·08	
				5·87
				+ 12·94

$$\therefore Z^{\cdot}_{\cdot} = \frac{\times\ 12{\cdot}94}{3{\cdot}148} = +\overset{''}{4}{\cdot}12.$$

On the Determination of Terrestrial Longitudes.—In this section we shall explain the principal methods used in the determination of terrestrial longitudes:—

Longitudes by transits of the moon and moon culminating stars.

The first introduction of this method appears to be entirely due to the German astronomers. About the year 1820 it was agreed on by Gauss, Bessel, Struve, Nicolai, and others, to observe transits of the moon, and certain pre-arranged stars lying in its parallel, for the purpose of determining longitudes. The celebrated Schumacher gave his zealous co-operation by the immediate publication of the results in the "Astronomisch Nachrichten." In these kingdoms the method was cultivated by Dr. Brinkley and the late Mr. Baily, the latter astronomer having contributed to its advancement by his memoir on the subject in the Transactions of the Royal Astronomical Society, which led to the insertion of the requisite data in the pages of the "Nautical Almanac."

The stars with which the moon is to be compared are selected in the same parallel of declination, for the purpose of nullifying any small errors which may be left on the transit instrument; but notwithstanding all the care which was taken by these refinements, there are still uncertainties produced by irradiation and a peculiar personal equation of the limbs of the moon, which tend to throw doubt on the results. The former quantity, or "irradiation," may be eliminated by the results of the moon's 1 L. and 2 L.; but the latter is a variable and unknown quantity, in some instances affecting the transits of the first and second limb in a different proportion. In the following pages we will show the method pursued by the Astronomer Royal in the longitudes of the North American boundary, and also those of MM. Rumker and Struve. The method of the Astronomer Royal is to be used in cases where there are no corresponding observations, and the results will consequently be affected with the tabular errors of the moon's right ascension in the "Nautical Almanac," an approximation to which, however, may be made by observations on other days at any standard observatory. The methods of M. Rumker and Professor Struve are intended for corresponding observations; the former having been applied in the determination of the longitude of Port

Stephens, New South Wales, and the latter in the longitudes of several places in Turkey, during the years 1828 to 1832 inclusive.

The first method of determining longitudes is by the absolute right ascensions of the moon's limb, and is only to be used where there are no corresponding observations of moon culminating stars. It consists in making two assumptions of longitude, differing by any known quantity, and interpolating for these times the right ascensions of the moon's limb from the numbers given in the section of "Moon Culminating Stars" in the Nautical Almanac. These tabular right ascensions are given for the time of transit of the upper and lower meridian at Greenwich. By comparing the interpolated data with the result of the meridian observation at the place, the exact difference of longitude may be found by a simple proportion. The true difference of longitude being supposed to lie between two assumptions 1' different, the following will be the proportion :—

As the difference of the two interpolated R. A.'s : 60 sec. : : the difference between the R. A. of observation, and the computed R. A. : the correction of the first assumption.

The data from the "Nautical Almanac" are as follows :—

Moon's R. A. on preceding day.
R. A. at lower passage, preceding day.
R. A. of corresponding day.
Moon's R. A. at next lower passage.
Moon's R. A. at following day.

The following method of interpolation may be used, the fourth differences being considered constant :—

Suppose $a_h = a_0 + bh + ch^2 + dh^3 + eh^4$. In this, for h write $-2, -1, 0, +1, +2$, and take the differences. This gives—

	1st differences.	2nd diff.	3rd diff.	4th diff.
$a_{-2} = a_0 - 2b + 4c - 8d + 16e$	$+b - 3c + 7d - 15e$			
$a_{-1} = a_0 - b + c - + d + e$	$-b - c + d - e$	$2c - 6d + 14e$	$6d - 12e$	
$a_0 = a_0$	$+b + c + d + e$	$2c + 2e$	$6d + 12e$	$24e$
$a_1 = a_0 + b + c - d + e$	$b + 3c + 7d + 15e$	$2c + 6d + 14e$		
$a_2 = a_0 + 2b + 4c 8 + d + 16e$				$= \Delta''''$

Now take the differences of the terms a_{-2}, a_{-1}, a_0, a_1, a_2, thus—

$$\left.\begin{array}{lllll}
a_{-2} & & & & \\
 & +\Delta'_1 & & & \\
a_{-1} & & +\Delta''_{-1} & & \\
 & +\Delta'_0 & & +\Delta'''_{-1} & \\
a_{-0} & & +\Delta''_0 & & +\Delta'''' \\
 & +\Delta'_1 & & +\Delta'''_0 & \\
a_{-1} & & +\Delta''_1 & & \\
 & +\Delta'_2 & & & \\
a_{-2} & & & &
\end{array}\right\}$$

Comparing this with the foregoing difference, we shall have

$$24e = \Delta'''' \therefore e = \frac{\Delta''''}{24}$$

$$6d + 12e + 6d - 12e = \Delta'''_{-1} + \Delta'''_0 \therefore d = \frac{\Delta'''_{-1} + \Delta'''_0}{12}$$

$$2e + 2e = \Delta''_0 \therefore e = \frac{\Delta''_0}{2}$$

$$b+c+d+e+b-c+d-e=\Delta_0'+\Delta_1' \quad \therefore b=\frac{\Delta'_0+\Delta'_1}{2}-d$$

$$\therefore a_h=a_0+\left(\frac{\Delta'_0+\Delta'_1}{2}-d\right)h^2+\left(\frac{\Delta''_0}{2}-e\right)h^2+\left(\frac{\Delta'''_{-1}+\Delta'''_0}{12}\right)h^3+\frac{\Delta''''}{24}h^4$$

The interval between any two consecutive terms of the given quantities, as a_{-2}, a_{-1}, is supposed to be unity, and therefore h must be a fractional part of this unit. Thus, if this interval be 12h., and the time for interpolation 3h. 20m., then $h=\frac{3h.\ 20m.}{12h.}$ If the interval between the consecutive quantities be 24h., then we should have $h=\frac{3h.\ 20m.}{24h.}$; and so on for other intervals.

The following example will illustrate the whole process :—

Longitude of Port Essington, north-east coast of Australia, deduced from lunar transits observed there in 1839 by Captain Owen Stanley, R.N.

The following meridian right ascensions of the moon's 1 L. at Port Essington are found by the usual means, applying carefully the error and rate of the chronometer :—

	R. A. h.	m.	s.
1839, June 20, ☽ 1 L. . .	12	50	14·38
„ June 22, ☽ 1 L. . .	14	20	57·01
„ June 24, ☽ 1 L. . .	16	2	59·99
„ June 25, ☽ 1 L. . .	16	59	11·11
&c.		&c.	

By assuming two longitudes, 8h. 49m. east and 8h. 48m. east, and interpolating, with fourth differences, the right ascension of the moon's limb, from the data for upper and lower transit over the Greenwich meridian, I find for longitude 8h. 49m. east, right ascension of the moon's 1 L., 12h. 50m. 14s.·64; and for longitude 8h. 48m. east, right ascension of the moon's 1 L., 12h. 50m. 16s.·46. The correction to the tabular right ascension of the moon's 1 L. is — 0s.·27, from an observation made at the Royal Observatory, Edinburgh.

The observed right ascension of the moon's 1 L. is 12h. 50m. 14s.·38; therefore, by a proportion, the longitude from this observation is 8h. 48m. 59s.·67 east.

Proceeding in a similar manner, we find for the observation on 1839, June 25, for assumptions of longitude 8h. 49m. east and 8h. 48m. east, the right ascensions of the moon's 1 L. 16h. 59m. 9s.·95 and 16h. 59m. 12s.·86 respectively (a correction of + 0s·09 being applied from the Hamburg meridian observation).

The result of the meridian observation at Port Essington is 16h. 59m. 11s.·11; therefore the resulting longitude is 8h. 48m. 31s.·12 east.

The second method will include the use of moon culminating stars observed at other stations. The simplest way is to form differences at each observatory between the transits of the moon's limb and each moon culminating star (properly corrected for the rate of the clock in the interval). Comparing these differences with the respective differences at other stations, the increase of the right ascension of the moon's limb will be obtained, which will, however, differ from the increase in the right ascension of the moon's centre, in consequence of the change of the moon's geocentric semi-diameter in the interval of the two passages. This correction may be computed as follows:—If the geocentric radius of the moon culminating in A is called r, and the true declination d; and for a culmination at another station, B, these quantities are called ρ and δ respectively, the correction will be

$$I \pm \frac{1}{15}\left(\frac{r}{\cos d} - \frac{\rho}{\cos \delta}\right)$$

the true difference of the right ascension of the centre for both times of observation. The sign — applies to the western limb, and the sign + to the eastern limb.

But if the apparent right ascensions of all the objects observed are given at the stations, the more direct way will be as follows (Struve's method):—

Example.—1846, March 10. The following corresponding observations of the moon 1 L and moon culminators were observed at Greenwich, Oxford, Hamburg, and Georgetown (U.S.) Observatories. The differences of longitude of Greenwich, Oxford, and Hamburg are supposed to be accurately known; and it is proposed to determine the longitude of the Georgetown Observatory.

The longitude of	Oxford is	. .	5m. 2s.·6 W.
„ „	Hamburg	. .	39m. 54s.·1 E.
„ „	Georgetown	.	5h. 8m. 20s. + West.

1846, March 10. Apparent Right Ascensions Observed:—

	Greenwich.	Oxford.	Hamburg.	Georgetown.
	h. m. s.	h. m. s.	h. m. s.	h. m. s.
ξ Leonis	9 23 40·73	9 23 40·94	9 23 40·773	
o Leonis	9 32 57·99	9 32 58·08	9 32 57·996	9 32 57·995
☽ 1 L	9 44 4·21	9 44 14·22	9 42 44·850	9 54 13·876
Regulus	10 0 12.39	10 0 12·42	10 0 12·327	10 0 12·380
ρ Leonis	10 24 44·47	10 24 44·46	10 24 44·358	

Then, comparing each star with the Greenwich Observation, we have the following corrections:—

Name of Star.	Oxford.	Hamburg.	Georgetown.
	s.	s.	s.
ξ Leonis.	— 0·21	— 0·043	
o Leonis.	— 0·09	— 0·006	— 0·005
Regulus.	— 0·03	+ 0·063	+ 0·010
ρ Leonis.	+ 0·01	+ 0·112	
Mean.	— 0·08	+ 0·032	+ 0·003

When the above corrections are applied to the right ascension of the moon's limb, we shall be enabled to give its place independent of any errors, either in the assumed equinox, or of any small residual instrumental correction. Thus, the following sidereal times of observation are formed:—

	Greenwich.	Oxford.	Hamburg.	Georgetown.
	h. m. s.	h. m. s.	h. m. s.	h. m. s.
Sidereal time of observation	9 44 4·210	9 44 14·140	9 42 44·882	9 54 13·879
Longitude . . .	0 0·000	+ 5 2·600	— 39 54·100	+ 5 8 20·000
Greenwich sidereal time .	9 44 4·210	9 49 16·740	9 2 50·782	15 2 33·879
Greenwich mean solar time	10 31 13·45	10 37 26·94	9 50 6·77	15 48 50·95

Interpolating for these times, from the "Nautical Almanac," we find the following right ascensions, declinations, and semi-diameters of the moon :—

	☽'s R. A. of Centre.	☽'s Semid.	☽'s Geo. Decl.
	h. m. s.	″ ′	° ′ ″
For the Greenwich Merid. Passage	9 45 3·86	14 48·06	+ 8 22 53
" Oxford "	9 45 13·84	14 48·06	+ 8 21 48
" Hamburg "	9 43 44·87	14 47·95	+ 8 29 10
" Georgetown "	9 55 13·54	14 48·93	+ 7 33 40

Computing the moon's geocentric radius, by the formula $\frac{\text{☽'s semid.}}{\text{15 cos decl.}}$, we have for the four observations :—

	Observed R. A. of ☽'s 1 L.	☽'s Semid. / 15 cos. Decl.	Observed R. A. of Centre.	Seconds of N. A.	Corrections of N. A.	Means.
	h. m. s.	s.	h. m. s.	s.	s.	D. A.
Greenwich	9 44 4·210	59·843	9 45 4·053	3·86	+ 0·193	s
Oxford	9 44 14·140	59·843	9 45 13·983	13·84	+ 0·143	+ 0·067
Edinburgh	9 42 44·882	59·852	9 43 44·734	44·87	— 0·136	
Georgetown	9 54 13·879	59·782	9 55 13·661	13·54	+ 0·121	D. A.

The longitudes of the first three observatories are considered well known, and, by taking their means, we obtain + 0s.·067. The correction from the assumed longitude of Georgetown is + 0s.·121; therefore the difference, + 0s.·54, is to be converted into an error of longitude.

The motion of the moon in R. A. at the time of Georgetown transit, by the formula $d\,\mathrm{L} = \frac{d\,\mathrm{A}' - d\,\mathrm{A}''}{\mu}$, where p is + 115s.·08 in 1m., or + 0s.·03197 in 1 second of sidereal time—hence the correction of the assumed longitude is $\frac{+\ 0\mathrm{s}.\cdot 054}{0\mathrm{s}.\cdot 03197} = +\ 1\mathrm{s}.\cdot 69$, or the true longitude from this observation is 5h. 8m. 21s.·69 west.

The following additional example of the determination of longitude by observation of the moon and moon-culminating stars, is from Struve's "Geographical Investigations in Turkey."

1831, May 22, the following transits of the moon and moon culminating stars were observed :—

APPARENT RIGHT ASCENSIONS.

Object.	Schurscha.	Dorpat.	Cracow.	Greenwich.
	h. m. s.	h. m. s.	h. m. s.	h. m. s.
θ Virginis .	13 1 14·16	13 1 13·83	13 1 13·90	
☽ L. . .	13 46·60	13 40·93	14 34·05	13 17 10·37
ι^3 Virginis .	26 45·64	26 45·80	26 45·75	26 45·77
174 Virginis	35 8·90	55 8·61	35 8·87	35 8·81

Taking the mean of the apparent right ascension at Dorpat and Cracow, we obtain—

	h. m. s.
θ Virginis . .	13 1 13·86
1^3 . . .	26 45·77
174 . . .	35 8·74

The following table of reductions is formed :—

Object.	Schurscha.	Dorpat.	Cracow.	Greenwich.
	s.	s.	s.	s.
θ Virginis .	— 0·30	+ 0·03	— 0·04	
ι^3 Virginis .	+ 0·13	— 0·03	+ 0·02	— 0·00
174 Virginis	— 0·16	+ 0·13	— 0·13	— 0·07
Means. .	— 0·11	+ 0·04	— 0·05	— 0·03

By applying which to the observed right ascension of the moon's 1 L, the right ascensions of the moon's 1 L. are obtained, viz.—

	h. m. s.
At Schurscha . .	13 13 46·49
Dorpat . .	13 13 40·97
Cracow . .	13 14 34·00
Greenwich . .	13 17 10·34

Proceeding, then, with an assumption of longitude of Schurscha = 4, we obtain a correction of the ephemeris $d\,A'$, which includes both the error of the ephemeris, and also the motion of the moon in R. A. produced by an error in the assumed longitude. The error of the ephemeris having been determined by the comparison of the observations at Dorpat, Cracow, and Greenwich, the residual error at Schurscha will be entirely due to the erroneous assumption of the longitude of the latter place. Thus, if $d\,A$ be the correction of the ephemeris determined at the three standard observatories; and, if the observed right ascension of the moon at Schurscha be α', and that from the ephemeris A', so that $\alpha' - A' = d\,A'$, then the correction of the assumed west longitude—

$$d\,L = \frac{d\,A' - d\,A}{\mu}.$$

The longitude of the place deduced from the lunar culmination will be then $L = L + d\,L$.

μ is the increase in R. A. (in arc) for the time of observation at the required place.

Assuming L′ = — 1h. 34m. 0s.·0 from Paris, and computing the value of the geocentric radius of the moon, by the preceding method the following table is formed:—

Place.	Mean Time at Berlin.	Observed R.A. of ☽'s 1 L.	r sec δ.	R. A. of Moon's Centre.		Corrections.
				Observed.	From the Berlin Ephemeris.	
	h. m. s.	° ′ ″	′ ″	° ′ ″	° ′ ″	″
Schurscha	8 25 17·39	198 26 37·35	+ 14 54·49	198 41 31·84	198 41 53·37	*d* A′ = — 21·73
Dorpat .	8 21 38·48	198 25 14·55	14 54·50	198 40 9·05	198 40 9·46	*d* A = — 9·41
Cracow .	8 49 29·93	198 38 30·00	14 54·46	198 53 24·48	198 53 24·47	= — 0·01
Greenwich	10 11 45·25	199 17 35·10	+ 14 54·34	199 32 29·44	198 32 31·68	= — 2·24

The three observatories give an error of the ephemeris very accordant; the mean is $d\,A = -0''\!\cdot88$, with $\mu = 0''\!\cdot4744$.

$$d\,L = \frac{-21\cdot73 + 0\cdot88}{0\cdot4744} = \frac{-20''\!\cdot85}{0\cdot4744} = -44\cdot0 \text{ sec.}$$

or the longitude of Schurscha $= -\overset{\text{h.}}{1}\ \overset{\text{m.}}{34}\ \overset{\text{s.}}{44\cdot0}$ from Paris.

The comparison with the individual observatories gives—

$$\text{With Dorpat } d\,L = \frac{-21''\!\cdot32}{0\cdot4744} = -\overset{\text{s.}}{44\cdot9} \text{ longitude } -\overset{\text{h.}}{1}\ \overset{\text{m.}}{34}\ \overset{\text{s.}}{44\cdot9}.$$

$$\text{Cracow} = \frac{-21\cdot72}{0\cdot4744} = -45\cdot9 \quad . \ . \ . \ . = \quad 45\cdot9.$$

$$\text{Greenwich} = \frac{-19\cdot49}{0\cdot4744} = -41\cdot1 \quad . \ . \ . = \quad 41\cdot1.$$

$$\text{The Mean is} \quad . \ . \ . = -1\ 34\ 44\cdot0.$$

M. Struve then proceeds to investigate the differences of determinations of longitude by the moon's 1 L., and moon's 2 L., and finds it necessary, in order to reconcile the observations made at twelve stations, to increase determinations of longitude by the moon's 1 L. 7s.·2, and to diminish those of the moon's 2 L. by the like quantity.

The following additional examples of the determination of longitudes by moon culminators are extracted from Rümker's. "Längen Bestimmung durch den Mond," Hamburg, 1849.

The longitude of Hamburg, in three examples, being supposed to be well known, let p be the sidereal time of the earlier, and π that of the later culmination; t the mean time of the earlier culmination, and τ that of the later, deduced from the known longitude. Then for these times the moon's right ascensions, declinations, and semi-diameters (viz. a and α, d and δ, r and ρ respectively), are to be interpolated from the ephemeris. There will thus be obtained from the ephemeris the differences of the moon's right ascension $\alpha - a$; and there will be deduced from the observations the difference $\pi - p \pm (\rho \sec \delta - r \sec d) = A$, the *upper* sign applying to the *first*, and the *lower* to the second limb of the moon. Hence there will be this proportion—

$$a - a : A = \tau - t : \lambda - L;$$

consequently $(A - (a - a)\ \frac{(\tau - t)}{a - a} = c$, the correction of the assumed longitude. The westerly place in these examples being the first, the sign of c is to be changed.

The observations at Port Stephens were made by Captain King, R.N., and have been published in the monthly notices of the Royal Astronomical Society; those at Hamburg, by M. Rumker, are given in the twenty-second volume of the "Astronomische Nachrichten."

1843, April 12, Hamburg, East.

	h.	m.	s.
Moon's 1 L. .	11	43	39·797
η Virginis . .	12	11	55·612

April 13, Port. Stephens, West.

	h.	m.	s.
η Virginis . .	12	12	21·38
Moon's 1 L. . .	12	19	12·16

Then reducing the sidereal clock times of the star's passage to one standard, as Hamburg, we have the right ascension of the moon's 1 L. at Port Stephens, 12h. 18m. 46s·392.

	h.	m.	s.
Moon's 1 L. p	11	43	39·797
Sid. time at mean noon .	1	20	2·455
	10	23	37·342
Acceleration	—	1	42·165
Mean Hamburg time . .	10	21	55·177
Long. L.		39	54·000
Mean Greenwich time t . April 12.	9	42	1·77

	h.	m.	s.
Moon's 1 L. π	12	18	46·392
Sid. time at mean noon. .	1	22	25·662
	10	56	20·730
Acceleration	—	1	47·426
Mean time at Port Stephen	10	54	33·204
λ	10	8	7·000
Mean Greenwich time τ. April 13.	0	46	34·20

The right ascensions of the moon's limbs accurately interpolated from the hourly ephemeris in the "Nautical Almanac" are —

	h.	m.	s.	
	11	44	47·1369	$= \alpha$
And	12	19	54·1846	$= a$
		35	7·0477	$= a - \alpha$
			2107·0477	$=$

The moon's semi-diameters accurately interpolated are, respectively, $r = 16' \ 39''·414$, $\rho = 16' \ 41''·8745$. The declinations are respectively $d = 3° \ 47' \ 5''$; $\delta = 7° \ 37' \ 34''·5$.

Log. $r = 999·414 = 2·9997464$ $\rho = 1001·8742$ log. 3·0008132

Log sec. $d = 3° \ 47' \ 5'' = 0·0009482$ $\delta = 7° \ 37' \ 34''·5$ log. sec. 0·0038586

3·0006986 3·0046718

r sec. $d = 1001''·5985$ ρ sec. $\delta = 1010''·815$

$$r \sec d = 1001''·5985$$
$$\rho \sec \delta = 1010·8150$$

$$\frac{\rho \sec \delta - r \sec d}{15} = \frac{9·2165}{15} = + \overset{s.}{0·6144}$$

$$\begin{array}{rrrrrl}
 & \text{h.} & \text{m.} & \text{s.} & & \\
\pi = & 12 & 18 & 46{\cdot}392 & & \\
p = & 11 & 43 & 39{\cdot}797 & & \\
\hline
\pi - p = & & 35 & 6{\cdot}595 & = & 2106{\cdot}5950
\end{array}$$

$$\begin{array}{rl}
A = & 2107{\cdot}2094 \\
\alpha - a = & 2107{\cdot}0477 \\
\hline
A - (\alpha - a) = & +\,0{\cdot}1617 \\
 & \text{h. m. s.} \\
\tau - t = & 15\ 4\ 25{\cdot}027 = 54265{\cdot}027
\end{array}
\qquad
\begin{array}{rl}
\text{Ar. co. log} = & 6{\cdot}676326 \\
\log = & 9{\cdot}208791 \\
\log = & 4{\cdot}734520 \\
\hline
\log = & 0{\cdot}619637
\end{array}$$

$$\begin{array}{rl}
 & \text{s.} \\
\text{Correction} = & -\,4{\cdot}165 \\
\lambda = & 10\ 8\ 7{\cdot}000 \\
\hline
\text{Sine longitude} = & 10\ 8\ 2{\cdot}835
\end{array}$$

Meridian or Mural Circle.—The north polar distance of an object is determined by means of the meridian or mural circle. Previous to the introduction of the circle, the astronomical quadrant was used for this purpose; but it was discovered, about the beginning of the present century, that its results had been in a great measure vitiated by the expansion of the instrument, and consequent change of figure. The late Astronomer Royal (Mr. Pond), by aid of a circle only two feet in diameter, was enabled to detect and point out the errors of the Greenwich quadrant by a direct comparison of the results of the two instruments. Since that time circular instruments have been very generally employed. The improved method of graduation, first introduced by Troughton in 1806, and the previous invention of the micrometer-microscope by Ramsden, have allowed artists to give to this instrument the greatest degree of perfection. The first large instrument of this class appears to have been the Greenwich mural circle, which was completed by Troughton in 1811, and which, in several details, may almost be considered as a new instrument. Some years previously, the celebrated Ramsden made a circle of five feet in diameter, with which Piazzi framed his great catalogue. The idea, however, is an old one, as Roemer, although better known as the inventor of the transit instrument, used a meridian circle for several years similar in construction to the modern form. In his letter to Leibnitz, December 15, 1700, he mentions that he preferred an entire circle of four feet in diameter to a quadrant of eight feet radius. Of his observations with the meridian circle, extending from December 1704 to December 1710, only three days remain, the others having been destroyed by the fire which occurred at Hafnia, October 2, 1728. These three days' observations, from October 20 to October 23, 1706, have been carefully reduced by Dr. Galle, and contained in his "Inaugural Dissertation," printed at Berlin in 1845. The diameter of this meridian circle and the length of the axis was five feet and a quarter; the length of the telescope fixed to the axis was five feet. The circle was "read off" by two microscopes placed at an interval of 10°.

The mural circle consists of the graduated circle (Fig. 196) fixed in the meridian, and supported by a long axis, on which it revolves. This axis passes through a block of *stone* or solid masonry, the weight of the instrument being taken off the pivots by means of counterpoises placed at the back of the pier. The circle is generally graduated on its rim from 0° to 360°, and also into smaller subdivisions of 5′ of arc.

The instrument is furnished with a telescope, B B, firmly fixed to the axis, and perpendicular to its plane, with which it revolves. The divisions are read by micrometer-microscopes, F, generally six in number placed at equal distances around its circumference, their object being to eliminate, by readings of opposite diameters, errors arising from eccentricity, flexure of the axis, and expansion. The size of the instrument at the principal observatories varies from four to eight feet in diameter. The Greenwich circles were six feet; that at Cambridge is of eight feet in diameter. The number of microscopes made use of at Oxford and Edinburgh is four; at Greenwich and Cambridge six are constantly *read off*. It was formerly thought necessary to have two instruments at Greenwich for determining a star's place, namely, a transit instrument and Troughton's mural circle; but a larger transit circle has been constructed for the Observatory, under the direction of the present Astronomer Royal, which has been in use since 1851. The telescope of this instrument has an aperture of eight inches, and a focal length of eleven feet and a half, a length of axis between the extremities of the pivots of six feet, the diameter of each pivot being six inches. The circle is six feet in diameter and of cast iron.

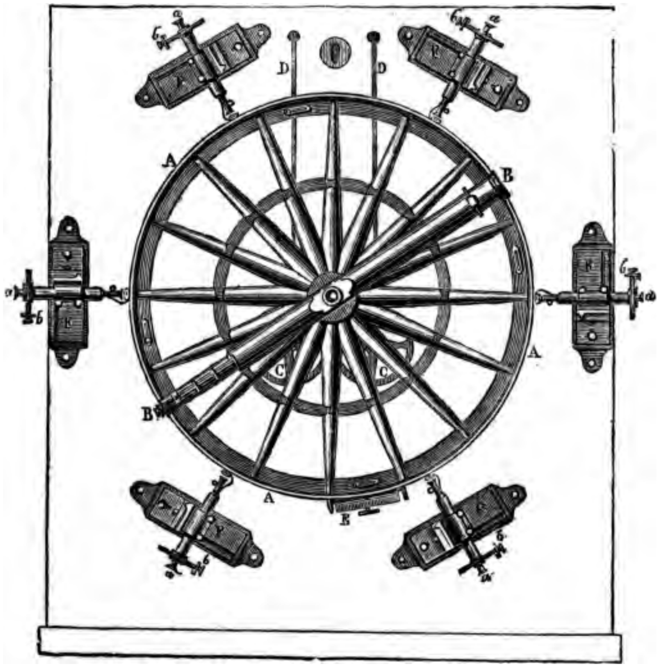


Fig. 156.

The axis of the circle is made horizontal by the aid of a plumb-line suspended in front of the circle, and viewed by two microscopes—one near the top, the other near the bottom of the circle; or, as this instrument is supposed to be used in conjunction with a transit instrument, the axis may be made horizontal by moving the adjusting screw, so as to make a zenith star pass the middle wire at the instant the star is passing the middle wire of the transit instrument.

The adjustments of the meridional position of the mural circle are made by the observations of certain stars in conjunction with the transit instrument. The reading microscopes should accurately describe 300″, or five revolutions between each graduated space on the limb of the circle. This can be approximated to by proper adjusting screws, which regulate the distance of the microscope from the limb; but, notwithstanding the accuracy with which the adjustment may be made, it is found that unequal temperature will alter these numbers considerably. The error arising from this circumstance is called the "correction for runs." It is the practice in observatories to determine the amount of this quantity frequently—which applies to a microscope reading of 5′—and to form corrections proportional for other circle readings. The following is an example (Cambridge Observations, May 3, 1834):—

	A	B	C	D	E	F	
Negative side .	23·5	18·7	19·2	24·7	19·0	27·3	Sums of
Positive. . .	25·1	19·4	20·3	26·5	20·3	26·1	Excesses.
	+1·6	+ 0·7	+ 1·1	+ 1·8	+ 1·3	—1·2	+ 5″·3

The telescope is furnished with a system of vertical wires similar to the transit instrument, and a fixed horizontal wire placed at right angles to the others. This wire is generally adjusted so that an equatorial star will continue bisected during its transit; but it is sometimes left with a small inclination which can be easily observed by comparison with the moveable horizontal wire. Several stars being bisected at the first and seventh vertical wires, the effect of inclination can be readily determined for the whole by comparison with the micrometer of the moveable horizontal wire. To determine the value of one revolution of this micrometer, any distinct terrestrial object may be bisected at one position of the micrometer, and the circle carefully "read off." The same mark is again to be bisected at another position of the micrometer, and the circle is again to be "read off." When the two circle readings have been properly reduced, and the runs of the microscope having been applied, their difference will be a value of circle reading in minutes and seconds equal to a certain number of revolutions of the micrometer. This operation being repeated at different micrometer readings, a current value of one revolution of the telescope micrometer can be obtained. The more distant and distinct the terrestrial object selected is the better.

A very convenient transit circle has been erected for the Observatory at Cambridge, Massachusetts, of which the accompanying engraving is a representation. With this instrument one observer can, at the same time, determine the right ascension and declination of a star with great precision. The telescope T (Fig. 197), has an object-glass of four and one-eighth inches of an aperture, and of four feet focal length. The length of the axes between the shoulders of the pivots is ninety-six inches; the pivots are of steel, and two and a half inches in diameter and the same in length. The eye-piece is provided with two micrometers, one having a vertical, and the other a horizontal movement. Besides the usual mode of illuminating the field through the axis, there are also facilities for illuminating the circus in a dark field. The circles are four feet in diameter, and cast in one piece, and both circles are graduated on silver from 0° to 360°, divided into five minute spaces. There are eight micrometer-reading microscopes attached to the granite piers, being four for each circle. Four of these are seen at A B C D, the other four being on the opposite side of the pier. These micrometers bisect diametrically both piers. The arm E, attached to the pier, supports an additional microscope, which serves as a pointer to indicate approximately the degrees and minutes. For leveling the axis a striding level is employed, and this, combined with a method of reflecting from quicksilver at the nadir point, affords means of ascertaining the amount of collimation of the middle wire without reversal of the pivots. There is, however, an apparatus for reversing the instrument when required. The object-glass of this circle is by Merz of Munich, the fitting by Simms of London.

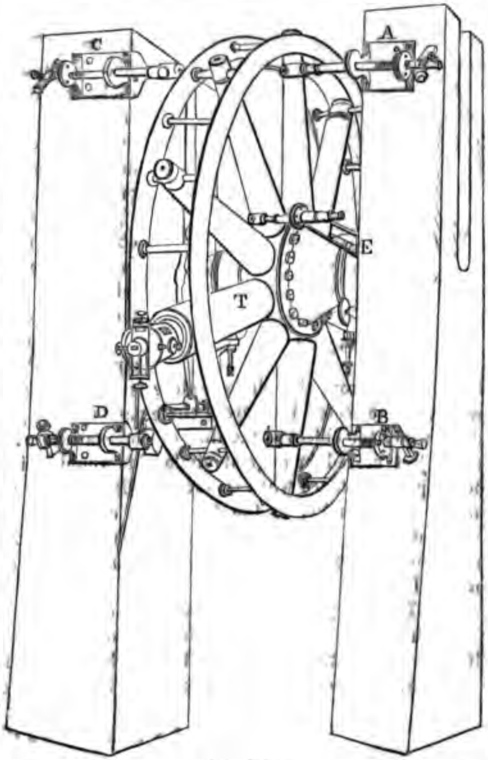


Fig. 197.

The mode of observation with the mural circle is as follows:—The telescope having been set approximately to the culminating star, the observer bisects the object by the moveable horizontal wire as it passes the meridional vertical fibre. He then "reads off" the points and the six microscopes, as well as the telescope micrometer, by a combination of which a concluded circle reading may be obtained. It may be necessary to mention that the instrument is furnished with a clamp and a slow motion screw, by which the horizontal wire may be brought on the star, after the telescope has been approximately directed to it. If, by inadvertence or other causes, the object is not observed at the meridian wire, it will involve a "correction for curvature," which is thus investigated:—

For the purpose of recording the position of every star within range by means of electro-galvanism, the telescope is firmly clamped to remain in its position, while the observer, sitting with his eye at the telescope, has but to press his finger upon at the instant a star is seen to pass the which for this purpose are divided int systems. Those for right ascension are t five in number, divided into groups c each, the intervals between the wires from two or three seconds. The wir difference of declination are also thirt and arranged in similar groups. In or prevent confusion between observatio right ascension and declination, the rule observe for right ascension on one set of first, and denote the magnitude of the s telegraphic symbol; and afterwards to o for declination on the inclined wires.

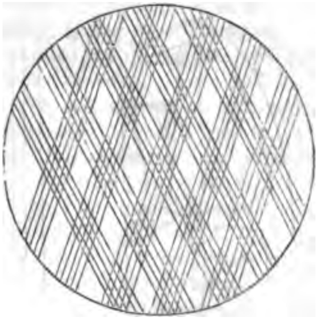


Fig. 198.

When an object is observed by the circle off the meridian, the north pola tance found from the circle reading is the N. P. distance of a point of the mer which is intersected by a great circle passing through the place of the object at the time of observation. If the object moved in a great circle, this would be its north polar distance when on the meridian; but as it moves in a parallel to the equator, a correction is required, which, as it is evident, will be positive for stars north of the equator, and negative for those south of it. It is thus computed:—

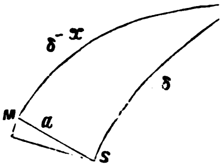


Fig. 199.

Let P be the pole; S the place of the object; $\delta =$ its north polar distance; $a =$ its distance from the meridian; $\delta - x =$ the observed N. P. D.; then we have

$$\cos\delta = \cos a \cos(\delta - x) = (1 - 2\sin^2 \tfrac{1}{2} a)(\cos\delta \cos x + \sin\delta \sin x)$$

$$\cos\delta = (1 - 2\sin^2 \tfrac{1}{2}a)(\cos\delta \times \sin\delta \times x \sin 1'')$$

$$= \cos\delta + \sin\delta \times x \sin 1'' - 2\sin^2 \tfrac{1}{2} a \cos\delta$$

$$- 2\sin\delta \sin^2 \tfrac{1}{2} a \times x \sin$$

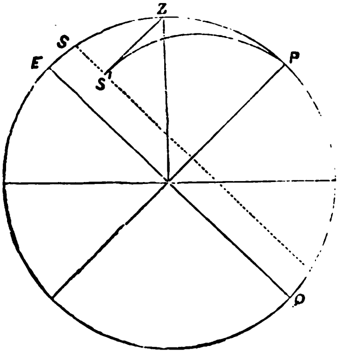


Fig. 200.

If a be small, the last term will vanis by making

$\sin \frac{1}{2} a = \frac{1}{2} a \sin 1''$, we have $2\sin^2 \frac{1}{2} a$

$\frac{1}{2} a^2 \sin^2 1''$

$\therefore \sin\delta \times x \sin 1'' = \cos\delta \times \frac{1}{2} a^2 \sin^2$

$$x = \cot\delta, \frac{\sin 1''}{2} a^2.$$

The distance between the wires is posed to be 20s. = one interval; and if the number of intervals, $a = n \times 20$;

$\therefore a^2 = n^2 \times 400$, consequently $x =$ co

$$\frac{\sin 1'' \times 400}{2} n^2.$$

$x =$ tan declination × (sin 1″ × 200)

In the case of a star near the pole, the following is the investigation for curvature of path (Fig. 200):—

Let $Z\,S' = Z$ = the observed zenith distance; $Z\,S - x$ = zenith distance on meridian; $Z\,P\,S = p^t$ the difference between the star's R. A. and the sidereal time of observation; d = stars declination; L = the latitude of the place of observation. Then in the triangle Z S′ P, we have

$$\cos z = \sin L \sin a + \cos L \cos d \cos p$$

$$= \sin L \sin d + \cos L \cos d \, (1 - \frac{p^3)}{2}$$

But $$\cos Z - x = \sin L \sin d + \cos L \cos d\text{; or, since } x \text{ is small,}$$

$$\cos Z + \sin Z \times x = \sin L \sin d + \cos L \cos d$$

Therefore $$x \sin Z = \cos L \cos d \times \frac{p^2}{2}$$

$$\sin Z \times x \sin 1'' = \cos L \cos d \times \frac{(15\,p)^2}{2} \sin^2 1''$$

$$\therefore x'' = \frac{\cos L \cos d}{\sin Z} p^2 \times \text{number whose log} = 6{\cdot}73673.$$

$$\text{or Log } x'' = \log \cos L + \log \cos d - \log \sin Z + 6{\cdot}73673 + 2 \log p$$

$$= \text{Log} \cos L + \log \cos d + \text{ar. co.} \log \sin Z + 6{\cdot}73673 + 2 \log p$$

For the purpose of obtaining a zero point of the mural circle, we make use of two different methods, of which the first consists in the observations of stars by reflection from a surface of mercury. The mode of operating is as follows: Point the telescope to any known star when it crosses the meridian, and record the reading of the circle; on the next night observe the same star as it crosses the meridian, by pointing the telescope upon the image of the star reflected from the surface of the mercury. As the surface of a fluid at rest must be horizontal, and as the angle of reflection is equal to the angle of incidence, this image will be just as much depressed below the horizon as the star itself is above it. The arc intercepted on the limb of the circle between the star and its reflected image is the double altitude of the star, and its middle point is the horizontal point of the circle, allowing for the difference of refraction at the moment of observation. It is evident that the image of a star seen by reflection will be as much depressed below the horizon, as the object is really above it; and by combining the two readings of the circle for the same star observed by reflection and direct vision, the horizontal point of the instrument may be readily determined. This method of obtaining a reflection and direct observation of a star at the same transit is daily practised at our principal observatories.

The second method is by the use of a Bohnenberger's eye piece previously explained. By causing the two images of the wire of the declination micrometer to coincide, and noting its readings, the *nadir*, and consequently the *zenith*, points of the circle may be determined by directing the telescope vertically downward upon a basin of mercury, the reflected image of the horizontal wire made to coincide with its direct image. The telescope is directed towards the *nadir*, which is distant 90° from the horizontal point, or 180° from the zenith point. As this observation can be made at any time, independently of the weather, it is a most valuable method, and in many observatories is the one exclusively employed. The horizontal point determined by direct and reflected observations should differ exactly 90° from the zenith point as determined by the collimating eye-piece. This eye-piece, first suggested by Bohnenberger, consists of an ordinary eye-piece with an aperture A (Fig. 201), cut in its side, and a plane perforated speculum B B inserted between the two lenses at an angle of 45° with the optical axis having a lamp held so as to throw a strong light on the speculum. The reflected images of the wires may be seen with great distinctness. Instead of a perforated opaque speculum, a piece of plane glass with parallel faces, and without any perforation, is sometimes used. The observer looks through the plane glass without difficulty, while sufficient light is reflected from the lower surface to render the lines visible. If the axis of the telescope be not horizontal, half the distance between the middle wire and its image, corrected for error of level, will give the error of collimation of the middle wire.

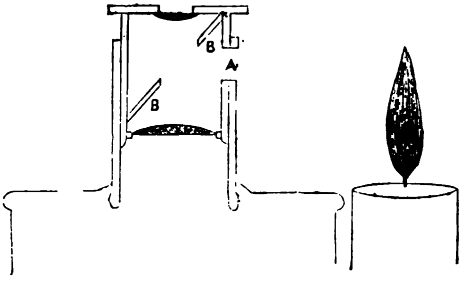


Fig. 201.

The tables of refraction made use of at the Royal Observatory, Greenwich, are those of Bessel, modified and expanded from the "Tabulæ Regiomontanæ." A numerical example of their application is given further on.

Example of a zenith point determination from the Greenwich Observatory, 1849:—

Day. 1849.	Name of Star.	Concluded Circle Reading. (Reflection.)	Concluded Circle Reading. (Direct.)	R + D 367° 5′	183° 32′
		North Paral.			
		° ′ ″	° ′ ″		
April 5	B.A.C. 3528	305 23 2·49	61 42 38·68	41·17	
	B.A.C. 3593	295 33 2·60	68 32 33·53	39·13	
	3 Ursæ Majoris	279 56 24·11	87 9 17·96	42·07	$\frac{R+D}{2}$
				3) 122·37	
				40·79	50″·40
		South Paral.			
April 3	Pollux	250 27 39·07	116 37 59·10	38·17	
3	26 Lyncis	270 1 15·66	97 4 24·29	39·95	
5	Regulus	234 46 57·72	132 18 41·52	39·24	
5	9 Cancer in Va	263 46 41·08	103 19 0·48	41·56	
				4) 158·92	
				39·73	49″·87

183° 32′
Horizontal Point = Mean 50″·14
or Zenith Point = 93° 32′ 50″·14

The adopted Zenith Point is 93° 32′ 49″·89

The annexed printed form will show fully the method of reducing observations of zenith distance. It is arranged for the reduction of the zenith distance observations with the transit circle, which differs from the above specimens principally in the valuation of the micrometer-microscopes. The manner in which the revolutions and parts of revolutions of the microscope-micrometer are converted into arc are explained as follows in the Greenwich Observations, 1852, page iv.:—

"I must premise that, as one revolution of each microscope-micrometer does not differ extravagantly from 1′ on the limb of the circle, we may consider each revolution as a nominal minute. Next, supposing the number of integral revolutions, as shown by each microscope to be the same for all, we ought, in order to obtain the mean of the fractions of a revolution, to add together the subdivisions as shown by the different micrometer-heads, and divide the sum by 6. Thirdly, as the subdivisions are in the decimal scale, we should then multiply this mean by 60 to reduce it to seconds. It

Zenith Point of Transit Circle for 1849.

Day and Hour.	Object.	Circle Reading. Direct.	Circle Reading. Reflection.	Sum of Seconds & Mean.	Half of Mean Sum.	Adoption of Zenith Point.
		North	Paral.			
		° ′ ″	° ′ ″			
Oct. 14 8	α Cephii	349 22 15·10	190 20 11·35	26·45		
16 9	B.A.C. 7851	325 58 47·99	213 43 40·48	28·47		
17 8	α Cephii	349 22 12·87	190 20 11·86	24·73		
17 8	β Cephii	341 25 18·15	198 17 8·48	26·63		
				4)26·28		
				26·57	13·29	× 4 13·16
			Nadir.			
Oct. 12 23	Wire R.		179 51 14·08			
13 8	”		14·51			
13 22	”		14·09			
14 8	”		14·12			
14 22	”		13·18			
15 6	”		13·72			
15 22	”		14·70			
16 6	”		14·56			
16 22	”		14·32			
17 6	”		13·43			
17 22	”		13·76			
18 7	”		14·45			
			12) 48·92			
			179 51 14·08	×	4 . .	16·32
		South	Paral.			
Oct. 16 8	ι Pegasi	26 41 52·07	153 0 38·51	30·58		
17 12	α Trianguli	22 28 17·15	157 14 15·10	32·75		
				3·33		
				31·67	15·84	× 2 11·68
						10) 41·16
						179 51 14·12

or Z P correction . . . 8 45·88

is evident thus that the number of nominal seconds to be attached to the nominal minute will be found by simply adding together the subdivisions on the micrometer-heads, and shifting the decimal point.

"The following were found by trial to be the *runs* of the six microscopes:—

"NUMBER OF REVOLUTIONS CORRESPONDING TO 5′ FOR MICROSCOPES.

	A	B	C	D	E	F
	r	r	r	r	r	r
Means..	4·912	4·882	4·882	4·879	4·886	4·878

"The results given above show that the screws of the micrometers are so sensibly equal that, in the reduction of the observations, it is sufficient to take the mean of the six readings, and to apply to it the mean correction for runs. To do this, the following process is employed:—The sum of the runs for the six microscopes above given is 29r·320. Now, if the correction by which each revolution of a micrometer may be converted into a minute of arc were exactly $\frac{1}{50}$ part of the reading, then the sum of

Example of the Reduction of an Observation with the Mural Circle.

	1848. Dec. 21.	
	Uranus.	
	° ′ ″	
Mic.	118 0	
A	3 37·7	
B	44·8	
C	65·8	
D	39·9	
E	42·3	
F	27·9	
	258·4	
	—0·2	
Correction for Runs — 9″ 3 for an arc of 5′	——6)258·2	
	118 3 43·03	Concluded Circle Reading.
Adopted Zenith Point ...	73 12 52·03	
Apparent Zenith Distance	44 50 51·00	
Add Refraction	1 0·73	
	44 51 51·73	
Subtract Parallax... ...	0·31	
	44 51 51·42	
Add Colatitude	38 31 21·80	
Geo. N. P. D. of Centre ...	83 23 13·22	

ROYAL OBSERVATORY, GREENWICH.

Observations of Zenith Distance with the Transit Circle in the Year 1851, and Computations of Geocentric North Polar Distance.

Approximate Solar Time.	Oct. 24.	24.	24.	24.	24.
Name of Object.	Wire (Nadir obs).	ζ Eridani.	ε Tauri.	ι Eridani.	Pallas.
Mode of Observation.	R.				
No. of Vertical Wire at which observation is made . . .	4.	4.	4.	4.	4.
Observer.	H. B.				
	° ′	° ′	° ′	° ′	° ′
Pointer . .	179 40	60 25	42 35	61 0	74 30
	r.	r.	r.	r.	r.
Micrometer reading of Microscope A . .	0·652	5·102	2·410	4·196	1·462
B . .	·637	5·046	2·385	4·137	·415
C . .	37 ·712	302 5·154	141 2·443	247 4·222	85 ·539
D . .	·414	4·823	2·146	3·940	·234
E . .	·497	4·872	2·221	3·973	·270
F . .	·775	5·168	2·498	4·252	·575
Uncorrected Mean of Microscopes. . .	179 40 36·87	60 30 1·65	42 37 21·03	61 4 7·20	74 31 25·25
First part of Correction for Runs . . .	0·74	6·04	2·82	4·94	1·70
Second Part, + 0″·381 for 100″ . . .	·14	1·15	·53	·94	·32
Micrometer reading, and equivalents . .	21·428|10 22·15 12·44 ·24	20·923|9 52·52 27·26 ·09	22·845|10 51 77 27·88 ·15	23·425|11 21·40 12·44 ·15	24·730|11 51·02 21·63
Correction for flexure and errors of division .	0·86	1·49	1·94	1·52	2·08
Circle reading at observation . . .	179 51 13·44	60 40 30·20	42 48 43·12	61 15 48·59	74 43 41·95
Add zenith point correction	8 46·25	8 46·25	8 46·25	8 46·25	8 46·25
Apparent zenith distance south . . .		60 49 16·45	42 57 29·37	61 15 48·59	74 57 28 20
Add refraction (from below)		1 45·37	54·94	1 47·96	3 36·54
True zenith distance south		60 51 1·82	42 58 24·31	61 24 34·84	75 1 4·74
Subtract parallax (from opposite page) . .					4·97
Geocentric Distance from astronomical zenith .					75 0 59·77
Add colatitude . .		38 31 21·80	38 31 21·80	38 31 2 1·80	38 31 21·57
Observed north polar distance . . .		99 22 23·62	81 9 46·11	99 57 44·60	113 32 21·57
Star's correction with sign changed . . .		11·79	9·95	11·38	
Mean N.P.D. Jan. 1, for stars . . .		99 22 35·41	81 29 56·06	99 57 55·98	
		in.	in.	in.	
From Appendix No. 1 to Greenwich Observations, 1836. Barometer and B (Table I.)		30 30 | ·03720	0·3720	30 29 | ·03706	·03706
Thermometer and T. (Table II.)		48°·5 | ·04257	·04257	48°·3 | ·04275	47°·8 | ·04319
Z. (Table III.)		1·94005	1·65823	1·95186	2·25086
Proportional parts .		·264 ·7	·178 ·12	·119 ·18	·341 ·23
t (Table IV.)		·20		·21	·80
Sum or log. Refraction in seconds . . .		2·02273	1·73990	2·03325	2·33555
Refraction . . .		105″·37	54″·94	107″·96	216″·54
Interior Thermometer.		53·0			52°·0

the runs would be $\frac{50}{51} + \overset{r}{30}\cdot000$, or $\overset{r}{29}\cdot4118$. It is not likely that the sum of the runs will, in ordinary cases, amount to this quantity; and the variable correction, after adding $\frac{1}{50}$ part to the mean of the readings of the microscopes, will therefore always be additive. To determine this quantity, let the sum of the runs be $\overset{r}{29}\cdot412 - x$. Then the true reading, in seconds of space, corresponding to a nominal reading of r'' (including the value in seconds of the nominal minutes), will be

$$r'' \times \frac{30\cdot000}{29\cdot412 - x}.$$

Or,

$$r'' \left\{ 1 + \frac{1}{50} + 0\cdot03468 \times x \right\}$$

Hence the correction, after adding $\frac{1}{50}$ part of the mean of readings, will be $+ r\,x \times 0''\cdot03468$. For the purpose of calculating this quantity easily, by the ordinary proportioned scale, it is convenient, in the first place, to compute its value, where $r = 100''$. This value, which is evidently $+ 3''\cdot468 \times x$, is tabulated for different values of x. Let X be the number taken from this table; then, for any actual reading r, the correction will be $\frac{r\,X}{100}$; a quantity which can easily be taken from the ordinary sliding-rule

Parallax.—The change which takes place in the position of a heavenly body on account of its having been observed from a point which is not the centre of motion, is called its parallax. All the heavenly bodies appear to move in the concave surface of a sphere concentric with the earth; hence the centre of the earth is considered to be the centre of motion. We shall now inquire into the change which would take place in the position of a body which has been observed at the surface of the earth, and its position as seen from the centre. Or, in other words, to find the correction which should be applied to an observation made at the earth's surface to reduce it to what it would be if made at the centre. Let C (Fig. 202) be the centre of the earth, O the place of observation on its surface, Z the geocentric zenith, S the object observed, then Z O S is the observed zenith distance, Z C S the zenith distance as seen from the centre, which is called the true zenith distance, because astronomers reduce all their observations to this point. The difference of these angles is the parallax; it is, therefore, the angle subtended by the earth's semi-diameter at the object observed.

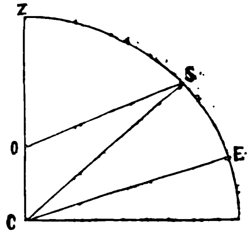


Fig. 202.

Put Z C S the true zenith distance = Z.

Z O S the apparent zenith distance = Z′.

C O the earth's semi-diameter = r.

C S the distance of the object from the earth's centre = D.

The parallax O S C = p.

Then $$Z + p = Z', \text{ or } Z = Z' - p.$$

This shows that the observed zenith distance is greater than the true zenith distance by the quantity p. Hence the true zenith distance is found by subtracting the parallax from the observed zenith distance.

It is evident that parallax exerts its influence in a vertical plane passing through the object; consequently it has no effect on the right ascension of an object when observed on the meridian; but if the observation be made on either side of the meridian, the right ascension will be affected by it, as it raises the object to an hour circle nearer to the meridian. Hence, if the object lay east of the meridian, the apparent right ascension is diminished, but, if west, it is increased by parallax. Refraction has a contrary effect.

Now we have $\frac{\sin Z'}{\sin p} = \frac{D}{r} \therefore \sin p = \frac{r}{D} \sin Z' = \frac{r}{D} \sin (Z + p) \ldots (1)$

The equation $\sin p = \frac{r}{D} \sin Z'$, gives $p = o$ when $Z' = o$, and $\sin p = \frac{r}{D}$ when $Z' = 90°$. As this is the greatest value of Z', $\frac{r}{D}$ is the greatest value of $\sin p$.

Therefore, in the geocentric zenith the parallax is nothing, whereas in the horizon it is the greatest possible. Put $\frac{r}{D} = \sin \pi$, then $\sin p = \sin \pi \sin Z'$. This equation may be put under the form

$$p \left(\frac{\sin p}{p}\right) = \pi \left(\frac{\sin \pi}{\pi}\right) \sin Z'$$

$$\therefore p = \pi \left(\frac{\sin \pi}{\pi}\right) \left(\frac{p}{\sin p}\right) \sin Z' \ldots \text{(2).}$$

The horizontal parallax π, or rather the horizontal equatorial parallax (for the moon), given in the "Nautical Almanac," is the angle subtended by the earth's equatorial semi-diameter.

Let r' be the semi-diameter of the place of observation, then $r' \sin \pi$ will be the sine of the horizontal parallax at that place, and we shall have

$$p = r' \pi \left(\frac{\sin \pi}{\pi}\right) \left(\frac{p}{\sin p}\right) \sin Z' \ldots \text{(3).}$$

This is the formula given in the Introduction to the Greenwich Observations for computing the moon's parallax when observed on the meridian.

The equation $\sin'' p = r' \sin \pi \sin Z$ may be written

$$p'' \sin 1'' = r' \pi'' \sin 1'' \sin Z'.$$

Or, $$p'' = r' \pi'' \sin Z' \ldots \text{(4)}$$

This will be sufficiently exact for the planets. In the case of the moon, a correction will be necessary, which is found thus—

$$\sin p'' = p'' \sin 1'' - \frac{p''^3 \sin^3 1''}{6} \text{ nearly; hence}$$

$$p'' \sin 1'' - \frac{p''^3 \sin^3 1''}{6} = \left(\pi'' \sin 1'' - \frac{\pi''^3 \sin^3 1''}{6}\right) \sin Z'$$

but $p''^3 \sin^3 1'' = \pi''^3 \sin^3 1'' \sin^3 Z'$, by neglecting powers higher than the third; hence, by substitution,

$$p'' \sin 1'' = \pi'' \sin 1'' \sin Z' - \frac{\pi''^3 \sin^3 1''}{6} (\sin Z' - \sin^3 Z')$$

$$= \pi'' \sin 1'' \sin Z' - \frac{\pi''^3 \sin^2 1''}{6} \sin Z' \cos^2 Z'$$

Consequently the correction is $-\frac{\pi''^3 \sin^2 1''}{6} \sin Z' \cos^2 Z' \ldots$ (5)

This correction is applied in the Greenwich Lunar Reductions from 1750 to 1830.

Again, since $\sin \pi = \frac{r}{D}$, for any other distance D', we shall have $\sin \pi' = \frac{r}{D}$; therefore, $\sin \pi : \sin \pi' \; \frac{1}{D} : \frac{1}{D'}$, or $\frac{\pi'}{D} = \frac{\pi}{D'}$.

If D' be expressed in parts of D taken as unit, we have $\pi' = \frac{\pi}{D'}$; therefore, if π be the sun's horizontal parallax at the mean distance from the earth, and D' the distance of any planet from the centre of the earth, expressed in terms of the sun's mean distance, the planet's horizontal parallax will be represented by $\pi \times \frac{1}{D'}$, and for a planet we shall have

$$p = \pi \times \frac{1}{D} \sin Z' \quad . \; . \; . \; . \quad (6)$$

It is by this method the parallax of the planets has been computed in the Greenwich Planetary Reductions from 1750 to 1830.

Equation (1) gives the means of finding the parallax when the true zenith distance is given, thus

$$\sin p = \sin \pi \sin (Z + p) = \sin \pi (\sin Z \cos p + \cos Z \sin p)$$

$$\tan p = \sin \pi \sin Z + \tan p \cos Z \sin \pi$$

$$\tan p = \frac{\sin \pi \sin Z}{1 - \sin \pi \cos Z} \quad . \; . \; . \; . \quad (7)$$

And by the usual method of development,

$$p = \frac{\sin \pi \sin Z}{\sin 1''} + \frac{\sin^2 \pi \sin 2 Z}{2 \sin 1''} + \frac{\sin^3 \pi \sin 3 Z}{3 \sin 1''}, \text{ \&c.} \quad (8)$$

We have seen that $\sin \pi = \frac{r}{D}$; hence, if D be very great, π will be very small, and, therefore, bodies at an extremely great distance, compared with the radius of the earth, have no sensible parallax.

Before proceeding any further, it may be well to remark that the value of $\sin p$ depends on lines drawn from the centre of the earth, or from what is called the geocentric zenith, which differs from the astronomical zenith for this reason:—All astronomical instruments used for determining the north polar distance of an object depend on a zenith point, or something equivalent to one, and this is nothing more than the instrumental reading corresponding to a vertical position of the telescope, and is determined by observing the same star by direct vision and by reflection. The zenith point is therefore the reading for a point perpendicular to the horizon, or, which is the same, a line drawn from the point thus determined in the heavens to the place of observation, is a normal to the curve by which the earth is generated, supposing it to be a surface of revolution. The earth is supposed to be a surface of revolution, generated by an ellipse revolving round its shorter axis; the ratio of the axis being 300 : 299. *Now, it is* well known that normals to an ellipse do not pass through its centre, hence

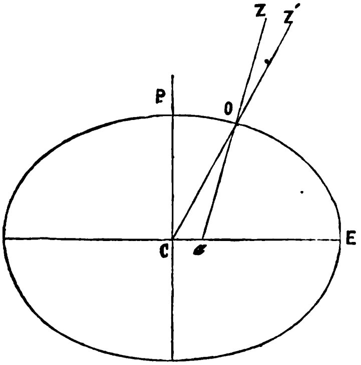


Fig. 203.

the lines drawn from the centre of the earth to the place of observation, which determines the geocentric zenith, will make a small angle with the normal to the earth's surface at the same place, which determines the astronomical zenith, and this angle is called the angle of the vertical. The geocentric zenith is nearer to the equator than the astronomical zenith by the value of this angle, and as all the bodies (some comets excepted) for which we have to compute parallax lie to the south of the zenith, the angle of the vertical is subtracted from the astronomical zenith distance, in order to have the geocentric zenith distance. This angle is easily determined, for it is the difference between the astronomical and geocentric latitude of the place of observation, and is thus found:—Let P (Fig. 203) be the pole of the earth, E a point of the equator, O the place of observation, Z' the geocentric zenith, Z the astronomical zenith, then Z' C E is the geocentric latitude $=$ L g, and Z G E is the astronomical latitude $=$ L a, and Z' O Z $=$ C O G is the angle of the vertical. Let a and b be the axis of the ellipse, the equation of the normal Z G is

$$y - y' = \frac{a^2}{b^2} \cdot \frac{y'}{x'} (x - x') = \frac{a^2}{b^2} \tan \mathrm{L}\, g\, (x - x');$$

therefore,

$$\tan \mathrm{L}\, a = \frac{a^2}{b^2} \tan \mathrm{L}\, g, \text{ or } \tan \mathrm{L}\, g = \frac{b^2}{a^2} \tan \mathrm{L}\, a \quad . \quad . \quad . \quad . \quad (9);$$

hence, L g becomes known. For an ellipticity of $\frac{1}{300}$, and the astronomical latitude of Greenwich, viz., 51° 28′ 39″, we find the geocentric latitude, 51° 17′ 29″, and consequently the angle of the vertical = 11′ 12″.

The value of r', by which the moon's horizontal parallax must be multiplied to have the horizontal parallax at the place of observation, is thus computed: C O $= r'$, O C E $=$ L g; then by the property of the ellipse $b^2 x^2 + a^2 y^2 = a^2 b^2$, we easily find

$$r^2 = \frac{a^2 b^2}{a^2 \sin^2 \mathrm{L}\, g + b^2 \cos^2 \mathrm{L}\, g}$$

$$= \frac{a^2 b^2}{a^2 - (a^2 - b^2) \cos^2 \mathrm{L}\, g}$$

$$r^2 = \frac{b^2}{1 - \frac{a^2 - b^2}{a^2} \cos^2 \mathrm{L}\, g} \quad \therefore\ r = b \sec \theta$$

$$\text{where } \sin^2 \theta = \frac{a^2 - b^2}{a^2} \cos^2 \mathrm{L}\, g.$$

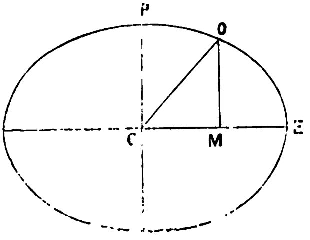


Fig. 204.

In the following section we shall give an investigation of parallax for the foregoing reductions, as well as its effects for occultations, &c.

The reductions from the apparent to the mean places are performed in a manner similar to that previously explained for right ascension.

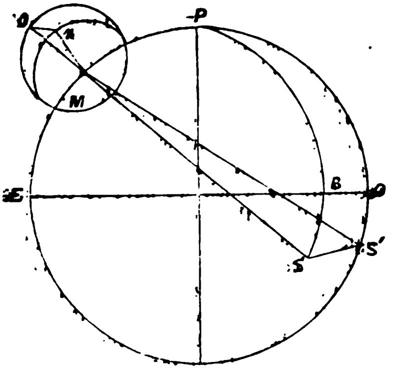


Fig. 205.

The other corrections for circle observations, viz., defective illumination of Moon and Venus, &c., are so fully explained in the Introduction to the Greenwich Observations, that a short investigation will only be necessary.

Correction for Defective Illumination.—The following is the investigation of the correction for the defective illumination of the moon in right ascension:—

Let S (Fig. 205) be the place of the sun, M that of the moon; if the sun were at S′ both limbs of the moon would be full 12h. ∽ apparent time of moon's transit = B P Q = arc B Q, but B Q × cos S B = S S = S M S′ = o M n = arc o n. This is the arc on the moon's disc, which is not illuminated, and its versed sin is the part of the surface not enlightened.

∴ Moon's semi-diameter × versed sine S S′ = correction.

Defective Illumination in N. P. D.—Moon gibbous.

Let E Q be the equator, P the pole, E M = E O M = S′ O B = D = the moon's declination. If the sun were at S′ both limbs of the moon would be enlightened. E P S = sun's hour angle = the difference between the times of passage of the sun and moon without regard to sign. Then

in O B = cot B O S′ × tan B S′.

∴ tan B S = sin O B tan D = sun (E P S — 90°) tan D =

— cos sun's hour angle × tan moon's apparent declination.

B S′ is called the new declination

New declination — sun's declination = angle required. When this angle is positive, the north limb is full.

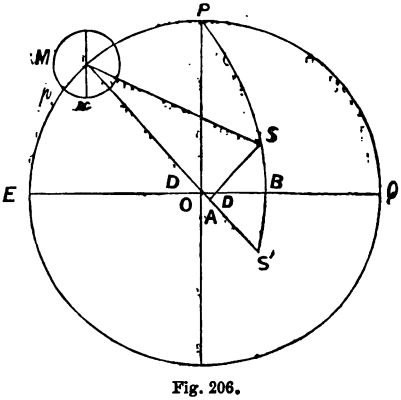


Fig. 206.

From S′ let fall the perpendicular arc S A, then cos B O S′ = cos B S′ sin B S′ O ∴ $\sin B S' O = \dfrac{\cos B O S'}{\cos B S'}$

sin S A = sin S S′ sin A S′ S = sin S S′

$$\sin A S' S = \sin S S' \times \frac{\cos B O S'}{\cos B S'}$$

= sin S S′ cos B O S′ sec B S′. S A is called the new angle, therefore sine new angle = cos app. decl. × sec. new decl. × sine angle required.

The new angle, or S A, is taken for the measure of S M A or p M q, the angle of defective illumination.

Defective Illumination in N. P. D.—Moon horned.

Mean time of passage 3h. to 6h.

Let S (Fig. 207) be the sun, M the moon on the meridian, *a c* perpendicular to S M. Join M O, D = E M = the moon's apparent declination, *d* = the sun's apparent declination. The angle M P S = T = the difference between the mean times of passage of the sun and moon; the angle S M O is the angle of defective illumination = 90° — P M S.

$$\tan \text{P M S} = \frac{\sin T}{\cos D \tan d - \sin D \cos T}$$

$$\tan \text{S M O} = \cot \text{P M S} = \frac{\cos D \tan d}{\sin T} - \frac{\sin D \cos T}{\sin T}$$

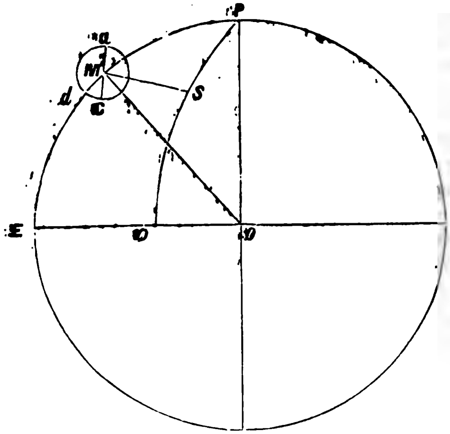


Fig. 207.

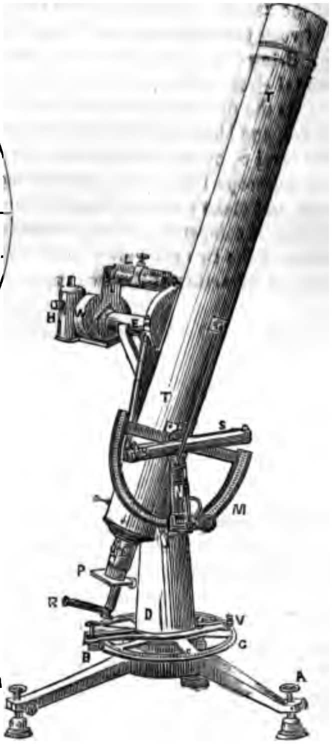


Fig. 208.

The Zenith Telescope.—The instrument employed in measuring the difference of the zenith distance and hour, called the zenith telescope, may be described here, although it is scarcely the proper place for it. A A (Fig. 208) are the screws which support the entire instrument, and by which the column carrying the telescope-screw is rendered truly vertical; C C is the horizontal circle twelve inches in diameter, graduated to 10′, and reading to 10″, by means of its vernier and microscope V. B is the tangent-screw for slow motion.

This circle serves to mark the position of the meridian when it has once been determined. It likewise enables the observer to turn the telescope promptly through 180° of azimuth. D is the vertical column which supports the telescope, and about which the telescope turns in azimuth. E is a horizontal axis, to one end of which is attached the telescope T T, which is counterpoised by the weight, W, at the other end; this axis

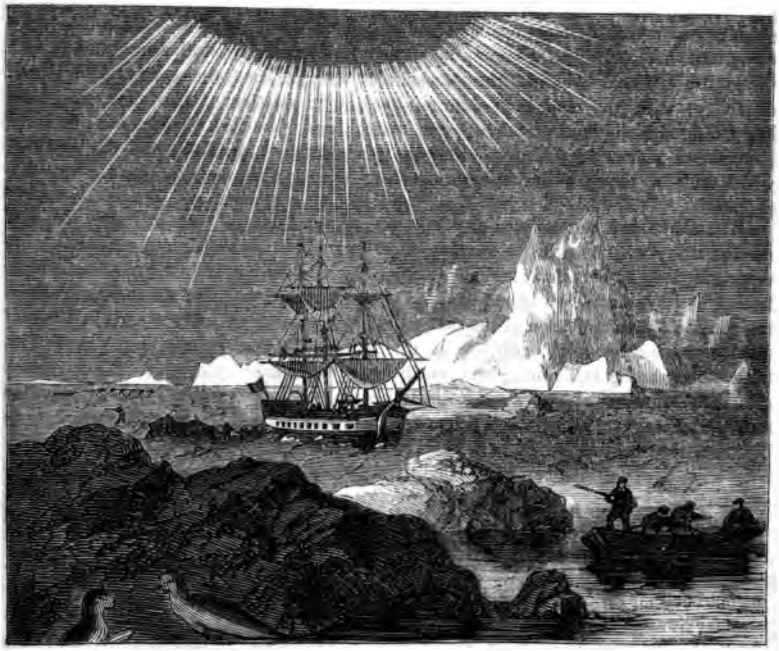
is hollow, and through it passes the light of the lamp, H, to illumine the wires
telescope. The telescope has a focal length of about forty inches, and an
three inches. L is the level by means of which the column D is
vertical, riding on the horizontal axis. M is a graduated semicircle
the telescope, and having a vernier, A, with a microscope. This semicircle
a finder for setting the telescope to the altitude of the stars to be observed.
a very delicate level attached to the semicircle. P is the parallel wire
for measuring small differences of altitude; and R is the diagonal eye-piece,
structed that the micrometer may not interfere with the field of observation.

The stars selected for observation by this instrument should be within a
range of the micrometer—say 10′—one culminating to the north and the other
of the zenith. Having levelled the instrument, set the telescope to an altitude
between the two stars, and bring the bubble of the level S to the middle of its

Bring the telescope into the plane of the meridian by setting the vernier of the
zontal circle to the point previously determined.

As the first star enters the field of view follow its image with one of the
wires, and bisect it at the instant it crosses the middle wire. Record the
the level S, noting the divisions corresponding to each extremity of the bubble,
the telescope 180° in azimuth, taking care to preserve the same inclination to
zon, and make a similar observation on the second star, bisecting it with the
wire. A comparison of the readings of the two micrometers will give the
tance of the two stars after correction by readings of the level, and also for
ence of refraction of the two stars.

◆



METEOROLOGY.

Definition and Limitations.—The term Meteorology, from μετεωρος elevated, was applied by Aristotle to signify phenomena occurring in elevated regions. It may be considered synonymous with the study of atmospheric phenomena, though all which concerns meteors proper is very nearly allied with astronomy.

In many respects Aristotle's opinions on meteorologic subjects display the usual acumen of that deep thinker; his remarks on the subject of dew are particularly interesting. Deprived, however, of the barometer and thermometer—deprived of optical instruments—the nature of electricity yet undeveloped, and the composition and functions of the atmosphere unknown, ancient speculations on meteorologic subjects were necessarily unsatisfactory and vague.

The meteorological writings of Theophrastus, Aristotle's pupil, were more diffuse than those of that great philosopher, and were long recognized as constituting a text-book. They constituted the groundwork of the Διοσημεια, or prognostics of Aratus, and were embodied in a versified rendering by Cicero in his youthful days. Portions of these attempts at versification are still in existence, and do but little credit to the great Roman orator in a poetical capacity.

Meteorology, in its most common and restricted sense, may be considered synonymous with *knowledge of the weather*. It therefore involves a full acquaintance with the nature and composition of the atmosphere; with the laws of gaseous and vapourous elasticity; with the conditions determining the production of fogs, dew, snow, and hail; also with the laws of atmospheric, optical, and electrical phenomena. It is the province of meteorology also to study the phenomena of aërolites, and the relations which subsist between atmospheric conditions and the development of organic species.

The above is a general outline of the scope and limits of meteorology. Its successful study will be seen to involve a pre-acquaintance with many sciences, more especially those of chemistry and electricity. There does not, indeed, exist any science having limits so undefined as meteorology. From a consideration of the theory of shooting stars to a contemplation of the mutual alliance subsisting between certain forms of disease and atmospheric conditions, or the relation between certain animal and vegetable tribes and given atmospheric conditions, the divergency is wide. Nevertheless, all these branches of study are intimately allied with meteorology; and perhaps the most delightful part of botanical science is that which seeks to establish connections between the localization of certain vegetable families in districts characterised by some peculiarity of meteorologic condition. Horticulturists have been too ready to overlook the influence of remote atmospheric conditions on certain vegetable families. Too frequently it has been considered that a vegetable surrounded by an atmosphere of temperature similar to that of its native region, and planted in a soil of similar chemical composition, must necessarily thrive. There are, nevertheless, meteoric conditions beyond these. Why is it that many species of the palm tribe refuse to grow very far away from their native regions, although transplanted to localities seemingly identical in all respects? Why is it that the cocoa-palm refuses to grow in regions very far distant from the sea? These questions involve meteorologic considerations of great interest; and not less interesting to a meteorologist is the partiality evinced to a restricted region by the cinchona tribe. An atmosphere very much rarefied and perpetually moist are so essential to their existence, that they cannot live without it.

The natural approach to meteorology is the study of the atmosphere, which admits of being contemplated under many aspects. It may be contemplated either as the atmosphere proper or theoretical, composed of two gases, oxygen and nitrogen; or as the practical atmosphere or mixture of the gaseous theoretical atmosphere with numerous vapours, extraneous gases, and fleeting undetermined miasmata. The atmosphere, too, admits of being regarded statically, *i.e.* at rest, and dynamically, *i.e.* in motion, the latter involving a study of the causes of winds. The atmosphere, lastly, may be considered in relation to the imponderable agents, to heat, light, electricity, and magnetism. I shall begin by investigating the nature of our atmosphere regarded chemically.

Chemical Constitution of the Atmosphere.—By the ancients air was considered to be an elementary substance. Chemistry at length demonstrated it to consist of two gases, oxygen and nitrogen combined, or rather mixed in the proportions of about eighty parts, by measure, of nitrogen, and twenty of oxygen; or, in other words, one volume of oxygen to four of nitrogen.

Considerable difference of opinion once existed on the question, whether the atmosphere be a chemical or a mechanical compound. To adduce evidence bearing on this discussion, would be foreign to the subject of meteorology. The generally received opinion is in favour of the mechanical constitution of the atmosphere. The law of the diffusion of gases perfectly accounts for the intimate mixture with oxygen and nitrogen in the atmosphere, without having recourse to the assumption of chemical union. An outline of the law in question it will be proper to give.

Gaseous Diffusion.—If two glass vessels be taken, as represented in the accompanying diagram (Fig. 1), the upper one being filled with hydrogen gas, the lower one with oxygen gas, placed in communication with each other by a capillary tube passing through the cork stopper of both, and allowed to remain at rest for about half an hour, perfect mixture of the oxygen and hydrogen gases will have ensued. Inasmuch as no chemical union takes place between the two gases thus circumstanced, and inasmuch as hydrogen gas filling the upper vessel is about sixteen times heavier than oxygen gas filling the lower vessel, some new cause of admixture has to be sought; it depends upon the mutual tendency of the two gases to become diffused through each other.

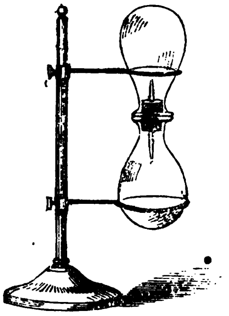


Fig. 1.

The above is an individual case exemplifying a general principle; any two gases might have been selected and mutual diffusion would have ensued. Oxygen and hydrogen gases have been here chosen because of the facility wherewith the circumstance of these having become diffused may be determined. It is well known that neither oxygen nor hydrogen gas, taken separately, will explode on the application of flame, whereas a mixture of the two readily explodes; hence the propriety of selecting these two gases for illustration of the principle in question will be obvious.

Faraday appears to have been the first to direct attention to the mutual diffusibility of gases. He noticed that bottles filled with gases, and corked or stoppered, or vessels filled with gases and inverted over mercury, in either case occluded to all appearance accurately, nevertheless almost always permitted mutual admixture of the air without and the gas within. Döbereiner, Mitchell, and Graham, more especially the latter, have since investigated this class of phenomena more narrowly, and Graham has succeeded in determining the law which regulates this diffusion. He finds that the relative diffusiveness of any two gases is expressed by the reciprocals of the square root of their densities. Thus, the density of air being one, its diffusiveness is one also. The density of hydrogen being 0·693, its diffusiveness is $\frac{1}{\sqrt{0{\cdot}063}} = \frac{1}{0{\cdot}2633} = 4{\cdot}56$; the density of ammonia being 0·5898, its diffusiveness is $\sqrt{\frac{1}{0{\cdot}5898}} = \frac{1}{0{\cdot}7681} = 1{\cdot}30$; and generally representing the density of a gas by d, its diffusiveness is $= \sqrt{\frac{1}{d}}$. Applying this rule to practice, it appears that, supposing hydrogen and ammonia placed under circumstances promoting their mutual diffusion, 456 volumes of hydrogen will become mingled with 1·30 of ammonia.

It will be remarked that the atmosphere has hitherto been treated of, in a theoretical sense, as a mere mixture of nitrogen and oxygen gases. Practically, however, the atmosphere is far more complex. It invariably contains portions of carbonic acid (about one part in a thousand), also extraneous gases, besides aqueous and other vapours. A mixture of all these things may be termed, distinctively, the actual or *practical* atmosphere. They will hereafter come under our notice *seriatim*; but even a theoretical mixture of oxygen and nitrogen gases is subject to remarkable variations of property, its chemical composition remaining unchanged; portions of its oxygen are subject to be converted into *ozone*.

The change of common oxygen into ozone, furnishes an illustration of one of the most remarkable discoveries of modern science. It displays what is, perhaps, the most extraordinary example of the condition of *allotropism*, or the existence of one body under two different aspects; it promises to render evident some of these occult atmospheric causes which determine the progress of epidemics, and promote the existence of endemic diseases.

A summary of our knowledge relative to ozone may be briefly stated as follows:—Oxygen gas is susceptible of undergoing a change, the nature of which is altogether veiled in mystery. It is susceptible of becoming odorous, corrosive, and irritating when breathed; its chemical action is susceptible of being exalted and modified, so that whilst ordinary oxygen gas neither bleaches nor corrodes silver, nor decomposes iodide of potassium, the allotropic, or second form of oxygen gas, will accomplish all these results, and many more too numerous for mention here. The general conclusion to which it is desired to bring the reader is this: if causes can be proved to exist capable of changing atmospheric oxygen gas in its ordinary state to oxygen gas in its extraordinary state, how vast, how complicated must be the meteoric results determined thereby! That such natural causes do exist will be readily inferred from a consideration of the artificial methods to which the chemist has recourse for changing ordinary oxygen into ozone.

Methods of Ozonising Oxygen Gas.—The most ready method of ozonising oxygen gas is as follows:—Take a few sticks of phosphorus, scrape them free from all superficial contamination, place them in a wide-mouthed bottle containing a little water but not enough to cover the phosphorus. Let the whole remain at rest for about ten or fifteen minutes, and a considerable portion of the atmospheric oxygen will have been converted into ozone. The ozonized air thus generated will be at present mixed with vapours of phosphorous acid; washing will free the air from these, however, without removing the ozone. That the air thus treated has become considerably modified in some way, will, in the first place, be rendered evident by the smell; atmospheric air, when pure and in its ordinary state, is devoid of smell. But the atmospheric air, the product of the experiment just detailed, will be found to have a very peculiar odour. It will be found, moreover, to be capable of removing the colour cf sulphate of indigo, and other vegetable and animal colouring bodies. All this is due to the modification which ordinary oxygen gas assumed—due to its assumption of the allotropic state—to its conversion into ozone.

Another ready method of generating ozone is this:—Moisten the interior of a bell-glass receiver, or a large-mouthed bottle, with ether; then take a glass rod, heat it in the flame of a spirit-lamp, and plunge it into the bottle or bell-glass; under these circumstances ozone will be formed, provided the glass rod has not been heated to an inordinate temperature, for the circumstance has to be mentioned that ozone is reconverted into ordinary oxygen gas by contact with any body heated above a certain, but not very well-determined, point.

We have seen that the ordinary method of generating ozone consists in promoting the contact of phosphorus with atmospheric air (or oxygen) under certain conditions.

Many other substances besides phosphorus are capable of generating ozone by contact. Oil of turpentine, and many other essential oils, will accomplish this; and the fact in question cannot be too forcibly remembered by the painter, who may discover in the philosophy of ozone the reason why certain pigments fade, or are bleached, thus destroying the general effect which he desired to produce. Meteorologically considered, however, the most important source of ozone remains to be described; I refer to the production of ozone by means of electricity. Every person who has been much accustomed to work with the electrical machine must have noticed the generation of something powerfully odorous during the friction of the cylinder, or plate, against the rubber. This odour has, in point of fact, been called the *electric smell.* Now, if this electric smell be compared with the smell of the atmospheric air which has been treated with phosphorus, as just described, and washed, the two odours will be found to be identical, which is a presumptive evidence that electricity has in some way been concerned in the formation of ozone—an idea which extended experiment fully comfirms.

It has already been stated that the most prominent quality of ozone is its highly developed oxidising power. By taking advantage of this property, we are supplied with an easy means of recognizing it, by means of a test-paper, imbued with a mixture of iodide of potassium and starch. The chemical reader need not be informed that iodine colours starch blue, whereas oxide of potassium does not; therefore test-paper, imbued with iodide of potassium and starch, may occasionally be resorted to for indicating the presence of certain bodies which have the faculty of decomposing iodide of potassium, and liberating free iodine. Ozone is one of these, which fact remembered will render the following experiment intelligible:—If a piece of paper, imbued with iodide of potassium and starch, be held between the prime conductor of an electrical machine and the knuckle or a metallic ball, and electrical sparks transmitted through it, spots of blue discolouration will be seen on the paper, corresponding to each electrical spark.

The method of detecting the presence of atmospheric ozone is now readily indicated. If a strip of paper, imbued with solution of iodide of potassium and starch, turns blue, the existence of ozone is demonstrated. The experiment is very striking when performed near the sea. During the prevalence of a land wind, the test-paper will generally afford slight indications, or none at all; during the prevalence of a sea wind, however, ozone can generally be detected.

The reader will now be prepared to form some idea of the natural causes which may generate ozone. We need assume no other agency than that of electricity, to be assured that the production of ozone must be universal, and, looking on the world as an aggregate, continuous; and when we consider the potent nature of ozone, the irritation it produces when breathed, the facility with which it bleaches, corrodes, and destroys, we shall not be at a loss to understand that the consequences to living beings of its excess or diminution must be all-important. A most important function of ozone has yet to be indicated, it removes almost more rapidly than chlorine itself the bad odours resulting from the decomposition of animal or vegetable bodies. If a piece of putrid flesh be immersed for a few minutes in a bottle of ozonized air, the odour of decomposition is totally destroyed. With these facts before us, we may form some idea to ourselves of the important functions which ozone is designed to accomplish. Lessen the amount of atmospheric ozone, lower it below given limits, and increase the atmospheric temperature to the degree most congenial to organic decomposition, and the air will soon be charged with disease-bearing putrid odours.

It is in accordance with all that philosophy has been able to teach us in relation to the laws of epidemic and endemic maladies, that the presence of such gaseous odours of organic decomposition as are here assumed, must be the fruitful source of disease; and it is not possible, after having studied the qualities of ozone, to refuse assent to the proposition that the existence of this agent in competent amount must be followed by the destruction of the pestiferous odours of organic decomposition. If, however, ozone be naturally formed at any time in excessive amount, it is not difficult to foresee that other serious consequences must result to animal life. The inhalation of an irritating gas cannot but produce injurious effects on organs so delicate as the lungs, and perhaps many of the now anomalous and inexplicable effects of change of air to patients suffering from chest diseases may hereafter receive their solution in a more intimate acquaintance with the laws of ozone.

Physical Properties of Gaseous Bodies.—The word gas is of German origin, and was first employed by Van Helmont to signify the vapour which escaped from liquids undergoing vinous fermentation. At later periods the term was applied to designate every invisible substance disengaged from bodies by the application of fire. Macquer, a celebrated French chemist of the eighteenth century, extended the meaning of the term gas to signify every kind of air besides atmospheric air; and modern chemists have extended the meaning of the term still further to indicate elastic fluids, whatever their colour may be, which are not readily condensible. At one time it was erroneously imagined that gases did not admit of condensation; in accordance with this belief a gas was defined as being *a permanently elastic* fluid, thus distinguishing this class of bodies from mere vapours, which, so far from being permanently elastic, are very readily condensible. Modern discovery has proved this distinction to be untenable. All the known gases, except oxygen, nitrogen, hydrogen, nitric oxide, carbonic oxide, and coal gas have been liquefied, and a great number of them solidified, by subjecting them to extreme cold and pressure.

Law of Marriotte—The Volumes of Gases are inversely to the Pressure applied.—This is a very celebrated law, and one that intimately concerns the meteorologist; it may be otherwise termed the law of compressibility of elastic fluids.

It will be recognized that the law in question, according to the exposition of it just given, is a general law applying to a vast number of gaseous and vaporous bodies. For a long time its absolute truth remained unquestioned, but more recently M. Regnault and others have demonstrated it to be not of such universality.

Even atmospheric air and nitrogen do not rigorously conform to the law; and carbonic acid, and liquefiable gases generally, are so little amenable to the law, that, as applied to them, it cannot be regarded as approximately correct. Even the rate of compressibility of hydrogen is not strictly accordant with the law, although the deviation in this case is in the opposite direction to the deviation when atmospheric air is concerned; for it suffers less compression than, according to the law, should take place. Carbonic acid and nitrogen, when compressed by a force of forty-five atmospheres, only fill seven-tenths of the space they ought to occupy according to the law.

Now, inasmuch as the philosophy of estimating the height of mountains barometrically, is intimately associated with the law of Marriotte, it is well to indicate that this law is not quite correct; nevertheless, as regards atmospheric air, it is so nearly correct that we may accept it without demur.

The experiment by which the law of Marriotte was deduced is as follows:—Into a strong glass tube, of equal diameter throughout, bent on itself (as represented in Fig.

2), open at the long and closed at the short extremity, a little mercury is poured in such a manner that it shall be perfectly level in the two legs, as represented by the line A B. Under these circumstances it will be evident that the air inclosed between B C and A D will be equally compressed. Let us assume the amount of compression to be represented by the weight x. Let us furthermore assume that the weight of a column of mercury between O and D to be $= x$, and let us call it m. If, then, on filling the long arm of the syphon with mercury, the column of air originally extending from B to C be diminished to the column extending from B′ to C, that is to say, to one-half,

$$\text{then we have } x + m = \tfrac{1}{2}\,BC, \text{ or } 2\,x = \tfrac{1}{2}\,BC\,;$$

proving that the compression of air within the limits of the experiment is inversely to the pressure applied. In like manner, when the pressure is triple, the volume of gas is reduced to one-third; when quadruple, to one-fourth, etc. M. Arago has experimentally determined that the law rigorously applies to atmospheric air up to a pressure of twenty-seven atmospheres.

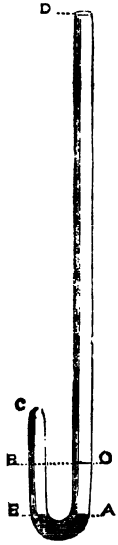


Fig. 2.

The Atmosphere as a Ponderable Agent.—Owing to the equality of pressure which the atmosphere exercises on every side, we are not ordinarily conscious of its possessing weight. Nevertheless, its weight is no less definite, under proper limitations of demonstration, than the weight of any solid or liquid. The weight of the atmosphere may be contemplated under two conditions: firstly, as the equivalent weight of an atmospheric column of known sectional area but unknown height; secondly, as the equivalent weight of a known atmospheric volume under proper limitations, presently to be indicated. Both these investigations will now have to be considered.

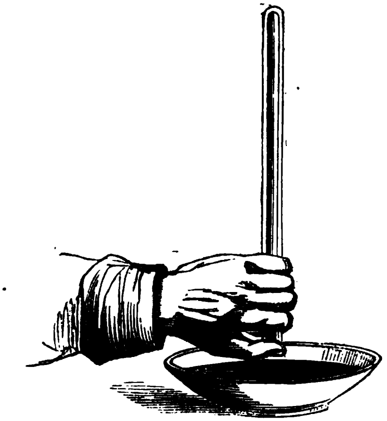


Fig. 3.

Determination of the Weight of an Atmospheric Column of known Sectional Area but unknown Elevation. The Barometer.—*Experiment* 1.—If a piece of glass tube be taken equal in bore throughout, having a length of some thirty-three or thirty-four inches, closed at one extremity and open at the other; if it be filled with mercury, then closed temporarily by the thumb, and inverted in a basin of mercury, as represented in the accompanying sketch (Fig. 3), we shall obtain a barometer of perhaps the most simple form this useful instrument can assume. We will proceed to study its philosophy. Firstly, let it be observed that though the tube *was* quite full of mercury, it does not remain quite full. No sooner is the restraining thumb removed, than a portion of the mercury sinks into the basin. If, instead of holding the inverted tube in the hand, as represented, some permanent support be devised for it; if the point corresponding with the present elevation or level of the mercury be marked on the tube; and if the tube be examined from day to day, the observer will soon find the level to fluctuate; some times it will rise, at other times fall. He will find, moreover, that the mercury will become depressed previous to stormy weather.

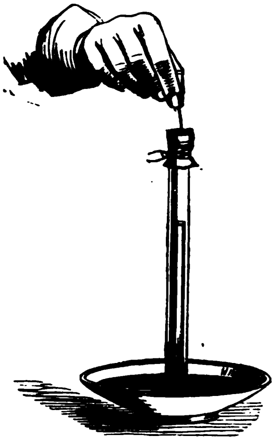


Fig. 4.

The instrument thus roughly extemporized is, in point of fact, a measurer of the weight of an atmospheric column of known sectional area (*i.e.*, the area of the interior diameter of the tube), but of unknown elevation. It is a barometer; and collaterally—inasmuch as depression of the mercurial column usually precedes stormy weather—the roughly-extemporized instrument is a weather-glass.

Demonstration.—It is desirable occasionally, when treating of natural phenomena, to violate the mathematical rule of accepting no fact until it has been demonstrated.

Thus, in the present instance, I have taken the fact for granted that the cause operating to maintain a mercurial column in the closed tube, is atmospheric pressure; and the cause to which variations of the height of that column is referable, is fluctuation of atmospheric pressure. The first proposition shall now be demonstrated, when the second will be accepted by inferential reasoning.

Experiment 2.—If, instead of a tube closed at one extremity, an open tube be taken and one end be occluded by tying securely over it two or three strong pieces of moistened bladder: if the tube thus prepared be filled with mercury, as before, and inverted in a basin of mercury, we shall be in a position to demonstrate the proposition that it is owing to atmospheric pressure—and that alone—that the mercurial column is supported. The operator has simply to prick the bladder, and let in air, when the whole column suddenly descends to the level of the mercury in the basin (Fig. 4).

There is another form of demonstration, as follows; but it is not so simple as the last, inasmuch as it requires the aid of the air-pump :—

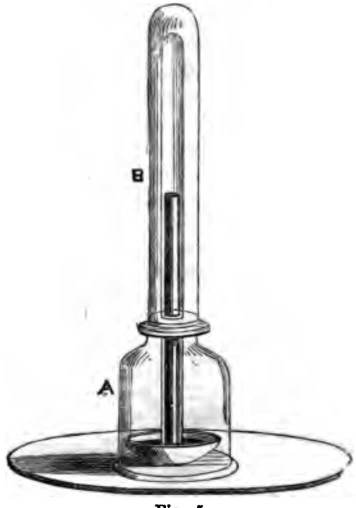


Fig. 5.

A is a glass air-pump receiver (Fig. 5), through the neck of which the tube B passes, inclosed in an exterior tube, which may be regarded as a prolongation of the receiver. C is the plate of an air-pump, on which the receiving jar is laid.

Let us assume that the barometer tube has been filled with mercury as before, plunged in the basin containing mercury underneath, the receiver A slipped over it, and finally extension made by the long glass sheath. Let us assume now that the air-pump is worked, and exhaustion gradually effected. Under these circumstances, the mercurial columns will be seen to fall, and it will fall by jerks, each jerk corresponding with a stroke of the pump-handle. Hence, by the two preceding experiments it is demonstrated that to atmospheric pressure, and atmospheric pressure alone, is the variation mercurial column due. It is also proved, inferentially, that the fluctuations of height witnessed from day to day in a barometric column are due to variations of atmospheric pressure. It appears, then, that by the barometer we actually weigh a column of atmospheric air equal in length to the whole elevation of the atmosphere, whatever that may be, and equal in area to the internal sectional area of the barometer tube. The barometer, however, gives us no information in terms of cubic dimensions of the barometric mercury. Thus, supposing the barometric tube employed to have a sectional area of one square inch, and supposing the mercurial column to be thirty inches high, then we should be correct in averring the pressure of an atmospheric column a square inch in sectional area, and extending the whole height of the atmosphere, to be equal to the weight of thirty cubic inches of mercury. Now the weight of thirty cubic inches of mercury will be about fifteen pounds; hence the atmosphere is said to exert a mean pressure of fifteen pounds on every square inch at the level of the sea.

Influence of Elevation on the Barometric Column.—It will be evident, on reflection, that the atmospheric pressure must decrease with every increment of elevation above the level of the sea. Founded on this principle, the barometer is frequently employed to determine the height of mountains; and, reversing the application, it might also be employed to determine the depth of mines and wells. At the elevation of about thirty-six miles, the pressure of the atmosphere cannot amount to more than 0·001 of an inch of the barometric column; and conversely, at a depth of about sixty-six miles, the density of the atmosphere would be about 100,000 times greater than at the surface of the earth, being six times more than the density of gold and platinum; so that, supposing either of these metals to be plunged into such an atmosphere, they would actually float.

If the atmospheric density were uniform, a barometric fall of one inch would correspond to 11,065 inches, or 992 feet of air. But it is not uniform, as we have already seen; therefore such deviation from the standard of uniformity, and indeed many other circumstances, must be taken cognizance of before we are enabled to employ the barometer as an indicator of atmospheric elevations.

The law of Marriotte teaches that the dilation of a volume of air is proportional to its density, so long as the temperature to which it is exposed is constant; whence it follows, that the density of the atmosphere diminishes from below upwards in geometrical progression. Reasoning on this basis, it appears that, assuming any particular elevation above the point of observation, its numeral exponent may be regarded as the logarithm of the density of the lowest atmospheric layer, or, in other words, of the barometric column. A consideration of the mutual relation subsisting between numbers and their logarithms will render this evident, for logarithms are nothing more than numbers increasing by arithmetical progression, corresponding to other numbers, the increase of which is also in geometrical progression. Being possessed of a table of logarithms expressing the densities of atmospheric layers, one might calculate the height of a mountain by two observations made at two stations; but the same result may be arrived at by using a common table of logarithms, and multiplying them by a constant factor. According to Deluc, the factor is 10,000.

Many other considerations have yet to be taken cognizance of before the barometer can be accurately applied as a measure of mountain elevations—they are temperature, latitude, relation subsisting between the specific gravities of air, and mercury $= \frac{1}{1046}$, and dilation of mercury for each degree of the thermometer $= 0 \cdot 0001$ for each degree of Fahrenheit's scale. All these general considerations have been embraced in tables.

Although the circumstances necessary to be taken cognizance of when employing the barometer as an indicator of mountain elevations are numerous and complicated, nevertheless the results obtained are susceptible of a considerable degree of accuracy. Subjoined is a table of comparative results between trigonometric and barometric observations. The difference, it will be seen, is only trifling.

COMPARISON OF TRIGONOMETRIC AND BAROMETRIC MEASUREMENTS OF MOUNTAIN HEIGHTS.

Observers.	Place of Observation.	Latitude.	Longitude.	Trigonometric height in feet.	Barometric height in feet.
Webb . .	Gunna Nath, Stockdale	29° 45′ 56″	79° 30′ 29″	6,828	6,831
Borda . .	Bagha Ling, Temple . .	29° 47′ 30″	80° 2′ 27″	7,646	7,635
Von Buch.	Teneriffe	28° 30′ 0″	16° 13′ 0″	12,188	....
	Ditto	....	....	....	12,131
Buckle .	Sugar-loaf, Sierra Leone	8° 29′ 40″	13° 15′ 0″	2,493	....
Sabine .	Ditto ditto .	....	....	....	2,521
Sabine .	Spitzbergen	73° 0′ 0″	10° 0′ 0″	1,644	1,640

When an approximate result is alone required, and the height is inconsiderable, a fall of $\frac{1}{10}$th of an inch may be allowed for every ninety feet of elevation, or $\frac{1}{1000}$th of an inch for every foot. This rule suffices for the small differences of elevation at which barometers are hung, and enables the observer to institute a comparison between them. Correction for temperature must, however, not be omitted. The ratio of expansion for mercury, glass, and brass—the materials employed in the manufacture of barometers—will be pointed out hereafter; meantime I may as well indicate that perhaps, after all, it is well in practice to ignore these complex elements, and to consider $\frac{3}{1000}$ths of an inch as the allowance for mercurial expansion for every degree above 32, and *vice versa*. Applying the above corrections for temperature and pressure to practice, let it be required to know the altitude at the level of the sea and at 32° Fah. of 29·565 inches of mercurial column at a place 150 feet above the level of the sea, and at temperature 55° Fah.

	Inches.
Actual height of mercurial column	29·565
Deduct for 23° of temperature above 32 $\frac{3}{1000} \times \frac{23}{1} = \frac{69}{1000}$ = . .	·369
Altitude of mercury . . . . . . = . .	29·496
Add for elevation ·001 × ·150 . . . . = . .	·150
Altitude at level of sea, at temperature 32° Fah. . . = . .	29·649

The reader will remember that the previous remarks have reference to the barometer as affected by air at rest, this being the simplest atmospheric condition which theory can assume. Hereafter we shall discover that the barometer, when influenced by the atmosphere in motion—by winds, in other words—is subject to influences from that cause.

Further Improvements of the Barometer.—The barometer in its simplest form, as already described, is a more perfect instrument than many in which simplicity of form is departed from, in deference to portability; nevertheless it is not quite correct. To be absolutely correct it is indispensable that the mercurial level in the basin should bear a constant ratio to the mercury remaining in the tube, a condition which evidently cannot be obtained in the instrument just described. In proportion as mercury descends out of the inverted tube, the level of the mercury in the basin will be elevated, and to the extent of such elevation the indications of the instrument will be prejudiced. Various means are had recourse to for lessening, or absolutely removing, this evil. It may be lessened by increasing the width of the basin to such extent that the ratio of elevation of the mercurial surface may be so greatly diminished that it will practically cease to impart errors. It may be absolutely removed by one of two devices. One consists in mounting the receiving basin on a screw, which, by elevation and depression, regulates the quicksilver to any desired level. Such is the contrivance of M. Fortin, whose construction of a barometer is here subjoined (Fig. 6); but more usual is it to attach to the barometer a long scale having a slide motion, so that the lower end or commencement of the scale may be made to coincide with the level of the mercury in the basin. Practically, however, the basin is usually dispensed with, the reservoir for mercury being a mere extension of the barometric tube, in some cases bulbed, and in others quite plain. Both these forms of construction are annexed (Figs. 7, 8).

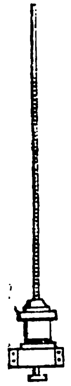


Fig. 6.

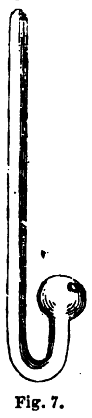


Fig. 7.

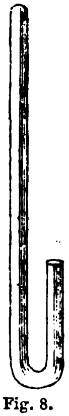


Fig. 8.

The Weather-glass.— The primary and only *direct* function of the barometer is as I have described. Its mercurial column indicates, by rising and falling, the varying weight of the superincumbent atmosphere. Very frequently this direct function is taken no account of; variation in the state of the weather being all which the observer desires to make himself acquainted with. Subserviently to this intention, all direct rise and fall of the barometric column is lost sight of, and the indications of a dial-plate with moveable hand substituted. Such an instrument is termed the dial weather-glass, the construction of which is as follows:—T (Fig. 9) is a barometer tube, W is a small float attached to one end of a cord, the other extremity of which is attached to a small weight, N. From this arrangement it will be seen that every rise and fall of the real barometric column in the long arm of the tube, will correspond with a parallel fall and rise of the mercurial column in the short arm. It will be seen, moreover, how the small float, *w*, is raised and lowered, how the pulley M will be caused to revolve, and the index-hand to traverse the dial plate of the instrument. The exterior of the wheel barometer is represented in Fig. 10.

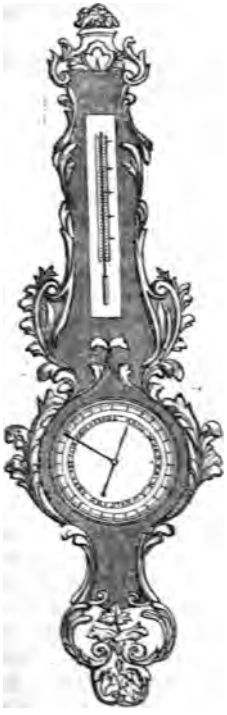


Fig. 10.

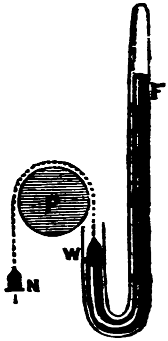


Fig. 9.

I need scarcely indicate that the wheel barometer is considered, *barometrically*, a very imperfect instrument. Not only is the varying ratio between the mercurial column and the level of the mercury in the reservoir here a necessity, the very index motion depending upon it; but the presence of the float, *w*, tends also to embarrass the free ascent and descent of the column of barometric mercury.

Manufacture of a Correct Barometer.—Not to render the principles concerned in the barometer complex, I have hitherto assumed that the act of charging a tube with mercury is simple and free from difficulties. Practically this is not so; many precautions have to be taken, otherwise the resulting barometer will be anything but correct. Firstly, the tube selected must not be too small; many instruments are rendered incorrect owing to neglect of this precaution. The internal diameter of the barometer tube should scarcely be less than a quarter of an inch; it may be even more with advantage. A small barometer tube prejudices the correctness of the instrument in two respects. Firstly, the variations of expansion and contraction of the mercury, due to variations of temperature, are more considerable; secondly, the motion of the quicksilver up and down is impeded by friction against the glass.

The Mercury must be Pure and deprived of Atmospheric Air.—Mercury, as commonly existing, is generally impure. It contains uncertain quantities of tin, lead, and sometimes zinc. Of course these admixtures damage the mercury for barometric purposes. The observer desires to read off his atmospheric pressures in terms of inches of mercury, not in terms of inches of a mercurial compound. It is indispensable, therefore, that the impurities be discharged or extracted. Various processes are used to this end, but the process usually followed by makers of barometers consists in agitating the mercury to be purified with dilute nitric acid, which gradually dissolves out the extraneous metal and leaves the mercury pure.

Far greater difficulties are encountered in discharging atmospheric air from the mercury employed. This is accomplished by boiling the mercury after it has been poured into the tube. The operation requires great delicacy, and the instrument is frequently broken in the operation.

Method of Reading off Barometric Indications correctly.—It is not possible to read off by referring to a common scale of inches and parts of inches the various small elevations and depressions of a barometric column. Two methods are had recourse to for obviating this difficulty: one is the diagonal tube, a contrivance altogether peculiar to the barometer; the other is the vernier or nonius scale, employed for the general purposes of facilitating the reading of minute scale divisions.

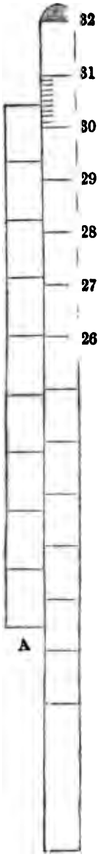


Fig. 12.

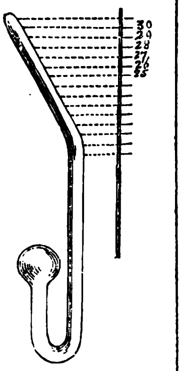


Fig. 11.

The diagonal-tubed barometer is represented by the accompanying diagram (Fig. 11).

The Nonius or Vernier Scale.—This is an ingenious contrivance for measuring small linear divisions by means of larger divisions, and consequently more easily recognizable by the eye than larger divisions would be. Thus, for example, by means of a vernier graduated in divisions of an inch and one-ninth we can read off tenths of an inch, as will be seen by reference to Fig. 12.

Let A be a scale sliding in proximity to B. Let each of the divisions on B be = one inch, and each of the divisions on A = one inch and one-ninth. From these considerations it follows that nine divisions on A are equal to ten divisions on B. Directing the eye to the upper limit of A, it will be seen that its edge corresponds to thirty inches, and *something more* on B. In this case the observer would have no difficulty in recognizing the amount over and above thirty inches on B to be equal to four-tenths of an inch. Our scale divisions are so large that a vernier scale is not required for conveying that information. But assume the tenths division between 30 and 31 to be obliterated, still we should be able to discover the overplus beyond 30 inches to be four-tenths by means of the vernier scale A, inasmuch as the number of tenths will be equal to the number of whole parts on the vernier scale A above the first line of coincidence between it and the scale B. Now the line of coincidence in question is at 26, counting upwards, from which, to the extremity, we have four divisions, which indicate a fraction of four-tenths of an inch over and above 30 inches.

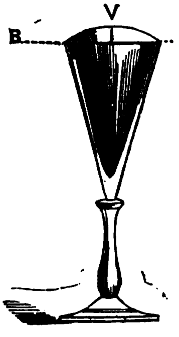


Fig. 13.

Correction of the Barometric Column for Capillarity.—If mercury be poured into a glass vessel, it will not furnish a perfectly level surface, but will be elevated, as in the accompanying diagram (Fig. 13); or, in the language of philosophy, it will constitute a meniscus; and if the line V be dropped perpendicular to the line B, joining the two corners of the meniscus, the line V will constitute, in the language of trigonometry, the versed sine of the meniscus. The convexity of the meniscoid surface of mercury will vary in proportion as the diameter of the tube or other vessel containing the mercury varies. Hence the allowance to be made for diminution of the height of a mercurial column, owing to capillarity, is determined by two considerations—the diameter of the tube, and the length of the versed sine of the corresponding meniscus.

The necessity for such allowance does not apply, as will hereafter be seen, to barometers of every form; neither is it imperative, so long as the indications of one barometer are to be compared amongst themselves; but it is indispensable if we would compare the indications of barometers with each other.

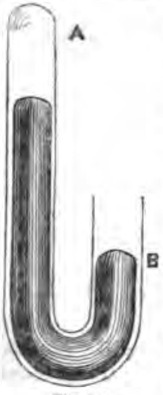


Fig. 14.

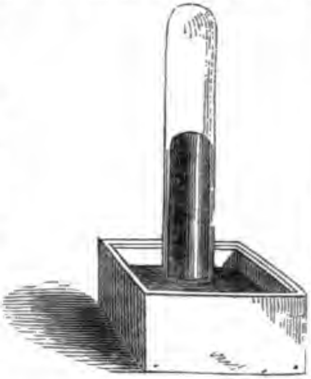


Fig. 15.

First, let us consider the cases to which the correction for capillarity does not apply. It does not apply to any barometer, the reservoir of which constitutes part of the tube itself. The accompanying diagram (Fig. 14) represents a barometer of great tubular diameter. Two meniscoid surfaces of mercury are there apparent—one at A, another at B. Now it is evident that the columnar interference at A will be exactly equal to the columnar interference at B, provided the tubular diameter be equal throughout. Hence, whether the degree of elevation be counted from the summit of the lower to the summit of the upper meniscus, or from the base line joining the two corners of the lower meniscus to the base line joining the two corners of the upper one, the two resulting columnar estimations will be strictly equal and comparable. But it is different when the observer has to do with barometers constructed on the original, or Toricellian, plan of a separate reservoir; in this case the meniscoid elevation of the mercury in the reservoir is practically ignored, being so inconsiderable that it amounts practically to nothing, as will be seen by reference to the accompanying diagram (Fig. 15).

Annexed is a small table of depressions due to capillarity. Larger and more elaborate tables have been calculated, but the one given will suffice for, perhaps, every occasion. The meteorological student cannot have the fact too strongly impressed upon him, however, that the barometer is confessedly a very imperfect instrument; therefore correct results from barometric observations are to be looked for as the mean resultant of a number of accumulated observations, rather than from any elaborate mathematical tabulations of principles, correct enough in themselves theoretically, but which do not admit of being realized in practice.

In the record of some barometrical observations, we find cognizance taken of thousandths of inches. Until the errors which attach to the principle of the barometer greatly diminish, a record of thousandth of inches cannot be otherwise regarded than as a kind of philosophic affectation.

DEPRESSION DUE TO CAPILLARY ACTION.

Diameter. Hundredths of inches.	Depression in Decimals of Inch.		
	Ivory.	Young.	Laplace.
5	·2949	·2964	....
10	·1404	·1424	·1394
15	·0865	·0880	·0854
20	·0583	·0589	·0580
25	·0409	·0404	·0412
30	·0293	·0280	·0296
35	·0212	·0196	·0216
40	·0154	·0139	·0159
45	·0112	·0100	·0117
50	·0082	·0074	·0087
60	·0043	·0045	·0046
70	·0023	....	·0024
80	·0012	....	·0013

Thermal Expansion.—I have already adverted casually to the condition of thermal expansion as an interfering cause in all estimations of true barometric columnar heights; and, by anticipation, I have already furnished an approximative means of making allowance for it. We will now proceed to examine more narrowly the function of thermal expansion, which not only intimately concerns the barometer, but is the fundamental basis of the thermometer, and besides it enters as an element into so many meteorologic calculations, that a thorough investigation of its laws cannot be omitted. I shall, therefore, embody the laws of thermal expansion in a few propositions for successive demonstration.

Heat may be regarded in the two senses of signifying temperature, or that sort of heat which is recognizable to the sense of touch, and which affects the thermometer; and heat which is devoid of these manifestations, which neither creates the sensation of warmth nor is amenable to thermometric demonstration. The former we may express by the term sensible heat, and the second by the term latent or insensible heat.

On the supposition that all the functions of heat, sensible as well as insensible, are referable to a real physical agent, the term caloric has commonly been applied as the representative of such agent; but though the term be in general use, it is perhaps objectionable—modern science leading us to infer that the functions of heat are due to a condition of matter, rather than to a separate agency. It is to evident heat, recognizable to the touch, that I shall now direct the reader's attention.

I. *Heat affects the Volume of all Bodies.*—The general effect of heat on bodies is to cause their expansion. So general is this rule, that we shall do well to consider it as universal; treating all deviations from it hereafter as so many exceptions.

II. *The Volume of all Solids is increased by increase of Heat.*—This proposition is demonstrated by so many instances commonly occurring, that specific experiments are hardly required. The wheel-wright takes advantage of this property to bind tightly together the wood-work of his carriage-wheels. He heats the annular tire, by which he expands it; he then slips the tire over and around the wood-work, and, allowing the tire to cool, the wood-work is tightly braced together by an indomitable force.

Some years since the walls of the *Conservatoire des Arts* et Metiers, at Paris, were found to be diverging from the perpendicular. They were restored to their original lines by the following beautiful expedient:—They were perforated transversely, copper bars were thrust through the perforations, each bar at either extremity being supplied with a nut and screw. Every alternate bar was now heated by means of a spirit-lamp flame; being heated the bars expanded, and the screw-nuts being now turned close up to the wall on either side, the bars were allowed to cool. By cooling they contracted, pulled the walls to some extent together, leaving the ends of the unheated bars protruding; their screw-nuts were now turned close up to the wall on either side, and the heating process repeated. Thus little by little the walls were restored to their original position.

An exemplification of the expansion of iron by heat sometimes occurs to the laundress. Occasionally she is surprised to find that the heater of her Italian iron will not enter its corresponding sheath when red hot, though it enters readily enough when cold. This is attributable, as the student will perceive, to the effect of thermal expansion on the iron (Fig. 16).

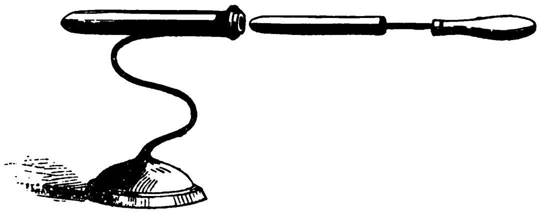


Fig. 16.

Thousands of familiar instances might be cited, all illustrative of the same property. I shall leave their consideration to the reader, concluding my remarks on this part of the subject by bringing before his notice a common lecture experiment, illustrative of solid thermal expansion. Let G (Fig. 17) be a guage, into which the metallic bar, A, accurately fits, whilst cold; it will be found that when the bar A is heated—moderate heating will suffice, such as may be accomplished by means of the flame of a spirit-lamp, or a basin of hot water—the heated bar will no longer fit into the guage. In this way the student may demonstrate the fact that each different solid possesses its own definite rate of expansion for equal degrees of temperature.

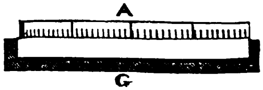


Fig. 17.

Investigations of the law regulating the thermal expansion of bodies under every condition are attended with extreme difficulty. They have been conducted by Regnault, Rudberg, and others, with great industry and much success; but, as in most cases where the investigation of natural phenomena through long ranges are concerned, the results are merely approximate. To give the reader a general notion of one of these difficulties, let it be assumed that A B C D stand for successive equal intervals of thermometric graduation. Let it be assumed that the rate of expansion of a substance from A to D be known to be equal to a quantity expressed by Y. It by no means, however, follows—nor do philosophers believe—that because the rate of expansion of the body between A and D is equal to Y, therefore the rate of expansion of the same body between A and B is equal to $\frac{Y}{3}$.

Further consideration of this matter may, however, be omitted in a treatise on meteorology. We merely want to be acquainted with approximate results of the law, in order to allow for practical discrepancies between the apparent and the actual indications of the instruments employed in the course of our researches.

To convey a general notion of this kind of knowledge to the meteorological observer, the student's attention may be directed to the fact that, supposing a barometer-scale to be made of brass, and supposing the tube to be filled as usual with mercury, then the amount of expansion of mercury by heat, and for which allowance has to be made, will be determined by the ratio $\frac{M}{B}$, if M stands for the co-efficient of expansion of mercury, and B with the co-efficient of expansion of brass. By the term co-efficient of expansion is meant the number indicating the amount of expansion peculiar to any body for given ranges of temperature.

III. *The Volume of all Liquids is increased by increase of Heat.*—Investigations prosecuted for demonstrating this law are attended with a difficulty which does not apply to the previous case. Liquids require vessels to hold them, and these vessels are themselves amenable to expansion. This difficulty has been very ingeniously avoided by MM. Petit and Dulong, who determined the expansion of liquids by a method founded upon the well-known hydrostatic principle, that the vertical heights of two fluids communicating by a horizontal tube are in inverse ratio to their densities. The accompanying apparatus (Fig. 18) was employed in their experiments. A, B, C, D is a tube bent twice at right angles, and enlarged at either extremity; the two vertical tubes are connected inferiorly by a horizontal tube of exceedingly fine bore. By virtue of the hydrostatic law just mentioned, it follows that if any liquid of homogeneous density be poured into the vertical leg of one side, it will rise to a corresponding elevation in the vertical leg of the other; and if the fluid in one vertical leg be now heated, and consequently expanded, its height will be in excess of the columnar height of the other by a definite quantity. By an easy train of mathematical reasoning, the expansion due to heat can be deduced from a consideration of the different levels and the different temperatures of the two vertical tubes. Our diagram represents each vertical tube surrounded with a cylindrical vessel. These vessels are for the purpose of commanding variations of temperature, one tube being filled with ice, whilst the other is filled with hot water.

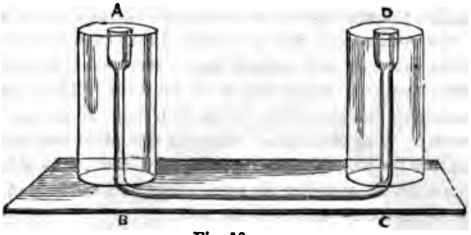


Fig. 18.

By an experiment of this kind, the co-efficient of thermal expansion of mercury from 0 to 100 of the centigrade scale is $\frac{100}{5550}$; whence, assuming its rate of expansion equal throughout, the expansion for every centigrade degree will be $\frac{1}{5550}$, or for every degree of Fahrenheit $\frac{1}{9990}$; in decimals = 0·000101; or, expressed in round numbers, one ten-thousandth part of its bulk.

IV. *The Volume of all Gases is increased by increase of Heat.*—Though the consideration of this law is not related, like the two preceding, to the *construction* of the barometer, it is intimately connected with the functions of that instrument, more especially as regards its application to the measuring of elevations; we shall do well, therefore, to consider it at once.

It has already been remarked, that the rate of expansion of all solid bodies for given increments of temperature is various; and a similar remark applies to liquids. The rate of expansion of gases was first closely investigated by the immortal Dalton, who arrived at the conclusion that all gases were equally affected by equal increments of heat, expanding to $\frac{1}{480}$th parts of their volumes at 32° Fah. for each of the 180°, between 32° Fah. and 212° Fah. As regards temperature above 212° Fah. and below 32° Fah., it was imagined by Dalton that the same law of expansion held good. Various theoretical reasons exist of a nature to create a doubt as to the large generalization of Dalton, and these doubts have been fully substantiated by M. Regnault. He finds that all gases do not dilate to the same extent between equal limits of temperature, neither is the dilatation of the same gas between the same limits independent of its primitive density. These are interesting facts: they prove that to assume one co-efficient of dilatation for all gases is obviously incorrect; nevertheless his experiments go to prove that such an universal gaseous co-efficient of dilatation may be adopted for convenience, without appreciable error, within the theoretical limits of ordinary experiment. We must not, however, continue to adopt $\frac{1}{480}$th as our working gaseous co-efficient of dilatation for every degree of Fah. between 32° and 212°, nor $\frac{1}{483}$, as subsequently adopted by MM. Petit and Dulong, but $\frac{1}{491}$.

The Atmosphere Actually or Practically Considered.—We have hitherto regarded the atmosphere as a compound or mixture of nitrogen and oxygen gases only, but the reader need not be informed that such an atmosphere is altogether theoretical. Besides nitrogen and oxygen gases, there always exists a portion of carbonic acid, also aqueous vapour, either visible or invisible. These are invariable components of the atmosphere, indispensable to its functions, adapting it to the purpose of this world's economy. The atmosphere contains also other materials, the results of local operations in nature no less than the operations of man. As the subject of our present investigations, let us consider the atmosphere as a mixture of the theoretical atmosphere *plus* aqueous moisture and carbonic acid.

Limits of the Atmosphere.—It follows, from a consideration of the laws of elasticity, that the atmosphere must vary in density for every difference of elevation. The atmospheric layer nearest to the earth must be pressed upon by the superincumbent atmosphere above it; whence the deduction follows, that when we speak of 100 cubic inches, or *any* definite measure of the atmosphere, weighing a certain number of grains, certain conditions and limitations are implied. Some of these have reference to the composition of the atmosphere chemically considered, others have reference to the atmosphere merely regarded as an elastic medium. To the latter consideration alone the reader's attention will be now directed.

Seeing that the atmosphere, in accordance with the laws of elasticity, must necessarily expand the higher we ascend above the normal level of the surface of our globe—or, in other words, the level of the sea—the first question which arises is this—To what extent do the atmospheric limits reach? does the expansion go on *ad infinitum* to the farthest realms of space, or are these limits definite? and if definite, what is the cause, or what are the causes, of limitation? Two theories have been adopted in reference to this question. According to one theory the atmosphere is illimitable; according to the other it is limited. Some of the arguments for and against I shall now proceed to give.

If the atmosphere be really illimitable, let us see what should follow, to be in accordance with recognized laws, to which all ponderable matter, or matter subject to gravitating influences, is amenable. Gravitation being directly as the mass of gravitating bodies, it should follow that, were the atmosphere illimitable, each of the heavenly bodies should be surrounded with an atmosphere proportionate to its mass—an assumption which astronomy disproves. Thus astronomy furnishes strong proof in favour of the finite extension of the atmosphere. A consideration of the laws of the atomic constitution of matter lends further, and perhaps stronger, proofs. Chemistry is full of evidence in favour of the atomic constitution of matter; or, in other words, is full of proofs that all material substances are composed of molecules or particles; to which extent they can alone be divided, and not beyond. The mathematical reasoning which has been employed against this atomic theory, as chemists term it, is specious at a first glance, but really untenable. To argue that the theoretical space occupied by any material particle may be supposed capable of division, and sub-division, *ad infinitum*, is really not to the point. Space is one thing, the matter occupying such space is another. The mathematical objection touches the space alone, not the matter; therefore the chemical evidence in favour of the atomic constitution of matter is unanswered, and is apparently unanswerable. Let us now regard the consequences of this assumption as it relates to atmospheric air. The late Dr. Wollaston was the first person who directed attention to the limitation which should theoretically be imposed on atmospheric expansion, supposing the assumption of its atomic constitution to be correct. The atmosphere, like other ponderable material bodies, is subject to gravitating influences, thus imparting a tendency of descent towards the earth. On the other hand, the atmosphere being elastic, its particles are mutually separated from each other by the operation of this force. Now, assuming the atomic constitution of the atmosphere to hold good, there must be some finite distance from the earth's surface, at which the force of elasticity would be counterbalanced by the force of gravitation, which distance would correspond with the farthest limits of the atmosphere. The mean distance of this limit is assumed to be about forty-five miles, though it must differ for every point north and south, being greatest over the equator, and least at either pole, as indicated by the accompanying diagram (Fig. 119).

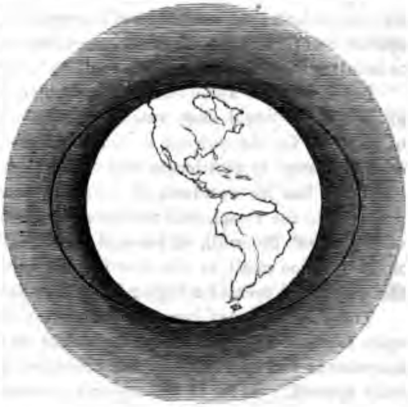


Fig. 19.

The reason of it being greatest at the equator is immediately referable to the diurnal rotation of our globe on its axis, thus generating a centrifugal force, which has determined the oblate spheroidal form of the earth. Material bodies will be affected by this centrifugal force, *cæteris paribus*, directly as their attenuation, whence it follows that the atmosphere, being a gas, must be affected to an extreme degree. It will be sufficiently evident, however, that the atmosphere is only affected by the earth's diurnal rotation intermediately, or by friction, the velocity of motion imparted to it by the earth being less considerable than the velocity of the earth itself. We shall hereafter find, when we come to treat of the trade winds, that these permanent aërial currents are not altogether referable to atmospheric motion, but in some degree depend upon the diurnal motion of the earth.

Determination of the Weight of a given Volume of Atmospheric Air.—Nothing can be more easy than the theoretical means of solving this problem; and though certain practical difficulties do interpose, we had better, for the sake of theoretical explanation, consider them absent.

The case under consideration is general, not specific. The determination of the weight of a given volume of atmospheric air is accomplished similarly to the determination of the weight of a given volume of any other gas; nor does it differ in principle from the process had recourse to when solids or fluids are concerned.

If, to take the simplest practical case, without reference to cohesive state, it were desired to ascertain the weight of a given bulk of copper or brass—say one hundred cubic inches—the operator's first care would be to obtain a solid of copper or brass having these cubic dimensions, which having been obtained, no vessel for the purpose of weighing it in would be necessary. This is the simplest case of bulk-weighing which can occur; nevertheless, a condition has to be regarded which the superficial observer might forget, or perhaps not be aware of. The copper or brass alters its dimensions for every variation of temperature. If heated, it will expand; if cooled, it will contract; so that, practically, under no two degrees of temperature has the mass of brass or copper the same size. Practically, so long as solids are concerned, these variations of size, dependent upon variations of temperature, are not of much consequence in ordinary operations of weighing. It is necessary, however, to estimate them for other reasons, and to tabulate these variations. We shall have occasion to refer to this tabulation hereafter.

Let it now be assumed that the problem before us is to determine the weight of a given bulk of liquid—say water. In this case we must have recourse to some vessel of capacity for the purpose of holding the water to be weighed, and further elements of complexity, in addition to that of temperature, are introduced. Firstly, the vessel employed has its own laws of expansion and contraction; secondly, the water, when poured into the vessel, will not have a perfectly flat surface; so that, except the mouth of the vessel be small, an error of considerable magnitude will be imparted; neither must it be too small, or the functions of capillary attraction will come into play, and the water will present a higher level than properly belongs to it.

The chief source of inaccuracy, however, which the operator meets with in operating upon solids and liquids, is that dependent on the variations in bulk referable to thermal increments and decrements; any alteration due to variations in pressure being practically ignored. So far as atmospheric pressure is concerned, which is the only kind of pressure we need take cognizance of as affecting our subject, it exercises so little influence on the dimensions of solids and liquids, that we may put it altogether out of consideration. Far different, however, is it when gases are concerned. Their attenuation and elasticity are such, that variations of atmospheric pressure exercise the most powerful influence over them; so that the degree of atmospheric pressure operating at the time of the experiment is, at least, of equal consequence with the degree of temperature.

We are now in a position to trace the theoretical steps necessary to be followed in effecting the weight of a given volume of any gas.

Necessarily, as in the previous case, a vessel of capacity is required; but, inasmuch as a gas does not admit of being poured into the vessel like water, some practical expedient for accomplishing this must be devised. We had better omit all consideration of this for the present, and assume that the vessel (which will be a globe or flask having a neck with stop-cock attached) already filled with the gas to be weighed, at a definite temperature and definite pressure. This accomplished, the operator has only to weigh his flask full of gas, deduct the weight of the flask from the total weight of flask and gas, and the result is gained. Practically, however, many points have to be considered.

The Gas must be pure.—Whether gas, or liquid, or solid, any body, the weight of which we desire to know, must be pure; but this precaution applies in the highest degree to gases.

The Gas must be either dry or its amount of moisture must be definite.—The property which gases have of taking up vapours, especially aqueous vapour, is well known; and it will readily be seen that to the extent of the presence of such vapour will the weight of a given bulk of gas and vapour mixed fluctuate. One of two processes has now to be followed: either the gas must be artificially dried by exposure to one of the hygroscopic bodies used by chemists for that purpose; or, it must be saturated with moisture to the fullest capacity at some given temperature.

The further steps of the calculation are based upon a consideration of the ratio between the specific gravity of steam or vapour, and the specific gravity of dry gas; and, lastly, the amount of vapour which a gas absorbs at a definite temperature. According to Gay Lussac, the ratio between the specific gravity of aqueous vapour and air under similar conditions of temperature and pressure is

$$\frac{0{\cdot}620}{1} = \frac{\text{vapour}}{\text{atmospheric air}}.$$

The amount of aqueous vapour which an unit volume of gas can absorb at given temperatures, has been ascertained and tabulated.

Applying this knowledge to practice, let us assume that 100 cubic inches of moist air, at 60° Fah. and 30 inches barometer, weigh 31 grains, it is required to know how much 100 cubic inches of dry air would weigh.

We begin by turning to a table indicating the quantity of vapour present in a gas saturated with vapour at any given temperature. Dalton's table gives this quantity for 60° Fah. as 0·524. We next perform the following calculation—

$$30 : 0{\cdot}524 :: 100 : 1{\cdot}747 = \text{the volume of vapour in 100 cubic inches of moist air at } 60^\circ \text{ Fah.}$$

And as 100 cubic inches of aqueous vapour weigh 19 grains, 1·747 cubic inches weigh 0·3368th of a grain.

Weight of 100 cubic inches of moist air . .	31
Deduct	0·3368
	30·6632

Therefore the weight of 100 — 1·747 = 98·253 cubic inches of dry air = 30·6632 grains, and 98·253 : 30·6632 :: 100 : 31·214 grains.

Whence it follows, according to the foregoing calculation, that the weight of 100 cubic inches of dry air at 30 inches barometer and 60° Fah. is 31·214 grains.

Such, then, are the practical operations by which the weight of a known volume of gas is determined. I have chosen atmospheric air as the subject of illustration, but the processes are identical whatever the gas may be.

Although the result of the calculation just effected gives 31·214 grains as the weight of 100 cubic inches of atmospheric air at 30 inches barometer and 60° Fah., and although the number may be accepted for all purposes of meteorologic calculation, nevertheless it must not be viewed in an implicit sense. In point of fact, the exact weight of atmospheric air is not yet made out. Probably the determinations of MM. Dumas and Boussingault are most reliable. According to their experiments 1 litre or 61·02791 cubic inches of air at 0° centigrade and 0·76 metres barometer weigh 20·065 grains; whence it follows that 100 cubic inches, under the same conditions, must weigh 31·093 grains at 60° Fah.

Barometric Pressure at the time of Experiment must be an Element of the Calculation.—A consideration of the laws of pressure, as influencing the volume of gases and vapours, will have made the student aware that due allowance requires to be made for variations referable to this cause. Now we are acquainted with amount of expansion and contraction dependent on variations of pressure. This information is conveyed by a study of the law of Marriotte, which proves that a rule of proportion will furnish the information required. Thus, for example, suppose we have 100 measures of any gas at a pressure of 29 inches of the mercurial barometric column, and it is required to ascertain what volume the gas will fill at 30 inches of the same—this being the normal pressure to which all calculations as to the volume of gases are reduced—then we say

$$\text{As } 30 : 29 :: 100 : 96{\cdot}66$$

In other words, the 100 volumes of gas under those conditions would contract into 96·66.

Temperature must be an Element of the Calculation.—When it is considered to what extent gases and vapour suffer expansion and contraction by variations of temperature, the necessity of this calculation will be obvious. I have already explained the ratio of thermal expansion to which gases and vapours are subjected by variations of temperature. *Practically*, we have seen at page 466 that this ratio may be considered identical for all of this class of bodies, and to be equal to $\frac{1}{491}$th part of their bulk at 32° Fah. and 30 inches barometer for every degree of temperature between 32° Fah. and 212° Fah. Let us now apply this information to practice.

If the temperature of the gas be above 32° Fah., multiply its total volume by 491, and divide the product by 491 *plus* the number of degrees that the temperature of the gas exceeds 32° Fah. The numeral result of this operation gives us the correct volume the gas in question would occupy at 32° Fah.

For example, we have 100 cubic inches of gas at 50° Fah.; it is required to know what volume this gas would occupy if raised to 60° Fah.: thus

$$\frac{103 \times 491}{491 + 18} = 96{\cdot}46 = \text{the volume at } 32^\circ \text{ Fah.}$$

$$\text{And} \qquad 96{\cdot}46 + \frac{96{\cdot}46 \times 28}{491} = 101{\cdot}56 = \text{the volume at } 60.$$

Recapitulation and Deductions.—It appears, then, that the operation of weighing a gas demands in all cases that due allowance should be made for variations of heat and of pressure; and if the gas be charged with vapour, due allowance has to be made for moisture also.

The Thermometer.—In the course of our preceding investigations relative to the atmosphere, we have seen that temperature is an important element. Not only are the chemical functions of the atmosphere intimately related with temperature, but without being able to take cognizance of the expansion produced by increments of heat, the meteorologic observer is unable to comprehend some of the most ordinary physical conditions of the atmosphere. We have already seen that the general effect of heat, as regards alteration of dimensions, is expansion. If the amount of this expansion be determined for any particular range between fixed points, and the linear extension or space thus intercepted be divided into smaller spaces, each of these becomes a representative of heat or temperature. Supposing we assume the temperature at which water freezes to be our starting point or first limit, and the temperature at which water boils to be our other limit; and supposing, furthermore, we assume the space between the two to be divided into any given number of parts—say for example 180—then we may describe any third body the temperature of which is between the temperature of freezing and the temperature of boiling, to be one, two, or any number of parts above the former or below the latter. Thus, by applying these principles, we should have constituted the thermometer or heat-measurer. If the rate of expansion of any one body for given increments of temperature were regular and well determined, there would be no theoretical difficulties in the way of making a thermometer; practical difficulties there would be, but the hypothetical part of the task would be sufficiently easy. It so happens, however, that the number of expansive agents capable of employment for this purpose is limited.

The earliest thermometer was that of Sanctorini. Its construction is represented in the annexed diagram. A glass stem, open at one extremity, is terminated at the other by a bulb (Fig. 20). Into the bulb and a portion of the tube is poured a coloured fluid, which being done the stem is inverted into the lower vessel. By virtue of the ordinary laws of hydrostatics, the level of the fluid in the stem will remain constant for every constant temperature; but inasmuch as every increment of heat will cause the air contained in the bulb to expand, so will it necessarily cause the coloured fluid—which latter merely serves as an index—to descend in such manner that were the ratios of successive equal linear measures of descent equal, the instrument would be a no less delicate measurer of variations of temperature than it is a delicate indicator of the same. For certain reasons, now to be described, it is not a delicate heat-measurer. Its successive lineal columnar measurements are not comparable among themselves; whence it follows that the instrument is not a thermometer or heat-measurer, but a thermoscope or heat-indicator.

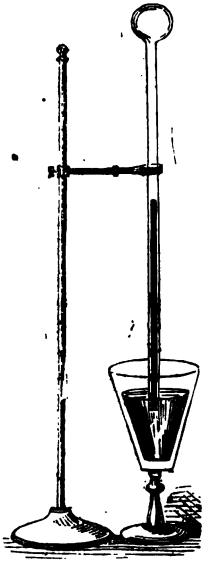


Fig. 20.

Concerning the reasons wherefore the instrument just described is not a perfect instrument, they readily admit of being made evident. They are immediately referable to the fact that the coloured fluid, which, according to the necessities of the experiments should be dynamically passive, is really active. Its activity, moreover, is a variable quantity. If the coloured fluid were merely an index having no dynamical power of its own, then the total increments of expansion and contraction of the air contained in the bulb and part of the stem, would be proportionate to the increments of heat and cold within so small a deficit of the truth (see page 458) that the error need not enter into calculation; but examination of the structure of the instrument will show wherefore this cannot be so. *Actually* the total expansion of the air in the bulb is the resultant of two forces—the force of aerial elasticity due to heat, and the force of pressure or downward tendency of the columnar liquid. Inasmuch, therefore, as the columnar height of the liquid in question varies for every temperature; and inasmuch, moreover, as the rising and falling of the liquid in the reservoir or lower receiving vessel also confuses the result, the reason will be sufficiently evident wherefore the heat-determining instrument of Sanctorini is not a correct measurer of temperature.

The Differential Thermometer.—Although various forms of air-thermometers are occasionally used in conducting certain specific experiments, their use is rare. Almost the only form of air-thermometer in frequent use is the differential thermometer, an instrument the function of which is to determine the difference between the temperature of any two adjacent bodies, or of the adjacent parts of any one body, without informing the observer concerning the actual temperature of either.

The differential thermometer is represented by the accompanying diagram (Fig 21). It consists of a glass tube having a bulbular expansion at each extremity, and joined by a stem bent on itself twice at right angles, so that the two bulbs look upwards. During the process of manufacturing this instrument, whilst one of the bulbs was yet unclosed, liquid was poured into the instrument just enough in quantity to reach a little way up the vertical part of the stems; each of these vertical parts were now supplied with a scale-division of equal lengths or degrees. Looking at this instrument, the observer will readily see that, supposing the tension or expansive force of the atmosphere in each bulb to be equal, the index fluid in the stem will stand at precisely the same elevation in both vertical stems; but supposing the air in one bulb to become heated to a higher degree than the air in the other; supposing, in other words, its expansion to be greater, the corresponding columnar height will be diminished, and necessarily the opposite columnar height will be raised. Hence the difference between the two columnar heights will be that of the temperature of the two bulbs expressed in equal parts of columnar measurement.

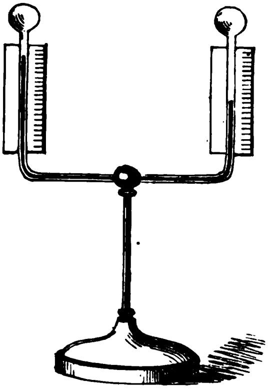


Fig. 21.

Thermometer Scales, and Ordinary Thermometers.—The liquids ordinarily employed in the manufacture of thermometers are mercury and alcohol. The former is preferred to all others, when its use is practicable, on account of the comparatively equal expansion to which it is subject for equal grades of temperature. In the manufacture of thermometers, however, intended to be employed for the measurement of temperatures below the freezing point of mercury, that fluid is necessarily inapplicable. Spirit, or alcohol, has never been frozen by the most intense cold yet produced, therefore it is substituted for mercury on such occasions. I shall now proceed to detail the successive steps in the manufacture and graduation of thermometers.

Under the head of *Barometer* the inconvenience was pointed out of using a glass tube of small diameter, because of the interference resulting from the expansion and contraction of mercury by heat. Now that which is a cause of embarrassment in the barometer, is the function on which the action of the thermometer depends. It follows, therefore, that in proportion as the bore of a thermometer tube is more small, so is the resulting instruments more delicate, because greater linear increments of expansion will be generated for given amounts of temperature. Necessarily, however, it happens in practice that if the diameter of the tube and the diameter of the mercurial column contained in the tube be smaller than certain limits, it is difficult to be seen. It is hardly necessary to indicate, moreover, that the length of linear expansion may be increased to any given limit by increasing the dimensions of the corresponding bulb, or mercurial reservoir. Practice alone can determine the proper relation which should subsist between the bore of a thermometer tube and its corresponding bulb.

The following directions for the manufacture of a thermometer are not intended to cause the meteorological student to usurp the functions of the mathematical instrument-maker; on the contrary, they are intended to make known to him the defects which he should look for in a thermometer, and which, if discovered, should cause the thermometer to be rejected.

The tube must be equal in bore throughout.—If the bore or diameter of a thermometer tube be not equal throughout, it is evident that the amount of linear expansion cannot be equal, and that the instrument will be absolutely worthless. A very easy means of guaging this equality is the following:—Having selected a piece of thermometer tube open at both ends, tie on to one extremity a rigid bottle of india-rubber, and dip the other end in mercury, thus (Fig. 22). Pressing the bottle, a portion of the atmospheric air will be expelled; then allowing the bottle to expand, some mercury will enter, forming a mercurial column, the length of which admits of being measured. Let it be measured by means of a pair of compasses; then let the mercury be driven to various parts of the tube, and the measurements repeated. If the tube be of equal bore throughout, the mercurial column will necessarily be of equal length throughout: this is evident. If the tube stand this test, it may be considered good. Instead of the india-rubber bottle the breath might be employed, but it would be attended with the disadvantage of moistening the tube. However, it is possible to use the breath without prejudice, if the operator take the precaution of blowing through some material absorptive of moisture. The next step in the manufacture of a thermometer consists in fusing one end of the tube, and blowing it into a bulb; this, again, should be effected by means of the caoutchouc bottle, lest moisture be introduced. Mercury has now to be introduced, which is accomplished as follows:—

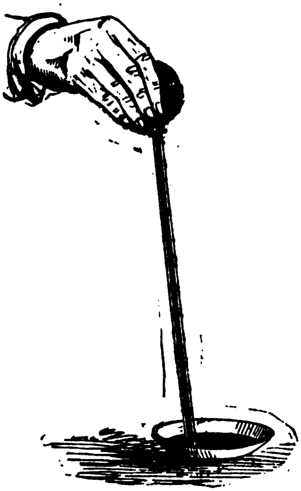


Fig. 22

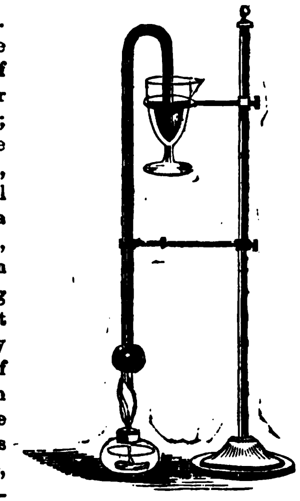


Fig. 23.

The thermometer tube having been bent as represented (Fig. 23), its open extremity is immersed in a vessel of mercury. Heat being now applied to the bulb, the air therein contained is expanded, and the heat being removed a partial vacuum results, to fill which mercury rushes in. By repeating the operation, the mercury already contained in the bulb is vapourized, and the vapour expanding drives out all the remaining atmospheric air, so that on the removal of heat the whole—tube, bulb, stem and all—becomes filled with mercury. The bent part of the tube is now broken off, and the final quantity of mercury duly apportioned to the tube. The amount of this apportionment can be only determined by practice; but, in general terms, it may be described as being such a quantity that, at the boiling point of mercury, it shall nearly extend to the extremity of the tube.

The next process is one of extreme delicacy. It has for its object the sealing or melting the open extremity of the thermometer tube, without admitting the slightest portion of atmospheric air, the presence of which would materially interfere with the delicacy of the instrument. The operation is conducted as follows:—The open end of the tube having been melted in the blow-pipe flame, is drawn out to a fine termination, thereby diminishing still further the internal bore of the tube, and rendering the final occlusion of its orifice more easy. The mercury in the bulb is now heated once more, and at the very instant when it is seen to fill the tube and to be in the act of overflowing the capillary extremity, the tube is closed by the fine jet of blow-pipe flame *b* (Fig. 24). It only now remains to graduate the thermometer.

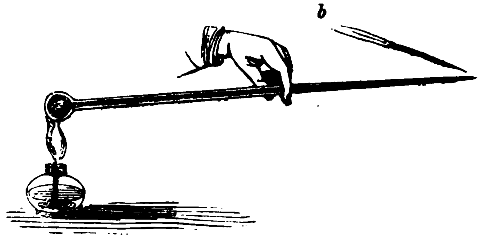


Fig. 24.

Graduation.—The instrument just described is, in its present condition, a thermoscope, or indicator of heat; but it is by no means a heat-measurer—to convert it into which it requires to be graduated into a determinate number of equal parts. These parts are usually called degrees; but it must be remembered that the so-called degrees have no ratio whatever to any particular unit. They are not like the degrees on the circumference of a circle, each one of which is related to the trigonometrical ratio, or the ratio of the circumference to the diameter. Thermometric degrees are altogether arbitrary, except in so much as certain usages in this respect obtain. This much is, however, invariable and universal—the divisional parts or degrees must be established between certain limits fixed by nature. The limits usually imported into the manufacture of thermometers are the boiling and the freezing of water, which phenomena always, under similar conditions, take place at similar respective temperatures. Founded on the bases of these limits three principal scales of graduation have been devised. They are the centigrade scale, or scale of Celsius, the scale of Reaumur, and the scale of Fahrenheit. The latter is mostly used in this country; the former is chiefly adopted on the Continent.

Let us commence our illustrations with the centigrade scale, as being most easy. The term centigrade seems to be significant of a hundred divisions. In point of fact, the centigrade scale has for its 0, or zero, the temperature at which water freezes, and for its 100 the temperature at which water boils; the intermediate space being divided into 100 equal parts. And if it be desired to carry the graduations above or below the guage limits, this is accomplished by measuring off equal parts by means of a pair of compasses.

Reaumur's scale has also its zero point at the elevation of mercurial column corresponding with the freezing point of water, but at the other extremity of its scale the boiling point of water is considered to indicate 80; hence the intermediate space is divided into 80 equal parts. The appended diagram (Fig. 25) will render evident the peculiarities of the ordinarily employed thermometric scales.

Conversion of One System of Graduation to Another.—This conversion of thermometric scales is frequently necessary. The rules for effecting the conversion are evident on reflection; nevertheless it is well to reduce these rules to general formulæ. If all these scales counted their zero from the same point, the method of converting one scheme of graduation into another would be still easier than we find it. Actually, some little confusion at first arises from the circumstance that Fahrenheit's zero is placed not at, but below the freezing point of water.

Conversion of Fahrenheit to Centigrade Degrees.—The proposition is evident that one Fahrenheit degree is equal to five-ninths of a centigrade degree, inasmuch as the number of Fahrenheit degrees between the freezing and the boiling point is to the number of centigrade degrees for the same space as unity to five-ninths, as is seen to be established by the following proportion:—

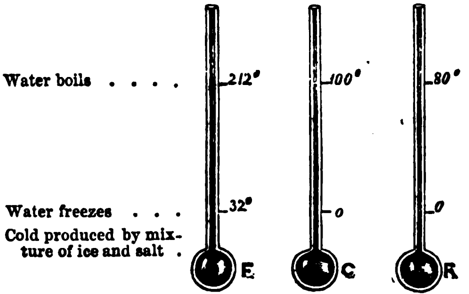


Fig. 25.

$$\begin{array}{ccccc} \text{F.} & & \text{C.} & \text{F.} & \text{C.} \\ (212 - 32) = 180 & : & 100 :: 1 & : & \frac{100}{180} = \frac{5}{9}; \end{array}$$

whence it appears that we may convert Fahrenheit degrees into their centigrade equivalents by first subtracting 32°, which leaves 180; then multiplying this number by five, and dividing by nine, as in the following proportion:—

$$212 - 32 = \frac{180 \times 5}{9} = \frac{900}{9} = 100.$$

Conversion of Centigrade to Fahrenheit Degrees.—Inasmuch as each centigrade degree is longer than a degree of Fahrenheit in the ratio of $\frac{9}{5}$ to one, therefore the former may be reduced to the latter by multiplying by nine, dividing by five, and adding thirty-two. The truth of this operation may be readily demonstrated by working on the number 100, which should give, if the rule just enumerated be correct, 212:—

$$\frac{100 \times 9}{5} = 180 + 32 = 212^\circ \text{ F.}$$

The examples just given illustrate the process of calculation when positive degrees or degrees above zero, thus (+), are concerned. Exactly the same rule has to be followed when negative degrees (—) are in question, although the rule, when stated in common terms, appears to be different, inasmuch as following the diction of arithmetic the operator must be told to subtract 32. An example will render this more evident. Suppose we require to represent 5° below zero, or — 5° of C., by its equivalent F. Now the number 32, with 9 subtracted, gives 23° for remainder. Viewing all the steps of the calculation involved algebraically, it will be found that the rule of adding 32 has been implicitly followed; a negative 5 however (— 5) yields in the following operation a negative 9 (— 9), which, being added to + 32, is equivalent to subtracting a positive 9 (+ 9). For example—

$$\frac{-5 \times 9}{5} = \frac{-45}{5} = -9 + 32 = +23.$$

The various steps for the reduction of one system of thermometric degrees to another, are comprehended in the appended formulæ:—

Fahrenheit to Centigrade, $\frac{F. - 32 \times 5}{9} = C.$

Centigrade to Fahrenheit, $\frac{C. \times 9}{5} + 32 = F.$

Fahrenheit to Reaumur, $\frac{F. - 32 \times 4}{9} = R.$

Reaumur to Fahrenheit, $\frac{R. \times 9}{4} + 32 = F.$

The Register Thermometer.—The greatest difficulty attendant upon the use of the thermometer for meteorologic observations, is referable to the necessity of frequent examination. To obviate in some measure the necessity for this, the instrument called the *Register Thermometer* has been devised—an instrument which depends for its action on the traversing of two steel bars in the bore of the tube, each pressed forward by the expansion of a liquid column. An instrument of this kind is represented in Fig. 26. The register thermometer is accurate enough in its indications for some rough purposes, but it is by no means adapted to supersede the use of thermometers of ordinary construction.

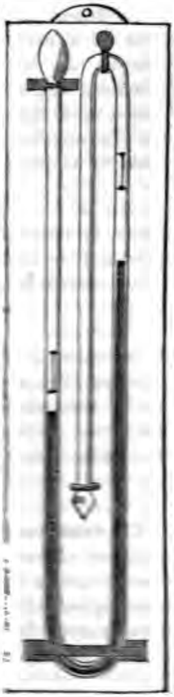


Fig. 26.

The Thermometer of Breguet.—A very delicate thermometer has been invented by M. Breguet. It differs from all which I have hitherto described, in the fact of its dispensing altogether with mercury, or other expansive liquid, and utilising the expansion or uncoiling of a compound metallic bar.

Fig. 27.

The illustration of the different rates of expansion by heat of two different metals (for instance, iron and brass) is often made by the following contrivance:—a (Fig. 27) is a compound bar of this kind, perfectly straight when cold; but if this same bar be heated, it becomes curved, as represented by Fig. 28. Breguet's thermometer is merely an amplification of the preceding experiment; instead of a straight compound bar, a compound bar twisted into the spiral form is employed. One end of the spiral is fixed, the other end is free, and is attached to an index. The mode of

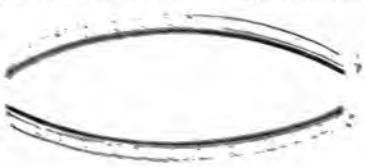


Fig. 28.

action of the instrument will be obvious. Variations of temperature producing variations of curve, will cause the spiral to unfold, or contract, according as the variations are towards the direction of increased heat or increased cold. The metals employed in making the spiral of Breguet's thermometers are platinum, gold, and silver. Experiment has demonstrated that the needle of this instrument travels over equal arcs for equal increments of temperature; hence Breguet's thermometer is not only comparable with itself, but with all other instruments on the same construction. Unfortunately this delicate instrument has no great range of application, its indications being limited between the freezing and the boiling points of water (Fig. 29).

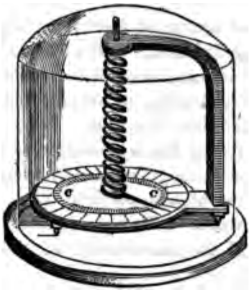


Fig. 29.

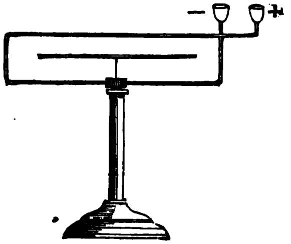


Fig. 30.

The Thermoscope of Nobili.—By far the most delicate indicator of minute increments and decrements of temperature is an instrument founded on principles totally different to any already described. The electrical thermoscope of Nobili admits of being thus described:—The action of the instrument is based upon the fundamental fact of electro-magnetism, viz., that a magnetic needle, freely suspended and placed in

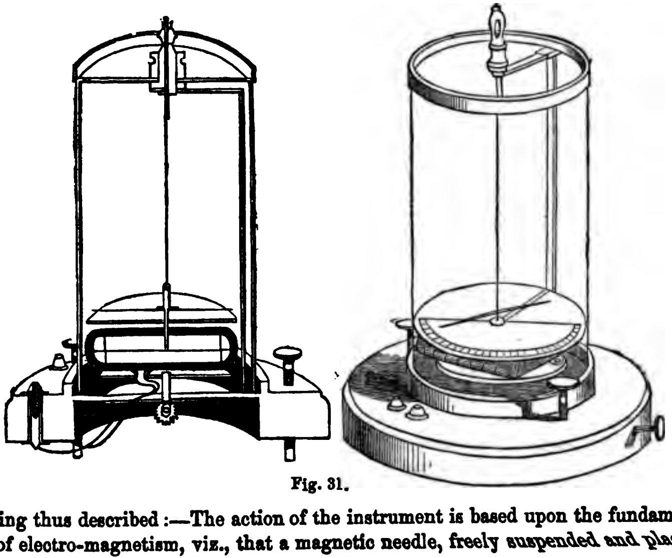


Fig. 31.

the vicinity of an electric current, finally arranges itself at right angles to that current. Hence the deflection of a magnetic needle becomes indicative of the existence of such current. Founded on the consideration of this fact, we have the instrument termed the galvanometer, which, in its simplest form, is represented by Fig. 30; and a still more delicate construction of which is represented in the woodcut on the previous page (Fig. 31).

It remains now to remark that heat is a fruitful source of electricity, especially when heat is applied to one end of a mechanical arrangement of alternate bars of two metals, such as bismuth and antimony, as represented in the accompanying diagram (Fig. 32).

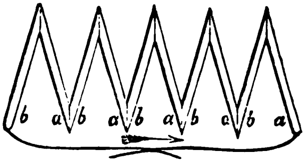


Fig. 32.

In order to render the combination of metallic bars more compact, they are usually arranged in a bundle as represented below (Fig. 33).

Very few words will now render comprehensible to the reader the structure and functions of the thermoscope of Nobili. A bundle of bismuth and antimony reduplications, as just described, being placed in communication with a galvanometer, the magnetic needle of the latter is ready to be deflected on the first occurrence of an electric current, and such electric current is a direct consequence of the application of heat to one extremity of the system of compound metallic bars. The annexed diagram (Fig. 34) represents the thermoscope of Nobili, as employed to indicate small increments of radiant heat evolved from the little lamp on the left, and transmitted through the central diaphragm.

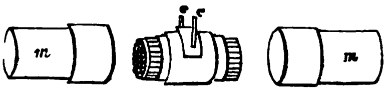


Fig. 33.

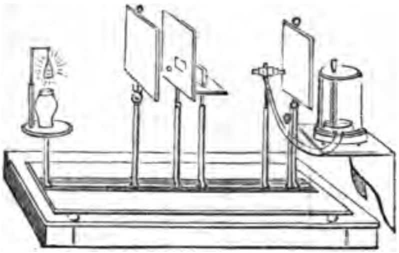


Fig. 34.

Considerations Respecting the Use of Thermometers.—Much that is incorrect passes current respecting the functions of the thermometer—the accuracy to which it is susceptible, and the spontaneous deteriorations to which the instrument is subject. A few words, therefore, on these matters may be advisable.

Let us assume the thermometer under consideration to be a mercurial thermometer; let us assume that the mercury used was absolutely pure, that the tube was proved to be of equal bore throughout, that the process of sealing was accomplished without permitting the ingress of the slightest amount of atmospheric air; let us, finally, assume that the tube has been accurately graduated—such an instrument may be regarded as free from all errors of construction.

Nevertheless, it is found that a thermometer thus unexceptionable at first is liable to deteriorate by time, the guage points—*i.e.*, 32° and 212°—corresponding with higher portions of the tube than they should do, and necessarily, all other degrees. Most probably this result is referable to a gradual contraction of the sides of the tube and of the bulb, the contraction being determined by continuous atmospheric pressure.

Looking at this contraction, the propriety is suggested of retaining thermometer tubes filled some time previous to graduation.

The Kind of Heat indicated by Thermometers.—The term heat is commonly held to be synonymous with *temperature;* but philosophy accepts it in a more extended sense, as comprehending not merely one effect (temperature), but the cause of many effects. The philosophy of latent and specific heat is almost too purely physical for extended examination here; hence a slight reference to these conditions will suffice. The thermometer is not adapted to take cognizance of heat in this latent or specific form. It is only indicative of evident heat or temperature; nor are its indications in this narrow field so complete, nor the information it conveys so extensive, as is frequently supposed. The thermometer does not even profess to indicate the *quantity* of calorific heat, but only its *degree;* terms which are totally distinct, as will soon be perceived. Let us take the following as an illustration of the difference:—A pint of boiling water is as hot as a quart of boiling water, the temperature of both being 212° Fah.; hence the thermometer, if appealed to, will indicate this identity of temperature. But, necessarily, a quart of boiling water must contain twice as much calorific heat as a pint of the same: the deduction is too obvious for comment.

Again, strictly speaking, the thermometer cannot be said to present us with the *correct* temperature of anything, inasmuch as the degree of columnar expansion is not the degree corresponding with that of the thing with which the barometer is brought into contact, but the mean of the thing touched, and the bulb which touches it. This objection attains its minimum when the atmosphere itself is the medium, the temperature of which we desire to investigate; but it becomes of practical importance in al other cases, and its consideration teaches the thermometric observer the necessity for employing instruments the bulbs of which are as small as compatible with well-marked amounts of columnar expansion. This remark especially applies to cases in which the bulk of liquid or solid operated upon is small. As concerns the graduation of thermometers, the most correct instruments are those of which the graduations are effected on the tube of the glass itself. If the graduation be made on a scale of brass, the resulting indications will be very incorrect, except the relative expansibility of brass and glass be taken into consideration, and duly allowed for; this, however, is so troublesome that it will be too frequently evaded, and errors will creep in. Far better than brass is box-wood; and slate and ivory are better still.

Correspondence between Thermometers.—Scarcely any two thermometers exactly correspond, even though the same materials be employed in their construction, so numerous are the points which require attention. Between thermometers constructed with different materials, between air thermometers and mercurial thermometers, for instance, or either of these, and spirit thermometers, the discrepancies are still more considerable.

The experiments of M. Regnault relative to the discrepancies subsisting between mercurial and air thermometers are amongst the latest on this important subject. He found that between 32° and 212° Fahrenheit, there is an almost absolute coincidence between the degrees of the air and the mercurial thermometer. From 212° to 482° Fahrenheit, the mercurial and air thermometers remain pretty equal; but after the latter point the mercurial gains on the air thermometers.

Hitherto I have treated of the atmosphere, statically considered, in a condition of repose; but perfect atmospheric quiescence is unknown in nature—it is always agitated more or less; hence we are led to the consideration of atmospheric currents or winds.

So variable are winds in these northern latitudes, that their incertitude has passed into a proverb; primary atmospheric currents, nevertheless, are constant in their direction, and are referable to variations of temperature simultaneously existing in different parts of the world.

Heat in its non-latent condition, or, in other words, that condition of heat which is recognizable by the thermometer, has a tendency to equalize itself. Hence, if two bodies *a* and *b*, of which *a* is hotter than *b*, be situated in proximity to each other, there is an immediate tendency to equalization of temperature as between the two; but the thermal conditions which regulate the production of winds will be most readily appreciated by reflecting on what takes place when a heated solid is suspended in the atmosphere by a small chain or wire; we shall find, on investigation, that a heated solid thus circumstanced gradually becomes cool by the operation of three distributive influences—*conduction*, *radiation*, and *convection*.

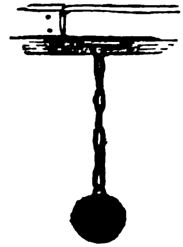


Fig. 35.

Conduction.—If the chain by which the heated cannon-ball is represented to be suspended in the annexed diagram (Fig. 35), be examined from time to time by the thermometer, or even by the fingers, it will be found to increase in heat evidently because it removes a portion of temperature from the heated cannon-ball by conducting it away. This function of heat is so well understood and so commonly exemplified, that no further consideration of it will be necessary here.

Radiation.—If a thermometer be held even at the distance of some feet from the heated cannon-ball, the mercurial column will be sensibly affected, thus demonstrating the transmission of heat. But how transmitted? By contact with the atmosphere? Clearly not, inasmuch as the result just indicated still occurs if the cannon-ball be placed in the vacuum of an air-pump. By experiment it has been determined that the temperature, in the case under consideration, has been given off in the condition of rays, precisely as light is evolved, and hence the propriety of the term radiant heat.

Convection.—Independently of the two processes of heat-distribution already described, there is yet a third. Referring to the suspended hot cannon-ball, we shall find that, if suspended in a room from the ceiling, the air near the ceiling becomes hotter than the air below: hence a portion of the temperature of the hot cannon-ball must have become accumulated there, by reason of some cause besides those of conduction and radiation, both of which distribute the temperature equally in all directions, through a homogeneous medium—such as the atmosphere for example, in our assumed experiment. The process of heat-distribution known as convection, is the necessary result of the expansion of liquids and fluids by heat; for when expanded they are specifically lighter; and, when specifically lighter, they must necessarily ascend. It is by virtue of the process of heat-distribution, termed convection, that the child's soap-blown bubbles ascend instead of at once falling to the surface of the earth.

Temperature and pressure being equal, breath evolved from the lungs is heavier than atmospheric air, because it holds more carbonic acid. Nevertheless, inasmuch as it is evolved from the lungs hotter than the surrounding atmosphere, soap-bubbles blown with it ascend. Presently, however, they descend, because the heat acquired from the lungs being evolved, and equalization of temperature with that of the surrounding atmosphere having ensued, the great specific gravity of the gas wherewith the bubble is blown causes the latter to sink to the surface of the earth.

It is to the process of heat convection, that we owe the salutary draughts in our chimneys, and also unsalutary draughts in our apartments. No sooner is fuel lighted in a fire-place, than the superincumbent air becomes higher and specifically lighter; it therefore ascends, and cold air rushes in to fill its place. Thus we have, in point of fact, a local wind; and the causes which determine that wind are exactly comparable to the causes of winds which take place in the grand economy of nature, as will soon be rendered manifest.

The Trade Winds.—Applying the facts just developed, let us now regard the surface of our globe in the aggregate, with reference to the localities of maximum and minimum temperature, and the consequence of such difference of temperature in originating an aerial current. It is evident that the hottest portions of our globe's surface are comprehended within the tropics, and the coldest portions are the arctic regions—north and south.

These circumstances being premised, we are now in a condition to anticipate the direction of the aerial currents or winds which must necessarily ensue. Firstly, an ordinary current of heated air should rise aloft in the tropical regions, then diverge and pass north and south to either arctic circle, thus constituting what may be termed the upper trade current. This current, as it proceeds north and south, gradually becomes cold, in which condition it is rendered specifically heavier, falls to the surface of the earth, and floats along towards the equator, thus generating two principal currents—one north, the other south. Whilst yet in the frigid and the temperate zones these primary currents encounter so many interferences that their primary or fundamental direction is masked or veiled; but still proceeding north and south, the persistent directive tendency of the trade winds is at length developed. But the currents no longer flow directly north and south. By the operation of a cause which will presently be rendered evident, the directive tendency of either current has acquired a certain impulse towards the west; or, in other words, both currents come more or less from the east. But at length they blow almost from due east, and finally cease altogether; so that the equator, and a certain space north and south of the equator, are comprehended within what is termed the region of calms. The north trade wind meeting the south trade wind, they are mutually destructive of each other. Two conflicting aerial forces by mutual impact come to rest, just as two billiard balls, each rolling gently from an opposite point, become quiescent.

No part of the earth's surface, however, is subject to such capricious and such violent tempests as the so-called region of calms. This is a result which theory would lead us to suspect. The cause has now to be explained wherefore the trade winds do not blow directly north and south. If our globe were at rest, or if its only motion were motion in its orbit, such would be the result; but it revolves on its axis also from west to east, and this circumstance fully explains the deviation from northness and southness of the trade winds. If the lower or returning aerial current which constitutes the trade winds ceased to exist altogether, then our globe's diurnal rotation would generate a current in the apparent direction of east to west. I say *apparent direction*, because, in point of fact, we may regard the atmosphere under these circumstances as being tranquil or passive, and our globe revolving in the midst of it, in the direction of west to east. Inasmuch as the force representing this apparent east wind, and the force representing the north and south atmospheric currents, flowing towards the equator from either pole, are simultaneously operating, there occurs a resultant, which is the trade wind. The appended diagram (Fig. 36) roughly illustrates the points which have been described. Towards the extreme north and extreme south the great aerial currents, ultimately destined to become the trade winds, are represented fluctuating and variable; gradually, however, they acquire a northern and a southern directive tendency respectively; lastly, they come from the point of almost due east, and then cease altogether.

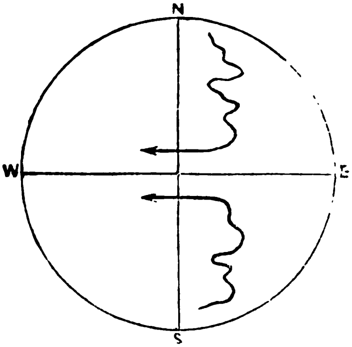


Fig. 36.

A consideration of the influences which determine trade winds leads to a facile explanation of many aerial currents of less extent. The heating influence referable to the geographical position of the tropics is not the only influence of this kind. A peculiar condition of the earth's surface, taken cónjointly with a favourable condition of the sun's rays, may bring about similar results. In this manner the Mediterranean etesian winds, or aerial currents from the north, may be accounted for. Looking at the geographical condition of localities south of the Mediterranean, we find the Sahara, or Great Desert, a region whose surface is strewed with masses of pebbles and sand, and almost totally devoid of vegetation. Such a surface must necessarily become elevated by the solar rays to a high temperature, an atmospheric column must ascend, travel to the north, then fall, and at length return from the north to the Sahara, whence it came.

Land and Sea Breezes.—Many countries near the sea are subjected to winds of diurnal periodicity, known as land and sea breezes. About eight or nine A.M., an aerial current begins to flow from the sea towards the land, and persists until about three P.M., when a current in the reverse direction, or from the land towards the sea, takes its place, and continues throughout the night until sunrise next morning, when it ceases, and a calm ensues until the completion of a period of twenty-four hours from the occurrence of the preceding land breeze. These currents, in reverse directions, can be easily accounted for when we consider the heating agency of the sun. Necessarily land becomes hotter than water under an equal power of luminous rays; whence it follows, that the surface of the ground becoming heated after sunrise, determines the ascent of an atmospheric current vertically; thence, proceeding oceanward, the same current returns from the sea to the land. No sooner does the sun set, than this current is reversed.

In the preceding explanations of the cause of winds as being referable to inequalities of temperature, it will be observed that reference has alone been made to the calorific effects of the sun's rays. This is, in point of fact, the only source of heat which has to be taken cognizance of in meteoric considerations; for, although the earth's own temperature gradually increases as we pierce downwards below the surface, so imperfect are the conducting powers of the materials of which the crust of our planet is composed, that all consideration of them may be safely omitted in accounting for the present phenomena.

Process of Reverse Atmospheric Currents.—Although the existence of atmospheric currents proceeding in a direction reverse to those we meet with on the earth's surface is forced upon the mind by inferential reasoning, and the fact must be accepted, even though no further evidence of it could be adduced, nevertheless direct proofs are not wanting. The direction of upper layers of clouds afford their testimony to the truth of the opinion. Where the trade winds prevail, the higher strata of clouds may be seen taking a direction opposite to that of the wind itself; and travellers, during their ascent of high mountains, have frequently proved the existence of a superior wind pursuing a course opposite to the wind below. This has been especially obvious on the Peak of Teneriffe, a mountain situated in the belt of the trade winds. Here, on the summit of this peak, it has been always found that a south-west wind prevails, whereas the trade wind at the base of the same mountain blows from the north-east.

A similar remark has been made by travellers who have ascended Mona Kea, in Owyhee, the height of which is 18,000 feet. But perhaps the most striking illustration is the following:—The island of Barbadoes lies eastward of St. Vincent, and between the two the trade wind continually blows, and so forcibly that it is only with difficulty, and by making a long circuit, that a ship can sail between the latter and the former. Nevertheless, on one occasion, during an eruption at St. Vincent, dense clouds formed over Barbadoes, and large quantities of ashes fell on the island. A similar result was observed after an eruption of the volcano of Cosoguina, on the shores of the Pacific, in Guatemala, in January, 1835, some of the volcanic ashes falling in Jamaica, more than 800 miles in a direct line distant, and directly opposed to the prevailing lower current. At the same time another portion of ashes was carried westward, or in an opposite direction, falling on Her Majesty's ship "Conway," in the Pacific, more than 1200 miles distant.

The trade winds are, for the most part, only recognizable at sea; the solid material of land developing local aerial currents of their own. The extent of prevalence of the trade wind is various. In the Atlantic it prevails from 8° to 28° or 30°, but in the Pacific only to 25° N. L. In the southern hemisphere, the extent of the trade wind has been less accurately determined. When first the phenomena of trade winds were noticed by Columbus and his associates, they caused the greatest consternation. Accustomed to the fluctuating and irregular breezes of Europe, they regarded the continuance of a wind from the east as emblematic of their perpetual banishment from their native shores. The early Spanish navigators, however, very soon learned to appreciate the value of trade winds, by the aid of which treasure-laden galleons could, setting out from Acupulco, manage to arrive at Manilla almost without changing a sail.

As respects the upper current proceeding from the equator to either pole, it varies, as might be anticipated, in different localities. Travellers, who have ascended the Peak of Teneriffe, inform us that this upper current is found in that locality at an elevation of 9000 feet; but Humboldt, during his explorations on the Andes, discovered the eastern trade wind to be blowing at an elevation of 8000 feet above the level of the sea. As the upper, or equatorial current loses its heat, its specific gravity becomes greater, and it sinks lower and lower, no longer manifesting any well-marked directive tendency.

On the ocean, and between 30° and 40°, there is a prevalence of westerly winds, especially in the southern hemisphere. In the Atlantic a similar tendency is manifest; whence it follows, that the voyage from Europe to America occupies more time than a voyage in the reverse direction. It is difficult to say what may be regarded as the prevailing wind in these isles—probably, however, a south-western wind, as stated in the following table:—

Table Representing the Relative Prevalence of Winds in different Countries for a Period of 1000 days.

Countries.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.
England	82	111	99	81	111	225	171	120
France	126	140	84	76	117	192	155	110
Germany	84	98	119	87	97	185	198	131
Denmark	65	98	100	129	92	198	161	156
Sweden	102	104	80	110	128	210	159	106
Russia	99	191	81	130	98	143	166	192
North America	96	116	49	108	123	197	101	201

The direction of winds is found, taking the average of many years, to vary according to the season. In Europe south winds are more prevalent than any others during winter; east winds belong more especially to the spring; west and north winds to the summer, and towards October the wind usually veers round to the south. Usually the wind is more strong in February and March than at any other, and at all seasons the wind is usually strongest at noon.

Storms.—Whenever the air, from any cause, is thrown into violent commotion, the result will be a storm. The philosophy of storms, notwithstanding the attention which has been devoted to the subject, is by no means well understood. In point of fact, the causes of storms are numerous and complex. If we reflect on the agency of temperature on the air, one prevalent cause of storms will at least become manifest. If one portion of the atmosphere be suddenly heated, violent commotion must arise—there must follow a storm. The laws of latent heat demonstrate that whenever water is suddenly condensed, the surrounding air must be raised in temperature; and thus we have one of the most frequent local causes of storms. According to modern observations storms are, for the most part, circular whirlwinds progressing in a north-eastern direction from the south to the north of the tropic of Cancer. In proportion as a locality is devoid of mountains and near the sea, so is it more liable to be subject to storms. Perhaps the most violent of all European storms are those which occur in the south of France during the prevalence of the north-east wind termed *mistral;* but the most violent storms occur in and near the tropics, and are termed *tornadoes, trovadoes, hurricanes, typhoons,* &c. Hurricanes are essentially tropical; the West Indies suffer from them more than any other region. Hurricanes are of yearly occurrence in the West Indies, but the islands of Trinidad and Tobago, being protected by mountain elevations, usually escape them altogether. The wind, during a hurricane, frequently makes an entire circuit, blowing from every point of the compass; and it is by no means an unusual occurrence for the wind to cease awhile altogether, and then commence blowing again. Perhaps the most violent hurricane on record is the one which occurred in 1780. It destroyed the fleet of Lord Rodney, and a vast number of merchant ships. It killed no less than 9,000 individuals in Martinique alone, and 6,000 in St. Lucia. It totally destroyed the town of St. Pierre in Martinique, and almost as completely the town of Kingston in St. Vincent, only fourteen houses of the latter being left unmolested. Not a few of the West Indian hurricanes extend their ravages northward to the United States; usually, however, with a violence greatly diminished.

The eastern, western, and southern coasts of Africa are also subject to storms of almost equal violence with the West Indian hurricanes; these storms, however, in the localities under consideration are called tornadoes. At Sierra Leone and the adjacent parts two or three tornadoes usually usher in the dry season; they are sometimes accompanied with rain, and sometimes without; when of the latter kind, they are called white tornadoes. The eastern coast of Africa, especially towards the south, is also very subject to tornadoes. One of the most violent storms on record occurred in the Mozambique Channel in 1809, extending to the islands of Bourbon, Mauritius, and Rodrigues. The violence of storms at the Cape of Good Hope is proverbial; the early Portuguese navigators, therefore, called the southern cape of Africa the Cape of Storms.

Typhoons may be described as hurricanes of the Chinese and Japanese seas; like hurricanes, they have a rotatory motion, but they are more localized in their action, having no distinct rectilineal progression.

Hot Winds.—Although heat may be regarded as primarily the cause of all winds, it does not follow that all winds must be hot; indeed, we know that the result is the direct opposite—that many winds are very cold. The temperature of a wind is for the most part totally independent of the temperature which caused it, and is determined by the nature of the surface over which it blows. The principal hot winds are those denominated the simoom, the harmattan, the chamsin, the sirocco, and the solano. The term simoom, or *samiel*, means poisonous, and is derived from a belief of the Arabs that the devastating effects of this wind are attributable to some poisonous emanation which it bears. There is no foundation, however, for this notion. The terms chamsin and harmattan are little else than Egyptian and negro appellations respectively for the simoom: The Egyptian term *chamsin* means fifty, and has reference to the duration of the wind fifty days—from April 27 to June 18. The simoom is the terror of desert caravans. At its approach the horizon grows dark, the sun's rays scarce penetrate with lurid gleam the atmosphere charged with particles of burning sand. The wind blows with fitful violence, scattering death and desolation in its track, withering the trees and shrubs which it encounters, suffocating animals, and burying them under waves of sand. The camels no sooner perceive the advent of the simoom, than rushing to the nearest tree or bush, or seeking the spur of some projecting rock, they place their heads in the direction opposite to which the wind blows, and endeavour to screen themselves from its violence. The traveller throws himself on the ground on the lee-side of the camel, and screens his head from the fiery blast within the folds of his robe. Too frequently all these precautions are unavailing, both man and beast falling a prey to the terrible simoom. The idea, however, of the wind being poisonous is not founded on fact. In the western parts of Asia, more especially in Arabia, the simoom only blows in the summer months, and with maximum violence in July. It occurs only in the day time, and for the most part only lasts a few hours. In Lower Egypt, the direction of the simoon is from the south-west; in Mecca, it comes from the east; in Surat, from the north; in Bassora, from the north-west; in Bagdad, from the west; and in Syria, from the south-east; in every case proceeding from the neighbouring desert where the air has suffered rarefaction. The simoom, far from being poisonous, is in some localities beneficial to health, by drying up aqueous exhalations, which, if not removed, would give rise to fevers and other diseases. This is particularly the case on the western coast of Africa. It is, nevertheless, always injurious to vegetation.

The Italian sirocco and the solano of Spain may be regarded as European continuations of the harmattan or simoon of Western Africa. The sirocco, although usually restricted to Malta, Sicily, and southern Italy, sometimes extends into Germany and Switzerland: in the latter locality it is denominated the *föhn*. The föhn, although prejudicial to trees, develops to a surprising degree the vegetation of young plants, and can hardly be regarded as a calamity. It is most prevalent in Switzerland, near the Lake of the Four Cantons. Its period of duration does not usually exceed a few hours, though sometimes this period is exceeded, and it rarely occurs in winter.

The southern part of Australia is subject to a hot north wind, presenting such a marked resemblance to the sirocco that geographers are led to the natural inference that the unknown interior of the Australian continent is a desert of sand and rock, like the Sahara and the wilds of Arabia Petræa.

Cold Winds.—These winds are less noticeable and fewer in number than those already mentioned. Their low temperature is usually referable to the circumstance of their passing over mountain ranges covered with snow. The most considerable winds of this kind exist in Mongolia, Beloochistan, and the Russian steppes. To this class also belongs the *mistral*, a north-east wind prevalent in southern France, and which is exceedingly prejudicial to vegetable life.

Whirlwinds.—When two violent winds meet, the result is a whirlwind, so called from its rotatory character. If a whirlwind occurs at sea, or over water, it elevates a large column of water aloft, sometimes to the height of many hundred feet, thus giving rise to the meteoric phenomenon termed a *water-spout*. If a whirlwind occurs on land it lifts up dust, boughs, the roofs of houses, and other solid matters, producing a column of well-defined shape. These whirlwind columns, whether they consist of water or solids, present the same general formation and contour. They consist of a hollow cone, sometimes straight, but more frequently curved or horn-shaped, its upper portion proceeding from a cloud; its lower part consisting of an aggregation of water or of sand and dust, according to the locality. The upper and lower portions of these columns are so much denser than the remainder, that they are generally opaque, whereas the middle portion is generally transparent. The tint of these colours is various — sometimes gray, sometimes brown or nearly black, and occasionally fiery red.

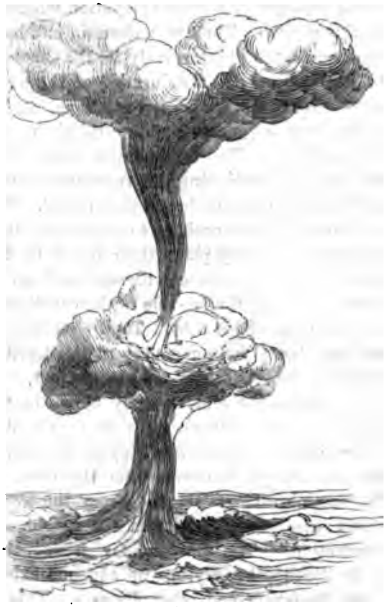


Fig. 37.

Independent of the circular or axial motion of these whirlwind columns, they pursue an onward course, sometimes straight, at other times curved. The velocity of this course differs within wide limits. Sometimes a man on foot can readily keep pace with it, whilst at other times they proceed at the rate of nine or ten miles an hour, sometimes more.

Whirlwind columns, whether they eventually become water-spouts or not, always originate on land, or in the vicinity of land where the winds and temperature are mutable. They are usually attended with thunder, lightning, and other electrical phenomena; and they constitute the centre of an aerial commotion, all around the focus of which a profound calm prevails. Bodies which they have taken up are not readily deposited, but carried along in their onward course. Sometimes they are quite in the clouds, at other times on the surface of the earth or water, and their formation may be prevented. Even when already formed, they may frequently be destroyed by some violent aerial commotion, such as that produced by the discharge of a piece of ordnance—a fact well known to seafaring men. The size and height of these whirlwind currents is various; occasionally they present a diameter of no more than two feet, while the diameter of some has been estimated at two hundred, or even more. Again, the height of some is no more than thirty feet, whereas others have been known the height of which was no less than three thousand feet.

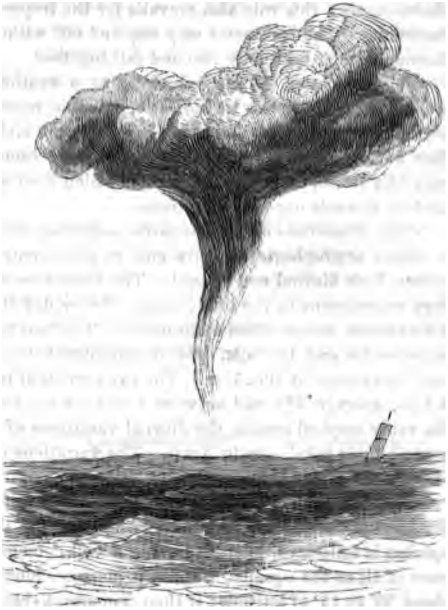


Fig. 38.

Water-Spouts.—Of these columnar whirls the water-spout is less damaging than the dry whirl, probably because the weight of fluid which it carries diminishes the violence of its rotatory motion. Not the least extraordinary amongst the many curious circumstances relative to water-spouts, is the well-attested fact that, although occurring at sea, they have been occasionally known to break and deluge a ship with a torrent of fresh water.

Influence of Wind on the Barometer.—Although the barometer has hitherto been considered in reference only to the pressure of a tranquil column of air, its variations are influenced by many other circumstances, which we must not omit to consider. Amongst the most important of these are winds. Having regard to the ultimate cause of winds, it will be evident that the existence of a wind bespeaks the condition of different temperature in two different places. Hence, every wind necessarily varies to some degree the temperature which would have subsisted at any given place under a perfectly tranquil atmosphere. Now, inasmuch as the atmosphere expands by heat and contracts by cold, varying to a corresponding degree its density or specific gravity, so it follows that the height of the barometric column will be influenced by winds. In Europe it will be generally found that a fall of the barometer corresponds with a rise in the thermometer; this rule also prevails for the tropics, nevertheless it is subject to many variations. The barometer may rise and fall without any corresponding change in the thermometer or both may rise and fall together.

The application of the barometer as a weather-glass is altogether collateral and secondary, nevertheless its indications in this respect are, for the most part, worthy of confidence; generally the barometric column sinks the day before rain occurs, and rises during its prevalence. The barometric column is much agitated during the existence of a storm, owing to the conflict which then ensues between atmospheric currents tending towards opposite directions.

Other Variations of the Barometer.—Besides the elevation of barometric mercury due to direct atmospheric pressure and to aerial currents, there exists other fluctuating causes, both diurnal and annual. The former are scarcely noticeable in temperate, but very conspicuous in the torrid zone. Every day the barometric column twice attains a maximum, and as often a minimum. The two periods of maximum elevation occur between 8½ and 10½ A.M. (say an average of 9h. 37m.); and between 9 and 11 P.M. (say an average of 10h. 11m.) The two periods of minimum elevation are between 3 and 5 A.M., average 3¾; and between 3 and 5 P.M., average 4h. 5m. During winter and the rainy tropical season, the diurnal variations of the barometer are least, and they assume their maximum in April. The variations are much less on elevated mountains than in the plains below.

Mean Barometric Condition of a Place.—It was formerly assumed that everywhere at the level of the sea the barometric condition for the same time was identical. This opinion is fallacious, latitude having a well-determined influence in this respect. It is least of all at the equator, whence it increases north and south, attaining its maximum about 30° or 43° of latitude; it then decreases to between 60° and° 70. Within the polar circle it would appear to reascend, but further experiments for this locality are a *desideratum.*

Longitude also appears to exert some influence over the elevation of the barometric current. It is greater in the Atlantic than in the Pacific, by a small but readily perceptible quantity.

Causes of Periodical Barometric Variations.—Various opinions have been advanced to account for these periodical barometric variations. To say they are attributable to difference of temperature is to advance a cause too remote from the result. Many philosophers have attributed these variations to the existence of veritable atmospheric tides; but the most plausible explanation of diurnal barometric variations would seem to be that of Dove, who assumes them to depend upon the varying amount of aqueous vapour. Aqueous vapour and atmospheric air are possessed of different specific gravities, and the barometric height of a column of mercury for any time will be the sum of pressure of dry atmospheric air and associated moisture; as the relative amount of the two varies, so will vary the height of the barometric column.

Atmospheric Moisture and its Derivatives.—When treating at page 469 of the means to be employed for weighing a gas, the facility wherewith gaseous bodies absorb moisture was adverted to. Some idea then may be gained of the amount of moisture present in the atmosphere, seeing that the latter is ever in contact with large expanses of water. The atmosphere, in point of fact, is never dry, or in any way near dryness. Even when the air seems parching hot, drying the skin and withering vegetables, it is easy to demonstrate, by the aid of chemical agents, the existence of aqueous moisture; without the presence of which neither the functions of animal or of vegetable life could be maintained. Even when the air approaches the condition of dryness, within very remote limits, breathing is difficult and symptoms of feverish restlessness speedily sets in. The natural craving of the lungs for moisture is demonstrated by the presence of a close stove in a small room. The sensations, which are very unpleasant, can always be alleviated by placing a small dish of water on the stove, so that evaporation may go on continuously. It is of the utmost importance, then, to be enabled not only to demonstrate the existence of atmospheric moisture, but to determine its quantity. A few experiments for effecting this demonstration I shall now detail.

Experiment 1.—The accompanying diagram (Fig. 39) represents a balance or pair of scales, into one pair of which there has been placed a small dish of oil of vitriol, and into the other a counterpoise. If the apparatus be exposed to the air, even when the earth is hottest and dryest, nevertheless the equilibrium of the pair of scales will soon be destroyed. Some ponderable increase will have been acquired by the pan containing the oil of vitriol, and analogy demonstrates the increase in question to be due to the absorption of water. Founded on this property, oil of vitriol is frequently employed by the chemist for desiccating substances which could not be heated without damage. Accordingly, if a pan of oil of vitriol and a moistened sheet of paper be enclosed together, under an inverted glass, the paper will in course of time become dry. Far more rapid and powerful is the operation of the oil of vitriol when, instead of being placed together with the substance to be dried, and in a mere bell-glass, the two are placed under the receiver of an air-pump, and the air exhausted. Under these circumstances an atmosphere of aqueous vapour alone soon fills the air-pump receiver, and the absorptive operation of the oil of vitriol being continuous, the water is speedily evaporated.

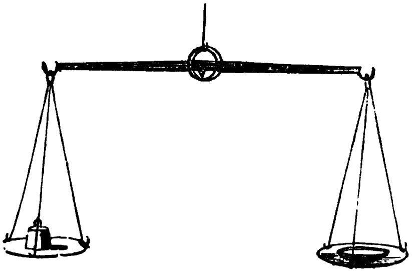


Fig. 39.

Experiment 2.—Instead of oil of vitriol, carbonate of soda, or chloride of calcium, and, in an inferior degree, common salt (chloride of sodium) may be used; for these bodies are all hygrometric—that is to say, they have the property of absorbing water from the air. Of the three bodies mentioned, chloride of calcium is the most hygrometric, and is of constant application by the chemist. Founded on the hygrometric quality of common salt, and other saline materials contained in sea water, is the property which certain sea weeds have of becoming moist in damp weather, and of indicating by their dry crispness an opposite atmospheric condition.

Although aqueous vapour be always present in the atmosphere, it is not always visible. Frequently it is quite transparent, and only demonstrable by the process of getting it out; but at other times it aggregates, becoming vesicular, and forming clouds, fog, dew, rain, snow, hail, or sleet.

Dew.—Although the philosophy of dew is now perfectly well understood, no atmospheric phenomenon before the happy researches of Dr. Wells was more imperfectly explained and involved in greater mystery. The formation of dew is immediately referable to the function of radiation, concerning which it will be proper to make a short explanation in addition to that which has been already stated at page 480.

In that place the general indication only has been made that a heated body—for example, a cannon-ball—if suspended in space, darts off heat cognizable on temperature under the condition of rays. It remains now to be stated that the function of radiation is determined as to its extent by the surface of bodies: rough metallic surfaces radiate more than those which are smooth; glass surfaces radiate more than metallic surfaces; plants radiate more than the earth; grass and leaves more than bushes and trees; loose gravelly land more than hard soil.

To demonstrate the effect of surface on radiation, many instructive experiments may be performed by means of the differential thermometer and a cubical canister of tin plate. If such a canister be taken, and one side of it scratched, another polished smooth, another painted white, and the fourth black, a mixture of lamp-black and size being used by preference for the latter purpose—if the canister be now filled with hot water and held between the two bulbs of a differential thermometer as represented in the accompanying diagram (Fig. 40), each side of the canister will represent and indicate a different amount of radiating influence, as shown by the complementary disturbance of the two mercurial columns. It will be found that the polished side has the minimum, and the blackened side the maximum, radiating effect. It will soon be perceived that these deductions concerning the property of radiation are intimately connected with the philosophy of dew.

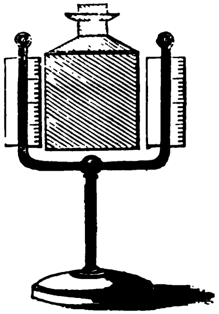


Fig. 40.

Until the experiments of Dr. Wells, which will be soon adverted to, the most erroneous notions prevailed concerning the theory of dew. According to some it fell from the sky, according to others it rose from the ground, both which theories are altogether untenable.

It is a sufficient answer to the proposition that dew falls from the sky, to say that dew never occurs when nights are cloudy; and it is a sufficient answer to the statement that it rises from the ground, to remark that a slight screen thrown on the ground, or elevated above the ground, is incompatible with the formation of dew.

The theory of dew is hardly explained by a consideration of the laws of radiant heat. The starting point of the investigation is the atmosphere. Now the atmosphere always contains moisture, as I have already explained, and the amount of this moisture will, *cæteris paribus*, be correlative with the degree of atmospheric heat at the time. If, then, the atmosphere being raised to its fullest point of saturation for any given degree of temperature, that temperature should by any chance fall, the result will necessarily be a deposition of moisture. Let us now apply these principles to the conditions of a heat-radiating surface of the earth and a clouded sky. In this case no dew occurs, nor, according to theory, should any occur, inasmuch as the clouds perform the functions of a second radiating surface. The earth radiates heat owing to the clouds; but the clouds in their turn radiate heat back again to the earth; whence it follows that the earth practically does not lose heat, and its temperature not falling below the temperature of the circumambient atmosphere, no atmospheric moisture can be deposited; in other words, no dew can occur.

For these facts we are indebted to the late Dr. Wells. They are demonstrated by thousands of natural conditions, and bear the test of any properly-devised experiment.

The following diagram (Fig. 41) is intended to show the manner in which a screen will prevent the occurrence of dew. Two plates of glass are represented as supported over an expanse of grass. Underneath the glass plate not the slightest dew will be found, though the grass around will be dewed heavily.

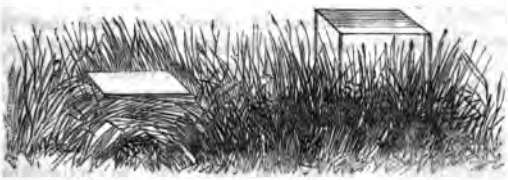


Fig. 41.

A very pretty illustration of the conditions which regulate the formation of dew, will frequently be supplied by a sheep lying down on the grass, on a clear, tranquil, cloudless night, when, to use a popular but incorrect expression, dew is falling; it will be found that the upper part or aspect of the wool of a sheep, is completely drenched with dew, although the under part or aspect of the animal is dry, as represented in the accompaning diagram (Fig. 42).

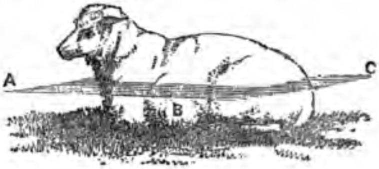


Fig. 42.

The explanation of this phenomenon will be so obvious that no further remark concerning it is necessary.

Consideration of the laws of radiant heat will render manifest the reason wherefore some surfaces are more bedewed than others. The amount of dew will depend, *cæteris paribus*, on two circumstances—firstly, on the kind of surface; and secondly, at its angle of inclination. Reference has already been made to the comparative facility wherewith certain bodies found in nature favour the deposition of dew upon them, and the most

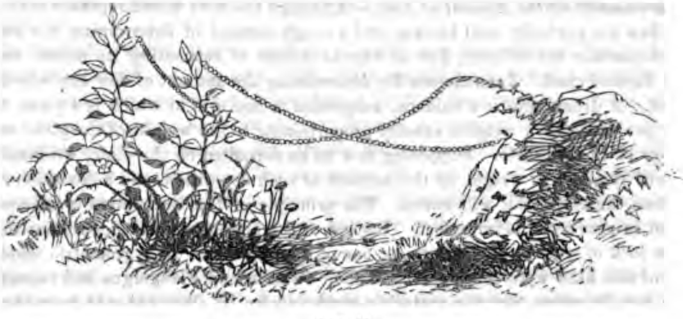


Fig. 43.

casual observer cannot fail to be struck with the difference. In all cases, the bodies which indicate heat are the most favourable to the deposition of dew upon them. Few, if any, objects naturally occurring are so solicitous of dew as spiders' webs; and no object present the phenomena of dew under a guise so beautiful. Not unfrequently a thin filament of cobweb, so small that it would be invisible to the naked eye, presents itself to the vision on a dewy morning as if it were strung with little pearls (Figs. 43, 44).

Fig. 44.

That angular inclination of a body should influence, and be intimately connected with, the function of dew formation directly follows from a consideration of the laws of radiant matter; and it may be readily illustrated by a diagram. Let us be assured that a screen of glass be supported over the radiating surface of the earth—in one case horizontally, in the other case at an angular inclination, as represented by the accompanying diagrams (Figs. 45, 46). It will be evident that the horizontal glass in Fig. 45 will radiate back more heat than the diagonal glass in Fig. 46.

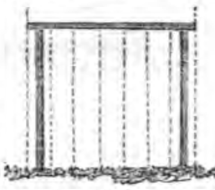


Fig. 45.

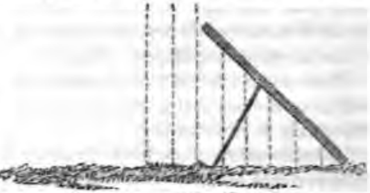


Fig. 46.

Determination of the Amount of Dew.—Although the laws which regulate the formation of dew are perfectly well known, and a rough method of determining the amount of dew deposition not difficult, yet no correct means of estimating its actual amount has yet been devised. Instruments for determining the amount of dew are called drosometers. A drosometer is a balance, suspended to one arm of which is a plate, to the other a pan containing weights exactly proportionate to the weight of the plate, so that both may be in equilibrio. Supposing dew to be deposited on the plate, evidently the latter will increase in weight by the amount of such deposition, and a deviation of the beam from the horizontal will ensue. The principle of the instrument is unimpeachable, but in practice it is imperfect. Instead of the plate as described, recourse may be had to a lock of wool or eider-down, or one of a large choice of hygrometric materials. Bodies of this kind were employed by Wells. Wilson and Flangergue had recourse to a plate, but it seems that the materials employed by Dr. Wells should have the preference.

A very instructive experiment relating to dew formation, and one which may be regarded as presenting a summary of the whole matter, is as follows:—

If on a clear, cloudless night, when dew is being deposited, a glass ball (Fig. 47) be suspended in the open air some height from the ground, dew-drops will form on the ball; not, however, equally on all portions of its surface. Firstly, its upper aspect will be bedewed, then its sides; but only rarely, and in extreme cases, the inferior aspect of the glass globe becomes covered with dew-drops, and when they do occur they are very small; indeed, a complete gradation of size is manifest, the dew-drops decreasing in size as the upper aspect of the globe is departed from.

Generalizations.—The following generalizations relating to the phenomena of dew may now be appended; they will, for the most part, be seen to be directly deducible from a consideration of the laws of radiant heat:—Clouds and brisk winds are both inimical to the formation of dew; the former, because of their own radiating power; the latter, because of the removal of cool air, its place being supplied by air already warmed. If the night be cloudy, and the wind still, very little dew results. If clouds and wind occur together, dew is totally absent. Screen-like objects interposed between the sky and the radiating surface produce an effect identical with clouds, hence bodies freely exposed to the atmosphere, *cæteris paribus*, are most freely bedewed. Morning and evening are not the times, as commonly supposed, when dew is formed most copiously. It is deposited at all hours of the night, but most copiously rather after midnight. It sometimes occurs even before night, late in the afternoon. Dew is not deposited with equal readiness in all parts of the world, but attains its maximum in warm lands near the margin of the sea, rivers, or lakes; as, for example, near the Red Sea, the Persian Gulf, the coast of Coromandel, at Alexandria, and in Chili. It is quite absent in very arid regions, in the interior of continents—such, for example, as central Brazil, the Sahara, and Nubia; neither does it frequently occur at sea, because of the bad radiating quality of a surface of water.

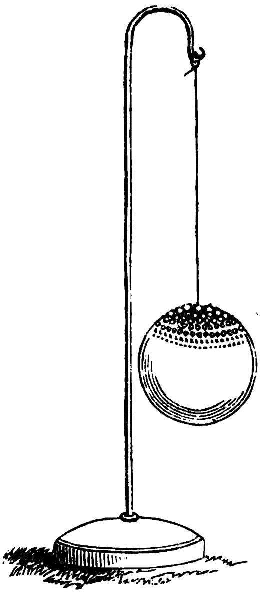


Fig. 47.

The imperfect radiation of a surface of water is well illustrated by the following striking experiment:—Glass is a good radiating surface; whence a piece of glass freely exposed in an atmosphere when dew is forming soon becomes covered with dew. If, however, the glass have its surface wetted previously to exposure, instead of becoming more wet it becomes dry, simply because radiation is impeded, and evaporation takes place unchecked. *Cæteris paribus*, the amount of dew produced will be proportionate to the amount of aqueous vapour present in the atmosphere, and thus readily explains the fact that a copious production of dew is frequently the precursor of rain.

Honey-Dew.—Occasionally a sweet, damp, sticky moisture attaches itself to leaves during the night, and does not disappear throughout the day. The term honey-dew has commonly been applied to it, though, in point of fact, it is not dew at all, being merely an excretion from certain insects termed *aphides.*

Hoar-Frost.—Hoar-frost differs only from dew in the circumstance of temperature. One is deposited and remains uncongealed; the other, becoming consolidated by the agency of freezing cold, is converted into ice. Every meteorological observer knows that the existence of hoar-frost is held to be indicative of coming rain, and in most instances the opinion is verified. The phenomena of hoar-frost are even more beautiful than the corresponding ones of unfrozen dew. Upon leaves and vegetable stems the deposition of hoar-frost is particularly beautiful. If hoar-frost be examined microscopically, or sometimes even attentively by the naked eye, a crystalline structure will be evident. The crystals belong to the same crystallographic system (the rhombohedric) as those of snow, but their general appearance is somewhat different.

Fogs.—Fogs may be regarded as clouds which form close to the earth's surface; hence we might discuss their peculiarities under the general head of clouds. They are characterized by some peculiariaties, however, chiefly dependent upon the vesicular aqueous vapour of which they are composed, embracing and retaining volatile particles evolved naturally or from the operations of man. In this way it is well known that London fog is anything but pure aqueous matter. One of its very important constituents is the condensible part of smoke. As regards the production of fog it is usually referable to one of two circumstances: either non-visible aqueous vapour may be converted into the visible or vesicular form by decrease of atmospheric temperature immediately, or by the cooling agency of the earth's surface depressing the temperature of the layer of air next to it below the dew point. In this country, and in Europe generally, fogs are of most frequent occurrence in spring and autumn. There are regions, however, in which fogs prevail throughout the year. The coasts of California are almost constantly veiled in fog; and the same remark applies, in a minor degree, to the western coast of the American continent, even so far south as Peru. Newfoundland, Nova Scotia, and Hudson's Bay, are all subject to frequent and dense fogs, attributable in these localities to the condensation of vapour which arises from the hot gulf-stream by contact with neighbouring and colder air. Fogs do not occur so frequently on level plains as on mountainous regions. In Arabia and the arid table-land of Persia they are almost altogether wanting. London and Amsterdam have acquired a somewhat evil character for fogs; but this meteoric condition applies to many other European localities with an almost equal amount of propriety. At Antwerp fogs are very prevalent, and the navigation of the lower and middle Rhine is sometimes impeded for weeks together by the occurrence of this pest of the sailor. Neither can Paris boast of much immunity from fogs; they are somewhat less dense and less frequent than our own London fogs, it is true, but are, nevertheless, far from contemptible. Amongst the regions which are likely in future to be celebrated for the prevalence of fogs, the Black Sea may be enumerated. Until recent events that locality was comparatively unknown to us; and since the conveying of stores to our troops in the Crimea has necessitated its continuous navigation, the embarrassment of fogs has only been too apparent.

Dry Fogs.—Under this name has been described a dull opaque appearance which the atmosphere of certain regions occasionally assumes, deadening the fiery beams of the sun, and dulling that luminary so that he may be looked at without pain by the naked eye, and embarassing respiration.

The dry fog is most common in certain parts of North America, during the period known as the *Indian Summer*. It also occurs in Germany, and more rarely in England. There can be no doubt that many atmospheric opacities, different in character as well as in cause, have been summarily classed under the denomination of *dry fog*. When the phenomenon occurred locally, it can generally be traced to such causes as the burning of extensive districts of turf or of forests. When its prevalence is more general, the most rational explanation would seem to be that which attributes it to volcanic eruptions. Some meteorologists have invoked electricity as the cause of this phenomenon, but it is not easy to see in what way the assumed cause could produce the effect in question. Electricity has long been to meteorologists what the class *radiata* was to Cuvier; namely, the receptacle for things unknown or unexplained.

Clouds.—Clouds are perhaps the most beautiful of all aerial phenomena. All the charms of changeful variety of colour, of form, and of motion are theirs; nor is their utility inferior to their beauty. Without clouds there would be neither rain, nor snow, nor hail; the consequences of this deprivation may be anticipated, or they may be readily learned, by turning to the geography of countries where rain, and snow, and hail are unknown. All regions thus circumstanced, provided irrigation be impossible, and that the altogether exceptional condition of copious dews be absent, are, despite the most favourable conditions of climate and of soil, barren wastes.

Notwithstanding the thousandfold varieties of clouds—their protean shapes, their manifold colours, and other distinctions—when the observer comes to regard them with a scrutinizing eye, he will not fail to recognise broad distinctions between them, admitting of being made the basis of a philosophic classification. Thus, some clouds are devoid of outline, their edges merging away into circumambient air; some are black and massive, almost conveying the idea of a hard substance; some are white and fleecy; others extended like a pennon. All these are forms of cloud which present manifest distinctions amongst themselves. Mr. Howard was the first who effected a regular classification of clouds. This classification is now generally adopted, I shall, therefore, present the reader with an outline of his system. According to this meteorologist, there are three elementary and four secondary forms of cloud.

Primary Forms.—The first primary form is the *cirrus*, consisting of feathery expansions, and which is only seen in clear weather.

The second primary form of cloud is the *cumulus*, composed of large hemispheroidal masses superiorly and apparently resting below on a horizontal base. This form of cloud chiefly occurs in summer.

The third primary form of cloud is *stratus*, composed of horizontal layers, the smaller layers being underneath. It is this form of cloud, more than any other, which presents itself under a variety of beautiful colours. It chiefly appears at sunset.

Secondary Forms.—The secondary forms of clouds are—(1) *Cirro cumulus.* It is a mixture, as its name indicates, of cirrus with cumulus, and is made up of an aggregation of small white clouds, which have been compared to a flock of sheep. (2) *Cirro stratus.* A compound cloud, which is formed of the two primary clouds embodied in its name. (3) *Cumulo stratus.* This compound cloud chiefly appears towards night in dry windy weather, and is of a leaden colour. (4) *Nimbus, or rain cloud.* This cloud is seen in greatest perfection during a thunder-storm. All the varieties of clouds described are represented in the appended diagram (Fig. 46).

It will be readily anticipated that clouds are frequently so mingled and confounded, that they are not always susceptible of the precise classification just announced; nevertheless, a prevalence of one type of cloud over another will be generally seen to prevail.

Relation between Clouds and the Weather.—People who are in the habit of narrowly studying the phenomena of clouds, are enabled to draw conclusions of much accuracy respecting the coming weather. Thus cirro cumuli are for the most part indicative of serene fair weather; the prevalence of wind subsequently to the appearance of much extended and highly-coloured stratus clouds, is a matter of popular experience. The appearance of nimbus clouds proclaims the advent of rain; and the cirro stratus which sometimes colours the sky as with a veil, all well-defined form being absent, is almost a sure forerunner of bad weather.

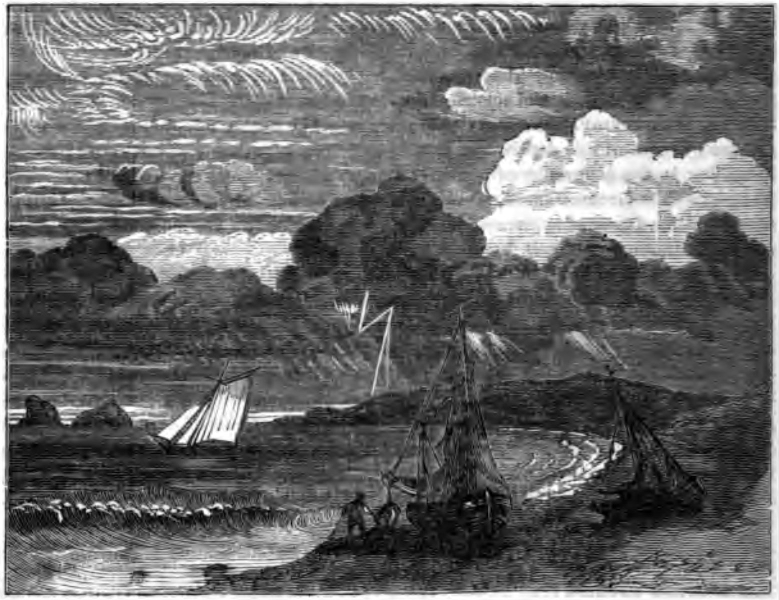


Fig. 48.

Composition of Clouds.—That clouds are composed of water in some condition does not require to be demonstrated; but some explanation must be given of the circumstances enabling water, a material so much heavier than atmospheric air, to remain suspended frequently in very elevated regions, where the atmosphere, thin though it be on the earth's surface, is still more attenuated. This is a matter which it must be confessed is still veiled in considerable obscurity; but perhaps the most rational explanation of cloud formation is the following:—Firstly, aqueous vapour is diffused, or rather absorbed, invisibly throughout the air. The laws of the absorption or diffusion are perfectly well known. The amount differs, as we have already seen, for different temperatures, being proportionate to the temperature. Assuming, then, that the upper regions of the atmosphere at any time are saturated with atmospheric moisture in its invisible condition, let us now contemplate the effect of cooling that atmosphere by any cause. It is not difficult to furnish reasons for these cooling agencies; they are numerous and varied—such as sudden variations of electric condition, sudden variations in the direction of winds. If, then, from any cause the atmosphere is cooled below its capacity for holding vapour in the invisible form, aqueous deposition occurs. If this deposition take place on the earth's surface, the result is dew; if aloft in the air, we have a cloud.

Thus far the steps of each succeeding change are evident; but the remaining points of cloud-formation are more obscure, the circumstance of chief difficulty being to find an explanation of the aerial permanence of clouds, seeing that the material of which they are composed is so much heavier than air. Probably the most consistent explanation is this:—Atmospheric moisture, when it changes from the invisible to the visible form, assumes the physical condition of spheroids or vesicles—minute bubbles of water, in point of fact, each bubble filled with air. If these vesicles be exposed to the sun's rays, it is evident, from consideration of known laws, that they must become specifically lighter than the surrounding medium; and thus affected, they would float, for the same reason that a soap-bubble floats whilst it is yet warm, notwithstanding that the air which it contains is heavier than the surrounding atmosphere. We seem, therefore, in a position enabling us to account for the upper part of cloud strata. Directing our attention now to the lower part of these strata, it seems rational to assume that the vesicles of which they are composed should descend. Probably they do so; continuing their descent until they come in contact with an atmosphere sufficiently warm to dissolve their aqueous coating, and convert their water once more into the vaporous or invisible form. Thus it may be, and most probably is, that a cloud which looks permanent to the eye, is really exposed to the continued operation of resolution and re-formation: or, rather, that the two opposing causes, which are here spoken of as producing active changes, balance themselves, and give rise to a condition of equipoise.

Although it has hitherto been taken for granted that clouds are formed of unfrozen water, we know that such is not invariably the case. If the temperature of a nimbus cloud sinks to freezing point, or 32° F., its contents freeze, and snow is the result. Many philosophers, indeed, but more especially Kaemtz, are of opinion that the cirrus—the cloud which soars in the highest regions, frequently at an elevation not less than 20,000 feet above the earth's surface—consists of particles of snow or ice. Assuming this to be the case, it is not easy to advance the reason of such molecules remaining aloft in a medium so attenuated as is atmospheric air in a position so elevated.

Position of Clouds.—It is somewhat remarkable that every known form of cloud assumes more or less the horizontal position. Vertical clouds are very rare; and if we choose to except water-spouts, not recognizing them as clouds, perhaps we may say unknown. The horizontality of clouds, warrants their being spoken of as composed of strata; generally, indeed, these strata are very well defined. MM. Paytier and Hoffard have carefully examined the thickness of these cloud-strata on the Pyrenees, and have found their average variation to be between the limits of 3400 and 1600 feet. The maximum observed thickness was 5000 feet, although this measurement is undoubtedly in some cases greatly exceeded.

Height of Clouds.—Several meteorologists, amongst whom Riccioli, Wrede, Kaemtz, and Arago must be particularly mentioned, have set themselves to the task of discovering the height of clouds. The methods by which these investigations were made have been various. Riccioli determined their height by placing two observers a certain and known distance apart; Wrede by making use of their shadows, and then reducing the computation of height to the solution of a problem in trigonometry. Riccioli states the maximum height of clouds to be 25,000 feet, and Lambert takes their minimum height at 13,000, whilst their maximum height, according to the same, another is from 15,000 to 20,000 feet. Gay Lussac, when he acquired in his balloon an elevation of 21,600 feet, perceived small clouds floating still much above him. Perhaps the statements of

Kaemtz, relative to the height of clouds above the earth, are the most trustworthy. He believes the usual range of cumulus to be from 3000 to 10,000 feet; of cirrus from 10,000 to 24,000 feet; of nimbus, or thunder-cloud, between 1500 and 5000 feet. That very accurate physicist Pouillet, as the result of certain experiments performed in 1840, states, that he has proved the existence of clouds at an elevation from about 22,300 to 38,000 feet, and probably, as the general result of all recorded trustworthy observations on the elevation of clouds, we may arrive at the conclusion that cirrus does not descend below 2000 or 3000 feet, whilst nimbus occasionally descends so low that it approaches the earth within a few hundred feet of surface. The maximum mean elevation of clouds seems, in low latitudes, evidently on account of the greater capacity of the atmosphere to absorb and dissolve aqueous vapour. It should here be remarked that the elevation of a cloud cannot be determined by reference to its apparent place in the sky; and, except the distance be known, neither can its actual size. These remarks are illustrated by the accompanying diagram (Fig. 50), where the same cloud will be seen at different times under very different angles by the same observer, as reference to the angles A E C and B E D will testify, whence the height would differ for two different statures.

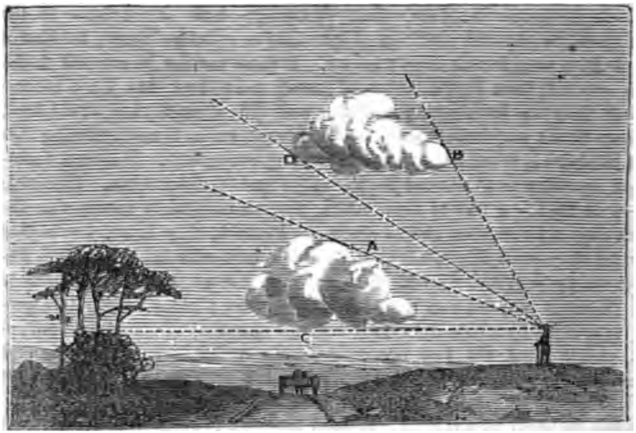


Fig. 49.

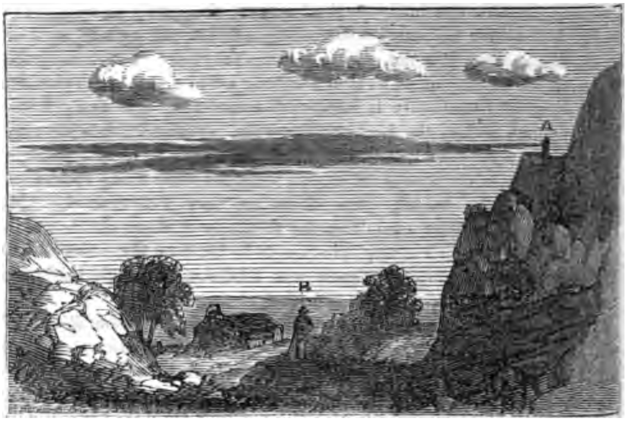


Fig. 50.

And again, in the subjoined diagram it is demonstrated that the observer at A will see clouds which are quite invisible to another observer at B.

Rain.—When from any cause cloud-vesicles aggregate into drops, and these drops fall, the result is rain. Rain would appear, therefore, to be necessarily dependent on a cloud; and in a large majority of instances we find this to be the case. Still, the phenomenon of rain without clouds is well attested. The amount of rain which falls at any place, and at any stated interval, may be readily estimated by means of an instrument termed a pluviameter, or rain-gauge. The principles on which this instrument is founded are of the simplest kind. If, for example, a cup or basin, of known cubic capacity and known orifice, be exposed so that it may receive the fallen drops of rain, the cup or basin would be a rough rain-gauge. Were it not that the collected water thus exposed would be continually evaporating, thus apparently diminishing the total fall of rain, no better rain-gauge need be desired; but in practice it is necessary to provide against such evaporation; and it is usually accomplished by forming an instrument of such construction that one part may be destined to collect the fallen rain, and another part to diminish, within the smallest limits, the amount of evaporation. In expressing the amount of rain which falls at any particular spot, it is necessary to express the height also at which the observation is made. The annual fall of rain increases as land acquires elevation; nevertheless, at one and the same place the amount of rain decreases with elevation, for the reason that each drop of rain throughout its descent goes on collecting moisture and becoming larger. Dalton appears to have been the first to notice this fact. He proved, on comparing two sets of observations,—one set made at the base, the other at the summit, of a high tower,—that the amount of rain at the top to that at the bottom was as two to three. Similar observations made at the Paris Observatory have led to similar results. This variation between the amount of rain at different elevations in one locality, is the more considerable as the point of aerial saturation is more nearly attained; for which reason it is less in summer than in winter. The heaviest rains usually occur in the tropics, and during the hot season. There the fall of rain is enormous, sometimes amounting to an inch per hour; nay, Humboldt has related that in South America no less than five inches of rain fell in one hour. In these islands we can hardly say that any one season merits the designation of the rainy season; but in tropical regions, except the belt of calms, and in the subtropical regions, the separation between the dry and rainy seasons is well marked. In the continental portions of the torrid zone, the rainy season sets in when the summer heat attains its maximum, and continues during four or five months, the atmosphere being clear and bright throughout the remaining portion of the year. Near the equator there are two wet seasons, sometimes separated from each other by a totally rainless period, but at other times demarcated only by periods of maximum and minimum fall of rain. Dutch Guiana furnishes the well-known illustration of a country having two well-marked rainy seasons: one, and that the chief, commences in April, and lasts till June; the other, or minor rainy season, commencing in the middle of December lasts till the middle of February. The drops of tropical rain attain a magnitude never seen in the tamer showers of these northern regions; their weight is so considerable, and the force with which they descend so great, that their splash or stroke leaves a smarting sensation on the skin. The region situated between the influence of the two trade-winds, and commonly known as the region of calms, is devoid of periodic rains, although the fall of rain there is frequent and heavy.

Rainless Portions of the Earth.—There are some localities in which rain never occurs: for example, Egypt, the Desert of Sahara, the table-lands of Persia and Mongolia, the rocky flat of Arabia Petræa, &c. Rain is generally the most abundant near mountain ranges; but there are exceptions, one of the most remarkable of which is presented by the part of Spain south of the Sierra Nevada.

Condition of Europe with regard to Rain.—Europe, considered in relation to the prevalence of rain, admits of being divided into three districts—the South European, the Middle European, and the Swedish. In Portugal and the larger portion of southern and central Spain, there is an almost total absence of rain during summer; but north of the Pyrenees, rain occurs at variable times throughout the whole year. All the portions of Europe north of the Alps and Pyrenees are subject to the Middle European and the Swedish pluvial conditions. The characteristic of the Middle European climate as regards rain is, that the latter chiefly occurs during westerly winds; whereas the Swedish climate is characterized by the prevalence of rain during both easterly winds and westerly winds, which bring rain to the whole of Central Europe, and deluge our isles with wet, leaving the bulk of their moisture by the mountainous Scandinavian range which separates Norway from Sweden. St. Petersburg and Moscow cannot be said to belong either to the Central or Northern European climate; these places lie on the confines of both; hence neither westerly nor easterly winds are there prevalent. In England, the maximum number of rainy days throughout the year occurs in Cornwall and Devonshire; passing thence east into Central Europe, the total number of rainy days per annum continually declines. If we assume the annual amount of rain which falls at St. Petersburg to be nearly three, the corresponding annual amount for the West of England will be 2·1; in Central England, 1·4; in Central Germany, 1·2. This statement assumes an average of some special localities to have been taken into consideration: special places present many deviations. In describing any place as subject to rain, or *rainy*, distinction must be made between the actual quantity of rain per annum which falls, and the total average number of rainy days. Understanding by the latter term every day on which rain, much or little, falls, the number of rainy days increases in Europe from south to north. The mean average for Southern Europe may be taken as 120, in Central Europe as 146, and in Northern Europe as 180. The following statement indicates the total number of rainy days per annum for a few places specifically named:—

Buda	112	Ratisbon	115
Warsaw	138	Rotterdam	187
Germany, average of	150	Paris	160
Carlsruhe	174	Poitiers	99
Tagernsee	170	St. Petersburg	168
Munich	149	Moscow	205
Stuttgardt	127		

Annual Distribution of Rain.—The time of year at which rain is most prevalent, is subject to much variation for different countries. Throughout Central Europe rains are most prevalent in summer, but in Southern Europe the preponderance is on the side of winter rains. Norway is subject to copious winter rains; whilst in Sweden they are almost entirely wanting. Sweden, in point of fact, although placed so near the sea, has a climate altogether like that prevalent in continental regions. The reason wherefore Norway is subject to winter rains, and Sweden is deprived of the same, hinges upon the explanation already made that western winds (which predominate in Scandinavia during winter) lose their moisture in passing over the Scandinavian range. Although summer rains are in many places rare, yet when they occur they are generally more copious than rains at any other period.

Rain which falls in summer at different places, taking the rain which falls on a winter's day at the corresponding place as unity :—

England	1·07	Germany	1·76
Western France	1·03	St. Petersburg	2·17
Central France	1·57		

from which statement it appears that the prevalence of summer rain increases towards the east.

Peculiarities of Rain-Water.—Fresh-falling rain-water, collected far from towns or other sources of local contamination, is very nearly pure; nevertheless, modern chemical observation has succeeded in discovering the presence in rain-water of many substances present in small quantities. Nitric acid and nitrate of ammonia are by no means unusual constituents; and iodine has been frequently recognized. As regards the sources of these and other extraneous bodies, much still remains to be discovered. Nitric acid is most probably formed in the atmosphere by the agency of electricity; and the ammonia may be referable to exhalations from decomposing matters on the earth's surface. Many of the extraneous bodies, especially salts, sometimes recognized in rain-water, are unquestionably due to the action of winds upon finely-divided ocean-spray.

Showers of Fishes, Stones, &c.—Instances are on record of whole shoals of fishes, and numerous collections of other animals—also stones, &c.—being cast on the earth by showers. At one time these phenomena were regarded mysteriously, and referred to occult causes. At present they are deprived of their mystery, and referred to the previous elevation of the fishes, &c. by aerial currents, whirlwinds, and water-spouts.

Snow.—If the temperature of a cloud should fall at any time to 32° F. or lower, instead of rain the result is snow. Much that is beautiful and beneficent is seen in this divided form of frozen water. In our own temperate clime we do not comprehend, except by reflection, the true value of snow in the economy of nature. Its fall amongst us is uncertain and exceptional; we know not when it is to come, or how long it is to remain. We therefore make no provision for it—regard it as a condition to be tolerated—regret that it interferes with our locomotion—that it impedes our railway trains, and wets our feet; and wish it away. Nevertheless, even in these isles, the farmer, from experience, is not insensible to the value of snow. He says it keeps his winter crops *warm;* and the thoughtless passer-by, wrapped in his own self-conceit, laughs at him for making a statement so apparently grotesque. The philosopher, however, who is aware of the low heat-conducting power of snow, and who can appreciate the evil consequences of frost on vegetation, indorses the farmer's statement.

If we would desire to recognise the full benefits of snow, we must direct our attention to northern climes—to Sweden, to Russia, and Canada. There the advent of snow is looked forward to as a blessing; and when it comes, the period of its duration admits of being predicted with tolerable accuracy. No sooner is the ground covered with sufficient snow, than wheeled-carriages, which but yesterday were sticking up to the axletree in mud and wet, are put aside, and sledges supplied in their stead. Market-places, which before the snow had fallen were naked and unworthy, now teem with good things brought from hundreds of miles away. Snow has all at once laid down a far-stretching railroad, over which the sledges glide almost with the ease and velocity of a railway-train.

Form of Snow Flakes.—In certain conditions of temperature snow falls as a pulverulent body, in other conditions as a flaky amorphous mass; but if very dry snow be microscopically examined before it has been broken up, indications of crystalline structure will be recognizable. Sometimes these crystalline snow-flakes attain such large dimensions, that they are quite evident to the naked eye. The crystalline forms thus developed are numerous, but they are all referable to one crystalline system, the *rhombohedric* or *rhombohedral;* the characteristic of which is that crystals belonging to it have three axes crossing each other at the angle of sixty, and one axis at right angles to these. Scoresby, who has minutely examined these snow-flakes, describes five principal forms of snow crystals:—1st, crystals having the form of thin plates, which are the most abundant; 2nd, surfaces or spherical nuclei, with ramifying branches in different planes; 3rd, fine points, or six-sided prisms; 4th, six-sided pyramids. The latter form is the least frequent of all. The accompanying diagram (Fig. 51) represents the principal crystalline varieties of snow-flakes.

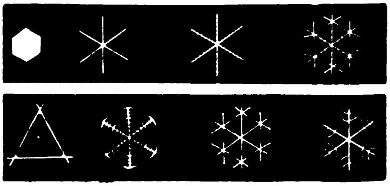


Fig. 51.

The Snow-line.—Inasmuch as the upper regions of the atmosphere are intensely cold, there is an elevation for every latitude at which atmospheric moisture is changed into snow. This elevation corresponds with what is termed the *snow-line.* At the equator the snow-line is elevated from 11,000 to 12,000 feet above the sea-level. As we proceed towards the north, the elevation of the snow-line will evidently be lower. Snow does not fall on level ground in Europe farther south than Central Italy; but in Asia and America the region extends nearer to the equator. Through Florence passes the isothermal line of 59° F., and it may be regarded as the southern limit of the region in which snow falls on level places. Snow does not usually fall at the time of maximum cold; some meteorologists say it never does—but this is an error. After snow has fallen the weather generally increases in severity. We are usually in the habit of assuming that the total quantity of snow which falls increases as we reach either pole —an assumption, however, which is only correct within certain limits. Thus, taking the northern hemisphere, for instance, the fall of snow increases from the isothermal of 59° F. to the isothermal of 41° F., which latter cuts the town of Drontheim, in Norway. Passing still further north, the quantity of snow goes on diminishing, evidently because in the polar regions the temperature of the air is too cold to retain much moisture, and atmospheric moisture must necessarily be the antecedent to either rain or snow.

The atmospheric condition during the fall of snow may vary from the limits of almost complete tranquillity, to the other extreme of most violent perturbations. In Germany, and other countries having a corresponding latitude, the fall of snow is usually tranquil, except during the months of February and March. In high latitudes snow usually occurs during violent tempest-gusts, almost equal sometimes to the West Indian hurricane or the Chinese typhoon. In Norway these storms are very frequent, also in Kamtschatka; in which latter region they are called *purga.* They are veritable thunder-storms, as is completely proved by the intense electrical condition of the atmosphere. On mountainous elevations snow-storms are commonly prevalent, irrespective of latitude.

Coloured Snow.—Red and green snow have been frequently described by travellers. The cause of these phenomena is now referred to the presence of minute algæ—the *protococcus nivalis* in the so-called red snow, and the *protococcus viridis* in the green variety.

Hail.—Frequently cloud-vesicles become aggregated and frozen into lumps of various sizes and shapes, sometimes opaque, sometimes transparent, and occasionally, though not very often, containing neuclei of solid foreign matters. Meteorologically, aggregations of this kind constitute hail. In most parts of the world where rain occurs hail is known, but certain localities are particularly subject to hail-storms. Generally hail falls by day; indeed an opinion prevails that hail-storms are unknown by night. This supposition is, however, erroneous. The form of hail is various, though for the most part they assume a spherical, spheroidal, paraboloidal, or pyriform contour; and still more frequently they are rounded, flattened, or angular. According to Delcross, the most common form of hail is that of a three-sided spherical segment, resulting from the comminution of larger spheres.

The diameter of hail-stones at a mean latitude is, according to Muncke, not usually greater than one and a-half or one and three-fourths of an inch, although on some occasions blocks of ice of enormous dimensions have fallen. For example, in 1719, there fell at Kremo, hailstones weighing not less than six pounds; and at Namur, in 1717, others weighing not less than eight pounds. Again, it is stated that, in 1680, masses of ice fell in the Orkneys twelve inches thick; and in 1795, hailstones fell in New Holland from six to eight inches long and two fingers thick. It is recorded, but on doubtful testimony, that there fell in Hungary, on May 28, 1802, a piece of ice three feet square by two feet thick, and the weight of which was 1100 pounds. But even this is much exceeded by a statement that, in the latter part of the reign of Tippoo Saib, a lump of ice fell at Seringapatam as large as an elephant. The *size* of the elephant, however, is not mentioned.

With regard to foreign substances existing in hailstones, they are described as various. In 1755 there fell in Iceland hailstones containing sand and volcanic ashes; others which fell in Ireland in 1821 contained a metallic nucleus, which proved, on analysis, to be iron pyrites (sulphide of iron); a similar phenomenon occurred in Siberia in the year 1824. The presence of small pieces of straw in hail has been frequently demonstrated.

The largest hailstones fall in summer during thunderstorms. Storms of this kind are most frequent in June and July; they are more rare in May, August, and September, and still more so in April and October. They usually occur at the close of long periods of calm, sultry weather. Hail-clouds are much lower in the sky than rain-clouds; and are generally recognisable by a peculiar ragged or jagged contour, and by their lower portions being marked with white streaks, the other portions of the cloud being inky black. Previous to the occurrence of hail, the barometer sinks very low; and, what is unusual before rain, the thermometric column suffers a corresponding depression. The thermometer during a hail-storm has even been known to sink through 77° F. A peculiar rustling sound in the air is also indicative of speedy hail, by a darkness resembling that dependent on an eclipse of the sun. Hail-storms are very seldom of long duration, usually lasting a few moments only—seldom longer than a quarter of an hour. The rapidity with which hail-storms travel is very great: one which occurred in Central France in 1788 travelled at the rate of forty miles an hour. The force of hailstones is sometimes dangerously great, not only breaking windows and shattering tiles, but killing herds and men and animals, cutting off branches of trees and herbage, and, in short, desolating all save the largest vegetable growths. The hail-storm in France of 1788, already adverted to, extended its devastations over 1039 parishes, destroying property to the amount of 25,000,000 of francs. Although hail-storms often extend very far in a linear direction, their breadth is usually inconsiderable,—often but a few hundred, or at most a few thousand feet,—though the linear extension has been known to exceed four hundred miles.

It has been already mentioned that wherever rain-clouds rest, hail may occur; nevertheless, latitude and local conditions determine the frequency of the phenomenon. Rain seldom occurs on the level land of tropical countries; and it is rare in the extreme north. The hail-belt, pre-eminently so considered, is comprehended between 30° and 60°, and to elevations less than 6000 feet. Even within this belt, and below the limit of elevation just assigned, there are certain localities where the occurrence of hail is a very rare phenomenon. Certain of the Swiss valleys may be cited as a well-known illustration of this remark; more especially in the Valais and its allied dales. It has also been well determined that hail more rarely occurs at the base of mountains than in localities a short distance removed. Perhaps no country, upon the whole, is more subject than France to the ravages of hail-storms, and in no country are the effects more serious. It has been ascertained that the average annual number of hail-storms in France is about fifteen. They are especially prejudicial to the vine and the olive, sometimes laying whole districts under desolation. Having regard to the highly excited electrical condition of the atmosphere as the rule during the occurrence of hail-storms, great hopes were once entertained that they might be prevented, or their ravaging power diminished, by means of suspended conductors. The idea of using such conductors appears to have been first suggested by Guenaut de Montbeillard in 1776; and hail-conductors have been extensively tried, but hitherto without any amount of practical benefit to justify their longer continuance. In 1820 a peculiar kind of hail-preventor was suggested by La Postolle, and subsequently by Thollard. The instruments consisted of straw ropes in which a metallic wire was interwoven, and suspended by means of pointed rods similar to lightning-conductors; but, like instruments having a similar object, and which preceded them, they were found to be unavailing.

Methods of Determining the amount of Atmospheric Moisture.—Having described the numerous forms under which aqueous moisture may exist in the atmosphere, it now remains to indicate and describe certain instruments which have been devised by different experimenters for determining its amount. These instruments, founded on different principles, as will be seen, are termed hygrometers.

It is a matter of common experience that many bodies are affected as regards their dimensions, more particularly their linear dimensions, by mutations of atmospheric moisture. Wood is a very common example of this property; more particularly a stick of wood cut transversely to the grain. Founded on this property of wood, the late Mr. Edgeworth constructed a very ingenious toy, which, though it be not a hygrometer, inasmuch as it does not measure the amount of atmosphere prevalent in the air, is at any rate a *hygroscope*. It is related that the somewhat eccentric philosopher just named once laid a wager that a certain toy—a wooden horse—constructed by himself, should, after the lapse of some time, walk across his room. The horse was accordingly made, and placed at one end of a chamber; the door of the chamber was then locked, and the key deposited in safe keeping. In process of time the horse did indeed arrive at the other end of the room; and the manner in which this was accomplished will now be made evident.

Underneath each hoof was a claw, long enough to stick into the flooring, and there take hold. The horse itself was made out of a piece of wood cut transversely to the grain; the consequence was that when the weather was dry, the linear dimensions of his back contracted, and when the weather was wet his back again elongated. Now, bearing in mind the construction of the feet of this toy-horse, it is evident that these alternate contractions and expansions must necessarily result in a forward motion.

Again, the condition of human hair illustrates the effect of varying amounts of moisture in the atmosphere. Every lady knows that she cannot retain her hair in curl during wet weather so well as when the weather is dry, because of the moisture present, which causes the hair spirals to relax and unfold. In point of fact, each hair contracts and elongates alternately by every mutation of dryness and moisture; so that if only the exact ratio of contraction and expansion could be determined and applied, the meteorologist might construct a hygrometer, having for its basis of actuation a human hair. The hair-hygrometer of Saussure takes advantage of this principle.

To construct this hygrometer, a soft human hair is boiled for a short time in a solution of sulphate of soda, afterwards for a few minutes in pure water; it is then well washed to free it from all adhering salt, and dried in a shady place. Next, one extremity of the hair is fastened to the extremity of a little tongue, and the other end is wound round a small pulley having two grooves. The second groove is for the purpose of retaining a filament of silk, from which a weight is suspended for the purpose of retaining the hair in a constant state of tension. To the pulley is fastened an index, traversing a graduated arc, whenever the pulley turns in any direction by the contraction or elongation of the hair. The graduation of the instrument is thus effected:—It is placed in a receiver holding chloride of calcium, or concentrated oil of vitriol, the air is exhausted from the receiver, and the place where the index then stands is marked. This mark corresponds with the point of greatest dryness, or 0 of the scale. We have next to determine the point of greatest saturation, which is thus effected:—The instrument is next placed in a receiver containing a dish of water; so that as the hair elongates the index turns, and finally coming to rest, the point at which it stands is marked. This mark corresponds to the maximum of moisture, which of course may be indicated by any number arbitrarily selected; this number being usually 100. Finally, it remains to divide the interval between the two extremes into 100 equal parts, and the instrument is complete. Notwithstanding all the care which may be devoted to the construction of this instrument, it is, after all, scarcely deserving the name of a hygrometer—it is little better than a hygroscope.

Occasionally certain helical vegetable fibres have been used as hygrometers in a way which the accompanying diagram will render manifest.

Let A represent a circular card or other flat disc, and B a vegetable filament helically coiled—it is evident that the helix will unravel in a damp, and tighten its coil in a dry, atmosphere. An instrument of this kind has been sometimes employed for the purpose of determining whether a bed be moist or dry. The instrument is a very good hygroscope; but, inasmuch as the coiling and uncoiling of the helix is most comparable with equal arcs, the instrument can hardly be termed a hygrometer.

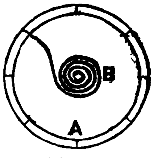


Fig. 52.

We are under obligations to the ingenuity of the Dutch for another hygrometer, or rather *hygroscope*—for, like the instrument just described, it is not a true indicator of the *quantity* of moisture present in the atmosphere. The instrument is of this kind.—A piece of catgut is suspended from one extremity, and to the other, or lower extremity, is fixed transversely a little horizontal bar. On one extremity of the bar, a lady in gay summer attire is represented, on the other a man dressed appropriately for a rainy day; finally, the catgut and its toy-appendages are surrounded with a case having two openings, and in such fashion that only one of the toy images can be visible at a time. Now, it is evident that the fibres of the suspended catgut will partially untwist under the influence of moisture, and re-twist as the atmosphere becomes dry; whence it follows that the lady will appear under the latter circumstances, and the man under the former. This instrument, though something more than ingenious, for it is a good *hygroscope*, does not merit the dignified term of *hygrometer*.

The Dew-point Hygrometer of Daniell.—If a wine-bottle be taken from its bin, it will frequently be found covered with moisture; and in proportion as the air is saturated with moisture, so will the depression of temperature be at which this moisture begins to be deposited on the bottle. In this manner, if we had the means of regulating the temperature of the bottle at our will, depressing it at pleasure, we might ascertain the exact temperature at which moisture would begin to be deposited; and thus noticing the variations of temperature at different times, we might establish and tabulate a correspondence between each particular temperature at which moisture was deposited, and the corresponding amount of moisture contained in the atmosphere. Now, the degree of temperature at which moisture begins to be deposited in this way, is called the *dew-point;* and hence the propriety of the appellation *dew-point hygrometer* which has been given to the instrument presently to be described. It consists of a doubly-bent exhausted glass-tube, each end terminating in a bulb. One bulb is covered with a coating of thin gold or platinum foil, the other with a fine linen rag. The former bulb is partially filled with ether, and holds a small thermometer, the graduated portion of which passes up the tube. If ether be dropped on the second bulb, evaporation rapidly ensues, and the bulb is cooled, thereby condensing the vapour of ether which it contains, and permitting a new evolution from the ether in the bulb. This evolution of ether cools the bulb, and causes dew to be deposited on its surface. The inclosed scale indicates the dew-point. The following reasoning explains how the determination of the dew-point can indicate the amount of aqueous vapour in the atmosphere:—In proportion as the temperature of the air is elevated, will it be capable of holding more moisture. Hence, by cooling the air, its power of holding moisture is diminished; a portion of moisture, therefore, becomes condensed in the form of dew-vesicles. The greater the moisture contained in air, the more readily will condensation ensue for a given reduction of temperature.

The dew-point hygrometer of Daniell, though a great advance upon the rude instruments just described, is attended with some imperfections. Its construction has been improved upon by Döberemer and by Regnault; but the instrument, in its most perfect form, still leaves much to be desired. Not only does its employment necessitate the use of a large amount of ether; but, what is of more consequence, when the weather is extremely dry the deposition only takes place with great difficulty. A far more effective instrument, though based on different conditions, is that now about to be described.

The Psychrometer.—The psychrometer consists of two thermometers mounted on the same frame, the bulb of one thermometer being naked, whilst the bulb of the other is enveloped in muslin or other similar absorbent texture, from which there extends a wick-like absorbent stem, terminating in a cistern of water. From a consideration of the structure of this compound instrument, it will be evident that the mercurial column of the naked or uncoated bulb will stand higher than the mercurial column of the second or wetted bulb. The reason of this is obvious. The process of evaporation lowers the temperature: and it follows, that under one condition, and *only one*, can the readings of the pair of thermometers which constitute the psychrometer correspond—namely, when the atmosphere is saturated with moisture to such an extent that it is unable to take up more. By an extension of this reasoning it will be now evident that the mercurial readings of the pair of thermometers will continually vary, according to the amount of dryness or moisture of the surrounding atmosphere. The variation, in point of fact, is in an inverse ratio to the amount of moisture; so that by means of formulæ we can easily connect the indications of the psychrometer with the dew-point.

Diurnal Variation of Atmospheric Moisture.—The amount of moisture present in air varies at different times of the day. There appears to be two maxima and two minima. The first maximum occurs about 9 A.M., the second at 9 P.M.; the first minimum shortly before sunset, the second about 4 A.M. Popularly, the air is said to be most damp at sunrise, and in the sense of dew or palpable moisture the popular expression is correct; but, provided the air be hot enough, we have already seen that it can absorb large quantities of moisture, retain it invisibly, and impart no sensation of moisture; indeed, pure steam is no more wet than a pure gas is wet.

Monthly Variation of Atmospheric Moisture.—The fact needs no comment, that all months throughout the year are not equally moist. It appears that at London, Paris, Geneva, and Great St. Bernard, the absolute amount of vapour in these places attains its maximum in January, and its minimum at the end of July or the beginning of August; but the relative moisture is greatest at London, Paris, and Geneva in December, and least in May.

Like the true aerial atmosphere, atmospheric aqueous vapour continually varies as to its amount of tension or elasticity. According to Dove, the amount of tension is less during north and south winds than during eastern and western winds, an observation which has also been confirmed by Kaemtz. Necessarily, too, the direction whence the wind blows must influence the quantity present of aerial vapour. North and north-east winds are, at least in these latitudes, less moist than winds blowing from the opposite direction.

Inasmuch as the aqueous vapour dissolved invisibly in air assumes the condition of vapour whenever the air is cooled below the dew-point, the influence which mountain ranges exercise in robbing winds of their moisture and producing rain will be readily evident. By an extension of the same reasoning, it will be also evident that winds which have reached into continents far distant from the ocean, and lost considerable bulks of water, must be necessarily dry. Such are the indications of theory, and observation fully confirms them.

The Meteorologic Relation of Imponderable Agents.—In our preceding investigations, the meteorologic relation of the imponderables has been almost unnoticed. Incidentally, some few of the leading properties of heat have been treated of, but no general statement of the meteorologic relations of these agents has been offered, this being a subject of such vast importance that it merits a treatment of itself.

The expression, imponderable agents, or imponderable forces, is now commonly applied to indicate the cause or causes, whatever they may be, which give rise to the phenomena of light, heat, electricity, magnetism; we must now also include actinism, or the radiant influence of the sun, being neither light nor heat, to the operation of which photographic pictures are due.

The imponderable agents have always presented a field of great interest to the student; but especially interesting is the study at this time, seeing that it is a tendency of modern philosophy to refer all these imponderable agencies or forces to various modifications of one grand cause. The correlations between light and heat, electricity and magnetism, is so intimate and so well marked, that assent can hardly be refused to the assumption that they must all be due to a modification of one common agent. Nevertheless, even though the cause of heat and light, magnetism and electricity, be granted as one and the same, the functions of these four results are so diverse that no generalization of treatment will include them all; they must be held distinct, and treated each under its own head.

Light.—This treatise not having chemistry for its primary object, but merely embracing chemistry as a collateral adjunct, it may be sufficient to comprehend under the general appellation *light* all non-calorific radiant influences. Strictly speaking, we ignore by this arrangement the existence of a peculiar radiant influence termed actinism; but so long as the exclusion be noted, and a reason assigned for the omission, no prejudice to scientific truth will result. A sufficient reason is, that the function of actinism concerns the meteorologist in a minor degree; that it has already been discussed in the treatise on Chemistry; and that all its meteorological relations may be with convenience included under the general treatment of light.

Theories of Light.—Except for the completeness of description, it is rarely worth while to quote the opinion of philosophers of Greece and Rome on any matter of natural science. The wonderful acumen, the quick perception, the subtle reasoning faculty of the Greeks—though the source of a noble literature, of a sculptured embodiment of all that is beautiful in living forms, of a terse logic and wonderful geometry—was perhaps disadvantageous to the development of experimental science. Minds that could venture so far in the region of speculation, were not the most likely to invoke the tedium of protracted experiment which the cultivation of physics demands; accordingly, all that has reached us relating to this branch of knowledge was crude and unreliable. The first theory of light which is on record is, I believe, that of Plato, who assumed that light consisted of certain emanations evolved from the eyes of animals. Subsequently the prevalent notion was that light consisted of emanations from luminous bodies,—a theory which was adopted by Newton, and has been designated the theory of corpuscles, for a reason which will speedily be evident. Even previous to the time of Newton, a theory of light, called the undulatory theory, had been advanced; but at that period its sway was short, though subsequently the theory has been revived, and is at the present time accepted almost universally. Of the corpuscular theory, the remark may be made that it affords a rough and gross explanation of the greater number of common luminous phenomena, for which reason it was long universally accepted; it is totally incompetent, however, to deal with some of the uncommon and very interesting phenomena of light, especially those of double refraction and polarization. The undulatory theory of light (so called from *undula*, a little wave) assumes that the property of function which affects the optic nerve, and which we agree to call light, is a result of the vibrations of certain waves occurring in an alternated medium far too subtle for chemical analysis, and which is conventionally termed ether. Now in strict truth it must be admitted that there is something opposed to the Baconian code of induction, in beginning with the assumption that a fluid ether of the kind indicated exists; and it must also be admitted that the direct evidence in favour of the existence of such ether is but slight. There does exist, however, a presumptive evidence favouring the existence of the agent, independently of the arguments analytically deduced from the *petitio principii* that light is really the result of waves. Astronomers have remarked that the motions of the heavenly bodies in space are subject to certain retardations, indicative of their travelling through some impeding medium; and this is, perhaps, the strongest argument which can be produced. If such medium be a reality, we have the *petitio principii* granted, which the undulatory theory of light demands.

The study of the agencies of light involves a consideration of less theoretical reasoning than is demanded by a corresponding study of the other imponderable agents. In studying the phenomena of electricity and of magnetism, we can hardly make one step without continually employing theory of some kind as a rallying-point for our ideas, and a stepping-stone by which we rise to our deductions. Not so in the matter of light. Whatever be the ultimate cause of this agency, whether corpuscular or undulatory, the function or impression of light is exercised in straight lines, except in a few special cases, which will be brought under notice hereafter.

Velocity of Light.—Astronomical observations of two distinct kinds furnish us with very precise observations, relative to the velocity of light. Its rate of travelling is, firstly, deduced from certain phenomena of Jupiter's satellites; secondly, from the aberration of light. As the result of both these kinds of investigation, light is found to travel at the rate of about 192,000 miles in a second of time. The rapidity, as will be seen, is enormous; yet it is far exceeded by the rapidity of the passage of electricity.

Determination of the Velocity of Light by the First Method.—Astronomical calculations inform us when each particular eclipse of Jupiter's satellites should take place, and the instant any such eclipse does occur may be seen by observation. Now, considering that the diameter of the earth's orbit is 190,000,000 miles, it is evident that our planet must at one time be 190,000,000 miles nearer the planet Jupiter than at another time. Hence, the visual indication of such an eclipse must come to us through a path at one time 190,000,000 miles nearer than at another time, inasmuch as 190,000,000 miles is the measure of the diameter of the earth's orbit. From comparative observations of this kind, it is determined that light occupies about sixteen minutes and twenty seconds in traversing the distance of 190,000,000 miles, which gives the velocity of light per second at about 192,000 miles.

Determination of the Velocity of Light by the Second Method—Aberration of Light.—It will be convenient now to adopt the common expression "ray," as indicative of a luminous agency exercised rectilinearly. Whatever theory of light be adopted, this definition will hold good.

Having premised this definition of the term luminous ray, we may now say that luminous bodies are rendered evident to us by reason of rays of light darted off in straight lines. Provided, then, that a luminous body darting off these rays, and the observing eye which receives these rays, be both at rest, and provided that all interfering causes were absent, the eye would refer the luminous body to its true point in space. If, however, the case be varied by assuming the observer to be in motion, or the luminous body to be in motion, or both, then the eye would not refer the luminous body to the correct point in space, because of what is termed the aberration of light. Inasmuch as our planet is in motion, and the heavenly bodies are in motion, we never see the latter in their true positions, but in the positions which they respectively occupied at some anterior period proportionate to the time occupied by light in travelling from them to the eye. From these considerations it will be evident that if we know the position in space of a heavenly body at any instant of time, and the space travelled over by the observer in a similar time, also the direction of motion in the two cases, the necessary elements are furnished for calculating the velocity of light.

The following diagram (Fig. 53) is intended to illustrate the foregoing proposition.

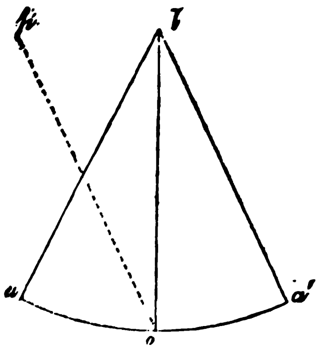


Fig. 53.

In the diagram *l* is a luminous body in space, *a o*, *o a'* represent portions of the earth's orbit travelled over by the observer in given equal portions of time—seconds, for example. The direction of luminous rays is indicated by *l a*, *l o*, *l a'*. It is now demonstrable that if light do *not* occupy time in travelling, the luminous object *l* will appear in its true position for each time of observation, however much the observer may have progressed. If, on the contrary, time be really occupied by light in travelling, then the observer's eye, supposed to move in the direction *a a'*, will not see the luminous body *l* by reason of the rays which emanate from it at the period of observation, but by reason of the rays which started on their journey at some anterior period; and, seeing that the direction, or angular position, of luminous objects is determined by taking cognizance of their luminous rays, it follows that *l* will never be seen by the moving observer in its true position. Let us suppose the observer's eye to be at *a*, then *l* will not appear in its true position,—namely, the position indicated by the line *l a*, but in some antecedent position, which the diagram does not represent. Supposing the observer to have arrived at *o*, the apparent position of *l* would correspond to *p* (*i.e.*, parallel to *l a'*, as indicated by the dotted line *o p*). It follows, then, that the distance of a luminous body being known, also its true position and its apparent position, the rate of travelling of light can be determined by trigonometric calculation.

Consideration of Primary Optical Laws.—The meteoric relations of light being almost exclusively optical, it will be proper to enumerate a few primary optical laws.

Ray of Light.—Definition: *A ray of light is a rectilinear agency of the luminous essence for any given transparent medium.* This definition is equivalent with the ordinary

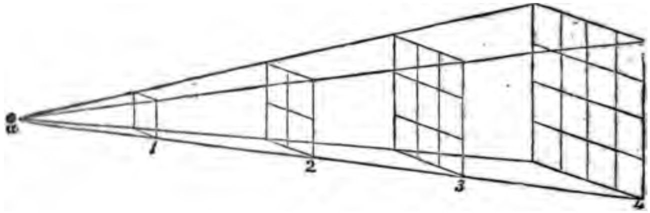


Fig. 54.

expression that light travels in straight lines—so far as that expression is correct. Taken without limitation, however, the expression is not correct. The agency of light in straight lines is only maintained for one homogeneous transparent body, and not those in some peculiar cases.

Law I.—The intensity of light varies inversely to the square of the distance. The operation of this law is illustrated by the preceding diagram, wherein the figures 1 2 3 4 represent four distances from a luminous object; the intensity of light at 2 will be one-fourth the intensity of the same at 1; at 3, one-ninth; and at 4, one-sixteenth. If *a* in the preceding body be conceived to stand for an opaque screen, having determinate square divisions—say one foot—and 2 3 4 other opaque screens, having the respective dimensions of four, nine, and sixteen feet, then at position 1 the one-foot screen will intercept all the light, at 2 the four-feet square screen, &c.

Law II.—When a ray of light falls on a reflective surface, the reflected and the incident ray are both in one plane. Thus, in the annexed diagram, *m i p* represents a reflective plane, *d* an incident ray, *n* a reflected ray, and *i* the point of impact; thus the rays *d* and *n* lie in one and the same plane.

Law III.—The angle of incidence and the angle of reflection are equal. Thus, referring to the subjoined diagram (Fig. 55), where *d i* represents the incident ray, impinging at *i* and reflected at *n*, the ray *d i* makes the angle with a line *b i* perpendicular to the reflecting plane *m i p*, which is equal to the angle made by the reflected ray *i n*.

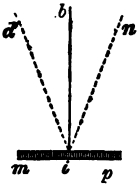


Fig. 55.

Law IV.—When a ray of light passes from one transparent medium into another of different density to the first, refraction ensues. If the density of the second body be more considerable than the density of the first, it is refracted in a direction towards a perpendicular to the plane of the refractive surface. If the order of relative density be reversed, the ray of light is refracted from the perpendicular plane of the refracting surface.

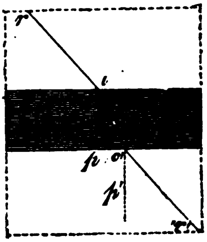


Fig. 56.

The annexed diagram (Fig. 56) is intended to illustrate the law just enunciated. The diagram represents two refractive media. The upper one is indicated by four dotted lines, including a rectangle; the lower one by four plain lines, also including a rectangular space. We may assume, for the conditions of our argument, the upper rectangular space to be filled with atmospheric air; the lower rectangular space corresponding with a block of glass. Assuming, now, *r* to stand for a ray of light passing in the direction *r i d' r'*, we remark that on entering the glass the ray bends towards the perpendicular *p*, because glass is the denser medium. On leaving the glass, the ray bends from *p'*, and assumes its original course. Hence, the law is satisfied. Hereafter we shall discover that a full appreciation of the action of lenses depends upon a previous cognizance of the law.

Many natural instances continually present themselves in exemplification of this law. The salmon-poacher well knows, that if he would succeed in spearing the fish which he sees lying on the bed of a river, he must not strike in the apparent direction of the object, but he must make allowance for the refraction of light caused by the water.

A very common experiment, illustrative of the refraction of light, is performed with a basin containing some small object, which latter, for a certain position of the observer, is only rendered evident when the basin is filled with water. Thus, in the annexed diagram (Fig. 57), the basin is supposed to contain a boy's marble. The latter would be invisible to the eye at E by a ray of light passing in the direction of E K, although visible by the same ray when bent in the direction of O, as would be the case provided the vessel were filled with water.

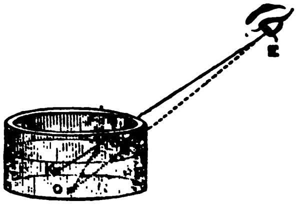


Fig. 57.

Far more important to the meteorologist is the following illustration of the law of luminous refraction by the atmosphere. We have already seen that the atmosphere is to be considered physically as made up of concentric shells of elastic matter of varying density. Practically, then, each successive atmospheric layer (to assume the existence of layers where the blending is complete) has a different refractive power for a ray of light.

The accompanying diagram (Fig. 58) is intended to represent the effect of atmospheric luminous refraction. For the purpose of illustration, three atmospheric shells or zones are depicted. Let us now trace the refractive effects of these zones on a ray of light.

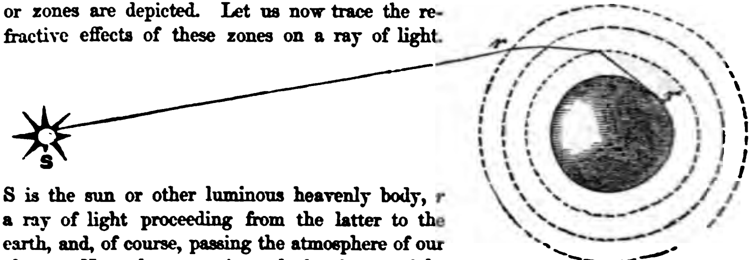


Fig. 58.

S is the sun or other luminous heavenly body, *r* a ray of light proceeding from the latter to the earth, and, of course, passing the atmosphere of our planet. Now the ray *r*, instead of going straight on as it would have done if the transparent medium—the atmosphere—were of one uniform unvarying density, takes the course *r r'*. The outermost zone of air refracts it a little; the second zone being denser, refracts it still more; and the third, or innermost zone, denser still, effects a third and final amount of refraction; so that the sun would be rendered visible to an observer at *r'*, although really below the horizon.

Chromatics.—Hitherto, light has been treated of without reference to colour; but certain phenomena of light, involving the production of colour, are especially interesting to the meteorologist. The rainbow—the tint of clouds, and aurora or morning dawn—the manifold hues of sky and sea, may be cited as familiar examples.

To the immortal Newton we are indebted for the first consistent theory of colours; a theory which, slightly modified, is generally accepted at the present time. The fact is almost too familiarly known for comment, that Newton effected the decomposition of light by means of a triangular prism, causing a white ray to split into coloured rays, and demonstrating that white light consisted of the prismatic colours blended together.

According to Newton, the primary or prismatic colours were seven, as follow:—1. Red; 2. Orange; 3. Yellow; 4. Green; 5. Blue; 6. Indigo; 7. Violet;—violet rays being most, and red rays least refrangible. Subsequent experimenters, however, have reduced the number of primary rays to three,—namely, *red*, *blue*, and *green*. This demonstration cannot be satisfactorily arrived at by prismatic decomposition; but by employing glasses of different colours, and absorbing certain tints of light, the demonstration is easy.

The subject of meteorology scarcely demands that I should enter upon the discussion of spherical or chromatic aberration, still less on the consideration of double refraction and polarized light. I shall, therefore, conclude this short theoretical exposition of the functions of light by presenting a summary of the arguments for and against the corpuscular or emissary, as well as the undulatory theory.

It has already been remarked that the wave theory of light—*i.e.*, the undulatory theory—originated at a period anterior to the corpuscular theory introduced by Sir Isaac Newton. The great objection to the wave theory was this:—if light be the result of waves, it was argued, how is it that light will not travel round a corner after the manner of sound? which latter was known to be the result of vibrations in the air or other medium. This question was deemed to present an impossibility to the comprehension of the advocates of the wave theory of light. Curiously enough, the modern philosopher may accept the corner-test as one of the strongest proofs in favour of the wave theory of light.

As the most striking similarity prevails between light and sound in many of their relations, let us establish a comparison between the two, in respect of turning a corner. Every one knows that sound *can* turn a corner; but the fact is scarcely less patent that, in the act of turning a corner, a portion of the soniferous impulse is deadened or lost. The proof of this assertion is so common, that we scarcely require the performance of an artificial experiment. Who is there who has not noticed the sudden deadening of the rumbling noise of carriage-wheels, immediately the carriage has turned to the right or left down a side-street? Who that has been present during artillery practice—sometimes standing near the muzzle of a cannon, sometimes behind it—has not been made cognizant of the difference between the violence of the report of the cannon's discharge? Thousands of instances of this kind must have presented themselves to everyone.

The following experiment simply, yet satisfactorily, illustrates the same propositions. If a tuning-fork be struck and held at arms-distance from the ear, its prevailing note will be heard with a certain distinctness. If, now, a card be interposed between the tuning-fork and the ear, the former may be brought very near the latter without making the listener cognizant of a sound equal in intensity to that which he heard when the tuning-fork was more distant, but when no card intervened. Indeed, whatever test be devised for demonstrating the deadening of sound by necessitating its travelling round a corner, the result is invariably of one kind—it is affirmative of the proposition.

In endeavouring to establish the existence of a similar property in respect of light, due allowance must be made, of course, for the difference of degree between sonorous and luminous vibrations. I shall very soon demonstrate that, whatever may be the cause of light, each particular colour corresponds to a certain size of *something*, either of wave or of particles. Assuming these measures to be the measures of waves, then we are in the condition to prove that the difference between the size of sound-waves and of light-waves is enormous; and being enormous, we cannot expect that light should be able to turn a corner to a similar extent and with the same facility we find sound to do. But that it can turn a corner within certain limits is unquestionable; therefore, we establish an analogy in an important particular between light and sound, and furnish an answer to the argument which Newton thought to be unanswerable.

The simplest demonstration of the fact that light turns the corner is furnished by the phenomena of shadow. What artist is there who does not perfectly well know that the edge of a shadow is somewhat more illuminated, or less dark, than the centre of the shadow? and granted that the fact be so, does it not prove that light and sound, in the matter of turning a corner, present a complete analogy?

Another illustration, not so familiar as the last, but equally expressive, is this:—If a minute perforation be made in a card, of course a certain amount of light can be caused to pass through the perforation. If, now, an object be viewed through the aperture, the object will be represented magnified, just as it would if a magnifying lens were fixed in the aperture. Now, on the assumption of the passage of light through the aperture, merely subjected to the law of diminished intensity, inversely as the square of the distance, we cannot account for this magnifying power; but if we grant that the light, by friction (to use a comprehensible but not unexceptionable term) against the edge of the aperture, is bent outwards, then the result is accounted for.

Interference of Light.—Some of the most beautiful arguments in favour of the undulatory theory have reference to what is called the interference of light. What is meant by this term may be thus summarised. It is possible, by certain optical arrangements, to produce darkness by causing one ray of light to impinge against another ray in a certain distance; whereas, varying the distance, it is also possible to make the luminosity of the first ray more powerful by the extent of the luminosity of the second. How beautifully, how completely does this accord with the phenomena of musical sounds! Let the following experiment be performed by a person whose ear is moderately sensible to harmony, and the effect noted:—Let two musical instruments—wind-instruments are the best (and perhaps two organ pitch-pipes are preferable to any other)—be caused to produce simultaneously discordant notes. That there is discord the ear can hardly fail to recognize; but to recognize the cause of discord will require a little attention. The two series of pulsations will be heard clashing mutually against each other; all idea of musical tone will have departed; and even the sound, regarded as a mere noise, will have become less than the sum of the two sounds.

If, however, the two organ pitch-pipes be now set in harmony, as the musicians term it; and, more especially, if one be set to produce a tone one octave above the other, and both simultaneously sounded; not only will the result be harmonious, satisfying the musician's ear, but the power of the sound will be equal to the sum of the power of the two notes.

Now the scale-value of musical notes can be demonstrated to correspond with, and be dependent upon, waves of definite size for any particular medium; and it admits of demonstration that musical harmony depends on an accordance between sonorous vibrations; moreover, that discord results when soniferous waves clash in different phases of their vibrations. All this, however, will be more evident if we seek an illustration in a case involving the production of visible waves; as, for example, the illustration furnished by the waves which can be made to arise on the surface of water. If a stone be thrown upon the surface of water in a pond previously unruffled, a series of concentric waves will be developed, originating in the centre, or point of impact of the stone, and extending outwards concentrically. If, now, a second stone be thrown on the surface of the pond, evidently a second series of concentrically expanding rays will be produced, which, meeting the first series, will give rise to one of two opposite effects, according to the manner in which the waves strike each other. If the crest or swell of one wave happens to correspond with the crest or swell of a second, then the two will blend, and the result will be a wave larger than either. If, however, the crest of one wave happens to strike the depressed curve of another, the result will be a diminution of the size of both waves—nay, the absolute destruction of both, provided they were originally of precisely the same dimensions.

The meaning of the expression, "*correspondence of phase of vibration,*" will now be evident. If the crests of two waves strike and coalesce, both curves pursuing the same direction, they are described as meeting in the same phase of their vibration; if they meet, the curves of each wave pursuing poposite directions,—that is to say, one rising while the other is falling,—then the waves are said to meet in opposite phases of their vibration.

I have selected the surface of water as furnishing a visible illustration of wave interference, in every way comparable to the interference of sonorous waves as demonstrated; and to the interference of luminous waves as assumed on the strongest grounds of probability.

Let us proceed now to develop the principles from the consideration of which the size of an unseen wave may be demonstrated. Sonorous waves are of this kind.

Firstly, supposing atmospheric air to be the medium of soniferous waves, we require to know the velocity of sound through this medium. This has been determined to be about 1,120 feet per second at a mean temperature and pressure. Considering then that the velocity with which sound travels has been determined, and that each particular tone of the musical scale corresponds with a determinate number of undulations or vibrations in a given tone, we may ascertain the dimensions of sonorous waves corresponding with any particular tone, provided we know the number of vibrations for any given tone. M. Savart accomplished this by a very ingenious instrument, the construction of which will be best introduced by the following reference to a circumstance frequently occurring. Perhaps it may have happened to the reader that on some occasion when briskly passing along near a long range of iron or wooden railings, he has unconsciously touched them with the end of his cane, which necessarily will have struck each bar of the railing with lesser or greater amount of velocity, according to the rate at which the pedestrian may have been walking along. Now it will scarcely fail to have been noticed that for every degree of rapidity of impact, there will have been produced a different sound, the tone becoming more and more shrill in proportion as the velocity of impact is greater.

On this principle is founded the instrument of M. Savart—a spiked wheel, capable of being set in motion with a determinate velocity. Inasmuch as this machine furnishes a known velocity for a known sound, we have two of the data for ascertaining the size of a sonorous wave; the remaining datum is the velocity of sound, already mentioned. The size of a sonorous wave, corresponding with any particular musical note, may now be determined by application of the subjoined formula:—

Let $S =$ velocity of sound per second,
$N =$ number of vibrations per second necessary to produce any given note,
$W =$ length of wave corresponding to that note;
then $\frac{S}{N} = W$.

From the application of this formula are determined the lengths of organ-pipes, corresponding to different notes, as given in the following table:—

NUMBER OF VIBRATIONS PER SECOND PERFORMED BY WAVES OF AIR CORRESPONDING TO CERTAIN MUSICAL NOTES, AND LENGTH OF THE RESPECTIVE WAVES.

Notes of the Organ.	Length of Pipe.	No. of vibrations per second.	Length of Wave.	
Lowest C	32	16	70	or $\frac{1120}{16}$
C^1	16	32	35	or $\frac{1120}{32}$
C^2	8	64	17·5	or $\frac{1120}{64}$
C^3	4	128	8·75	or $\frac{1120}{128}$
C^4	2	256	4·375	or $\frac{1120}{256}$
C^5	1	512	2·1875	or $\frac{1120}{512}$

Determination of the Size of Sonorous Waves.—Having illustrated some of the phenomena of waves by reference to a surface of water, and shown in what manner the size of sonorous waves may be determined let us now proceed to apply a parallel course, of reasoning to a determination of the size of luminous waves, corresponding with any particular colours,—assuming, of course, as we are obliged to assume, that the sensation of light be referable to the existence of waves. The accompanying diagram (Fig. 59) will illustrate the manner by which this can be effected: it represents a plano-convex lens, laid with its convexity downwards upon a flat glass surface. If this be done, and if the two be pressed together with a certain degree of force, the space of air lying between the flat glass and the convexity of the lens at every point, with the exception of the centre, will be tinted with the primary colours. The outermost band of colour, represented in the diagram by the letters *a a'*, will correspond with red light; and the innermost or central band, circumscribed around *b'*, will correspond with violet light; and between the two all the prismatic tints will appear in their ordinary scale of gradation. Let us contemplate the beautiful deductions which flow from this simple experiment. Firstly, it is evident, that for each coloured band there is a corresponding definite thickness of air; so that whatever the cause of light may be—whether particles, or waves, or anything else—the space corresponding with each particular colour is the measure of that on which the colour depends, to the same extent that the cleft between two rocks, into which a fish has become fixed, is a measure of the size of the fish: so if we can ascertain the size of this aerial space for any given point, we can ascertain the size of the cause of light; and granted that waves of different dimensions are the cause of different colours of light, we can speak confidently of the size of each particular wave. These measurements do not admit of being determined by rough direct measurement of rule and line; but they can easily be measured by obvious trigonometrical calculations, inasmuch as the curve of the lens is the solid formed by the rotation of an arc, of a circle of known diameter.

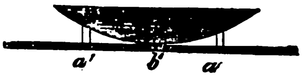


Fig. 59.

Knowing the velocity of light, and knowing that it travels at the rate of 190,000 miles per second, we are now in a position to determine that a wave of the extreme red of the solar spectrum has a length of ·00000266th part of an inch, and that it vibrates 458 million times per second; that a wave of extreme violet light has a length of

·00000467th part of an inch, and accomplishes 727 millions of vibrations in a second. The steps of the calculation, as will be observed, are the exact parallels of the steps already given above for the calculation of the size of sonorous waves, and are based on a knowledge of the velocity of light per second, and the actual space corresponding to each particular tint. Inasmuch, however, as the subject of light is somewhat abstruse, perhaps the following parallel cases will render the train of reasoning, by which the size of the waves of light are determined, more obvious:—

PARALLEL CASES.

Light travels at the rate of 190,000 miles per second.	A man travels at the rate of 60 yards per minute.
Length of waves of red light, ·00000266th part of an inch.	(Assumed) length of the man's strides, 1½ yard each.
Query.—How many vibrations does a ray of red light make per second?	*Query.*—How many strides does the man make in a minute.
Formula for solving the query.	*Formula for solving the query.*
$\frac{\text{Velocity of light per sec.}}{\text{Length of wave of every colour}} = \text{No. of vibrations per sec.}$	$\frac{\text{Velocity of man per min.}}{\text{Length of stride}} = \text{No. of strides per min.}$
Or,	Or,
Answer.	Answer.
·00000266)190,000(458 millions of vibrations per second.	1½)60(40 strides per minute.

Atmospheric Decomposition of White Light.—Coloured light has been demonstrated to be a component of white light; and, indeed, there is very little white light in nature. Owing to the various decomposing agencies to which light is subject, the result is generally coloured. The tints of natural objects are generally determined by their quality of luminous absorption. If a surface absorb red and blue light, merely reflecting yellow rays, then the tint of the body in question will be yellow. If it absorb yellow and red, merely reflecting blue rays, then the tint of the body will be said to be blue; and so on for the remaining case. An absolutely white body should of course reflect the rays of every colour, but absolute whiteness is rare. An absolutely transparent body, again, should transmit rays of all colours; but this absolute transparency can be scarcely said to exist. Even the atmosphere, transparent though it seem to be, obstructs much light, though the blue colour of the atmosphere is not owing to the blue colour of its particles, but to the reflection of blue rays.

Water, again, is said conventionally to be transparent; but we have only to look through a mass of water, or to look upon the surface of a mass of water, to be convinced that this liquid has the property of absorbing some colours more than others. The consequences which flow from the imperfect transparency of the atmosphere are very curious and important. If it were perfectly transparent, the heavens would appear black to us, and the heavenly bodies would be seen brilliantly shining as if in a framework of jet. By its absorptive power the atmosphere becomes to a certain extent visible; the light of the heavenly bodies is mellowed and softened down; and the beautiful phenomena of twilight and morning dawn are determined. Were the atmosphere perfectly transparent, the surface of the earth would be illuminated in a way totally unadapted to our necessities. Wherever the direct rays of the sun might fall, the surface would be illumined with a blaze of light; but all other spots, provided light did not chance to be reflected upon them, would be quite dark. Aeronauts, and travellers who ascend elevated mountains, confirm the indications of theory relative to the agency of the atmosphere in diffusing or scattering luminous rays. In proportion as the elevation above the earth's surface is greater, so does the sky appear more dark.

Magnetism.—The magnetic-needle, when freely poised on a pivot or freely suspended, is known to assume a directive tendency; and that tendency is popularly described as being north and south. Actually, however, the direction of north and south is only correct for certain parts of the earth's surface. In by far the greater number of places, the line of true north and south is more or less departed from. In certain localities the departure is very great; thus, for example, in Greenland the magnetic-needle actually points east and west; and Parry even found that in one part in the west of Greenland the north pole of the magnetic-needle actually turns to the south. In France the magnetic meridian, or line in which the freely-suspended needle comes to rest, is nearly coincident with the astronomical meridian, only differing from it by 22° towards the west; thus giving rise to what is called magnetic declination, or variation. A few centuries ago there was an absolute coincidence between the magnetic and the astronomical meridians—there was, in point of fact, no magnetic deflection or variation.

Independently of its northern and southern directive tendency, the magnetic-needle is subject to the influence of what is called the magnetic-dip. The dipping-needle consists of a magnetic-needle poised in such manner that it can move in a vertical plane, when the influence of dip will be rendered manifest in a way best illustrated perhaps by the following case:—Premising that a new magnetic steel-needle may be readily magnetized by contact with a magnet, it is evident that such a non-magnetized needle may be so poised on a pivot, or suspended by a string, that it shall lie in a perfectly horizontal direction. If, whilst lying thus, it be suddenly magnetized by contact, and if the operation be performed anywhere in the northern hemisphere, the horizontality of the magnetic-needle will be immediately departed from, and the northern extremity or pole will be depressed. If the experiment be performed anywhere in the southern hemisphere, then the horizontality of direction will also be departed from; but it is the southern extremity now of the magnetic-needle that will be depressed. The amount of the depression or dip is various for different parts of the world. At a certain line, not quite correspondent with the terrestrial equator, the needle lies horizontally—there is no dip; this line corresponds with what is called the magnetic equator. The average amount of dip for this part of Europe is about 70° north. Proceeding towards the north, the dip continually increases; and it attains its maximum at the north magnetic pole, which, however, is not coincident with the terrestrial north pole, but some distance removed from it.

As the northern magnetic-dip goes on increasing as we pass towards the north, so does the southern magnetic-dip go on increasing when we travel in the opposite direction; but the magnetic phenomena of the southern hemisphere have not been so accurately studied as the phenomena of the northern. Here it should be remarked that some confusion has arisen for want of accurately defining the meaning of the term magnetic pole. Sometimes it has been held to signify the two points of greatest magnetic energy of the whole earth, independently of the collateral energy due to the operation of local causes; at other times it has been held to be synonymous with the point exhibiting the greatest influence of these local causes. Adopting the latter idea, some writers describe two northern magnetic poles—one in Siberia, the other in North America; but this seems an unphilosophical application of the term, polarity. Accepting the term magnetic pole in its first sense, it will be easy to see that one of the northern magnetic poles, at least, corresponds with the point of greatest cold.

Variations of Terrestrial Magnetism.—These may be divided into regular and irregular. The former are dependent on determinate causes; and though the causes of the latter be not determinate, analogy furnishes us with a plausible explanation of them. Referring to magnetic variations of the determinate kind, they will be found to present a correlation with variations of temperature. Some time during the morning the north polar extremity will have attained its greatest variation towards the east, and shortly past noon it will have deviated towards the west of its normal meridian; it then returns eastward again, and about midnight it will have assumed a deviative position almost similar to that which it had in the morning. These oscillations have not the same amplitude at all times of the year; they are greater in summer than in winter—greater on a warm and cloudless, than on a cold and clouded day: circumstances which point to the thermal origin of the variations.

The amount of dip also manifests horary variations; a circumstance which leads to the inference that the magnetic poles are not fixed spots like the geographic poles of the earth, but are subject to deflections. This is exactly what should result, if the thermal theory of terrestrial magnetism be adopted.

The regular variations of the magnetic-needle are so clearly referable to the normal effects of solar heat operating upon the surface of our globe, that philosophers have universally agreed to refer them to that cause. But the influences of solar heat are occasionally abnormal; besides which, specialities of locality, the effect of casual flows of tepid oceanic water, the influence of volcanoes, &c., furnish a sufficient basis for an *a priori* assumption that other magnetic variations besides those which admit of being predicted will come into operation. Hence, other phenomena of magnetic aberration will be determined—they have been studied more especially by M. Gauss; and their dependance upon what may be termed an abnormal-thermal condition of our globe is rendered the more probable that they are coincident with irregular variations of the barometer.

Three systems of lines are employed to represent on charts the three different values of magnetic declination, inclination, and intensity. These lines were denominated by Humboldt *isogonic*, *isoclinic*, and *isodynamic* lines.

Isogonic lines are such as connect those parts of the earth possessing equal magnetic declinations; and charts are formed in accordance with these lines, called declination charts. Inasmuch, however, as the amount of magnetic declination for any one place is never permanent, but continually changing, these charts are only reliable for short successive periods, and require to be frequently altered. There are, nevertheless, a few points on the earth's surface where the amount of declination is permanent, or very nearly so; the most remarkable of these are Spitzbergen and the wertern parts of the West Indies. The most important and most remarkable of all the isogonic lines is that coinciding with places where the magnetic and astronomical meridians exactly coincide; and where, consequently, the needle points due north. In the year 1657, this line passed through London, and two years later through Paris.

Isoclinic lines are such as connect parts of the earth which are characterized by the same amount of magnetic inclinations. Charts of these lines exist under the name of inclination charts. The most important of these lines is the one corresponding with the magnetic equator; and at which, as before explained, the dipping-needle has no inclination. The magnetic and the terrestrial equators cut, as was before explained, and the points of intersection continually vary. In 1825 one point of intersection of the two equators occupied a position near the Island of St. Thomas, on the western coast of Africa, being distant about 188½° from the South Sea node; between 1825 to 1837 the former node shifted 4° westward. On the Brazilian coast the magnetic is situated 15° south of the terrestrial equator. About one-fifth of the magnetic equator cuts the surface of the ocean.

Places at which the intensity of magnetic force is equal are said to be isodynamic, and on charts are represented as being cut by isodynamic lines. Generally the intensity of magnetism may be said to increase from the equator to either pole; nevertheless, isodynamic magnetic lines neither run parallel to the magnetic nor to the terrestrial equator. The minimum of intensity occurs near the coast of Brazil. The maximum of magnetic intensity is about twice the minimum.

Cause of Terrestrial Magnetism.—Reference has already been made to the influence of heat in causing perturbations of the magnetic-needle. We shall presently find that the very existence of terrestrial magnetism is ultimately referable to heat.

When treating of the thermoscope of Nobili, some mention was made of the function of thermo-electricity, and the conditions necessary for bringing it into operation. It will now be desirable to discuss the properties and conditions of this function more closely.

Nothing can be more certain than the correlation which subsists between heat, electricity, and magnetism. It is impossible to produce an elevation or a depression of temperature without producing at the same time a manifestation of electrical excitement. This will be demonstrated hereafter under the head of Electricity. For present purposes it will be sufficient to establish the law of electro-magnetic excitation, and to show in what manner it is in some cases ultimately referable to the operation of heat. Long before the precise dependance of magnetism upon electricity was known or suspected, the connection subsisting between them was assumed. Pieces of steel were frequently rendered magnetic during thunderstorms; the polar tendency of magnets already existing was reversed by the same cause; and at a later period in the progress of electrical experiment the discovery was made that a small steel-needle might be converted at pleasure into a magnet by subjecting it to the influence of a wire helix, acting the part of an electrical conductor.

The experiment in question is very easy, and as follows:—Within the coils of a wire helix (see Fig. 60) a needle, enveloped in paper or some equivalent imperfectly-conducting material, is laid. An electric discharge then being transmitted through the conductor, and the needle removed, it will be found to have acquired magnetic properties, indicated by its quality of attracting iron-filings at either end; by its north pole or end repelling the north pole or end of a known magnet, attracting the opposite, and *vice versa.*

Looking at the conditions of this experiment, it will be perceived that, whatever connection between the producing electric current and the produced magnet there may be, the electricity has been acting tangentically to the long axis of the needle. This is an important point, for we shall hereafter see that the same tangential relation continues to subsist in all subsequent experiments resulting in the formation of electro-magnets.

Common or frictional electricity does not furnish the operator with a means of proceeding far in his examination of the relations subsisting between electricity and magnetism. The aid of voltaic electricity is required, as in the following experiments:—If a metallic wire be twisted into a helix, as represented in the following diagram (Fig. 60), and a bar of iron, previously enveloped in paper, inserted within the helix; if now a current of voltaic electricity be transmitted through the wire, the iron bar will for the time-being be converted into a magnet. This experiment is, indeed, similar to that already indicated, with the exception that an iron bar instead of a steel needle is employed; the fact being that, though steel, when once rendered magnetic, retains the magnetic influence with much permanence, iron is more readily amenable to the same influence, and is therefore commonly employed in these and similar experiments. Instead of using a plain wire, it is better in practice to employ a wire covered with flax, or cotton, or silk fibre, or with a layer of gutta-percha, when the necessity which previously existed of enveloping the bar in paper ceases. Coated wire of this description is extensively prepared by manufacturers of gutta-percha.

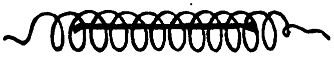


Fig. 60.

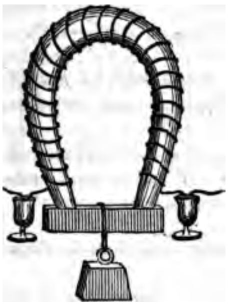


Fig. 61.

The influence of voltaic electricity in determining magnetism is still more evidently manifested when the iron bar, instead of being straight, is bent into the horse-shoe form as represented in the accompanying diagram (Fig. 61); and the power of the developed magnet is greater if the number of helical turns be increased. Usually, therefore, in practice, several wires are soldered together at either extremity, as represented in the following woodcut (Fig. 62). In all these instances, the circumstance will not fail to have been noticed that the magnetic power is developed at right angles, or tangentically to the direction of the passing electric current.

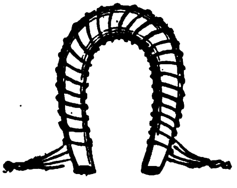


Fig. 62.

Voltaic Conducting Wires are themselves Magnetic.— If a wire conducting voltaic electricity be brought in proximity to iron-filings, it attracts the latter just as a magnet would do; thus begetting a strong *a priori* assumption that such conducting wire, for the time being, is itself magnetic. Now, if the assumption be well founded, of course it should exercise the usual influences of attraction and repulsion when a freely-poised or freely-suspended magnetic-needle is brought into its vicinity. Such influence is found to be manifested; and thus we shall presently see that all the hitherto anomalous tangential influences are lucidly explained. Firstly, let us not discard the illustration presented by the attraction of iron-filings with the conducting wire, without deducing from these phenomena an important corollary.

Assuming A B (Fig. 63) to represent such a wire, and assuming, as is really the fact, that the iron-filings are attracted all round the surface of the wire, it is evident that the magnetic polarity is developed in lines at right angles to A B —that is to say, in the direction of *c d;* whence it follows that such conducting wire is a magnet at right angles to its length.

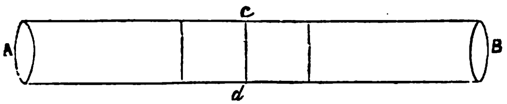


Fig. 63.

This point is very strikingly illustrated by the following contrivance:—Z C (Fig. 64) are respectively plates of zinc and copper, communicating with a wire in such manner that a current of electricity passing from one plate to the other must necessarily pass along the wire. Now, the instrument here represented may be caused to swim, by means of a cork-float, in a basin of dilute acid, when it becomes an active voltaic combination; and, inasmuch as the liquid support admits of the free motion of the instrument, allowing it to turn in any direction, the vertical plane cut by the wire ring should, provided magnetism be developed, assume the direction of the magnetic meridian, *i.e.* north and south. This it is found to do—again illustrating the proposition, and more plainly than before, that a wire conducting voltaic electricity is magnetic at right angles to its long axis. Not only does this remark hold good, but the two magnetic polarities have a definite relation to the transverse section of the wire; one definite side of the plane always pointing north, whilst the other necessarily points south. If the conducting wire, instead of being bent into a mere ring, be twisted helically, as represented in Fig. 65, the magnetic conditions developed will be still more evident.

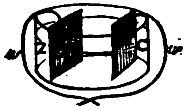


Fig. 64.

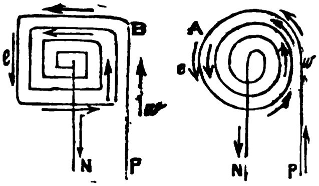


Fig. 65.

This was a form of wire devised by Amperè, for the purpose of demonstrating the magnetic character of wire in the act of transmitting electric currents. The forms of apparatus in question may be delicately suspended or converted by means of pieces of cork and metallic plates into floating arrangements. Thus arranged, the flat spirals will always arrange themselves in the direction of north and south; one definite side of the coil always corresponding to one invariable direction of the electric current. In conformity with principles already enunciated, each coil of the flat helices may be regarded as a separate magnetic pole.

If, now, we carefully investigate the law of this developed magnetism by means of a freely-poised magnetic-needle, it will at a first glance appear to be altogether anomalous. Thus, if a voltaic conducting-wire be held in successively different relations to a magnetic-needle, the needle will seem to be deflected according to no recognizable rule; but further examination demonstrates not only the existence of a law, but demonstrates the seemingly irregular motions to be still due to the operation of the tangential force already described.

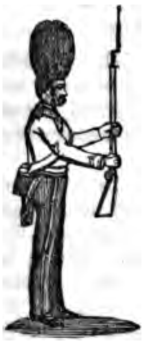


Fig. 66.

The following simple rule will, under all circumstances, serve to fix the direction of the polarity of a magnet developed by electrical agency in the mind:—Let the reader assume that he, himself, is the electrical conductor; that the one fluid theory of electricity is adopted; and, finally, that the current passes in at his head and emerges at his feet. Let him now conceive that he holds in his hand, and directly in front, a freely-poised magnetic-needle, under which circumstances the north pole of the magnet would always be deflected towards his right hand. In accordance with these facts, and in illustration of them, the following diagram (Fig. 66) has been devised. It is a soldier, who holds a musket in his hand; the bayonet of which musket is assumed to stand for the north magnetic pole. Supposing, then, a current of electricity to pass in at his head, and emerge at his feet, the bayonet would invariably be directed towards his right hand.

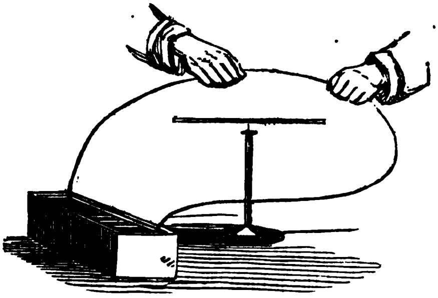


Fig. 67.

We have next to consider the effect of causing an electric current to encircle a freely-suspended magnetic-needle in a plane coincident with its axis, as represented in the accompanying diagram (Fig. 67). Reflection on the circumstances of this case will render manifest, that each part of the wire thus arranged will tend to produce the same general result—namely, deflection of the needle to a position at right angles to the vertical plane in which the wire lies.

If instead of one turn the wire be caused to encircle the magnetic-needle twice (Fig. 68), then the needle will be deflected with an energy double that effected by the previous combination; and *generally* in proportion as the number of coils is greater, so will the deflecting power be more considerable.

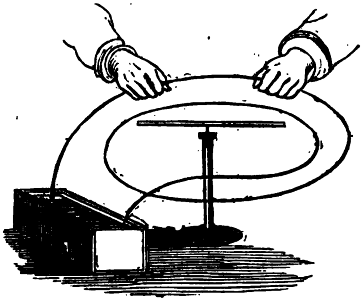


Fig. 68.

On an appreciation of these principles depends the instrument termed the galvanometer, which teaches the force of a voltaic current in motion, by causing a deflection of a greater or lesser axis of the suspended magnetic-needle.

The galvanometer, in its simplest form, is given at page 277, Fig. 30. It will be seen to be nothing more than a close mechanical adaptation of the principles developed in the two preceding operations. By increasing the number of coils in a galvanometer, it necessarily follows that its power of deflecting a suspended magnetic-needle will be increased also. Accordingly, delicate galvanometers are always formed with a compound coil, and are, moreover, covered by a glass shade, as represented at page 277, Fig. 31.

In all these experiments I have assumed, for the sake of not complicating the matter needlessly, that an ordinary single magnetic-needle has been employed. The use of such a needle, however, is attended with this important disadvantage,—namely, the earth's magnetic tendency is a force to be overcome by the magnetic energy artificially established. For example, the tendency is, as we have constantly seen, that a freely-poised magnetic-needle shall place itself at right angles to the direction of a passing electric current. If then the electric current should be caused to pass in the direction of north to south, the magnetic-needle should, in accordance with the principles developed, arrange itself east and west. Such is the tendency, and such is the direction the magnetic-needle would assume, provided the voltaic current be sufficiently powerful; but it is not difficult to conceive a case where, the electric current being weak, the natural directive tendency of the needle—*i.e.* north and south—in obedience to the earth's magnetic influence, should overpower it. To obviate this interference, the astatic needle has been devised. The astatic needle may be described as being a double magnetic combination of two needles mounted on one pivot, the north pole of one needle being opposed to the south pole of the other, as in Fig. 69.

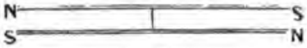


Fig. 69.

If the two magnets be of exactly equal power, then, evidently, the compound instrument would possess no natural directive tendency. In practice, this absolute balancing of forces is undesirable; therefore, usually one of the needles is rather stronger than the other, so that the slightest possible amount of directive tendency may be maintained.

Cause of the Directive Tendency of the Magnetic-Needle.—The experiments already described, and the principles deduced from them, furnish a rational explanation of the directive tendency of the magnetic-needle. Firstly, it is granted that any variation of temperature always develops a current of electricity. This proposition the reader will accept as authority for the present; but, hereafter, under the head of electricity, the demonstration will be made plain. Secondly, it is granted that wherever there is an electric current set up, there will always be a magnetic energy developed at right angles to the electric current. If we now assume, as the proximate cause of the magnetic-needle's north and south directive tendency, that it does so because the earth itself is a magnet in the direction of north and south, we have only to discover the cause of an electric current at right angles to this direction, and the mystery is explained. Now it is evident that our globe is diurnally heated in the direction of east to west by the sun's rays; whence, according to the result of artificial experiments, there should also exist an electric current in the same direction; and, this being so, the earth itself becomes a vast magnet, the one pole of which is northern, and the other southern.

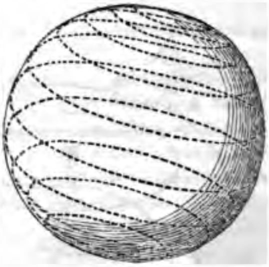


Fig. 70.

In illustration of this, the following experiment has been devised:—A hollow paper globe (Fig. 70) has been lined internally with a revolving copper wire, so arranged that it should serve as an electrical conductor. If electricity be passed through this copper wire, and a freely-poised magnetic-needle be placed in various successive positions on its surface, all the variations of deflection and dip which naturally occur may be indicated in the clearest manner imaginable.

Dia-magnetism.—For a long time it was imagined, and even in this day the idea is popularly entertained that iron alone, of all bodies, is susceptible of magnetism. Experimenters eventually admitted that the metal nickel participated with iron in the property in question, and the notion began to be entertained that yet other bodies might be included in the same category. It was by no means easy, however, to subject the opinion to the test of experiment, considering the known difficulty experienced in banishing every trace of iron from the materials operated upon. Magneticians even argued that the magnetism of nickel might be only apparent, the quality being attributable to the presence of iron. At length M. Biot set these doubts at rest definitively. He caused some nickel to be freed from all traces of iron by that distinguished chemist

M. Thenard; he then caused magnetic-needles to be made of this nickel; he not only determined the existence of their power of attraction, but their polar directive tendency; and he finally discovered the ratio of their polarity by comparison with an ordinary steel magnetic-needle. On the result of his inquiries he established the fact, that the directive force of the nickel was about one-third of that of the steel needle. The needles with which he conducted his experiments were eight inches in length by two-tenths of an inch wide, and the weight of each was about five grains. M. Cavallo followed in this line of demonstration, by proving that other substances besides nickel were susceptible of ordinary magnetism—brass, for instance; especially brass rendered hard by hammering.

The term *ordinary* magnetism requires to be explained, and the explanation will at once introduce the phenomena of *dia-magnetism*. The property of being subject to magnetic influence long known to be manifested by iron, and subsequently proved to be participated in by nickel and brass, is perhaps universal; but the kind of influence differs. Assuming iron to represent the normal kind of influence, let us consider what takes place if we suspend a needle of that metal in what is called the magnetic field—namely, the space between the two extremities or poles of a horse-shoe magnet. Under these circumstances, the iron needle would assume what is termed an axial position, one end being attracted to the north pole, the other end to the south pole of the horse-shoe magnet, as represented in the annexed diagram (Fig. 71).

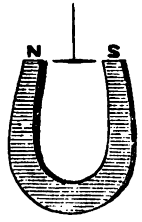


Fig. 71.

Now, M. Cavallo committed the following important mistake:—He fancied that he had demonstrated all the bodies proved by him as being magnetically endowed, to be magnetic in the same sense as iron and nickel are magnetic; that is to say, that supposing a needle of either of these bodies to be suspended in the magnetic field, one end of the needle should be attracted towards the north pole, the other end to the south pole of the magnet. If such were the case, the function now to come under notice as the function of dia-magnetism could have no existence. Before entering more into detail concerning the properties and functions of dia-magnetism, it must be premised that the position which an iron needle naturally assumes when hung in the magnetic field is said to be an *axial* position, or it is said to arrange itself *axially*. If it were to assume a position at right-angles to the same, its position would then be described as being *equitorial*. Now, M. Cavallo believed that all bodies endowed with a magnetic tendency, in any degree, would manifest that tendency by assuming the axial position when freely suspended between the poles of a horse-shoe magnet. This, as I have already remarked, is an error: some bodies arrange themselves axially under these circumstances, and are therefore said to be magnetic; whilst others arrange themselves equatorially, and are said to be dia-magnetic.

The knowledge of *dia-magnetism* may be said to have originated with Becquerel, though Coulombe and the Abbé Haüy had both laboured in the same direction. The investigation has been followed up with great ardour by Professors Faraday and Tindall in England, and Professor Plücker of Bonn, to whose published papers on this subject I must refer the student who desires further information relative to dia-magnetism than accords with the nature of this treatise. It must be remarked, however, that when bars are made of different substances, and submitted to the influence of the magnetic field, those of iron, nickel, and cobalt, point axially: most probably those of titanium, palladium, and platinum, are in the same category; but needles of all other metals assume the equatorial direction in the magnetic field, and are therefore *dia-magnetic*. The following table presents a list of magnetic and dia-magnetic bodies:—

MAGNETIC BODIES.		DIA-MAGNETIC BODIES.			
Iron	Cerium	Bismuth	Cadmium	Silver	Uranium
Nickel	Titanium	Antimony	Sodium	Copper	Rhodium
Cobalt	Palladium	Zinc	Mercury	Gold	Iridium.
Manganese	Platinum	Tin	Lead	Arsenic	Tungsten
Chromium	Osmium				

From a consideration of the preceding remarks, it will be observed that the functions of heat, magnetism, and electricity are intimately allied, more especially the two latter. Instead, therefore, of entering upon the discussion of meteorologic phenomena, due or attributed to magnetism, in this place, it will be desirable to present the reader with a short exposition of electrical science and phenomena.

Electricity. —*Definition and Derivation of Term.*—The term electricity is applied to comprehend a large class of phenomena, which are related in various ways with the operation of an invisible force, to which, founded on speculative considerations, the appellation *electric fluid* has been given; not that such fluid can be proved to exist, or that even it is at this time, by the majority of philosophers, supposed to exist, although, for the sake of convenience in illustration, the expression, electric fluid, is still popularly retained.

The science of electricity is one of the most recent; nevertheless, the primary phenomena on which the science is based are of very ancient date. Theophrastus and Pliny were aware that the substance amber, if rubbed with silk, flannel, &c., became endowed with the property of influencing the motions of certain light bodies, such as feathers, attracting them under certain circumstances, and repelling them under other circumstances; but there the investigation of this class of phenomena ended. About the middle of the last century, however, the subject was returned to, recommencing from the starting-point of Theophrastus and Pliny; and from the simple fact of the peculiar excitation of amber under certain circumstances of treatment, to build up the interesting and important science now known as that of electricity.

Development of Electricity.—The first development of electricity was accomplished by the friction of one particular substance—amber, as we have seen; but when the attention of modern philosophers was directed to the science, they soon found that amber only furnished one particular cause of a result far more general. Many other bodies, in addition to amber, were discovered which, on being submitted to friction, became electrically excited, or electrical; and to these the general term *electrics* was given. Furthermore, it was discovered that the bodies thus capable of electrical excitation were not capable of conveying away electricity; whence they were also called non-conductors of electricity. Great as was the advance thus made on the crude notions of Theophrastus and Pliny, it fell far short of the truth, modern electricians having proved that no real or functional difference subsists between conductors and non-conductors, only a difference of degree; consequently, bodies do not admit of division into the classes of non-conductors and conductors, except in a conventional sense, and as a matter of practical convenience. Experiments fully illustrative of the propriety of this view will be furnished hereafter.

Though friction be the earliest observed cause of developed electricity, and though it constitutes the principle on which the ordinary electrical machine is founded, nevertheless it is only one cause, and perhaps the least important, of those which the meteorologist has to take cognizance of as coming within the scope of his science. It is difficult to say what alteration of matter, chemical and mechanical, is *unattended* by the development of electricity. In all probability there is none of this kind, though it happens that in most cases special, and sometimes very refined, contrivances are necessary for rendering electrical excitation evident. A notable illustration of this is furnished by the hydro-electric machine, familiar now to many people by its exhibition at the Polytechnic. This machine constitutes the most powerful instrument for developing electricity by artificial means known; yet if the glass legs on which it stands were removed, the instrument would become inoperative, and the existence of all the vast force of electricity which it generates would remain unknown.

Having premised these general remarks concerning electricity, it will be desirable now, before taking cognizance of the operation of this force in nature, to present the student with the fundamental causes or propositions on which the science of electricity depends. In doing this, I shall avoid, as much as possible, having recourse to the electrical machine, or any complex electrical arrangements, which, though indispensable to the full illustration of secondary electrical facts, are rather perplexing than otherwise so long as fundamental principles alone are concerned.

Definition of the term Electric.—Any body which after having been rubbed acquires the property of attracting light substances, after the manner of amber, is an electric.

What bodies or class of bodies are Electrics?—Inasmuch as the act of friction will be involved in our experiments having reference to this demonstration, it necessarily follows that only one physical division of bodies—namely, solids—admit of being readily submitted to our notice; for though liquids and gases can be subjected to friction, yet the contrivances for effecting this, consistent with the requisite electrical demonstrations, are so complex that they cannot be taken cognizance of at present.

I shall assume that our present observations are limited to three bodies—glass, sealing-wax, and a metal; each, for convenience of manipulation, fashioned into the form of a stick. I shall assume, moreover, that the rubbers, or body wherewith friction is effected, are of flannel and of silk. By the employment of these simple materials some important results will be arrived at.

Experiment I.—If the stick of sealing-wax be briskly rubbed, and held at some distance (not too far) from a suspended feather, represented in Fig. 72, the latter will be attracted towards the sealing-wax—will attach itself to the latter: but the attachment will not be permanent. After a time it will leave the sealing-wax, and be repelled.

Experiment II.—If the previous operation be repeated with a stick of glass, the feather will be first attracted, and afterwards repelled, in a precisely similar manner as before. Hence, for aught we at present see, the kind of influence developed by friction on glass is similar to that developed by friction on sealing-wax.

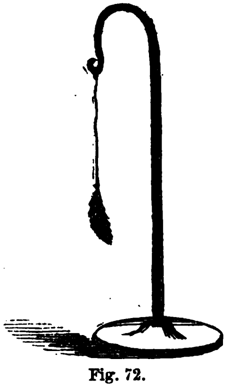


Fig. 72.

Experiment III.—Let the glass rod be excited by friction, and held near the feather as before. The feather will necessarily be first attracted, then repelled. If the rod of sealing-wax be similarly excited, and held near the feather, which has refused to be further attracted towards glass, it will, nevertheless, be attracted toward the sealing-wax, and *vice versa*. This experiment demonstrates that, whatever be the nature of electricity, this force is susceptible of two manifestations: it is, in point of fact, a *dual* or polar force, similar in this respect to magnetism. It admits of being demonstrated that, in either of the preceding experiments, the rubber or body wherewith friction is excited always assumes an electrical polarity different to that of the body rubbed.

Designation of the two Electric States.—As the two polarities of magnetic energy are designated respectively *north* and *south*, without which, or some equivalent designation, it would be impossible to describe the peculiarities of magnetic phenomena; so, in like manner, it will be necessary to designate the two electrical functions already proved to exist. Accordingly, the terms, positive and negative, or vitreous and resinous electricities, have been long employed. The words, positive and negative, have reference to the theory of Franklin, that all the phenomena of electricity depended upon the operation of one electric fluid. A certain class of electrical phenomena he assumed to depend upon an excess of this fluid—another, or opposite class of phenomena, to depend on a diminution of the same. The first class of phenomena he termed positive, the second negative.

The origin of the terms, vitreous and resinous, will perhaps have been anticipated from a consideration of the teachings of Experiments I. and II. Glass, it has been seen, when rubbed, gives rise to the development of one function of electricity, and sealing-wax of another. Now, glass and sealing-wax are, in this respect, only the types of all other bodies.

Conductors and Non-Conductors.—Referring to Experiments I., II., and III., three distinct stages of electrical condition may be observed. Firstly, the feather, before it has been subjected at all to the influence of the excited glass or sealing-wax, presented the condition of electric neutrality. Secondly, it presents two conditions of excitement, namely, attractive excitement and repulsive excitement. If, whilst the feather is under either of the latter conditions, it be touched with various substances successively, certain important results will be observed. If it be touched with the finger, all electrical excitation will be at an end. This effect is best demonstrated by touching the feather when in the repulsive phase of excitation. A similar result will ensue if, instead of the finger, the excited feather be touched with any metal or wood, or one of numerous other bodies to which the term electrical conductor is *conventionally* applied—I say *conventionally*, because the circumstance has been already indicated that the distinction between conductors and non-conductors is purely one of degree, not of kind. Nevertheless, in practice it is useful to divide bodies into electrical conductors, and electrical non-conductors. A table, representing this division, is appended. The terms non-conductor and insulator are, it is necessary to observe, synonymous.

CONDUCTING BODIES, PLACED IN THE ORDER OF THEIR CONDUCTING POWER.

All the metals.
Well-burnt carbon.
Plumbago.
Concentrated acids.
Dilute acids.
Saline solutions.
Metallic ores.
Animal fluids.
Sea water.
Spring water.
Rain water.
Ice above 13° Fah.
Snow.
Living vegetables.
Living animals.
Flame.
Smoke.
Vapour.
Salts soluble in water.
Rarefied air.
Vapour of alcohol.
„ of ether.
Earths and moist rocks.
Powdered glass.
Flowers of sulphur.

INSULATING BODIES, PLACED IN THE ORDER OF THEIR INSULATING FACULTY.

Dry metallic oxides.
Oils (the heaviest are the best).
Ashes of vegetable bodies.
Ashes of animal bodies.
Many dry transparent crystals.
Ice below 13° Fah.
Phosphorus.
Lime.
Dry chalk.
Native carbonate of baryta.
Lycopodium.
Caoutchouc.
Camphor.
Some silicious and argillaceous stones.
Dry marble.
Porcelain.
Dry vegetable bodies.
Wood that has been strongly heated.
Dry gases, and air.
Leather.
Parchment.
Dry paper.
Feathers.
Hair, wool.
Dyed silk.
White silk.
Raw silk.
Transparent precious stones.
The diamond.
Mica.
All vitrifactions.
Glass.
Jet.
Wax.
Sulphur.
The resins.
Amber.
Gum lac.

Gutta-percha is one of the most perfect insulators, but its exact place in the above table is yet undetermined.

The terms *conduction*, *non-conduction*, *insulation*, and, indeed, most other terms of electrical science, have reference to the idea of an electrical fluid or fluids; and, indeed, however much we are constrained to refuse our sanction to the probability of the existence of such fluids, nevertheless we cannot proceed far in electrical investigations without recognizing the convenience of many terms suggested by the assumption.

Division of Bodies into Electrics and Non-electrics untenable.—If the experimenter try to render a bar of metal electrical, as he succeeded in rendering a bar of sealing-wax and of glass respectively electrical, he would not succeed. The earlier electricians, having noted this result, termed the metals, and indeed *all* conducting bodies, non-electrics; but if the conducting property of the hand and of metals be considered, this division, so arbitrarily made, cannot fail to seem premature. No legitimate conclusion, as regards the electric or non-electric property of bodies, can evidently be arrived at until the body rubbed has been held by a non-conducting handle; for otherwise, even though electricity should be excited, it would readily pass away. If the experiment of rubbing a metallic bar be tried after such bar has been provided with a non-conducting handle, it is rendered electrical, its electrical excitation being made evident by the usual tests of attraction and repulsion. In conducting experiments of this kind, very satisfactory insulating handles may be made by enveloping one end of the bar to be operated on with a piece of gutta-percha, previously warmed by the fire, or softened by dipping into boiling water. The gutta-percha should be wrapped round and about the extremity of the bar, and trimmed whilst yet warm by a pair of *wet* scissors.

Induction.—In strict propriety of language, no one electrical function or set of functions can be referred exclusively to induction. Electricians of the last century were in the habit of thinking differently; they spoke of induction as if it were a function that might be exercised at will, whereas, in point of fact, no such exclusive electrical function exists.

If an insulated conductor, charged with either condition of electricity, be brought near to another conductor; the second conductor, if examined, will be found to be in the opposite electrical condition, which condition was sure to be induced. The term induced, though still employed, conveys a very different meaning to that formerly accepted; but a discussion of this point is hardly consonant with the requirements of this treatise; wherefore I must direct the reader who would know more concerning it to the volume of this series on Chemistry.

Electrical Generalizations.—In the few remarks on the imponderable agents already made, it is not proposed to present the reader with more than a faint outline of their general nature and correlations. All-important though they be to a meteorologist, that importance is paradoxical, though the statement may serve as a sufficient justification for treating very casually on them in a treatise like the present. The objects of meteorology are so numerous, and its topics so varied, that to devote more space to a consideration of the imponderable agents would be injudicious. Let us summarize, then, what has already been remarked concerning them, so far as relates to the subject of meteorology. Probably light, heat, electricity, and magnetism are all effects of one cause, differently modified. Between heat, electricity, and magnetism the alliance is marked; so is the alliance between light and heat.

Electricity is, perhaps, the most stupendous imponderable agent with which the meteorologist has to concern himself, and it is the one most amenable to human control. Not less wonderful than the energy of electricity is its universality: not a drop of water can be evaporated by the sun, not a current of water can flow, not a leaf can move or reed bend, not a breeze can skim the surface of the earth, without developing this wonderful force. Very short, indeed, is the task of specifying the material causes of electrical energy. We have only to include every known case of mechanical motion, and every known cause of chemical action, and the task is complete: it is one of universal inclusion.

In discussing the meteoric relations of the imponderables, it matters little with which we begin. Already certain meteoric functions of heat have been brought collaterally under the reader's notice; I purpose now considering the imponderable agents, not secondarily, but primarily.

Phenomena of Atmospheric Refraction.—A sketch of the laws of refraction has already been given, and what may be called the normal function of atmospheric refraction has been announced. Referring to that announcement, it will be seen that the amount of refraction is due to inequality of the density of the air, determined by pressure alone. But inequality of density, and therefore inequality of refractive power, may be the result of varying amounts of expansion, referable to the operation of varying degrees of heat; and thus arise what may be termed the abnormal effects of atmospheric refraction.

Every one must have noticed the peculiar, tremulous condition of the air in summer-time over an ignited brick-kiln or near a red-hot bar of metal, or even on the surface of the ground, provided the weather be sufficiently hot. This tremulous appearance is referable primarily to the expansion of air near a hot surface, and immediately to the diminished refrangibility attendant on such expansion. These local sources of heat set up local currents, each being composed of air of a different density from that of neighbouring currents, whence each has a different refractive power. That which an ignited brick-kiln, or a glowing metal bar, can accomplish on the small scale, is accomplished on a larger scale by many natural causes, giving rise to phenomena both striking and delusive. Pictures of ships and towns inverted, the vain semblance of lakes of water in the midst of burning sands where no water really exists, aerial cities, spectral forms of men and animals,—all these, and many more, are the phenomena of atmospheric refraction and reflection.

One of the most common effects of irregular atmospheric refraction is the twinkling of the stars. This appearance is strictly conformable with all the teachings of theory in reference to the laws of refraction, and is due to the fluctuations of variously-heated currents of air. When these small aerial currents, having different temperatures, are numerous, bad weather is likely to supervene; hence an explanation of the increased twinkling of stars before bad weather sets in,—a phenomenon which has been very commonly noticed.

Extreme examples of atmospheric phenomena are, for the most part, only seen in hot climates; but there they are frequent. The mirage is an atmospheric phenomenon, in part attributable to refraction and in part to reflection; it occurs in Egypt, and gives rise to the impression in a stranger's mind, of a lake or tranquil expanse of water, though the region is only a waste of sand. The explanation of the phenomenon is this:—The villages throughout Lower Egypt are usually built on elevated mounds; hence the houses are to some extent elevated above the general level of the earth, which level becoming intensely hot, imparts heat to the atmosphere placed in contact with it, and alters the refractive power of that portion of atmosphere. An optical illusion now ensues—the lower or heated atmospheric layer assumes a tremulous appearance, like the surface of a lake, on which the images of the buildings of the village are seen reflected, whilst the direct image of the village is still evident in its true position. The Egyptian mirage is so deceptive, that a stranger seeing it for the first time can hardly be convinced that the semblance of water is only an optical delusion. The term *mirage*[*] is peculiar to India; yet the phenomenon to which it refers is common in many other hot regions, especially in Central India and the Sahara.

The visual inversion of objects, a phenomena not at all uncommon in hot places, is partly due to refraction and partly to reflection,—for, in point of fact, the function of reflection may be demonstrated to be only an extreme case of refraction.

The *locale* is supposed to be a hot, sandy region, and a date palm is the object seen inverted, the explanation of which phenomenon is as follows:—The eyes of the observer being at p, will first see a direct image of the palm-tree by rays which come straight in the direction of the line $h\,p$; simultaneously he will see an inverted image of the palm-tree. Let us examine how this happens. Referring to the illustration, several parallel lines will be seen, $c\ c'\ c''\ c'''$. These are intended to denominate atmospheric layers of different amounts of density. Tracing the ray of light $h\ i$, let us now examine what becomes of it. Firstly, it impinges on the upper atmospheric layer, which is more hot, and consequently more expanded, than the next layer above; the ray $h\ i$ is, therefore, refracted *from* the perpendicular, according to the law mentioned at page 511. Passing on to the next layer, it is refracted still more *from* the perpendicular; and this refractive gradation is repeated on the ray $h\ i$ until it arrives at m, at which point, the tendency to fly from the perpendicular still remaining, this tendency is manifested not under the con-

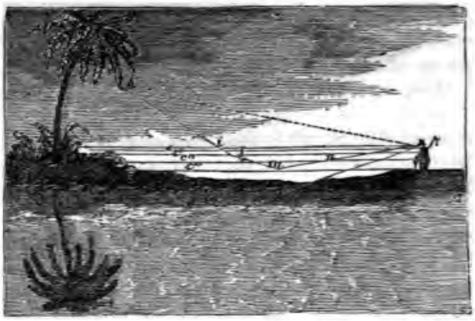


Fig. 73.

dition of refraction but reflection; consequently, the ray *hm* is directed towards the eye of the observer, and, along with the other rays upon which a similar operation has been effected, gives rise to the appearance of an inverted object. In order that the phenomenon should occur, however, there must be the following conditions besides those already mentioned:—Not only must the solar heat be considerable, but the air must be calm, so that the lower atmospheric layers may retain a density less considerable than the density of those above them.

As, under the peculiar circumstances just mentioned, refraction may pass into reflection, and the reflection may be excited upwards from below, so may the operation be reversed; in which case the inverted images of terrestrial objects will be seen in the air.

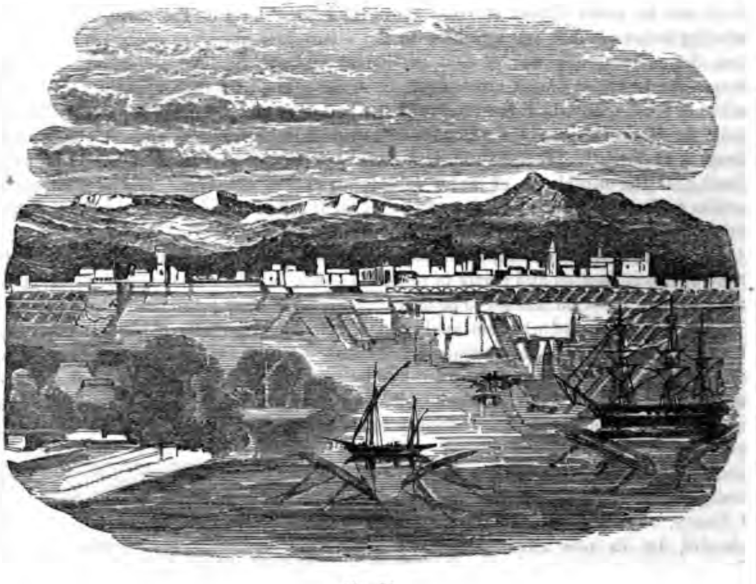


Fig. 74.

The celebrated *fata morgana*, sometimes observed on the Calabrian coast, and more especially at Reggio, is a celebrated example of this kind. At certain times the whole city of Messina and its environs are reflected downwards from an upper stratum of the air, thus presenting an appearance sufficiently curious, but by no means the striking and well-defined character which the records of early travellers would lead us to suppose.

The correlation between atmospheric refraction and atmospheric reflection, and, at the same time, a *rationale* of the peculiar aerial visions which may occur in certain atmospheric states, is furnished by the diagram (Fig. 75) suggested by M. Biot. The line *b t c* is supposed to be a ray of light proceeding from *b*, passing thence downwards to the point *t*, whence it is reflected to the observer's eye at *c*. Now the optical conditions of this arrangement are such that any rays proceeding from *b* below the ray *b t c* represented, would be invisible to the observer at *c*, whilst two images will be seen of all objects above this line. Supposing the object in question to be a man,—suppose, further, the man to be walking *from* the observer, he would be presented to the latter under the successive forms seen in Fig. 75.

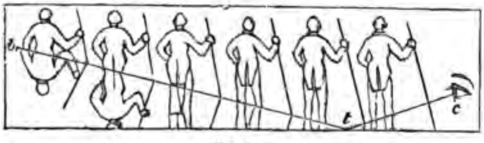


Fig. 75.

In these atmospheric optical delusions, involving the appearance of two images, one of them inverted, both natural and inverted images have occupied a horizontal plane. Occasionally, however, the reduplication of image has been projected on vertical planes, of which phenomenon the following is an example. On June 17, 1820, whilst MM. Soret and Turine were in the second story of a house on the lake of Geneva, they looked towards a ship two miles off, and making for the harbour. Immediately the vessel in question arrived at *q* she appeared reduplicated on a vertical plane; when she came to *r* the reduplication still continued, but the second image was further removed than before, an both were distorted; lastly, when the vessel arrived at the point *s*, the reduplicated image had receded to a distance still farther away, and both images, though distorted, presented an appearance of distortion very different from before; they were apparently drawn out, elongated both as to the hull and rigging, as represented in the accompanying diagram (Fig. 76). The following is the explanation of the phenomenon as suggested by the two observers above-mentioned, and there seems no reason to doubt its correctness.

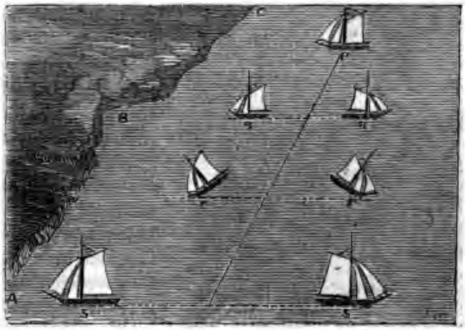


Fig. 76.

The letters, A B C, represent an outline of the eastern bank of the lake of Geneva; the air over that bank had, at the time of observation, been long under the shadow thrown upon it by the mountains of Savoy, whilst, contemporaneously the western bank had been strongly heated by the sun; hence, from the conjoint operation of these two causes, there were two vertical layers of atmosphere of different temperatures, and consequently of different densities; hence, they were of two different refractive and reflective powers.

All that is necessary to determine aerial reflections is a sufficient difference between the temperatures of any two adjacent atmospheric layers. In the instance already mentioned, this difference has been occasioned by portions of the ground being hotter than the strata of air with which they are in contiguity. The reverse of these conditions may, however, obtain, and phantastic atmospheric delusions may be the result. This latter case generally presents itself at sea, and by no means exclusively in warm localities. Thus, for instance, it is prevalent enough in the Northern Ocean. Sometimes the atmospheric delusion has merely the effect of prolonging the appearance of an object really below the horizon; sometimes not only is the appearance prolonged, but the body is seen double. Whatever the appearance, the class of atmospheric delusions now under consideration are usually seen near the horizon—a position where the optical powers of the atmosphere attain their greatest intensity.

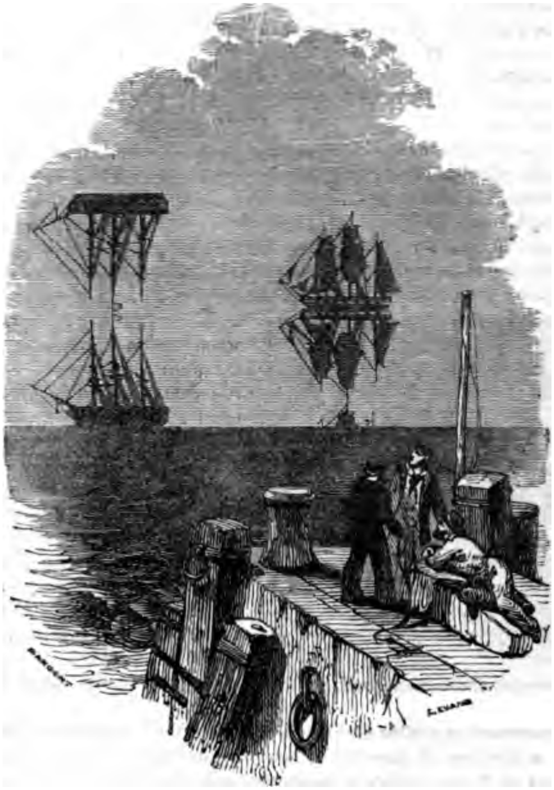


Fig. 77.

The Rainbow.—The most beautiful of all luminous meteorologic phenomena is the rainbow, which results from the decomposition of light by refraction through drops of rain, and subsequent reflection. Rainbows are of two classes, solar and lunar; the latter, however, are rare, and even when they do occur the bow is seldom coloured.

The chief conditions under which a solar rainbow may occur are the following:—The sun must not have less than 42° of angular elevation; the back of the spectator must be towards the sun, and rain must be falling from a highly illuminated cloud. The rainbow is usually double, and the theory of its formation may be thus explained. Let it be assumed that a straight line passes from the eye of the observer through the sun; then this line will constitute the axis of a cone, the base of which will be the rainbow, and its vertex the eye of the observer. If the bow be at the horizon, and the place of observation be a level plain, then the rainbow will appear as a perfect circle constituting the base of the cone. This complete circular appearance is, however, rare, the rainbow being far more generally, as its name implies, a mere arc of coloured light. The explanation is now evident of the fact that the rainbow cannot appear when the sun has attained an elevation greater than 45°. The rainbow, though still depicted, is depicted below the horizon, and is, therefore, invisible. It follows, then, that the size of the visible rainbow is inversely to the elevation of the sun above the horizon.

The annexed diagram (Fig. 78) illustrates the formation of the rainbow. The straight line A P is supposed to be drawn between the sun and the eye of the observer, passing through the latter. Through this line a vertical plane is supposed to be drawn. If the line A x be drawn through A so that the angle P A x shall amount to 42°, the rain-drops will reflect coloured drops to the eye. Assuming the line A x to rotate, a cone will evidently be generated, part of which, lying below the horizon, will be invisible. It is the surface of the cone thus generated which is the reflecting surface, and to which, therefore, the rainbow is due. Inasmuch as every colour has a refractive quality peculiar to itself, each drop only represents one tint to the eye. An arc having the breadth of about 2° is sufficient to include all the prismatic colours; 2° therefore is about the breadth of the rainbow.

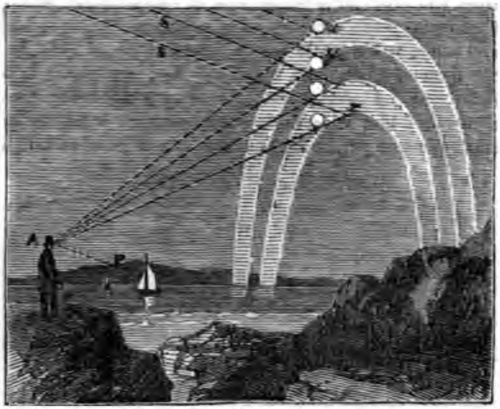


Fig. 78.

The colours of the rainbow are partly due to refraction and partly to reflection, as has been observed. The first effect of light on the drops of rain is refraction, by the operation of which white light arrives at the posterior side of each drop of rain, decomposed or dissected into the primitive colours of which it is composed. At the posterior aspect of each drop of rain the dissected colours are reflected unto the eye, and a coloured image is presented.

Such is an explanation of the theory of the primary rainbow—besides which, the surrounding rainbow requires to be noticed. The secondary rainbow is outside the primary, and is larger than it, but also much fainter. Its angular position is defined by the limits 50° 59′ and 54° 9′, measured with reference to the axis Ax. The secondary rainbow has all the colours of the primary, but less completely defined, and in a reverse order. Its existence may be explained by the statement that it is the result of light twice decomposed, whereas the true rainbow is the result of light only once decomposed. The secondary rainbow, then, is produced by drops of water very far off.

Inasmuch as any angular elevation of the sun above 45° is incompatible with the existence of a rainbow, it is evident that this beautiful meteor can never occur in the south. It may occur, however, either in the east, west, or north.

As concerns lunar rainbows, they are, as I have before remarked, exceedingly rare, and are very seldom coloured. Nevertheless, in northern latitudes, where the moon shines with a brilliancy unknown to us, coloured lunar rainbows are occasionally seen.

Halos and Parhelia.—These meteoric phenomena are far more rare with us than in more northern latitudes, where they are continuously visible for long periods of time, and give rise to phenomena of extraordinary beauty. The term *halo* is applied to a luminous circle occasionally seen around luminous bodies, more particularly the sun and moon, and is partly due to the refraction of light by vaporous water, sometimes in the form of true clouds, sometimes not; and partly to the properties of light termed diffraction and interference. As respects diffraction, the circumstance has been already announced that when light passes through a minute orifice—such, for example, as a small aperture punctured in a card—the edges of such light bend: this is termed diffraction. The term, interference of light, is used to explain the phenomena of colour, or alterations of luminous condition generally, which result from the assumed jarring impact of luminous waves meeting in different phases of their vibration.

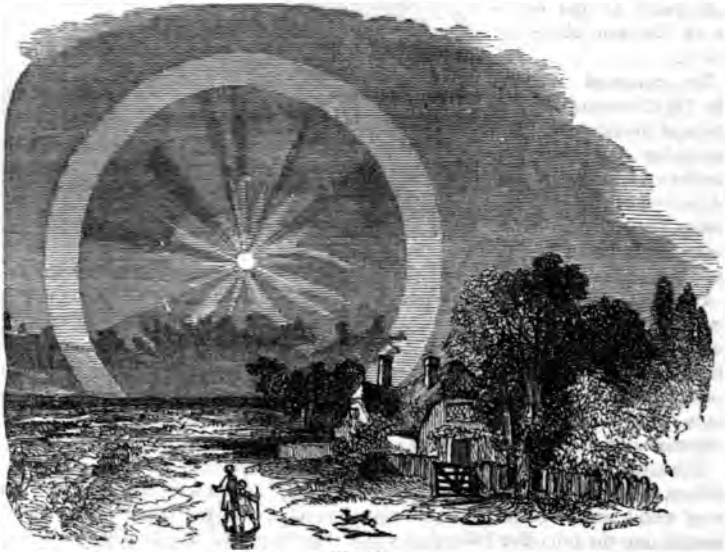


Fig. 79.

Solar halos frequently exist, though unnoticed, the sun's light being so powerful that the eye of the observer cannot withstand its impressions. By the aid of a sheet of glass rendered dull by smoke, these halos are frequently rendered visible.

Although halos, and also the phenomena next to be described, are referable to the action of atmospheric moisture on luminous rays, yet it is evident that the condition of that moisture will vary according to temperature; in other words, the aerial moisture which would be mere cloud-vesicles at temperatures above the freezing-point, would, if depressed below 32° F., be converted into snow, or spiculæ of ice. The alteration which these are capable of effecting on luminous rays being far greater than mere uncongealed vesicular water can effect, the resulting optical phenomena are far more brilliant and remarkable. Hence in northern latitudes the phenomena of halos and parhelia,—as arcs of light appearing near the sun, and sometimes intersecting each other, are called,—are brilliant and impressive beyond anything which corresponding phenomena occurring in this region would lead us to conceive. These luminous arcs, sometimes intersecting each other, are as often occasioned by the moon as by the sun. As the phenomena in question, when referable to the latter cause, are demonstrated *parhelia*, so when dependent on the former cause they are termed *paraselenæ.*

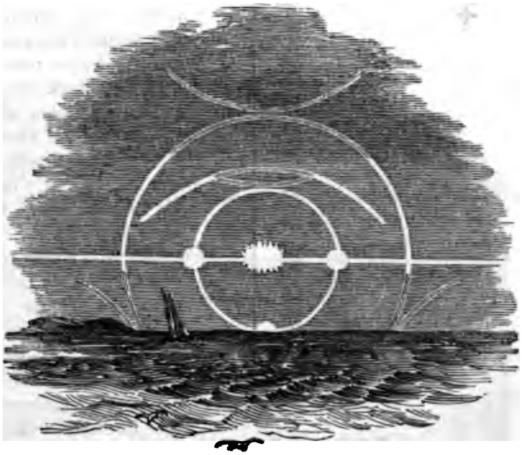


Fig. 80.

Frequently parhelia and paraselenæ consist, not only of the intersecting arcs just mentioned, but of circular luminous meteors to which the term *mock suns* are especially applicable. Associated with parhelia, and sometimes included under the same name, is a luminous band, passing horizontally through the sun, and not unfrequently making a circuit of the whole heavens. Where this luminous band and the inner parhelion cross, a mock sun usually appears, as represented in the accompanying diagram (Fig. 80).

Aurora Borealis.—It has already been indicated, under the head of electricity, that various circumstances materially operate to produce the condition termed electrical. By far the best-studied of atmospherical electrical phenomena are thunderstorms; but it is an error to suppose that the atmosphere contains at the time of a thunderstorm its maximum of electricity; the experiments of Faraday have sufficiently made out this point.

Reserving the consideration of thunderstorms for the present, I shall introduce here the subject of electrical phenomena by a description of the aurora borealis—a phenomenon sometimes said to be magnetic, inasmuch as the magnetic-needle is strongly affected during its prevalence, but which, nevertheless, seems more naturally to belong to electricity.

The term *aurora borealis*, or northern light, is applied when the phenomenon to be presently described occurs in the north; and the term *aurora australis* is applied when it occurs in the south. But the former has been seen as far south as 45° of southern latitude; and the latter has more than once been visible in Britain. Nevertheless, the beautiful phenomena of northern and southern lights are most prevalent towards the north and south poles respectively.

Northern and southern lights, when in their greatest perfection, consist of a well-defined arc of white light, and luminous streams of coloured light flowing therefrom. The arc is not permanent as in the rainbow, but bends and twists in all directions like a ribbon agitated by the wind. The intensity of the aurora varies within extensive limits: when faint, the light is only recognizable at night by careful examination; but when highly developed, the aurora borealis, or australis, can be seen during broad sunshine.

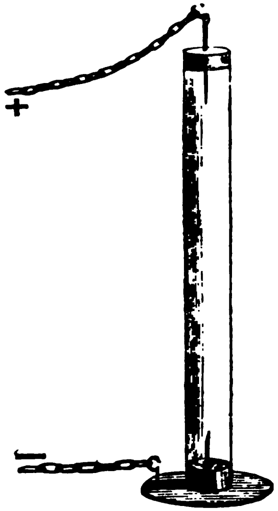


Fig. 81.

These phenomena may occur at any season, but they are most prevalent in the months of March, September, and October, or about the period of the equinoxes. The aurora borealis and australis are sometimes said to be magnetic storms; a more reasonable foundation is required for the remark than is supplied by the turbulent agitations of the magnetic-needle with which they are attended; but we have already seen that the functions of magnetism and electricity are so nearly connected, that it is impossible in some cases to distinguish between the two.

There is a very common electrical experiment which furnishes an artificial phenomenon very nearly resembling the northern and southern light, in all respects except in the shape of the illuminated body, which is a luminous arc.

The experiment consists in exhausting, by means of an air-pump, all the air out of a glass tube furnished with a metallic point at each end, looking internally, and placed in the electric current, as represented in Fig. 81. When a stream of electricity, of adequate intensity, is passed through the apparatus from + to —, the whole interior of the tube becomes illuminated with flashes of light, very similar in appearance to the flashes of the aurora.

The phenomenon of the aurora borealis was noticed by Aristotle and Pliny, although neither philosopher could have seen it to advantage. Gassendi first originated the term aurora borealis, to indicate the phenomenon of this kind observed by him on September 12, 1621. These phenomena appear to be subject to some laws of secular variation not yet understood. That they have appeared in certain years, and certain groups of years more than others, is certain. According to Mairan, twenty-six occurred between A.D. 583 and 1354; thirty-four between 1446 and 1560; sixty-nine between 1561 and 1592; seventy between 1593 and 1633; thirty-four between 1634 and 1684; two hundred and nineteen between 1685 and 1721; nine hundred and sixty-one between 1722 and 1745; and twenty-eight between 1746 and 1751.

After 1790, auroras became unfrequent, but since 1825 they have been on the increase. A very remarkable aurora borealis occurred in the autumn of 1847: it was conspicuous not only in England, but even so far south as Italy and Spain.

Height of the Aurora.—As a proof of the doubt which exists concerning the height of the aurora, they have been variously estimated from 3000 or 4000 feet to several miles. The probability is, that the conditions on which the aurora depends vary in the altitude of their operation; but the truth is, that, notwithstanding the electrician by his artificial experiments can imitate the light of the aurora borealis and australis, notwithstanding the prevalence of the phenomena in question near the magnetic poles seems to point to magnetic agency as the cause, our real knowledge concerning the aurora borealis and australis is very slight.

It may be as well here to present the reader with a summary of the various opinions which have prevailed at different times relative to the phenomenon in question. Many early writers referred the appearances presented to mere optical causes, considering them to be due to the reflection of the sun's light thrown upwards by a mirror of snow and ice, and a subsequent reflection downwards by atmospheric agencies. De Mairan, the observer who, perhaps more than anyone else, is entitled to be considered the chronicler *par excellence* of the phenomena of the aurora, attributed them to the penetration of our planet at certain periods into the solar atmosphere. On this supposition it will be remarked, that the solar atmosphere must be assumed to extend to the orbit of our planet, an hypothesis totally irreconcilable with the teachings of optics and astronomy. The celebrated Euler, the philosopher who could deal so satisfactorily with the abstrations of number and quantity, seems to have offered a most crude and improbable theory explanatory of the aurora. Adopting the molecular theory of light, he assumed that the solar rays, striking against the particles of our atmosphere, actually carried particles of the latter up into the heavens to a height of more than four thousand miles, the height at which Euler believed auroras to exist. Some philosophers, of whom Volta may be regarded the Coryphæus, adopted a chemical theory of auroras, referring them to the ignition of hydrogen gas spontaneously generated on the earth, and rising by its light specific gravity to the higher atmospheric regions. It was assumed by these philosophers that the evolution in question took place in the tropical regions chiefly, and that

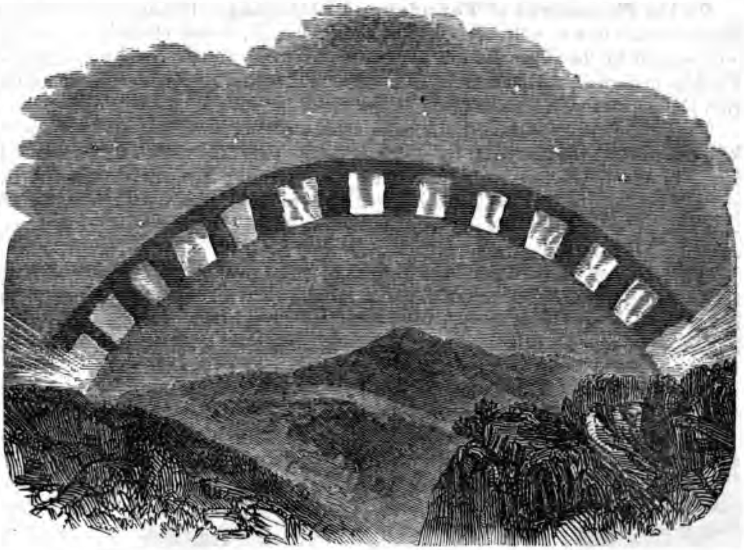


Fig. 82.

it was wafted by the upper current of air—treated of in connection with the trade-wind—to the north and south poles respectively.

Halley was the first, I believe, who suggested that the phenomena of aurora borealis and australis might be due to the passage of magnetism from one magnetic pole to the other; and the theory of Halley is so far retained, that the aurora is assumed to be in some way connected with electricity and magnetism, but in what manner is beyond the competence of observers to decide.

On the Phenomena of Thunder and Lightning.—Perhaps no meteorologic phenomena are now so well understood as these; though, before electrical science had been studied by the phlosophers of the last century, and the crowning experiment of Franklin performed, the phenomena of thunder and lightning were so mysterious, that even philosophers were content to refer them to the operation of an occult cause.

The intimate study of electrical science opens a field of somewhat abstruse matters for consideration; the field is far too wide and too abstruse to be dealt with satisfactorily here. In the treatise on Chemistry of the Imponderable Agents, belonging to this series, it has been treated somewhat in detail; and to this I must refer the reader who desires to know more on this subject than strictly belongs to the necessities of what I may term *practical meteorology.*

With this explanation I shall not hesitate to adopt the term *electric fluid*, although the reader has already been made aware that no such fluid is at all likely to exist. Let us now contemplate the phenomenon of that electrical excitation, the solution of which is lightning, under the simplest conditions that the phenomenon can assume. Let A and B (Fig. 83) represent two clouds, which, being made up of watery vesicles, are necessarily electrical conductors; and being surrounded by the atmosphere on all sides, are necessarily insulated. For the sake of our illustration, it will now suffice to assume that neither of the clouds here represented is electrically excited at this period of the description, and hence that the marks + and — are for the present misplaced. Let it now be assumed that the cloud A becomes positively electrified,—that is to say, charged with positive electricity, owing to some natural cause unnecessary here to explain; and let the results of this condition be traced out. Firstly, there is not in all nature, and there *cannot be*, such a condition as that of independent electric excitation; in other words, there cannot be one body *positively* excited without the co-existence of another body *negatively* excited. Hence, if cloud B were away and cloud A positively

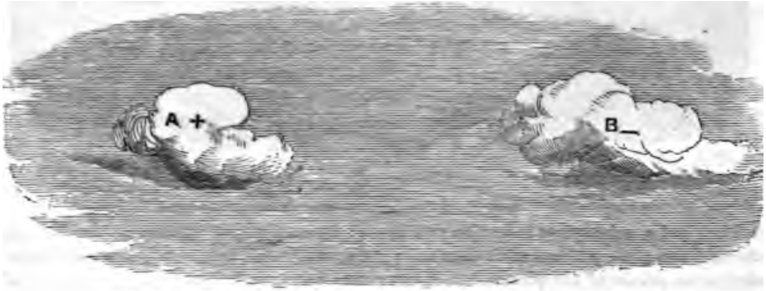


Fig. 83.

excited, the air circumjacent to A would assume the second or negative function; but if the cloud B is present, it therefore becomes negative, and the two clouds A and B are mutually attracted, because opposite electricities attract each other. Hence they approach until the space of air between the two is insufficient to restrain their mutual electric tension: this condition having arrived, a discharge takes place, precisely analogous to the discharge of a Leyden jar. Under the postulates of our experiment, the discharge, or lightning flash, takes place between the two clouds A and B.

It follows, however, from the consideration of known electrical laws, that just as the two oppositely electrified bodies may be two clouds as assumed, so also may they be one cloud, and the surface of earth or water, or conductors placed upon either one or the other, under which conditions a downward discharge will take place; and generally electricity will always take the nearest path between any two bodies oppositely charged, the conducting facilites being equal.

Lightning-Conductors.—It is almost unnecessary in these days to announce that Franklin, in the year 1752, first demonstrated the nature of lightning by drawing electric sparks from the string of a kite, previously caused to ascend into the region of a thunder-cloud. This experiment performed, the connection between lightning and electricity could no longer be doubted, and a means of drawing off a surcharge of the electric fluid by lightning-conductors was immediately suggested. The most important instruments were not adopted, however, until after numerous and varied conflicts. Firstly, the argument was adduced by some that lightning-conductors could not be adopted without impiety, being intended to contravene the will of Providence. An argument so fallacious was no sooner abandoned than lightning-conductors were exposed to another ordeal, founded on an erroneous practical estimation of a truth in theoretical electricity. I allude, as the electrician will perceive, to the contest between the advocates of spherical, and of pointed, terminations for electrical conductors.

Now, regarding the question of points or spheres abstractedly, it is easy to see that preference should be given to the former, inasmuch as points draw off and give issue to the electric fluid in silence, whereas spheres draw off and give issue to electricity in sparks; but, inasmuch as the largest spherical termination ever used, or ever likely to be used, for the upper extremity of a lightning-conductor, is virtually a point in comparison with the enormous surface of the smallest thunder-cloud, the dispute, though violent and prolonged, never had the practical significance which was at one time taken for granted.

The history of lightning-conductors furnishes a remarkable illustration of the difference between the mere knowledge of a fact, and the confidence or conviction resulting from that knowledge and justifying its practical application. The whole theory of lightning-conductors was almost as well known half a century ago as now; yet it is only within the last few years, and owing to the unflagging perseverance of Sir W. Snow Harris, that due effect has been given to the theory, and lightning-rods have been fearlessly applied.

Electrical Principles connected with Lightning-conductors.—The electrical principles on which the efficiency of lightning-conductors depend are few and simple; they all admit of being readily demonstrated by electrical experiments artificially performed, and they have been universally justified by the result of three practical applications:—

(1.) *Electricity cæteris paribus follows its course through the best conductors which happen to be in its path.*—Thus, for example, if a Leyden jar be charged, and the electric connection between its external and its internal coating be completed by three linear substances of equal length—say, for example, silk, wire, and linen thread—thin wire being the best electrical conductor of the three, will transmit the whole of the electricity to the exclusion of the silk and the linen thread. It is assumed, however, in the performance of this experiment, that the wire is sufficiently large to convey the whole electric energy.

(2.) *Provided the conductor be good, and its sectional area adequate, the electric fluid or energy is conveyed harmlessly away.*—No point in the whole of electrical science can be more satisfactorily established than this. It admits of being illustrated by numerous experiments, amongst which the following may suffice:—

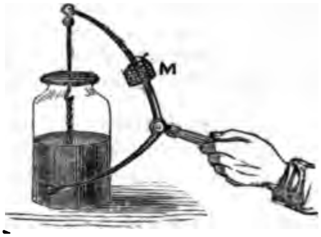


Fig. 84.

The diagram (Fig. 84) represents a common Leyden jar, represented in the act of being discharged by the ordinary discharging instrument. That instrument is, as usual, of brass, all save the handle, which is of glass, and therefore a non-conductor of electricity. Those who are conversant with the form and construction of the discharging instrument, are aware that its two terminal balls admit of being unscrewed. Assuming one of the balls, viz. the upper one in the diagram, to have been unscrewed, the liberated brass stem to be passed through a marroon, or box holding gunpowder, and the ball to have been again replaced, the conditions will have been fulfilled which the diagram represents. It is evident that the Leyden jar, as represented, will be discharged. It is, moreover, evident that the whole of the charge will be transmitted through the gunpowder contained in the marroon, yet that gunpowder will not inflame. If, however, instead of the conditions of the last experiment, a very fine metal wire (a steel wire by preference) be passed through the marroon, or rather through some combinations of explosive materials less potent than a marroon, which would now be dangerous, and electricity transmitted as before, the wire, not presenting a sufficient amount of transverse area of surface to convey the electricity, melts, and the explosive compound is inflamed.

(3.) *Lateral discharge must be provided against.*—The meaning of lateral discharge will be illustrated by the following experiment:—The diagram (Fig. 85) represents, as before, a Leyden jar readily arranged for being discharged through a metallic wire, one end of which has already been brought into contact with the outside of the jar, while the other end can be brought into contact with the knob communicating directly with the inside of the jar. The hand is represented in the act of holding a glass rod, around which one end of the wire is coiled, and the extremity of the wire is finished off with a ball. All these arrangements, the electrician will perceive, are necessary for giving effect to the efficient discharge of the Leyden jar through the conducting wire. The chief poin for observation, however, is the band of the wire, by means of which one part is caused very closely to approach another part, as represented at *a b*. Now it is possible by choosing a wire sufficiently small, and causing the two bands to pass sufficiently near, to determine the passage of an electrical spark from *a* to *b*, instead of proceeding through the entire length of the wire, from the internal to the external coating of the jar. This is what electricians call the lateral discharge, and it requires to be studiously guarded against in the construction and arrangement of lightning-conductors.

Application of the Foregoing Deductions.—Perhaps the deductions already arrived at will suffice for practical guidance in the matter of lightning-rods, though these deductions by no means exhaust the science of the subject. Firstly, the fact may be considered as proved, that all bodies, even the most dangerous and inflammable, all edifices, all living beings, may be shielded from the evil consequences of lightning, by the safeguard of lightning-conductors. The conductors, however, must present a sufficient external area; and the point has been made out by numerous trials, that a copper rod a square inch in sectional diameter will convey away the utmost fury of the most highly-charged thunder-cloud ever proved to exist. Copper is one of the best electrical conductors amongst metals; but, by providing a sufficiently increasing sectional area to compensate for inferior conducting power, any metal may be made to perform the function of copper. Whatever be the conductor, its upper extremity should project considerably above the edifice to be protected; and if pointed *theoretically*, all the better, though practically the bluntest termination could be only as a point by comparison with the enormous mass of a thunder-cloud. Far from preventing contact between the building to be protected and the conductor, as is sometimes done by the interposition of glass or earthenware guards, a lightning-conductor cannot be brought into too intimate metallic connection with every part of the edifice to be protected. The conductor should branch and ramify over the surface of the building, and should be brought into contact with every important system of metal line work, such as the iron pipe which frequently runs down the side of a wall; finally, the conductor should at its lower extremity be brought into contact as efficiently as possible with some good electric conductor, such as the system of gas or water-pipes which run underneath most houses, especially under the streets of most civilized towns. As regards the number of conductors necessary to be supplied to one building, that will depend, firstly, on the shape of the building, whether it be composed of many elevations, or whether, like a column, it has only one. Perhaps the best practical testimony on this point is gleaned from the fact, that a ship having three masts, one of which only was protected by a lightning-conductor, the unprotected masts have been shattered by the effects of a thunder-storm, while the other remained untouched. If a column be surmounted by a metallic statue, it is worse than useless to disfigure the head of the statue by a projecting metallic spike as the beginning of a lightning-conductor; nothing more is requisite in this case than to provide sufficient metallic conduction for the electricity downwards into the ground. Lightning-conductors, it should be remembered, *do not*, as they are commonly said to do, *attract* electricity. They no more attract electricity, than a gutter attracts water. They merely open a channel for electricity to pass through. Before the demonstrations of Sir William Snow Harris had taken effect, marine lightning-conductors were something more dangerous than lightning itself: consisting merely of chains, which were only elevated aloft after the thunderstorm had come on. Marine

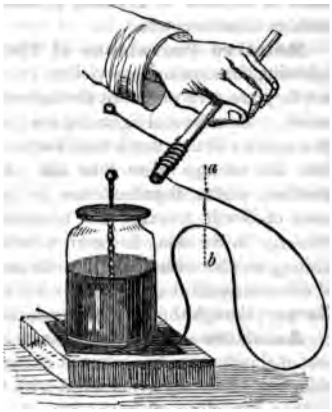


Fig. 85.

lightning-conductors are now fixtures on the masts; they are made of copper, running band-like down the mast, and embedded in the latter in such manner that, whether the masts be elevated or lowered, perfect metallic contact between any two pair of masts remains uninterrupted.

Relative Prevalence of Thunderstorms.—The phenomena of thunder and lightning are nowhere so violent or so frequent as in the so-called region of calms; but they are very prevalent throughout the torrid zone, more especially during the rainy season. Thunder and lightning are almost always absent in the polar regions; and even at a spot no farther north than Bergen, in Norway, the annual number of thunderstorms does not average more than six. As the rule, thunderstorms chiefly occur in hot weather, winter thunderstorms being comparatively rare. Iceland and the western coast of North America are remarkable for the predominance of thunderstorms in winter. In Sweden, however, winter thunderstorms are almost unknown; thus furnishing another example of the circumstance already noted, that the Scandinavian range of mountains effect a remarkable difference between the general climate of Sweden and Norway, though the two, geographically considered, are so close together.

Aerolites — Shooting Stars — Meteoric Stones.—The beautiful phenomenon of shooting stars is common enough; but at certain periods it is peculiarly remarkable, the whole sky being filled with these fleeting meteors. The beginning of August and the beginning of November are noticeable for their connection with shooting stars; more especially have they been recorded between the 9th and the 14th of August. The bouquet of shooting stars observed at this period in North America has been sometimes called the Shower of St. Lawrence.

For our knowledge respecting the periodicity of the phenomenon of falling stars, we are indebted to Quetelet Besenberg and others; but Muschenbroek, so long back as 1762, first directed attention to the so-called *shower of St. Lawrence.* In addition to the August and November phenomena of the kind under consideration, other periods have been noticed—for instance, in April, and from the 6th to the 12th of December; but these bouquets of shooting stars which thus occur are less considerable and less regular than the former. Besides these showers of shooting stars, the periodicity of which is well attested, single meteors of this kind are frequently noticed, and they occur at all seasons; therefore, whatever may be the cause of shooting stars, this cause must be regarded as continuously operating.

Sometimes the luminous meteor termed a shooting star attains a large magnitude; observers then speak of it as a fire-ball. The identity of shooting stars and fire-balls is now well established, though formerly they were treated of as distinct. Fire-balls are sometimes seen alone, but more frequently in connection with shooting stars. Their light and their bulk are frequently so considerable, that they can be seen in broad daylight. Their velocity through the heavens, or rather through the upper layers of our atmosphere, is various; but it generally exceeds that of the earth.

As regards the nature of shooting stars, we have only theory and analogy for our guidance; but our knowledge of fire-balls is far more accurate, and there seems no reason to doubt that their teachings illustrate the nature of falling stars. Numerous fire-balls dart through our atmosphere, become luminous, and disappear, no one knows whither. Others, though passing near to our atmosphere, fail to enter it, and therefore are not rendered visible. A third division not only come within our atmosphere, shine, and burst with a loud report, but they fall; yet, falling either into the sea or upon an unfrequented spot of land, the locality of their fall remains unknown. Whilst a fourth division of fire-balls may be seen to fall, dug out, and examined; thus supplying data for an investigation of fire-balls in general.

Masses of this kind are termed aerolites, and their connection with fire-balls has been placed beyond all doubt. Fire-balls have been seen to fall, and aerolites have been extracted from the place whereon they have fallen. Nevertheless, some aerolites have fallen upon the earth without the assumption of a previous appearance of luminosity. Cases, though rare, are well attested of an aerolite suddenly falling from a small cloud, attended with a noise resembling the discharge of cannon; others, again, have fallen silently, and out of the clear air, not the slightest trace of cloud being visible at the time.

Testimony concerning showers of stars and the fall of aerolites has been handed down to us from all periods, but it is only since the time of Chladni that the occurrence of these phenomena has been placed beyond doubt.

Amongst the best-attested examples of the fall of aerolites are the following:—On the 16th of June, 1794, a shower of stars fell at Sienna; and in the following year, December 13, an aerolite, weighing no less than fifty-six pounds, fell in England. Three years afterwards, and remarkably enough also on December 13, a fire-fall split up and discharged round stones. A very large shower of stones fell April 26, 1803, near Aigle, in France; the occurrence is particularly interesting on account of its having been noticed and verified by M. Biot. Ten such meteoric showers were observed in France in twenty-six years—*i. e.*, between 1790 and 1815. The meteoric shower at Aigle in 1803, which poured its contents over a surface of two and a half French miles long, by one in breadth, consisted of 2,000 fragments of different sizes, some weighing not more than two drachms, others near twenty pounds. Aerolites are sometimes much larger than this; one fell at Agram on the 26th of May, 1751, having a diameter not less than eighteen feet, and weighing seventy-one pounds; but the largest known aerolite fell in Mexico, and weighed between 30,000 and 40,000 pounds.

As to shape, aerolites are generally prismatic, or angular, rarely smooth; and almost always sheathed in a crust of pitchy blackness. Their specific gravity is various, some being sufficiently porous to absorb water with rapidity, others being dense and metallic. Looking at the specific gravity of aerolites in the aggregate, it may be said to vary between 1·94 to 4·28, thus presenting a mean of about 3·5. All the heavier varieties of aerolites are made up of iron, holding a little nickel; traces also of cobalt, manganese, chromium, copper, arsenic, tin, and other well-known elementary bodies, are found.

Origin of Fire-balls and Shooting-stars.—Various opinions have been advanced to account for these bodies. One of the earliest, if not the very earliest, of these hypotheses, originated in 1660, and assumed fallen aerolites to be mineral masses originally projected from lunar volcanoes; and calculations were made, having for their object to demonstrate what volcanic force might be sufficient to project aerolites of a given mass into the sphere of attraction of the earth's atmosphere. Unfortunately for the probability of this theory, the moon's surface appears to be altogether devoid of active volcanoes. Then followed the chemical hypothesis, according to which it was assumed that aerolites were nothing more than aggregations of metallic vapours, which had risen to the upper region of the atmosphere, aggregated there, and fallen. The opinion of Chladni is now, however, generally received; he regards aerolites to be of cosmical origin—to be so many planets, or planetary fragments, which revolve in orbits of their own, variously inclined to the orbit of the earth; that our planet encounters periodic shoals of these little worlds, some of which, becoming entangled in the earth's gravitating system, pass into our atmosphere, become heated by friction against its particles, and ultimately fall to the ground. Although the discovery of aerolites is comparatively rare, the meteors, of which they are the final result, are by no means so. It has been calculated that the average annual fall of aerolites is not less than 700, or about two daily.

Meteorologic Result of Occult Emanations.—When the utmost powers of a refined chemistry have been applied to the analysis of atmospheric constituents and conditions, much still remains to be unveiled. There are atmospheric causes, whatever they may be, of epidemic and endemic diseases, and perhaps other agencies which our philosophy little suspects or dreams of. I have ventured to include these undetermined agencies under the general expression *occult emanations*. It is not difficult to point out objections to this designation in some of its applications. Perhaps it is not strictly philosophical to speak of emanations thus hypothetically; perhaps this may be only a repetition of the error of assuming the existence of an electric fluid; perhaps the number of influences due to allotropism and to polarity is greater than we imagine; but, at any rate, the term "occult emanations" may be accepted as a rallying-point for a certain class of facts, until the time arrives when their true significance shall be correctly made out.

Without invoking the hypotheses of allotropism and polarity, there are undoubtedly some atmospheric agencies to which the expression occult emanations is applicable, and concerning which the only thing occult about them is the insufficiency of ordinary chemical examinations to demonstrate their existence, though that existence admits of being demonstrated by extraordinary chemical means. Thus, for example, it is a well-authenticated fact, that the atmosphere of localities in which fever is endemic, usually contains minute traces of hydrosulphuric acid, and an odorous animal matter—substances which ordinary chemical processes fail to detect, but which, nevertheless, by the adoption of refined methods of investigation can be proved to exist. The late Professor Daniell was of opinion that the much-dreaded fever of Western Africa was augmented by the diffusion, through the atmosphere of that coast, of minute traces of sulphuretted hydrogen. And the circumstances under which African fever originates, are perfectly consonant with the above theory. The disease only prevails now on the coast, its ravages being limited to a small belt, partly of land and partly of sea, Central Africa being comparatively exempt from its inflictions. Now Professor Daniell assumes the hydrosulphuric acid to be the result of decomposition, under a powerful sun, of matter borne seawards by the Niger and other great rivers, in connection with certain sulphates of sea-water. Be this theory true or the contrary, there can be no doubt as to the truth of the assumption which refers fever, when endemic in certain localities, to a vitiated condition of atmosphere; which vitiation may be generally summed up as consisting of minute traces of hydrosulphuric acid, and of undetermined animalized matter.

Amongst other occult emanations, we can hardly refuse to admit the cause, whatever that cause may be, of intermittent fevers. The ultimate, if not the proximate, cause of this class of disease is so well known, that we may almost produce or banish intermittent fevers at pleasure. Given heat and moisture continuously, ague almost invariably sets in, and continues its ravages as long as the conditions of heat and moisture coexist.

What is the occult emanation here? What is the proximate cause of intermittent fever? Are we to attribute the disease to the conjoined evaporation of moisture and heat *directly*, or to some further emanation to which these conditions give rise? Some pathologists have assumed that light carbonetted hydrogen, or marsh-gas as it is called, determines the disease; but the notion hardly coincides with known facts in relation to this disease. The horizontal demarcation of altitude above which the influence of ague cannot extend, is one of the most remarkable circumstances in connection with the disease; a difference of no more than ten feet in perpendicular height frequently corresponding with the region of fever and the region of salubrity respectively.

The mention of light marsh-gas naturally suggests the curious meteorologic phenomenon called Will-o'-the-wisp, or Jack-o'-lantern; of which gas ignited, or, according to some, phosphuretted hydrogen gas, it is believed to consist. The Will-o'-the-wisp is not a very frequent phenomenon anywhere; but it is chiefly seen in marshes and churchyards,—the latter locality apparently adding to the hypothesis that it is nothing more than ignited phosphuretted hydrogen gas.

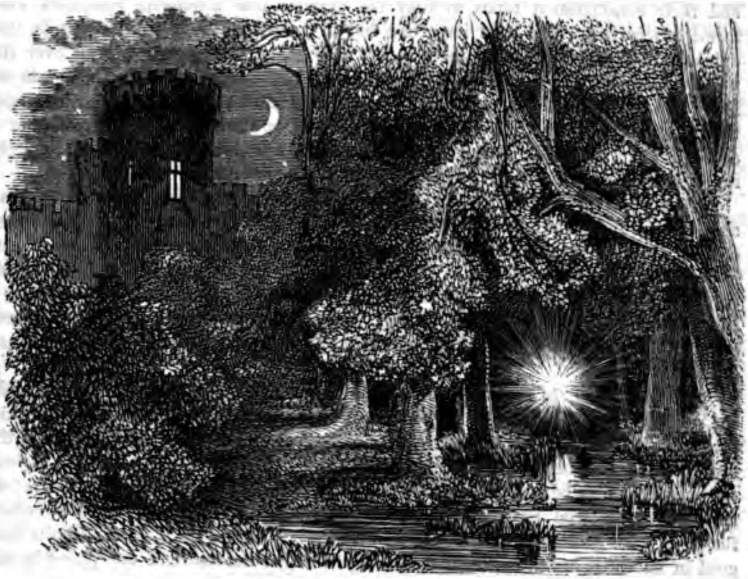


Fig. 86.

Scarcely less accurately demonstrated than the locality of ague, is the locality of yellow-fever, the focus of which may be considered to be Vera Cruz. Strange to say, the yellow-fever is totally unknown on the Pacific coast of Mexico; it extends north as far as New Orleans, but rarely further. This condensation, so to speak, of febrile energy, points to some local cause—most probably of atmospheric origin; but chemistry has been unable to determine its nature.

Reflections similar to the above are suggested by the contemplation of several contagious diseases; of which the plague is a notable example. This scourge, although capable of spreading from its normal focus into regions wide apart, is in certain spots determined conditionally by the existence of some unknown conditions; and these are most probably atmospheric. The plague never originates in very hot or very cold regions. Its focus of development is Egypt, Turkey, and the Levant, from which spots it can never be said to be absent altogether; but it only appears with violence at intervals of eight or ten years. Still more extraordinary than any of the preceding, are the varied conditions which give rise to Asiatic cholera. Unlike the plague and the yellow-fever, and intermittents, this fell destroyer seems independent of region, recognizable limits, or other conditions of demarcation. From east to west it has extended its ravages, under every vicissitude of season and of clime. Much as there is mysterious in this—absolutely ignorant though we be of the proximate influences by the operation of which cholera and other epidemics are generated, important facts have, nevertheless, been made out; and their consideration tends to allay the extreme fear wherewith epidemics were formerly regarded. The most important fact in connection with this subject is, that epidemic influences, whatever their nature may be, only as a rule prevail over the weak, exhausted, ill-fed, or mentally broken-down individuals of a community; whence it happens, that here in our metropolis the medical statician is enabled to lay his finger upon the regions of epidemic virulence, as he would on the locality of a mountain or a coal-field; and with equal satisfaction can he point to localities where epidemics formerly raged, but whence they have been banished by the art of man.

Climatology.—Many of the effects of heat, in its meteorological relations, have already been incidentally considered; but reference has not yet been made to the sources of heat, and to the means of its distribution over the surface of our planet. The term heat, as applied to the matter now under investigation, may be regarded as synonymous with elevation of temperature; and inasmuch as such elevation necessarily presupposes a condition of antecedent depression, we may, without impropriety, comprehend the meteorological effects of high and low temperatures (heat and cold) under one and the same generalization.

Central Heat of the Earth.—The hypothesis was first propounded by Leibnitz, that the whole of our planet was once a molten mass, which by the operation of cooling, uninterruptedly going on in successive ages, has become superficially encrusted over, the crust having become adapted to the necessities of animal and vegetable life. Various circumstances may be adduced in favour of this notion, more particularly the gradual increase of terrestrial heat downwards, the heat of deep springs, and the evidences of fusion in what geologists term the igneous rocks. Whether the idea of Leibnitz hold good in its entire acceptation,—that is to say, whether the centre of our planet be one molten mass or not,—there can be little doubt that all positions of the globe, at a sufficient distance below the surface, have at some period been submitted to fusion. Nevertheless, at this time, the earth's central heat may be altogether ignored as tending to influence, in any manner, the climatic temperature of our globe. Primarily, the sun's direct rays determine the climatic temperature; those portions of the world's surface being most strongly heated on which the sun shines at the greatest angle—a remark which of course applies to the tropics; while those are least heated on which the direction of solar rays is most oblique—a remark which of course applies to the arctic and antarctic regions. But the latitude of a region has less connection with its climatology than might at first seem probable. The varying conditions of insular and continental sea level, or elevated table-land, valley or mountain, and still more the influence of thermal oceanic currents, have much to do with the climatic result. The high table-land of Mexico is strongly illustrative of the effect of mere elevation. The traveller who disembarks first on the Atlantic coast, and wanders inland, soon finds himself amidst all the luxuriance of a tropical forest, and surrounded by all the dangers of tropical existence. Still wending his way inland, he ascends a mountain elevation, and finds himself suddenly transported to a region, where, on account of its elevation, the climate has totally changed, and with it the vegetation. So marked is the change, that the high table-land of Mexico is well adapted to the growth of wheat, which refuses to grow anywhere in tropical lowlands.

Of all the causes which influence the climate of a region, that attributable to oceanic currents has been hitherto least studied; yet there is none which deserves to be scrutinized more narrowly. Looking at the enormous amount of oceanic surface in comparison with that of the land—taking into consideration the mobility of water, its susceptibility of thermal impressions, and the effect of the configuration of capes, headlands, and lines of coasts—the contemplative observer soon arrives at the deduction, that the ocean presents to him, at least, as wide a field for investigation as the atmosphere, and one scarcely less interesting. "The fauna and the flora," says Maury, "of the sea are as much the creatures of climate, and are as dependent for their well-being upon temperature, as are the fauna and the flora of dry-land. Were it not so, we should find the fish and algæ, the marine insect and the coral, distributed equally and alike in all parts of the ocean. The polar-whale would delight in the torrid zone, and the habitat of the pearl-oyster would be also under the iceberg, or in frigid waters colder than the melting ice." The particles of water being mobile, the ocean, and indeed aqueous collections generally, are amenable to the same law of conviction as was described when treating of the cause of winds. Hot water, being specifically lighter than cold water, must necessarily come to the surface, and for every current in one direction there must be a counter-current in the reverse direction, precisely in the same manner as occurs in the development of a wind.

Contemplating the ocean in its relation to the effects of heat and motion, our original ideas concerning that vast collection of waters are modified and expanded. Instead of regarding the ocean as one shapeless aggregation of briny water, it presents itself to us as an assemblage of many streams,—a network of mighty rivers, each following its own course, each having its own temperature, its own flora, its own animals; and though devoid of palpable banks, scarcely less accurately defined on that account. Amongst all these oceanic currents, that denominated the gulf-stream is the largest in size, the most important in the functions it subserves. The gulf-stream is so far from being an imagining of mere theory, that the dark blue alone of its waters suffices to point out its limits and define its course.

All hypotheses as to the cause of the gulf-stream are as unsatisfactory as the direction of the stream itself, and its benign influences are evident. The first idea relative to the gulf-stream was, that it originated in the impetus given to the ocean by the disemboguement of the Mississippi; but placing out of consideration the inadequacy of this assumed cause, on account of the comparatively small amount of water which even a river so vast as the Mississippi can pour forth, it follows that, if really the cause of the gulf-stream, the whole Gulf of Mexico should, in process of time, be found to contain only fresh, or at the most brackish water. This, it is scarcely necessary to remark, is not the case. Franklin advanced the theory, that the gulf-stream is referable to the pressure of an inordinate amount of water against the coast of the Gulf of Mexico by the trade-winds,—an idea which is scarcely more tenable than the last.

Whatever the cause of the gulf-stream may be, the direction of its current is obvious. Setting out from the hot regions of the Mexican Gulf and the Caribbean Sea, it proceeds northward to the great fishing-bank of Newfoundland, and thence to the shores of Europe, yielding up its heat to the genial west winds, and thus transferring a portion of the superfluous heat of the tropics to our colder shores. The greatest heat of the oceanic water of the Mexican Gulf is about 86°, or about 9° above the ocean temperature due to latitude alone. After it has ascended to 10° of north latitude, the gulf-stream has still only lost 2° of the original heat with which it set out. Ascending northwards a distance of three thousand miles from its first origin, this mighty oceanic river still preserves the heat of summer even in winter time. It now crosses in a westerly direction, in a line coincident with about the fortieth degree of north latitude, spreads itself out, and imparts to Europe a genial temperature; which mere latitude could never give. The gulf-stream now pauses in its course; it is split into two divisions by the British Isles, and two gulf-streams are formed. Of these, one tends northward in the direction of Spitzbergen, whilst the second enters the Bay of Biscay—imparting temperature to each, and causing a soft mantle of vapour to arise, which, wafted landward, in its turn disperses the heat of the gulf-stream far inland. Very little is known concerning the depth to which the gulf-stream extends.

Lieut. Maury, of the United States naval service, assumes that depth to be two hundred fathoms; and arguing on this assumption, he calculates that the amount of heat led away from the Gulf of Mexico by this oceanic torrent raises, on a winter's day, the whole atmosphere which hovers over France and the British Isles from the temperature of 32° F. to about 79°; in other words, from winter-cold to summer-heat. But the genial influence of the gulf-stream on the British Isles is more than this. Every western breeze that blows towards us crosses the mighty gulf-stream, robs it of a portion of its heat, and comes towards our shores charged with warmth and laden with balmy moisture, clothing Ireland in a suit of green, and imparting a mildness to both England and Ireland which can be best appreciated when we consider that the coasts of Labrador, on the American side, and under the same parallel of latitude as England, are rigid with ice. In 1831, the harbour of St. John's, Newfoundland, was closed with ice as late as the month of June; yet the harbour of Liverpool, though 2° further north, is never ice-locked even in the severest winters. By referring to any chart of isothermal lines, the current of the gulf-stream, as just described, may be readily traced.

Although the climatic effect of the gulf-stream is so advantageous to Western Europe, more especially to these Isles, it is scarcely less advantageous to the regions whence it originates. If the gulf-stream be the channel along which an amount of heat so considerable passes westward, we may speculate on the consequences that would have arisen had the amount of temperature now conveyed away remained in the gulf itself. Even now the coast-line of this region is extremely hot and unhealthy; how much more hot and unhealthy would it have been had the gulf-stream not existed!

Under-Current of the Gulf-Stream.—As the trade-winds are only an under atmospheric current, passing in an opposite direction to a current above; so the gulf-stream is only the counterpart of an inferior current of cold water flowing back to compensate for that which has departed. Not only does theory proclaim this, but it is borne out by experiment. At a mean depth of two hundred and forty fathoms, an under-current of water flows into the Caribbean Sea; and the temperature of this current has been found as low as 48°, whilst the surface-water had a temperature of 85°. At the depth of three hundred and eighty-six fathoms the temperature had fallen to 43°; and at the very bottom of the gulf-stream the temperature was only 38°; hence the existence of the returning cold current is fully borne out. It comes, there is little reason to doubt, from the arctic circle; presenting the closest analogy to the lower aerial current which constitutes the trade-wind.

The course and extent of the gulf-stream were not generally known until the celebrated Dr. Franklin drew attention to the subject. The history of this event is worthy of narration, illustrating as it does the discriminating and logical mind of that extraordinary individual.

Happening to be in London in 1770, his opinion was demanded respecting a memorial presented by the Board of Customs at Boston to the Lords of the Treasury, stating that the Falmouth packets were generally a fortnight longer on their voyage to Boston than common traders were from London to Providence, Rhode Island: whence their request that the Falmouth packets might be sent to Providence instead of to Boston. "Franklin could not understand the reasonableness of this request, inasmuch as London was much further than Falmouth, and from Falmouth the routes were the same, so that the difference should have been the other way. Desiring a solution of his difficulty, he consulted Captain Folger, a Nantucket whaler, who chanced to be in London at the time. The whaler explained that the difference arose from the circumstance that Rhode-Island captains were acquainted with the gulf-stream, while those of the English packets were not. The latter kept in it, and were driven back sixty or seventy miles a-day; while the former avoided it altogether."—*Maury.*

The manner in which the old whaling captain had been made acquainted with the existence, the extent, and the direction of this gulf-stream, is curious enough in its way. His instructors were the objects of his search, *the arctic whales*—animals which, having a dislike to warm water, never enter the gulf-stream, though they swim close up to it on both sides.

Franklin having extracted this intelligence from the whaling captain, got him to draw a chart of the gulf-stream to the best of his ability. The chart was drawn, Franklin had it printed, and copies were sent to the Falmouth captains. They, however, were foolish enough to pay no heed to its teachings; nor did they profit for many years after by the knowledge of the gulf-stream. Though the date of Franklin's discovery was 1775, yet a knowledge of the gulf-stream was not generally diffused and acted upon until fifteen years later. Not the least extraordinary fact in connection with the gulf-stream is the sharpness of its line of demarcation. No river imprisoned between two scarped rocky banks could flow in channel more defined. "If," remarked the American author Jonathan Williams, "these strips of water had been distinguished by colours of red, white, and blue, they could not be more distinctly discovered than they are by the thermometer." "And, he might have added," remarks Maury, "nor could they have marked the position of the ship more clearly."

The notion prevails amongst sailors that the gulf-stream is the great storm-breeder of the Atlantic—the father of storms; and, indeed, the tempests which follow in its course or on its borders warrant that designation. What are the indications of theory in this respect? Had the Atlantic been still an untravelled waste, and the existence of a gulf-stream, such as we now know it, been propounded as the basis of discussion, would not the theorist have predicted that storms must originate in the meeting of the hot, moist atmosphere which hovers over the ocean tract of seething waters from the fiery shores of Mexico and the Caribbean Sea, mixed with the chilling blasts of the north? What torrents of water must result from the condensation of the tepid mists—what stupendous electrical force must be brought into operation!

If the gulf-stream, by its impulsive flow, sometimes impedes the mariner and drives his ship from the desired course, it nevertheless affords a compensation, not only in assisting to propel ships sailing in the direction of its course, but in affording a genial climate to the weather-beaten mariner, frozen and benumbed by the shivering blasts of the regions outside its channel. "No part of the world," says the writer to whom I am largely indebted for much that in these pages concerns ocean currents and ocean climatology,—"no part of the world," remarks Maury, "affords a more difficult or dangerous navigation than the approaches of our (the American) coast in winter." Before the warmth of the gulf-stream was known, a voyage for this reason from Europe to New England, New York, and even to the capes of the Delaware or Chesapeake, was many times more trying, difficult, and dangerous than it now is. In making this part of the coast, vessels are frequently met by shore-storms and gales which mock the seaman's strength, and set at naught his skill. In a little while his bark becomes a mass of ice, and his crew frosted and helpless. She remains obedient only to her helm, and is kept away for the gulf-stream. After a few hours' run she reaches its edge, and almost at the next bound, passes from the midst of winter into a sea at summer-heat. Now the ice disappears from his apparel; the sailor bathes his stiffened limbs in tepid water; feeling himself invigorated and refreshed with the genial warmth about him, he realizes out there at sea the fable of Antæus and his mother Earth. He rises up and attempts to make his port again; and is again as rudely met, and beaten back from the north-west; but each time that he is driven off from the contest, he arises forth from this stream, like the ancient son of Neptune, stronger and stronger; until after many days his freshened strength prevails, and he at last triumphs, and enters his haven in safety, though in the contest he sometimes falls to rise no more, for it is often terrible. Many ships annually founder in these gales; and I might name instances, for they are not uncommon, in which vessels bound to Norfolk or Baltimore, with their crews enervated in tropical climates, have encountered, as far down as the Cape of Virginia, snow-storms that have driven them back into the gulf-stream, times and again; and have kept them thus out for forty, fifty, and even for sixty days, trying to make an anchorage.

Nevertheless, the presence of the warm waters in the gulf-stream, with their summer-heat in mid-winter off the shores of New England, is a great boon to navigation. At this season of the year especially, the number of wrecks and loss of life along the Atlantic sea-board are frightful. The month's average of wrecks has been as high as three a-day. How many escape by seeking refuge from the cold in the warm waters of the gulf-stream, is a matter of conjecture. Suffice it to say, that before this temperature was known, vessels thus distressed knew of no place of refuge short of the West Indies; and the newspapers of that day—Franklin's *Pennsylvania Gazette* among them—inform us, that it was no uncommon occurrence for vessels bound to the capes of Delaware in winter to be blown off, and to go to the West Indies, and there wait for the return of spring before they would attempt another approach to this part of the coast.

The gulf-stream is the largest known oceanic current; it has been perhaps more fully studied than any other, and its teachings may therefore be appropriately regarded as the type of the rest. We have seen how powerful and extensive are its effects; we have seen a few of the purposes to which it ministers. The ocean is full of streams similar to this, each taking its well-defined course, carrying its own temperature, clad with its own fauna and flora, peopled with its own denizens. Thoughts like these prove how false and unfounded is the expression, *ocean waste*, so commonly applied. The ocean has its regions, its valleys, and its mountains—climates and varied inhabitants—no less than the earth.

Other Oceanic Currents.—*The Mediterranean.*—It has long been known that an upper or sailing current constantly sets into the Mediterranean through the Straits of Gibraltar. What, then, becomes of the water of the currents? That water must either be dissipated by evaporation, or there must be a second or back-current.

The existence of this under-current was first demonstrated very curiously in 1712. At that time, France being at war with Holland, M. L'Aigle commanded a French privateer, called the Phœnix of Marseilles. Near Ceuta this privateer gave chase to a Dutch ship bound to Holland, come up with her, delivered one broadside, when the Dutch ship immediately went down. A few days later, the sunken ship, with all her cargo of brandy and oil, came to light again; but this took place on the coast of Tangier, at least four leagues westward of the place where the ship went down, and in a direction quite opposite to that of the upper or navigable current. This well-authenticated case was communicated to the Royal Society in 1724.

Currents of the Red Sea.—Precisely similar to the Mediterranean currents, just described, are those of the Red Sea. The necessity of a free change of waters here is even more necessary than in the Mediterranean; the sea is not only shallower, and subjected to a more powerful evaporation, but its waters are not freshened by the afflux of any rivers. It has been calculated by Dr. Buist, that, taking into consideration the mean evaporation on every part of the surface of the Red Sea, a sheet of water, eight feet thick, and equal in superficial area to the whole extent of the surface of the Red Sea, will be raised in vapour annually. When this enormous rate of evaporation is considered, the necessity for a continuous interchange of water between the Red Sea and the external ocean will be evident. If the Red Sea outlet were choked up, so that ingress and egress were no longer possible, the evaporation of about a thousand years would, it is calculated, completely dry it up.

Currents of the Indian Ocean.—Many thermal currents originate in the Indian Ocean. Amongst the foremost of these is the Mozambique current; another of these currents, first escaping from the Straits of Malacca, and being swollen by warm streams from the Java and Chinese Seas, flows out into the Pacific between the Philippines and the Asiatic shores. Passing thence towards the Aleutian Isles it ultimately loses itself on the north-west coast of America. Meteoric conditions like those which mark the course of the gulf-stream also mark the course of this. Fogs and mists follow in its track, and storms are also generated on the bank of this oceanic river.

For a fuller account of oceanic currents than is consistent with the limits and objects of these pages, I must refer the reader to treatises which deal with the special matter, and above all to the work of Lieutenant Maury, of the United States service, to whom meteorologists are under deep obligations for his contributions to their knowledge of ocean phenomena.

In contemplating these oceanic currents, of which the gulf-stream may appropriately be considered the type, we cannot fail to be impressed with an evidence of design, where at first no design would seem to exist. These oceanic currents originate in, and are determined by, a peculiar conformation of the land. Now what can be more seemingly irregular or capricious than the shape of land? If dropped down into the ocean at random, or elevated by a subterranean power equally capricious, the crust line of islands and continents, the solid blocks of our planetary crust, could not well be more irregular than they are; and yet how practically harmonious is the relation between land and water: how well adjusted the powers of each, how well adapted to the mutual benefit of mankind. It appears an unimportant matter when in the map of the world we skim our eye over the southern hemisphere of the terrestrial globe; there we behold the limits of the African and American continents in their furthest extent; but how terribly would the locomotive faculty of mankind have been impeded had either continent expanded itself to the south pole, or even traversed much further south than is actually the case!

In connection with the subject of ocean currents, the effects of aqueous temperature on the ocean's denizens deserves a passing word of remark further than has already been devoted to it. In the Caribbean Sea, and other ocean caldrons, where stores of heat are accumulated for distribution in regions far away,—in the hot currents which originate in these tepid sources of oceanic rivers, there are a fauna and a flora: the fauna no less marked than we see on tropical lands. There grows the coral, there swims the shark; and thousands of shelled mollusca, of gorgeous colour and enormous size, revelling in oceanic forests of rank and bulky growth, represent the land forests and their denizens of corresponding climes. But though grandeur and beauty be the characteristics of these warm ocean spots—the thermal ocean-tropic (if the propriety of that expression be allowed)—it is the colder oceanic currents which support the form of animal life most useful to man. Strange though the circumstance may appear, it is no less true, that the fish of the hot parts of the world are always indifferent as food. This fact is strikingly illustrated in the Mediterranean. The temperature of the Mediterranean water is usually four or five degrees above the temperature of the external ocean, and what a difference in the fish! Whoever has compared the edible fish of the Atlantic with those of the Mediterranean, will be at no loss to admit the vast superiority of the former. The naturalist does not require to be informed, that not only are Mediterranean fish inferior to those of the Atlantic, but they are for the most part of different species.

Let us now take a glance at the locality of the principal fishing regions. Limiting ourselves to two of these, they would unquestionably be the fishing grounds of Newfoundland and Japan. The former is the better known of the two, though, perhaps, the latter is the more considerable, seeing that the natives of Japan are debarred from the use of animal food by their religion, although the eating of fish permitted, they are all ichthyophagi; and, notwithstanding the dense population of the Japanese islands, their inhabitants, though fish-eaters, are abundantly fed.

Between the gulf-stream and the coast is a narrow band of cold water: here are the fisheries of Newfoundland. Between the China current, as it is denominated, and flowing in an opposite direction, is the cold current, in which the Japanese fisheries are prosecuted. These are the two most prominent examples, and they will be found to present the type of many others.

Cold, and its Effects.—Under the heads of snow, hail, and hoar-frost, some of the meteorologic effects of cold have been already described. The phenomena due to this powerful agency are, however, so numerous and so important, that it may be well to make the subject of cold a matter of special contemplation. If, casting our eye over the field of nature, we endeavour to select the most prominent results of cold, they will be found to relate to the departure from an ordinary law, which Nature has made in the expansion of water during the act of freezing.

The term *freezing* is but a general expression for the act of solidification by cold. Popularly, the term is only applied to solidified water; but this is altogether a conventional acceptation of the word. We are justified, then, in comparing the act of solidification of water, with the act of solidification of any other fluid, and seeing to what extent the conditions which regulate one also regulate the other.

When the temperature falls to 32° F. water ceases to be liquid, and becomes ice; the weather is said to be frosty, and water is said to be frozen. Whatever water be contained in the atmosphere at the freezing temperature, is deposited in the solid form of hoar-frost, the particles not being irregular, but bounded by definite mathematical outlines—frequently giving rise to forms of great beauty, especially on window-panes, blades of grass, and leaves (Fig. 87). These forms are similar, in general contour, to

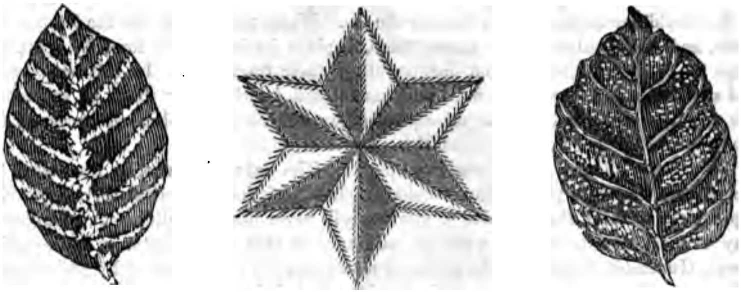


Fig. 87.

those of well-formed snow-flakes (Fig. 88), but far more beautiful, and, like snow-flakes, prove that frozen water is a crystalline body, and that it crystallizes in forms belonging to the rhombohedral system.

But the most important point connected with the freezing of water, and without

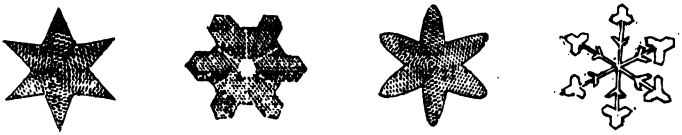


Fig. 88.

which our globe would cease to be habitable, is this:—Water, during the act of freezing, expands, and thus becomes specifically lighter. Ice, therefore, swims on water; it cannot sink. How stupendous are the consequences of this departure from the law of freezing! Had it so happened that frozen water, like frozen mercury, was more heavy than the corresponding liquid material, each frozen sheet of water would sink as soon as formed; and thus, being far removed from the melting influence of solar rays, the production of ice would have been accumulative, and the ocean, long ere this, would have been completely ice-locked.

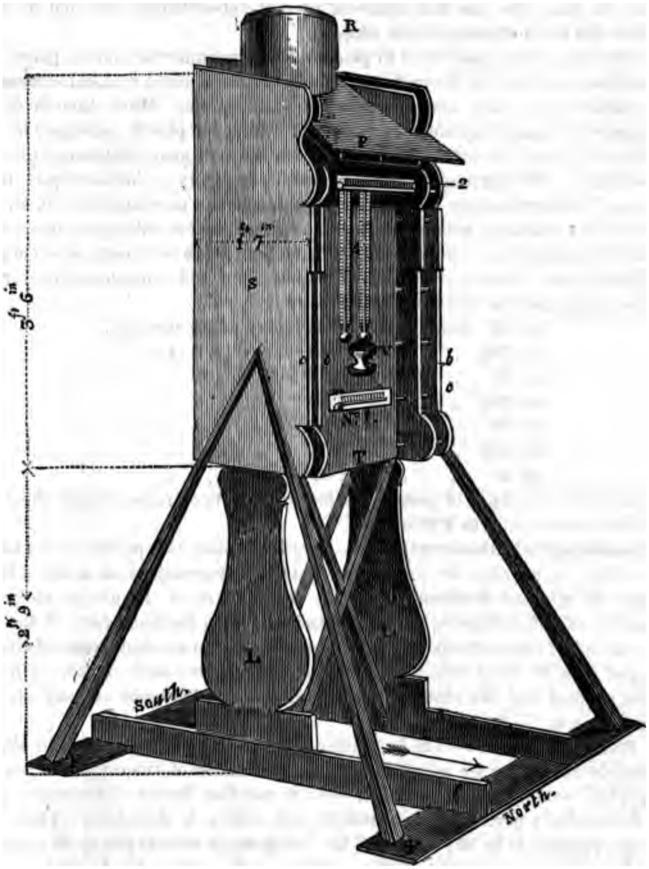
In tracing, hypothetically, an assumed aberration of Nature to its consequences, the mind is speedily entangled and lost in the chaos of jarring effects. The speculative meteorologist finds himself unable to thread the labyrinth of causation here involved. Having shown that the ocean would have been full of ice, had the ordinary law of solidification not been departed from in the case of water, it is perhaps unnecessary to follow the development of our hypothetical case further. That ocean life must have been destroyed is evident; that the sea's liquid highway would have ceased to be, is only a figurative expression for a frozen ocean. But would what is now the solid land have then served the purposes of animal life? Where could the rivers have flown, had the ocean been a block of ice? or would not the rivers have remained frozen too, seeing the vast cooling power of a frozen ocean? It is easy to see that, under such circumstances, our planet would have been totally unfit to be a resting-place for its present denizens had the freezing of water not assumed a departure from a law, though it be impossible to imagine all the consequences that would have resulted.

The same expansive force of water during the act of freezing, by the operation of which it is rendered specifically lighter than water, subserves many important purposes in the world's economy, besides floating the ice. Water percolating into the fissures of rocks, and being subsequently frozen there, displays its irresistible force by splitting large rocks into fragments, and disintegrating their fragments. In this way hard and sterile districts become covered with useful soil, in which low forms of vegetable life can take root; and by their subsequent decay, contribute the elements necessary to the support of higher forms of vegetation.

Relation of Climate to Organic Development.—The naturalist who, in his desire to see as in a dioramic picture the wonderful characteristics of animal and vegetable types, sighs to think that a vision so glorious will never pass in reality before his gaze, may at least console himself with the assurance of that great master of philosophic travel, Humboldt, that it is in the power of man's creative faculty, aided by philosophy, to imagine those striking types in the vividness of their truth, gladdening the closet with ideal images of the living features of Nature.

Even in the narrow region of European travel, the intelligent observer will not fail to see distinctive physiognomies. Passing from the cold green-sward and modest vegetation of our own Isles to the Mediterranean shore, a striking change in the aspect of nature meets the view. The sturdy oaks and elms of our own forests disappear; the absence of smaller grasses removes the green carpet of our meadows; tall graminaceæ spring up; the aloe and the prickly pear bespeak a mixed condition of heat and drought; and the date-palm, barely acclimatized, gives some faint notion of what the characteristics of a tropical forest must be. "It would be an enterprise worthy of a great artist," says Humboldt, "to study the aspect and the character of all these vegetable groups, not merely in hot-houses, or in the description of botanists, but in their native grandeur in the tropical zone. How interesting and instructive to the landscape painter would be a work which should present to the eye, first separately, and then in combination and contrast, their leading forms! How picturesque is the aspect of tree ferns, spreading their delicate fronds above the laurel-oaks of Mexico; or the groups of plantains overshadowed by arborescent grasses. It is the artist's privilege, having studied these groups, to analyze them: and thus in his hands the grand and beautiful form of nature which he would portray resolves itself, like the written works of men, into a few simple elements."

When the meteorologist has exhausted his knowledge in the laying out of climatic groups—when he has placed in correlation conditions identical, as he thinks, in every respect—the growth of vegetable forms demonstrates his inability to comprehend many hidden secrets of nature, which their delicate organization makes known. European olive-trees grow luxuriantly at Quito, but they bear neither fruit nor flowers; and a similar remark applies to walnut-trees and hazel-nuts in the Isle of France. In India, the bamboo flowers luxuriantly; but in South America, where it flourishes equally well, so far as general aspect of growth is concerned, so rare an event is the infloration of the bamboo, that during a four years' residence in South America, Humboldt was only enabled to obtain blossoms once. But perhaps a still more remarkable example of luxuriant growth without inflorescence is furnished by the sugar-cane. The West Indies have come to be considered as the region *par excellence* of the sugar-cane; yet it seldom bears flowers there—nor indeed does it in any part of the American continent; thus furnishing a strong presumptive argument in favour of the theory which asserts that no variety of the sugar-cane is indigenous to the New World.



LAWSON'S THERMOMETER STAND.

PRACTICAL METEOROLOGY.

So many amateur meteorologists are now springing up in every direction, that it may be desirable to point out the precautions and reductions necessary in order to render the raw observations useful to science. The present paper is, therefore, written in continuation of Dr. Scoffern's Meteorology, to enable the amateur, with as little trouble as possible, to reduce his observations to useful results.

To the non-meteorologist such terms as adopted mean temperature, elastic force of vapour, reduced barometric, &c., unexplained, serve rather to perplex than to enlighten, or point out that care has been bestowed upon the observations, and that the requisite reductions and precautions had been adopted.

The first duty of an observer is to procure good instruments, and to place them in such positions as may be deemed least likely to be affected by local circumstances, such as heat from a fire, draughts round a building, &c. Much time is frequently thrown away by using imperfect instruments. When a person purchases a thermometer, he is too apt to consider it correct; whereas, in many instances, it is far from being accurate. Mr. Hartnup, of the Liverpool Observatory, in his last report, mentions that among the thermometers used by the captains of the merchant service, an error of 4° or 5° is quite common; even a thermometer fitted up for taking the temperature of water at different depths, and professing to have been made with care, was found to be 8½° in error in one part of its scale. The most laughable instance, however, was a barometer which had the following errors:—

At 28 inches was — 2·22 inches of the standard.
At 28½ " " — 1·88 " "
At 29 " " — 0·73 " "
At 29½ " " — 0·30 " "
At 30 " " — 0·02 " "
At 30½ " " + 0·33 " "
At 31 " " + 1·07 " "

The whole yearly range of pressure seldom reaches two inches, whilst the range in error of this instrument was 3·29 inches.

Meteorological Observations.—In order to place this portion of the subject in as clear a light as possible, let us suppose that daily observations are made of the barometer and its attached thermometer, of the temperature of the air, of the wet bulb thermometer, of self-registering maximum and minimum thermometers, of the amount of rain, amount of ozone, temperature on the grass and in sunshine, state of electricity, amount and class of cloud, direction and force of winds, and state of the weather. Let us further suppose that two observations are taken daily, the hours of observation being 9 A.M. and 10 P.M.

The Pressure of the Air.—The barometer is the instrument by which the alterations in the weight of the air are ascertained. The gravitation of the earth exerts a certain pressure which would always be alike, were it not that lateral disturbances had the power of removing a portion of that pressure and adding it elsewhere. Thus, suppose the average pressure to be 29½ inches, if the barometer is seen to rise to 30 inches, it is certain that in another portion of the earth a corresponding fall has taken place to account for this change; however, as it is not our object to enter into the physical and local changes of the weather here, we shall at once proceed to describe the reductions that are requisite.

It is essential to correct observation that the barometer used be a standard instrument. The ordinary instruments are useless, owing to the friction of the mercury against the sides of the glass in small tubes, the impossibility of applying the reduction necessary to *correct* for the alteration of the height of mercury in the cistern with the common wheel barometer, owing to a rise and fall in the tube; and, further, because if errors occur in this barometer, they are increased by the circumstance of the index of the wheel barometer being a long-arm worked by a small wheel, thus multiplying all the errors.

The Standard Barometer should have a tube of not less than $\frac{4}{10}$ths of an inch in

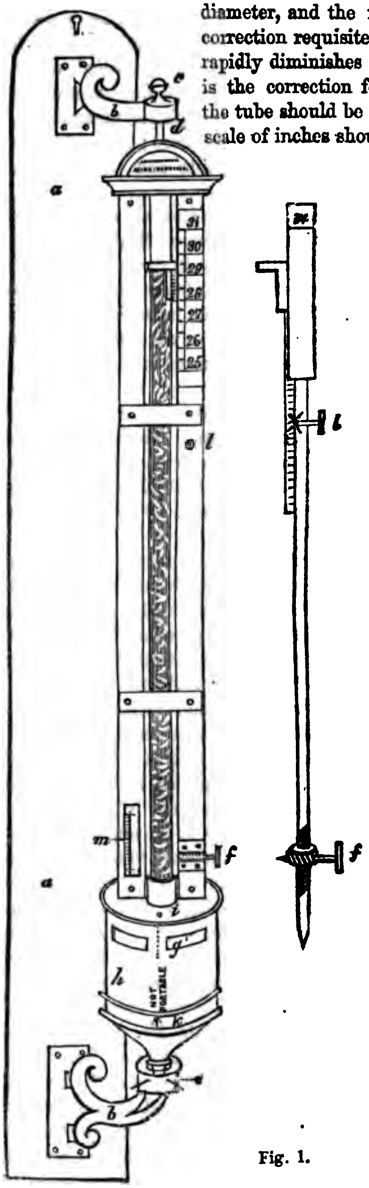


Fig. 1.

diameter, and the nearer it approaches to $\frac{8}{10}$ths the better, as the correction requisite for the friction against the sides of the glass rapidly diminishes with an increase in the size of the tube. This is the correction for capillarity, which is additive. The base of the tube should be plunged into a cistern of pure mercury, and the scale of inches should have a rod attached, terminating in an ivory point, so contrived as to be moved by rack-work until the point *just* touch the surface of the mercury. This is requisite, in order that the measurement may be made from the surface of the quicksilver in the cistern. Suppose we either neglect this operation, or the barometer is one not capable of having it applied; and let us further take a reading without reference to this correction after a sudden change in the barometer, this reading may be represented as 30·036 inches; on bringing the ivory point down to the mercury, this second reading may be 30·074 inches: thus exhibiting an error, from the want of this precaution, of ·038, or nearly four-hundredths of an inch. The thermometer fitted for use should have its bulb plunged into the cistern of mercury, otherwise it will not give the temperature of the mercury itself, but merely show the heat of the apartment in which it is placed. Such an instrument is made by Newman, the optician in Regent Street; it is the same as is made for the Royal Society, Greenwich Observatory, Admiralty, and the British Colonial Observatories.

Description.—*a* is the mahogany board to affix against the wall; *b*, brackets which support the barometer, between which it is capable of being revolved so as to observe the light on the surface of the mercury; *c*, a vase, which unscrews to allow of the socket *d* being removed to receive the upper end of the barometer; *e*, the adjusting screws for shifting the lower centre, by which the barometer is to be made exactly perpendicular: to accomplish this, the ivory point is to be adjusted to the surface of the mercury, the barometer gently turned between the two brackets; and if in any position the point should be elevated from the surface, or depressed into the mercury, the screws must be altered accordingly, until the point coincides in every position; *f*, the key by which the ivory point is adjusted—the ivory point being a termination of the brass scale marked off at the temperature of 32°, and which is adjusted by means of a tangent screw; *g*, the glass part of the cistern, through which the surface of the mercury and ivory point are seen; *h*, the cistern; *i*, the screw which is to be loosened when the barometer is fixed, to admit the atmospheric pressure; *k*, the moveable part of the cistern, on which the index *↑* is engraved; *l*, the key by which the vernier is adjusted; *m*, the thermometer dipping into and showing the temperature of the mercury.

This barometer is necessarily expensive. Modifications of this standard are made by Mr. Barrow of Oxendon Street, and Messrs. Negretti and Zambra of Hatton Garden; and these are more reasonable in price, answering remarkably well, though not equal to Mr. Newman's Standard. Having procured a barometer, it should be compared with the Standard, in order to ascertain its index error. The apartment in which it is kept must not be subject to great changes of temperature; one with a window facing the north is to be preferred; and the instrument should hang where there is a strong light, but an outer wall should be avoided.

Thermometers.—There are various constructions of instruments for ascertaining the temperature of the air; of these the mercurial ones are the best. Thermometers for comparison require to be placed upon a proper stand, as the errors arising from peculiarity of situation, from radiation, absorption of heat, &c., will alter the reading very materially. There are two forms of stand, the one constructed by the late Henry Lawson, Esq., F.R.S., the other by James Glaisher, Esq., F.R.S.

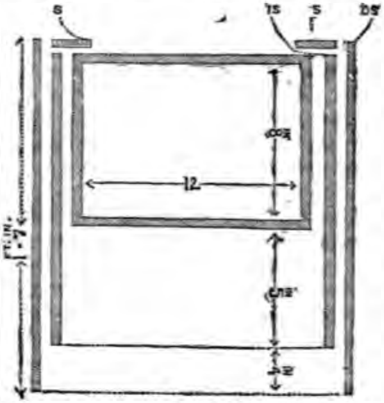


Fig. 2.

Thermometer Stands.—In comparing the readings of one thermometer with any other, it is requisite that each instrument should be placed, as much as possible, in a similar manner; without this uniformity no deductions can be drawn with any claim to accuracy. In looking to the situation of instruments used by persons who have not been aware of the necessity of providing themselves with a thermometer stand, some will be found facing the north, others the south, south-east, north-west, and, in short, every point of the compass. Again, some are placed from one to five feet from the ground, others ten to twenty feet; some in the angle of a large building, others exposed to the sun's rays either during the morning or the evening; some touching a wall, others at a distance from one; in short, the situations vary in as great a degree as the temperature deduced from the observations made by them. To obviate these sources of inaccuracy, the late Mr. Lawson constructed his "Thermometer Stand," a sketch of which is given at the head of this chapter. This stand consists of a frame, which has been found to answer the intended purpose very well. It is composed of *white deal* boards, and can be constructed by any carpenter. It consists of an oblong trunk T, 12 inches by 8 inches outside measure; to the opposite side of which trunk are nailed boards *b b*, at the distance of three quarters of an inch, and projecting about six inches from the trunk towards the north. Outside of these are nailed other thin boards *c c*, full half an inch distant, and projecting about four inches beyond the last-mentioned boards, also towards the north. These sides or shades being multiple, prevent the sun from heating the interior of the stand where the thermometers are placed. The top, or pent board, P, is made double, and the boards are placed full three-quarters of an inch distant from each other, and come forward so as to overhang, by a full inch, the Night Index Thermometer, placed immediately beneath, for the purpose of preventing rain or dew from falling perpendicularly upon the bulb of the thermometer. The legs, L L, of the stand are merely the continuation of the sides of the trunk. The board or feet, F F, are loaded or fixed to the ground, to sustain the force of the wind. The interior, T, is blackened to prevent strong reflections of light.

Fig. 2 is a ground-plan of the machine, which will prove sufficiently clear to any intelligent workman for its construction. The sides (and wood-work generally) are of half-inch white deal. The distance or space between the sides of the trunk T and the board or inner side, *i s* (Fig. 2), is three-quarters of an inch; and the distance from that board to the outer side, *o s* (Fig. 2), is full half an inch. The narrow boards, *s s* (Fig. 2), are to be nailed, with studs intervening, to the middle board or side *i s*, and are for the purpose of preventing the sun from shining between the trunk and the sides *o s* and *i s* when near the meridian. The sides are fixed, one upon the other, at the required distance (viz. three-quarters of an inch and half an inch), by numerous wooden studs, about three-quarters of an inch diameter; and the nails or screws passed through the sides and studs, fixing the whole firmly together. The whole is to be painted white, and no other colour, except the face of the trunk T, which may be black, to prevent strong reflections of light.

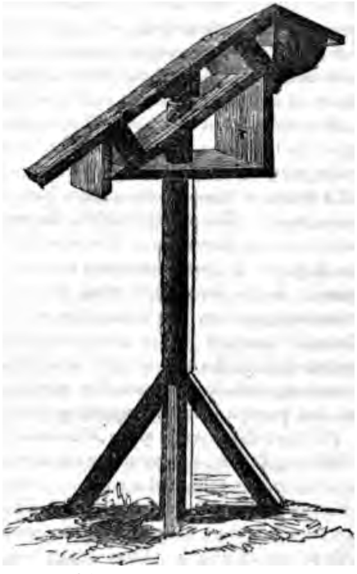


Fig. 3. Glaisher's Stand.

This thermometer-stand can be placed in *any* eligible spot that may suit the convenience of its owner; its four sides should face the cardinal points, commanding therefore a true north and south aspect. It can be visited on every side, and be free from all surrounding objects. The thermometers used can be read off with the greatest facility, and the whole will be at a known distance from the ground. Those instruments placed on the south face will have the meridian sun, and those on the north face will be always in the shade, in consequence of the projecting wings. It can be employed by any meteorologist, wherever residing; it is of a determinate form, height, and size; it is not costly, but firm, and can be placed on any open spot that may be thought eligible for its use. The instruments may be read off with the greatest promptitude, so as to prevent or reduce errors arising from the person of the observer being too long in the vicinity of the thermometers. By the general adoption of this stand, instruments placed upon it will all be used or observed under similar circumstances, and deductions from them be more correctly drawn than where there is no stand. It follows that observations made by individuals wherever residing, either in Europe, Asia, Africa, or America, if drawn from instruments thus similarly placed, can be compared one with the other with far less chance of error than has hitherto been the case.

Mr. Glaisher, F.R.S., has likewise constructed a stand differing from the one just described, but which is also an excellent contrivance.

Rutherford's Thermometer.—The mercurial thermometer of Rutherford's construction pushes a registering-pin before it as the mercury expands by heat; on the air becoming cooler, the mercury contracts, leaving the registering-pin at the point of maximum heat. One objection to this instrument will always be felt, *i.e.* the pin is not unlikely to become entangled with the mercury, especially if the mercury should oxidize.

It is very desirable to incline Rutherford's maximum thermometer from the horizontal position, so that the bulb shall be slightly the highest, in order that the thermometer may have the assistance of gravitation in pushing the pin forwards. Within the last fifteen years many of Rutherford's construction have at the Highfield House Observatory become useless, owing to the registering-needle becoming entangled with the mercury. Two thermometers, however, of this construction deserve to be noticed, the one purchased of Dollond and the other of Bennett: they have done their duty satisfactorily, and are at the present time in perfect working order.

Negretti's Patent Maximum Thermometer.—Too much praise cannot be given to this ingenious invention, which, from the circumstance of doing away with the registering-needle, prevents the possibility of the instrument getting out of order. Since the invention, half a dozen of these thermometers have been constantly employed here without any derangement. The principle of the thermometer is this:—A small piece of enamel is pushed into the thermometer tube near the bulb, and the tube is then bent so as to secure it in its place; if the thermometer act once, it will continue to act until the instrument is broken. As the temperature rises, the mercury will flow over this enamel; yet, on the air becoming colder, the contraction will only take place below this point, the whole column of mercury in the tube being left to mark the maximum heat. It is only requisite to turn the thermometer in a vertical position, and give it a gentle shake, when the mercury will descend and flow over the obstruction, which, in the horizontal position, had prevented it from returning into the bulb of the thermometer.

Phillips's Construction of the Maximum Thermometer.—This, the invention of Professor J. Phillips, F.R.S., also records without a registering-needle. A small bubble of air is passed down the tube, and a portion of mercury is made to remain above this bubble; as the air increases in temperature the whole column rises, yet, when it cools, the mercury only falls from below where the bubble of air is situated. It is an instrument liable to get out of order in travelling; yet, when once properly fixed, does its work well. In consequence of the bubble of air expanding with an increase of temperature, a slight error will be occasioned, and the thermometer will read a little too high in warm weather, and the reverse when cold.

The Minimum Thermometer, until lately, has been filled with *spirit* instead of *mercury*, with a slender glass pin floating in the liquid. This pin is carried down with it to the lowest point; on the temperature rising, this pin is left behind, and thus marks the coldest point. Like the maximum thermometer, gravitation should be allowed to assist the descent of the pin, which is accomplished by slightly lowering the bulb from the horizontal position. The best that I have received have been from Messrs. Negretti and Zambra; in these the pin is much longer and more slender, and they very rarely get out of order. A disadvantage will always be felt with spirit thermometers; their action differs from those filled with mercury.

The Mercurial Minimum Thermometer.—Meteorologists have for some time urged the opticians to invent a mercurial minimum thermometer, an invention which long seemed almost to be an impossibility. However, such an instrument now exists, thanks to Messrs. Negretti and Zambra. A thermometer with a large tube is placed in a vertical position, in which is a slender, pointed needle, which is brought down to the surface of the mercury; quicksilver being heavier than the needle, it is held above it; yet, the needle being pointed, plunges a small, but sensible, distance in the mercury, which it invariably does by the side of the glass of the thermometer tube. This being the case, the needle will descend to the lowest degree of cold; on the thermometer rising, the mercury presses the needle to the glass, and rises up by the side of it, instead of raising the needle. Four months' working of this thermometer has proved it to be a valuable invention. The instrument must come into general use amongst meteorologists.

The Solar and Terrestrial Radiation Thermometers are made entirely of glass, the scale being engraven upon the bulb itself. For thermometers continually exposed to damp, the swelling of the wood and the obliteration of the index thereon has been felt a great annoyance; consequently, the substitution of glass for wood has been hailed with pleasure by meteorologists, independently of its great improvement in a scientific point of view.

The white enamel placed along the back of the thermometer tube, an invention of Negretti and Zambra's, is now becoming generally adopted; the improvement is at once manifest on inspecting two instruments, the one with and the other without the enamel.

The Wet and Dry Bulb Thermometer.—The dry bulb is the ordinary thermometer, and the wet bulb differs only in having the bulb enclosed in a muslin bag, with a cotton-wick conductor to a cup of water, so that it shall always be wet from the capillary action of the cotton conveying the water constantly to it. If the muslin bag were attached to self-registering, instead of ordinary thermometers, the greatest heat and cold of the wet bulb would be obtained—an important addition to the meteorological instruments, yet one almost unknown. The muslin and cotton should be changed every month.

Bennett's Photographic Wet and Dry Bulb Thermometer is an additional evidence of the close relationship between heat and light. In this most ingenious contrivance *light* is incessantly writing down the *heat* of the air. It is simply requisite to place a sheet of paper upon the roller once a day in order to record every change in temperature of the wet and dry bulb for the twenty-four hours. This is, indeed, a triumph of science.

Evaporators.—Having had constructed gauges of various sizes, I am enabled to speak with confidence as to their working. The water in gauges under eight inches in diameter becomes too warm, owing to the small quantity that can be contained in them; consequently, an excess of evaporation results. There are two gauges to be recommended; the one Newman's, and the other Negretti's. These gauges work well. Newman's is a very convenient and ornamental instrument, having a graduated glass tube

It consists of a short cylinder twelve inches in diameter, having connected with it, by means of a stopcock, a glass tube graduated to hundredths, and terminating in a lower vessel, which will contain a sufficient quantity of water to be raised by artificial pressure into the upper one for exposure to the atmosphere.

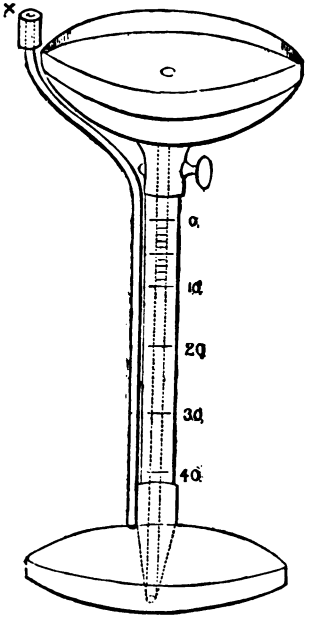


Fig. 4.

To use the apparatus, pour water into it until it rises to the zero in the glass tube, then by means of a syringe force the air through the tube × (Fig. 4) into the lower vessel, so as to raise the water into the upper one to any height you please. Now shut off the stopcock beneath to retain the water in the upper vessel; then, having exposed the apparatus for any length of time required, open the cock, the water will run into the lower vessel, filling it and part of the glass tube, the divisions of which will now indicate the quantity of water evaporated. Negretti's is the cheaper, being simply a cylinder of the diameter of eight inches, which fits into a wooden box filled with wet sand; this keeps the outside of the metal cool, and prevents that excessive evaporation which would result from the heating of the metal by the sun. Where the diameter of the gauge is large, and the water several inches deep, the effect of the sunshine on the metal sides is not felt.

Rain Gauges.—There are several constructions, yet none so good as Negretti and Zambra's, which is simple, and at the same time prevents any loss by evaporation,—an important point, which has been too much overlooked. Another contrivance is that of a cylindrical vessel of brass or zinc (Fig. 5); the latter would be the cheapest, and answer all purposes equally well. Into this cylinder, a funnel, with its tube bent, fits tightly; the diameter should be eight inches, and the tube about an inch in length. The object of this bended tube is to prevent evaporation taking place from the surface of rain collected in the rain gauge, for a few drops of water will hermetically seal the opening from the escape of vapour, and most frequently the evening dews will deposit sufficient moisture for this purpose, which the heat of the day will scarcely have time to dissipate before night brings a fresh supply.

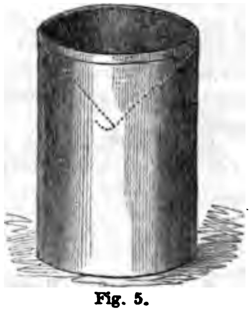


Fig. 5.

The readiest mode of measuring the amount deposited in the gauge, is by procuring another cylindrical vessel, or measure, which is exactly four inches in diameter, and four inches deep; this, when quite full, will just contain an amount equal to the deposit of an inch of rain as collected in the eight-inch gauge. Parts of an inch can be measured by plunging a rule (Fig. 6) perpendicularly to the bottom of the measure, the portion wetted by the water being the decimal part of an inch required. Thus, having made a rule exactly four inches in length, and divided it into ten equal parts, and each division being subdivided into ten others, a measure is obtained which will read off the hundredth of an inch; and as the divisions are tolerably wide, it is not difficult to estimate even to thousandths of an inch. Thus, for example, as used in the measure, the rule four inches long, divided into 100 parts, represents one inch of rain fallen; the score at twenty-five, or one inch, represents a fall of a quarter of an inch, and so on.

Electrometers.—Atmospheric electricity has been much neglected by meteorologists; it is an important item of meteorological investigation. There are several methods of studying the subject: the most simple is Glaisher's electrometer, which being portable, should become generally adopted. Where there are the conveniences for having exploring wires, properly insulated, the results are more satisfactory; and when this plan is adopted, the following electrometers should be used:—

1. De Saussure's Electrometer, which consists of two fine wires, each terminated by a small pith-ball, their expansion being measured by a graduated scale.

2. Volta's Electrometer, consisting of two thin stems of about two inches in length, and fitted to a metal rod by small rings.

3. Singer's Electrometer, consisting of two slips of gold-leaf. For stronger electricity, a pair of Dutch-gold leaves become a useful addition.

4. Zamboni's Dry-pile Electrometer. It is a single gold-leaf suspended from the conducting-rod between two dry piles, the negative pole of the one and the positive of the other being uppermost. This shows whether the electricity is positive or negative.

For powerful electric storms, the self-registering apparatus belonging to the atmospheric recorder is very useful; whilst a nicely-arranged electrical bell will be a means of warning the observer that his presence is required in the electric-room.

The effect of a thunder-storm, when three or four miles distant, as shown on these electrometers, is exceedingly interesting. It is not only possible to witness the instantaneous convulsion caused by a flash of lightning at least twenty seconds before the peal of thunder occasioned by it is heard; but it is also possible to know, several seconds before a flash takes place, that one is about to occur.

The beneficial effects of electricity on the vegetable kingdom are of a character so apparent, that any extended researches upon the branch of Meteorology calculated to throw additional light upon the subject, is very desirable.

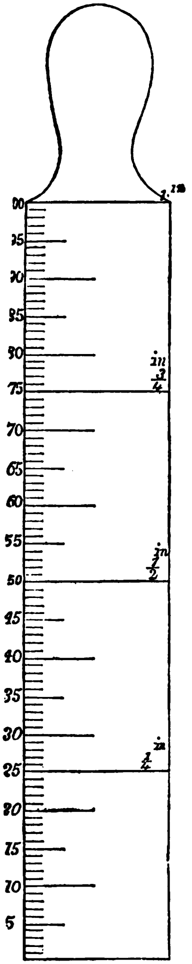


Fig. 6.

The Gimbal Vane.—This is a wind-vane exactly balanced and hung on gimbals, having vertical fans to carry it in the direction of the wind, and horizontal fans to enable it to tip *up* or *down*, to show the angle at which the air is blowing.

The Rain Angle is a simple contrivance for showing the angle at which the rain descends. It consists of a rod, which moves along a graduated arc.

The Distance Measurer.—A modification of the carpenter's rule, having an elevated eye-hole fixed vertically in the joint, and a slender pin screwed vertically on each leg of the rule. The rule itself slides on a graduated arc. In order to measure approximately the distance of a peculiar cloud, or other celestial phenomenon, from any fixed object, the legs of the rule are opened until one of the vertical pins covers the object to be measured, and the other the object to which it is referred. The distance will be seen on the graduated arc.

Ozonometer—A chemical contrivance for ascertaining the amount of ozone in the atmosphere. Ozone was discovered by Dr. Schönbein of the University of Bâle, in the year 1848. Test-papers are hung in a situation where there is a free current of air, but not in sunshine;—the north face of a thermometer-stand is a convenient situation. These papers will remain colourless if there be no ozone present, and will be more or less tinged with blue, as the ozone is more or less powerful. A scale of different tints, marked from 0 to 10, is furnished with the test-papers, to which the slips are referred after exposure to the atmosphere, immediately after immersion in water for the space of one minute.

These test-papers are made in the following manner:—

200 parts of distilled water,
10 „ of starch,
1 „ of iodide of potassium,

boiled together for a few seconds, and then slips of bibulous paper dipped into the solution; when dry, they are ready for use. If there be any ozone present in the air, it seizes on the potassium—the blue colour left being iodide of farina. Other test-papers have been prepared by Dr. Moffat of Hawarden, which do not require to be plunged into water in order to bring out the proper colour. Dr. Moffat's test-papers must be kept in the dark. They are placed in a box, the bottom of which is perforated with holes for the free passage of air. These test-papers, if kept in the dark, will last for several years, retaining the amount of discoloration.

The Wind Vane should be so contrived as to move with the least amount of friction; otherwise, in calm weather the changes will not be seen to take place in the direction of the wind at the proper time.

The Atmospheric Recorder, although too expensive for the ordinary meteorologist, still cannot be passed over in silence. Having had one in work for a year, I can speak of its value. Every change in the atmosphere is written down by pencils at the precise moment of occurrence. It is merely requisite to supply the cylinders with sufficient paper; after which, if the clock is occasionally wound up, and the pencils kept capable of drawing a clearly-defined line, no further care or trouble is needed. This machine is incessantly writing down the force of every gust of wind, the extent of every change in its direction, the commencement and termination of every shower, the quantity of rain which has fallen, and the amount of evaporation and electricity. The temperature of the air, the pressure and hygrometrical condition, are recorded every quarter of an hour. The discussion of these records cannot fail in being productive of much good to meteorologists. Mr. Dollond makes this machine.

The Actinometer.—An instrument for ascertaining the force of solar radiation. It consists of a large hollow cylinder of glass, soldered at one end to a thermometer tube, terminated at the other end by a ball drawn out to a point, and broken off so as to leave the end open. The cylinder is closed at the other end. It is filled with a deep blue liquid (ammonio-sulphate of copper). The cylinder is enclosed in a chamber which is blackened on three sides. This instrument requires careful manipulation, and is but little used.

Intensity of Light.—The gauge for ascertaining this consists of a long narrow box, with a window at one end facing the north. The box is painted black inside, with a white belt of a couple of inches in width, graduated up to 100. The brighter the day, the further can the figures be seen up this box.

The Transit Instrument is a useful addition. It has been well described by Mr. Breen in his treatise on astronomy. A very close approximation to correct time may be obtained by the use of Dollond's Portable Transit.

A good astronomical clock, with a mercurial pendulum, ought to be found in every observatory, and a good watch in the pocket of every observer; each should be provided with the seconds-hand. The watch in use here was supplied by Mr. Bennett of Cheapside; it has performed its work to my entire satisfaction.

Whewell's Anemometer, invented by the Rev. Dr. Whewell, is an ingenious self-registering anenometer, which gives the amount of horizontal movement in the air. A system of wheels is worked by windmill-sails, according to the velocity of the wind; and these carry a pencil, which is constantly recording the direction and velocity of the wind.

Osler's Anemometer constantly records the force and direction of the wind, and also the amount of rain. An admirable invention. Unfortunately, such instruments as this and Dr. Whewell's are too expensive for the majority of meteorological observers.

The Electrical Observatory at Kew, under the able management of Mr. Ronalds, has become a most important meteorological station, being, in fact, a collection of all requisite electrometers; a brief description of which will be found in Dr. Drew's "Practical Meteorology."

Daniel's Hygrometer.—An instrument for ascertaining from direct observation the temperature of the dew-point. This is an interesting contrivance, as a ring of dew is precipitated at the temperature of the dew-point. It consists of two glass balls, communicating with each other by means of a bent tube. The one ball is of black glass, and the other transparent. A thermometer is fixed with its bulb within the blackened ball; and as soon as three-fourths of this ball is filled with sulphuric ether, it is immersed. The air having been exhausted, the tube is hermetically sealed. The transparent ball is covered with muslin. A duplicate thermometer for ascertaining the temperature of the air is attached to the stem of the instrument. To find the temperature of the dew-point, all the ether must be made to run into the black ball; ether is then poured from a phial on the muslin; this produces rapid evaporation, and the temperature is cooled down to that of the dew-point; at this temperature, a ring of dew is formed round the black bulb, and at this instant the immersed thermometer must be read off: rapidity of observation being necessary, as the temperature will continue to fall below this point, and the ring of dew also to increase in width. In a few seconds the ring will gradually disappear, and at this moment the thermometer should be read a second time. The mean of the two readings will give the temperature of the dew-point. It is necessary that the ether should be very good.

The results obtained by actual observation and by calculation (from the wet and dry-bulb thermometers) are very nearly identical, as the following illustration, taken to-day at noon (May 17), will show:—

By Observation.

Daniel's hygrometer, temperature of air	56°·0
Daniel's hygrometer, temperature of dew-point when ring was formed	48°·1
Daniel's hygrometer, temperature of dew-point when ring disappeared	48°·2
Daniel's hygrometer, mean temperature of dew-point	48°·1
Daniel's hygrometer, temperature of dew-point below temperature of air	7°·9

By Calculation.

Temperature of dry bulb	56°·0
Temperature of wet bulb	52°·0
Calculated dew-point	48°·2
Temperature of dew-point below temperature of air	7°·8

Regnault's Hygrometer.—Another instrument for ascertaining the dew-point from direct observation. It differs considerably in its construction. Some meteorologists prefer it to Daniel's hygrometer. Messrs. Negretti and Zambra have constructed a modification of Regnault's hygrometer, and from them it can be procured at the same price as that usually charged for Daniel's hygrometer.

Connell's Hygrometer is a third instrument for ascertaining the dew-point from observation. In this the temperature is lowered to the dew-point by means of an exhausting syringe. The wet and dry bulb thermometer, however, answers every purpose, and to the ordinary observer is not so liable to be incorrectly read off.

The Cyanometer consists of a flat ring, divided into 53 equal parts, and numbered from 0 to 52, the 0 being white, and 52 very dark blue; the other numbers are painted every intermediate tinge from nearly white to deep blue. By its use the colour of the sky can be ascertained by observation.

General Directions.—The following occasional phenomena require a few words. The most uninstructed observer may give useful information on these subjects with comparatively little trouble to himself, by making known the particular features desirable to observe. It is desirable to note in

Thunder-storms, the direction in which they move, the point of the horizon in which they were first noticed, and that in which they disappeared; the time when thunder was first and last heard; the colour of the lightning; the number of seconds elapsing after a flash before thunder became audible (noted at different periods during the storm's continuance); the commencement and termination of rain; the direction of the wind before, after, and during its continuance; the time when the electrical breeze springs up (this is a peculiar violent breeze noticed in most thunder-storms); whether hail falls; and any other feature that may appear remarkable or be deemed desirable to be recorded.

Aurora Borealis.—Its position amongst the stars, whether merely a low auroral arch, or accompanied by coruscations. If a brilliant display, whether a cupola or dome is formed a little south of the zenith; and if formed, whether oscillatory among the stars. The hour of its occurrence when seen as a diffused light; if there be floating patches of luminous haze or cloud, &c.

Solar and Lunar Halos.—When visible, the quarter of the heavens where they appeared, and how long they remained visible.

Mock-suns and Complicated Circles of Light.—Their form and position with respect to the sun or moon; whether prismatic.

Meteors, or Falling-stars.—Their apparent size, shape, colour, path amongst the stars, velocity and duration; whether accompanied by a streak of light, or separate fragments; if large, whether a streak of light remained after the meteor itself had disappeared; and after bursting, whether any noise of explosion be heard—if so, how many seconds after the meteor itself had burst; the time of appearance, &c. These observations should more especially be attended to from the 6th to the 16th of August, and from the 9th to the 14th of November.

Gales of Wind.—Their direction, and estimated force; when they commenced and terminated; the height of the barometer during its continuance.

Snow.—Note when it fell, how deep in inches on the ground, and the form of the snow-crystals. To sketch the crystals, a magnifying-glass is requisite.

Hail.—The shape of the stones, &c.

The times of breaking up of long dry periods, and frosts; the termination of rainy periods, the commencement and duration of fogs, wind changes, &c.

Solar Eclipses.—During their continuance, and before and after, record the temperature in sun and shade repeatedly. Expose for ten seconds every five minutes slips of Mr. Talbot's sensitive-paper, to ascertain the effect of the diminution of sunlight on this paper.

Requisite Tables of Reduction.—1st. Glaisher's Hygrometrical Tables (2nd edition). These splendid tables enable the observer, with comparatively little labour, to calculate the temperature of the dew-point,—the elastic force of vapour, the weight of vapour in a cubic foot of air, the additional weight of vapour required to saturate a cubic foot of air, the degree of humidity; the whole amount of water in a vertical column of the atmosphere; the weight of a cubic foot of air, and also to separate the pressure due to vapour from that due to the gases, for any temperature from 10° to 100° F. On page 5 there is also a table for reducing the readings of the barometer to the level of the sea. Mr. Glaisher has calculated this table from the fact determined by M. Regnault, that air expands $\frac{1}{491 \cdot 13}$th part for every increase of 1° of heat.

2nd. Table of the Corrections for Temperature to Reduce Observations to 32° F. for Barometers with Brass Scales, by J. Glaisher, Esq. It is absolutely requisite to reduce the readings of the barometer to a certain acknowledged temperature, otherwise the true pressure of the air could not be ascertained; for it must be remembered that, besides the pressure of the air on the mercury, the mercury itself obeys the same law which is pointed out to us by the thermometer, *i.e.* expands by heat and contracts by cold. Thus, suppose the actual pressure of the air to be stationary, the barometer will be seen to rise or fall, if there be an increase or decrease in the temperature of the air. In like manner, the metal scale of the barometer is subject to expansion and contraction by an increase of heat or cold; and this, as Mr. Glaisher says, explains the apparent anomaly that, although the readings are said to be reduced to 32° (or the freezing-point), the point of no correction is 28½°. As metal bars will vary in length with every degree of temperature, it is apparent that a certain temperature should be determined upon at which the standard unit of measure must be referred to.

Now this temperature has been fixed at 62° F.; therefore above 62°, as the metal will expand, this will make the divisions on the scale of the barometer too large, and consequently the barometer will read lower than it should do; on the contrary, below 62°, the metal contracting, will bring the index divisions closer together, and the barometer will read too high, unless corrected. Thus, owing to this cause alone, at the temperature of 32° the barometer is made to read ·009 of an inch too high.

The great use of the reduction for temperature will be at once apparent when an example is given: thus, suppose a person has two barometers, one in a room heated artificially and the other as cold as possible:—

Reading of barometer at temperature of 90° .	= 30·200
Correction for temperature	— 0·165
Reading corrected for temperature . . .	30·035 inches.

Reading of barometer at temperature of 40° .	= 30·066
Correction for temperature	— 031
Reading corrected for temperature . . .	30·035 inches.

The difference ·134 inch between the two readings being due to the expansive action of heat.

These tables have been calculated from Schumacher's formula, which is here copied:—

$$-s \times \frac{m\,(t - 32^\circ) - s\,(t - 62^\circ)}{1 + m\,(t - 32^\circ)}$$

s = reading of barometer.

m = the expansion in volume of mercury for 1° F. = 0·0001001.

t = the temperature of the mercury and the scale.

s = the expansion of the brass scale in length for 1° F. = 0·000010434 (the normal temperature being 62°).

Thus the formula becomes—

$$-s \times \frac{0{\cdot}0001001 \times (t - 32) - 0{\cdot}000010434 \times (t - 62)}{1 + 0{\cdot}0001001 \times (t - 32)}$$

3rd. Table of Corrections to be applied to Meteorological Observations for Diurnal Range, prepared by the Council of the British Meteorological Society. These tables are of the utmost importance, as they enable an observer, from one, two, or three readings daily, to find from them the true monthly means; in fact, to make his observations represent a reading taken every hour day and night. Thus, if a reading of the barometer is made daily at 3 A.M. in March, the mean will be ·023 too low, or, at 11 A.M., ·015 too high. The necessity of this reduction becomes very evident from hourly readings of the thermometer; for, suppose the readings are made in June, at 4 A.M., the mean will be 9°·3 too low, or, if at 2 P.M., 8°·6 too high.

The only correction requisite for the reduction of meteorological observations not found in the three above-mentioned tables is that for capillarity—the capillary action of the tube of a barometer depressing the mercury by a quantity inversely proportional to the diameter of the tube. The following table will be found sufficient for this reduction: it is copied from the work published by the Committee of Physics and Meteorology of the Royal Society:—

Correction to be added to barometer-readings for capillary action.

Diameter of tube. Inch.	Correction for unboiled tubes. Inch.	Correction for boiled tubes. Inch.
0·60	+ 0·004	+ 0·002
0·50	+ 0·007	0·003
0·45	0·010	0·005
0·40	0·014	0·007
0·35	0·020	0·010
0·30	0·028	0·014
0·25	0·040	0·020
0·20	0·060	0·029
0·15	0·088	0·044
0·10	0·142	0·070

The following reductions for meteorological observations will supply examples of every reduction necessary:—

Barometer Reductions.—To find the mean pressure of the barometer for the month of February, 1856 (height above the sea-level, 281 feet).

Sum of all the readings made at 9 A.M., 867640; at 10 P.M., 867528.

Divide the above by the number of observations.

29)867640(29·918 inches.

58

—

287

261

——

· 266

261

——

· · 54

29

——

250

252

——

— · · 2

29)867528(29·915 inches.

58

—

287

261

——

· 265

261

——

· · 42

29

——

138

145

——

— · · 7

Sum of all the readings of the attached thermometer at 9 A.M., 13435; at 10 P.M., 13745.

29)13435(46°·3 temp. of mercury.

116

——

· 183

174

——

· · 95

87

—

29)13745(47°·4 temp. of mercury.

116

——

· 214

203

——

· 115

116

——

— · · 1

Mean pressure at 9 A.M.	= 29·918
Correction for temperature of 46°·3	= — 047
Mean pressure corrected for temperature	= 29·871
Index error	— ·002
Mean pressure further corrected for index error	29·869
Corrections for capillarity, the mercury being boiled	+ 002
Correct reading for 9 A.M.	= 29·871
Correction for diurnal range for February	— 008
Approximate mean pressure	= 29·863
Mean pressure at 10 P.M.	= 29·915
Correction for temperature of 47·4	= — 050
Mean pressure corrected for temperature	29·865
Index error	— 002
Mean pressure further corrected for index error	= 29·863
Correction for capillarity, the mercury being boiled	+ 002
Correct reading for 10 P.M.	29·865
Correction for diurnal range for February	— 007
Approximate mean pressure	29·858
Ditto ditto	29·863
Sum of the two observations	2)59·721
Adopted mean pressure for the month	= 29·8605

To reduce the mean pressure of the month to the sea-level, the adopted mean temperature of the air being 36°·0, and the cistern of the barometer 281 feet, the adopted mean pressure 29·860 inches.

In Table 2 of Glaisher's Hygrometrical Tables (page v.), showing the volume of a mass of dry air after expansion from heat for each degree of Fahrenheit's scale, it will be seen that a stratum of air 90 feet in thickness will balance a column of mercury 0·1 inch in height.

The factor for 36° is	1·008
Multiply this by 90 feet	90
	90·720

90·720 = 90·7)281·0(·3098 of an inch.
feet.
2721
··8900
8163
·7370
7256
·114

Adopted mean pressure for altitude of 281 feet	**29·860**
Correction to reduce to sea-level	**+ 310**
At sea-level the mean pressure is	**30·170 inches.**

$$\text{As } \overset{\text{in.}}{90{\cdot}7} : \overset{\text{feet.}}{0{\cdot}1} :: 281 : = \frac{281 \times 0{\cdot}1}{90{\cdot}7} = 0{\cdot}310$$

Another method, based upon the theorem of Sir George Shuckburgh and the calculations of Regnault, has been described by Dr. Drew, of Southampton, who has constructed a table showing the height, in feet, of a column of air equivalent in weight to a column of mercury one inch in height, at different temperatures, under a pressure of thirty inches of mercury. This table is copied from Dr. Drew's "Practical Meteorology."

Temp.	Feet,	Temp,	Feet,	Temp.	Feet,	Temp,	Feet,	Temp.	Feet,
30	865·1	40	882·8	50	900·5	60	918·2	70	935·8
31	866·8	41	884·5	51	902·2	61	919·9	71	937·5
32	868·5	42	886·2	52	903·9	62	921·6	72	939·3
33	870·3	43	888·0	53	905·7	63	923·4	73	941·1
34	872·1	44	889·8	54	907·5	64	925·2	74	942·9
35	873·9	45	891·6	55	909·3	65	927·0	75	944·7
36	875·7	46	893·4	56	911·1	66	928·8	76	946·5
37	877·5	47	895·2	57	912·2	67	930·6	77	948·3
38	879·3	48	897·0	58	914·7	68	932·4	78	950·1
39	881·1	49	898·8	59	916·5	69	934·1	79	951·8

For any temperature above that given in the table, the addition of 1·7 feet for every degree; and for any temperature below that given in the table, the subtraction of 1·7 feet for every degree, will give the factor required.

To work out the addition required to reduce to sea level, let T represent the tabular number opposite the temperature of the air, z the reading of the barometer at f feet above the sea-level, and x the correction required; then

$$x = \frac{f}{T} \times \frac{z}{30}$$

Dr. Drew gives the following practical example:—The barometer being 29·500 inches, the temperature being 50°, and the height above sea-level 60 feet?

$$x = \frac{60}{900{\cdot}5} \times \frac{29{\cdot}5}{30} = 0{\cdot}065 \text{ correction required.}$$

$\therefore$ 29·500 + 0·065 = 29·565 inches (the pressure reduced to the sea-level).

Thermometer Reductions.—To find the mean temperature for the month of December, 1855, from observations made at 9 A.M. and 10 P.M., and from self-registering thermometers.

Sum of all the readings made at 9 A.M. 10915; and at 10 P.M. 11017.

31)10915(35°·2 mean at 9 A.M.
93 + 0°·9 cor. for diurnal range.
161 36°·1 approx. mean for month.
155
65
62
3

31)11017(35°·5 mean at 10 P.M.
93 + 0°·5 correction for diurnal range.
171 36°·0 approx. mean for month.
155
167
155
12

Sum of all the readings of a self-registering minimum thermometer, 9365; and maximum thermometer, 12759.

31)9365(30°·2 mean minimum temp.
93 + 0°·2 index error.
65 30°·4 corrected for index error.
62

31)12759(41°·2 mean maximum temp.
124 — 0°·2 index error.
35 41°·0 corrected for index error.
31
49
62
— 13

30·4 mean minimum corrected.
41·0 mean maximum corrected.

2)71·4 sum of the two series.

35°·7 mean of the two series.
0·0 correction of diurnal range.

35·7 approximate mean from self-registering instruments.
36·1 approximate mean from hourly observations at 9 A.M.
36·0 approximate mean from hourly observations at 10 P.M.

3)107·8 sum of the three series.

35°·9 adopted mean temperature for December, 1855.

To find the mean temperature of the wet bulb thermometer, for the month of December, 1855, from daily observations made at 9 A.M., and 10 P.M.

Sum of all the readings at 9 A.M., = 10509; at 10 P.M., = 10658.

31)10509(33°·9 mean at 9 A.M.
93 + 0·6 cor. for diurnal range.
·120 34·5 approximate mean.
93
·279
279
···

31)10658(34°·4 mean at 10 P.M.
93 + 0·2 cor. for diurnal range.
·135 34·6 approximate mean.
124
·118
124
—..6

Approximate mean from observations made at 9 A.M. = 34 ·5
" " " " " at 10 P.M. = 34 ·6

Sum of the series . . . . 2)69 ·1
Adopted mean temperature of the wet bulb . . = 34°·5

To find the mean temperature of evaporation.

Adopted mean temperature of the air from dry bulb . . = 36·0
Adopted mean temperature of the wet bulb . . . = 34·5

Difference . . . . = 01·5
Adopted mean temperature of air . . . . . . = 35·9

Difference, or mean temperature of evaporation . . 34·4

To find the mean temperature of the dew-point, the mean dry bulb* being 35°·8, and the mean wet bulb 34°·5.

Example 1.—In the hygrometrical table for 35°,

The dew-point opposite to 34° wet bulb is . . 32·4 . . 32·4
The dew-point opposite to 35° wet bulb is . . 35·0

Difference on the increase in dew-point for an increase of 1° in wet . . . . . . 2·6
Proportional part of the increase for 0°·5 is . . . . . . + 1·3

Temperature of the dew-point corresponding to 35° dry, and 34°·5 wet, is 33·7
In the fourth column the decrease of dew-point for an increase of 1° in the dry bulb is — 1°·5, the proportional part for 0·8° is . — 1·2

The adopted temperature of the dew-point for 35°·8 dry, and 34°·5 wet, is = 32·5

Example 2.—To find the mean temperature of the dew-point for the month of December, 1865, dry bulb being 36°·0, and wet bulb 34°·5.

In the hygrometrical table for 36°,

The dew-point opposite to 34° wet bulb is . . 31·0 . . 31·0
The dew-point opposite to 35° wet bulb is . . 33·5

Difference . . . . 2·5
Proportional part of the increase for 0°·5 is . . . . . + 1·2

Mean temperature of dew-point . . . . = 32·2

The same tables, and the same manner of reduction, will find "the elastic force of vapour," "the weight of vapour in a cubic foot of air," "the additional weight required to saturate a cubic foot of air," and the "degree of humidity." To render the tables clear, one example of each will be worked out for a temperature of 35°·8 dry bulb, and 34°·5 wet bulb.

---

* The hourly readings of the thermometer, which are made at the same time as those of the wet bulb, are called the 'dry bulb.'

To find the elastic force of vapour of the temperature of 35°·8, the wet bulb being 34°·5.

In the hygrometrical table for 35°,

	Inch	Inch
The elastic force of vapour opposite to 34° wet bulb is	0·184	0·184
The elastic force of vapour opposite to 35° wet bulb is	0·204	
Difference, or the increase for an increase of 1° in wet	0·020	
Proportional part of the increase for 0°·5 is		+ 0·010
Elastic force of vapour corresponding to 35° dry, and 34°·5 wet is		0·190
In the 6th column, the decrease of the elastic force of vapour for an increase of 1° in the dry bulb, is — 0·010; the proportional part for 0°·8 is		— 0·008
Mean elastic force of vapour		= 0·182

To find the whole amount of water in a vertical column of the atmosphere, *i.e.* from the surface of the earth to the top of the atmosphere.

This is found by multiplying the elastic force of vapour by the constant 1383.

1383 constant.
182

2766
11064
1383

2·51706 inches.

Thus the whole amount of water in a vertical column of the atmosphere, if precipitated on the earth at one time, would be in this instance 2·5 or 2½ inches.

To find the weight of vapour in a cubic foot of air.

In the hygrometrical table for 35°,

	Grains.	Grains.
The weight of vapour in a cubic foot of air opposite 34° wet bulb is	2·1	2·1
The weight of vapour in a cubic foot of air opposite 35° wet bulb is	2·4	
Difference, or the increase in weight of vapour for an increase of 1° in wet is	0·3	
Proportional part of the increase for 0°·5		+ 0·1
Weight of vapour in a cubic foot of air corresponding to 35° dry and 34°·5 wet is		2·2
In the 8th column, the decrease of weight of vapour for an increase of 1° in the dry bulb is 0·2, the proportional part for 0°·8 is		— 0·2
The adopted weight of vapour for 35°·8 dry and 34°·5 wet is		2·0

To find the additional weight of vapour required to saturate a cubic foot of air.

In the hygrometrical table for 35°,

The dry bulb being 35°·8 and the wet bulb 34°·5.

	Grain.	Grain.
Additional weight of vapour required to saturate a cubic foot of air opposite 34° wet bulb is	0·3	0·3
Additional weight of vapour required to saturate a cubic foot of air opposite 35° wet bulb is	0·0	
Difference, or the decrease in amount required to saturate a cubic foot of air for an increase in 1° in wet bulb is	0·3	
Proportional part of the decrease for 0°·5		— 0·2
Additional weight of vapour required to saturate a cubic foot of air corresponding to 35° dry and 34°·5 wet is		= 0·1
In the 10th column, the increase in the weight of vapour required to saturate a cubic foot of air for an increase in 1° dry is + 0·3, the proportional part for 0°·8 is		+ 0·2
The adopted additional weight of vapour required to saturate a cubic foot of air for 35°·8 dry and 34°·5 wet is		= 0·3

To find the degree of humidity, the temperature being 35°·8 and the wet bulb 34°·5, complete saturation = 100.

In the hygrometrical table for 35°.

Degree of humidity opposite 34° wet bulb is	90	90
Degree of humidity opposite 35° wet bulb is	100	
Difference, or the increase of humidity for an increase of 1° in wet is	10	
Proportional part of the increase for 0°·5 is		+ 5
Degree of humidity corresponding to 35° dry and 34°·5 wet is		95
In the 12th column, the increase in the degree of humidity for an increase of 1° in the dry bulb is 9, the proportional part for 0°·8 is		— 7
Adopted degree of humidity for 35°·8 dry and 34°·5 wet is		= 88

To find the weight of a cubic foot of air, the mean pressure of the barometer being 29·742 inches, dry bulb 35°·8, and wet bulb 34°·5.

In the hygrometrical table for 35°.

	Grains.
In column 13, the weight of a cubic foot of air (the barometer being 29·0 inches) opposite 34° wet is	543·4
In column 14, the decrease of weight for an increase of 1° in dry is — 1°·1, the proportional part for 0°·8 is	— 0·9
	542·5
In column 14, the increase of weight for an increase in the reading of the barometer for one inch is + 18°·7 grains.	
Opposite 7 in the table, under 18°·7 in last column, is	13·1
Opposite ·04 in the table, in right-hand corner of the page, is	0·7
The adopted weight of a cubic foot of air is	556·3

To find the mean pressure of dry air, or that due to the gases when separated from the water contained in the air.

Subtract the adopted elastic force of vapour (which is the pressure of the water contained in the air) from the adopted pressure of the barometer.

Example.—If the mean pressure of barometer is . . . . = 29·742 inches,
And the elastic force of vapour is . . . . = 0·186 inch,

The pressure of dry air (of that due to the gases) will be . = 29·556 inches.

To find the monthly range of temperature, &c., deduct the coldest temperature from the hottest, observed during the month. Thus—

Hottest temperature . . . . . . 50°·8
Coldest temperature . . . . . . 12°·0

Monthly range of temperature . . . 38°·8

To find the mean daily range of temperature, deduct the mean of all the readings of the minimum thermometer from the mean of all the readings of the maximum thermometer.

Mean maximum temperature . . . 41°·0
Mean minimum temperature . . . 30°·4

Mean daily range of temperature . . . = 10°·6

To find the amount of terrestrial radiation, deduct the reading of a minimum thermometer placed on the grass from that of a minimum thermometer placed four feet above the grass. Thus—

Greatest cold four feet above the grass . . 36°·5
Greatest cold on the grass . . . . . 27°·8

Amount of terrestrial radiation . . . = 8°·7

To find the amount of solar radiation, deduct the reading of a maximum thermometer (with a blackened bulb) placed in full sunshine from the greatest heat in shade. Thus—

Greatest heat in sunshine . . . . . = 59°·3
Greatest heat in shade . . . . . . = 34°·1

Amount of solar radiation . . . . 25°·2

To find the greatest heat and greatest cold of the wet bulb thermometer. This is obtained by attaching the muslin and cotton conductor to *self-registering* thermometers, instead of to the ordinary thermometer. Thus—

Greatest cold of dry bulb .	34°·0	Greatest heat of dry bulb .	46°·7
Greatest cold of wet bulb .	32°·5	Greatest heat of wet bulb .	42°·8
Difference .	°·5	Difference	3°·9

To find the range of temperature of the wet bulb thermometer, deduct the greatest heat from the greatest cold recorded. Thus—

Greatest heat	42°·8
Greatest cold	32°·5
Range of wet	10°·3

To find the mean amount of cloud.

This, by practice, is accomplished very accurately by estimation, 10 being considered an overcast sky and 0 a cloudless sky. It must be remembered, in making the estimate, that, with a partially covered sky, the forms of the clouds, and the space they cover, are only correctly seen in the zenith; the nearer the horizon we approach, the more obliquely do we look upon them; and, consequently, near the horizon the sky will appear to be more overcast than it is in reality. The observer's judgment should, therefore, be more especially confined to the upper half of the sky.

Observations made at 9 A.M. and 10 P.M., December, 1855.

Sum of all the estimates at 9 A.M., 2158; and at 10 P.M., 2024.

31)2158(6°·96 mean at 9 A.M.	31)2024(6°·5 mean at 10 P.M.
186 — 0°·1 cor. for diurnal range.	186 + 0°·2 correction for diurnal range.
298 6°·86 corrected amount.	164 6°·7 corrected amount.
279	155
190	..9
186	
..4	

corrected reading at 9 A.M.	6°·9
corrected reading at 10 P.M.	6°·7
sum of the two series	2)13°·6
adopted mean amount of cloud . .	6°·8

Classes of Clouds.—Mr. Luke Howard, the well-known meteorologist, was the first to classify the clouds; and these classes have been very conveniently abbreviated by Mr. Glaisher. They are—

Cirrus = ci . . . feathery-looking, the most lofty cloud.
Cumulus = cu . . . mountainous-looking.
Stratus = st . . . the ground-cloud—forms at sunset and disappears at sunrise.
Cirro-cumulus = ci-cu . . rounded masses, or woolly tufts.
Cirro-stratus = ci-st . . horizontal masses.
Cumulo-stratus = cu-st . . an accumulation of cumuli, sometimes fungus-shaped.
Nimbus = ni . . . rain-cloud.
Scud = sc . . . broken, flying nimbi.

In recording the class of clouds, the interest is increased by also recording the direction in which the clouds are moving, the colour of the clouds, and their height and velocity. The height may be conveniently estimated by supposing 6 to represent very high clouds, and 0 those floating along the ground; he velocity, by supposing 6 to represent those moving at the greatest speed, and 0 those motionless.

To find the mean daily amount of rain. Whole amount of rain collected during the month of December, 1855, 0·764 of an inch.

31)0·764(0·025 of an inch being the mean daily amount.
62
144
155
— 11

To find the mean daily amount of evaporation for December, 1855. Whole amount evaporated, 0·610 inch.

31)0·610(0·0197
31
300
279
210
217
— · · 7 Therefore, 0·020 inch is the daily amount.

To find the amount of evaporation in rainy weather.

Suppose 1·000 inch of water is placed in the evaporator, and that a rain-gauge, of the same diameter as the evaporator, and placed at the same level, has recorded 0·510 inch of rain since last observation, whilst the evaporator is found to contain 1·444 inches. Then

1·444 inches the amount in evaporator.
0·510 amount of rain.

0·934 difference.
1·000 amount of water placed in the evaporator.

0·066 of an inch is the amount due to evaporation.

To estimate the force of wind.

Anemometers being expensive, the majority of meteorologists estimate the wind's force. It is recommended that 0 represent a calm and 6 a hurricane, for this estimate is easily converted into lb. pressure on the square foot; the square of the estimate will represent the lb. pressure on the square foot; viz.—

0 = 0 lb.
1 = 1
2 = 4
3 = 9
4 = 16
5 = 25
6 = 36

By practice a near approximation to the truth can be obtained.

Mr. Belville, in his Manual of the Barometer, gives the following concise table of factors for deducing the temperature of the dew-point, from the temperature of the air and that of the temperature of evaporation; this table originally appeared in the Greenwich Magnetical and Meteorological Observations for 1844. The dew-point deduced in this manner, is a close approximation to that obtained by Glaisher's hygrometrical tables.

Temperature.	Factor.	Temperature.	Factor.	Temperature.	Factor.
28° to 29°	5·7	34° to 35°	2·6	55° to 60°	1·9
29° „ 30°	5·0	35° „ 40°	2·4	60° „ 70°	1·8
30° „ 31°	4·6	40° „ 45°	2·3	70° „ 80°	1·7
31° „ 32°	3·6	45° „ 50°	2·2	80° „ 85°	1·6
32° „ 33°	3·1	50° „ 55°	2·1	85° „ 90°	1·8
33° „ 34°	2·8				

Multiply the difference between the wet and dry bulb by the factor corresponding to the temperature of the dry bulb, and subtract the product from the dry bulb, which will be the temperature of the dew-point. T the temperature, W wet bulb, f factor, x the product, and D the dew-point.

$$T - W \times f = x$$
$$\text{and } T - x = D.$$

Thus:—temperature 50 and wet bulb 45°,

$$50 - 45 = 5$$
$$5 \times 2{\cdot}1 = 10{\cdot}5$$
$$50 - 10{\cdot}5 = 39^{\circ}{\cdot}5 \text{ the dew-point.}$$

By Mr. Glaisher's tables this dew-point is 39°·7.

Recommendations and Precautions.—Preserve, as much as possible, the continuity of the observations. It is desirable not to change the positions of the different instruments, nor even to alter the method of reading and registering. As it is probable when two persons are employed in taking observations, that each will read slightly different, a series of simultaneous readings should be made, in order to find the personal error of the observers. If, from any cause, the continuity of the register should be broken, on no account attempt to fill the blank so caused by estimation. Be punctual to the hours determined upon for observations, and read off the instruments in the same way every day; by doing this, it is less likely that an observation of any one instrument shall be overlooked. It is convenient to rule and mark off the form of observation upon a slate, to be transferred to the observatory book, after the whole observations have been made. Before calculating the means, it is recommended to examine each column, to ascertain that no evident error of entry has been made;—an inch in the reading of the barometer is a common error. The maximum reading of the thermometer is also sometimes entered in the wrong column, being placed in that of minimum; and *vice versa.* Decimal arithmetic should always be used.

Before fixing the barometer, it should be ascertained that the space above the mercury is free from air. Incline the instrument slightly from its vertical position; if the mercury, in striking the upper end of the tube, produce a sharp rap, the vacuum is perfect; if the tap be dull, or not heard, there will be air above the mercury, which must be driven into the cistern by inverting the instrument and then tapping it gently. If the observer does not succeed in producing this sharp report by tapping, the instrument will require the aid of the maker. In fixing the barometer, adjust the tube vertically by the aid of a plumb-line. In reading the instrument, place the eye on the exact level of the top of the column of mercury, so that each side of the index, and the top of the column, shall be in the same horizontal plane.

The thermometers should be protected from rain; and in making a reading the observer should do it quickly, and whilst doing so avoid touching, breathing on, or in any way warming the thermometer by the near approach of his person.

Sir John Herschel recommends that every meteorologist should take an observation every hour throughout the twenty-four, on four stated days in the year; viz. March 21, June 21, September 21, and December 21, excepting when one of these days occurred on Sunday, then to substitute for this date the 22nd. The observations to commence at 6 A.M., and terminate at 6 A.M. next morning.

Thermometers should be frequently compared with a standard instrument, in order to ascertain whether the freezing-point has remained at the temperature as marked off on the scale. It is a well-known fact that the zero point moves, ascertained from the circumstance that after a great change in temperature, the glass requires a considerable time to enable it to return to its normal condition.

Newman's Standard Barometer form of Rutherford's thermometer differs in the following manner from that already described:—A platinum tube is drawn over a steel wire, which is said to prevent the index from fixing by oxidation. Not having had this instrument at work, I am unable to speak of its advantages from practice. An improvement has also just been accomplished with Negretti and Zambra's rain-gauge. I conceived it was liable to have the water in the canal surrounding it emptied into the graduated glass with the rain to be measured. At my suggestion this has been altered; the canal is now placed much lower down the gauge, and there seems no possibility of an erroneous measurement. The gauge may, therefore, be now said to be perfect.

Of late years two new barometers have appeared,—viz. the *Aneroid* and *Burdon's*. They are both good indicators of atmospheric pressure, but cannot take the place of a Standard barometer. In the first place, the metal is influenced by temperature; and in the second, there is a chance of the box of the *Aneroid* and the tube of the *Burdon* losing their vacuum. As a household instrument they are preferred to the common wheel-barometer; but I have not had an opportunity of testing their respective merits.

At the time of the Great Exhibition of 1851, when the jury were examining the meteorological instruments, a remark was made that a more perfect maximum and minimum thermometer was required, both of which should be *mercurial;* and on the counsel-medal being presented to Messrs. Negretti and Zambra, their attention was called to this suggestion. Not many months had elapsed before the *patent maximum* was produced, and within four years the *patent minimum*. The latter was sent for trial to a few meteorologists six months ago, and one of these instruments came to the Beeston Observatory. The experiments made with it have been perfectly successful; indeed, so much so, as to have astonished all who have used it. The ordinary minimum thermometers do not work uniformly with this new instrument, owing to the alcoholic vapour contained in the upper portion of their tube, and which is more or less developed according to the temperature. Two such important improvements have not taken place since the invention of the thermometer; and it is creditable to them that both should have emanated from the same persons who invented the enamelling of fine tubes.

Professor C. P. Smyth, the Astronomer Royal of Scotland, has caused the electric telegraph to work in meteorology. A wind dial at the one extremity of a wire is made to turn another simultaneously at the other extremity. The time will come when all large towns will have buildings devoted to these observations, and in which dials will be seen in every direction, some labelled Edinburgh, others Liverpool, Dublin, London, Paris, York, &c., and where the public will be enabled to see the direction of the wind at the same instant at most remote places. The benefit to the farmer and the navigator will be great from such an arrangement. Were such stations to be thickly scattered throughout the country, every change of wind, and every shower, could be traced and recorded, and a knowledge imparted, the benefit of which could not be sufficiently appreciated.

To be enabled to announce the approach of a thunder-storm, however, at a time when the sky is free from clouds, and to ascertain its speed so as to foretel when it may be expected in any given place, would afford the farmer an opportunity of so benefiting by the information that he would gladly pay a small rate in order to take advantage of it. The world is slow in appreciating any new invention until its usefulness is experienced; let us, however, hope that one or two such stations may speedily be established, and we venture to predict that, at no very distant period, every conspicuous eminence will have its *road station*, to impart the information collected at the principal establishments, where the electric wires are made to record the changes as they occur. Our knowledge of meteorology would then make rapid advances; laws of the weather would be unfolded; and predictions of coming changes, which are now mere guesses, as often wrong as right, would be based upon truth.

It is a common expression that nothing is more changeable than the weather; yet that all these changes are governed by certain laws, is as certain as that gravitation binds the heavenly bodies together. Were there no laws to keep the changes within certain limits, we should at one period experience cold, and at another heat so intense, that existence would be intolerable; the earth would be deluged with rain, and anon parched up with drought; the sky would be cloudy for months, perhaps years together, and then cloudless for as long a period; in short, the laws which govern the weather keep the extreme changes within proper bounds.

As the more complicated machinery of a meteorological observatory cannot be expected to be found except in our principal establishments, it will be requisite to mention what instruments are absolutely necessary for the ordinary observer. These are—

A standard barometer.
A wet and dry bulb thermometer.
A maximum and a minimum thermometer.
A rain-gauge.
An evaporator.
A thermometer-stand.
A wind-vane.

These would cost from £17 to £30, according to which barometer was selected. The following additional instruments are also desirable (the expense would be £4 or £5):—The solar and terrestrial radiation thermometers, Glaisher's electrometer, an ozonometer, and an extra rain-gauge.

Snow Gauge.—The gauge used here consists of a thin metal cylinder, eight inches in diameter and twelve inches deep, graduated upon one side to a quarter of an inch. This cylinder will penetrate through the snow, scarcely disturbing it, and the depth in inches is at once seen. By careful manipulation, if the cylinder is turned round, all the enclosed snow can be lifted from the ground. It is desirable to melt it in a wide-mouthed bath, being previously corked to prevent evaporation, as it frequently happens that snow is blown out of the mouth of the rain-gauge before it has had time to melt; consequently, the result of melted snow, as shown by the rain-gauge, will be too little in amount.

Calendar of Nature.—Every meteorologist should endeavour, as much as possible, to record the arrival and departure of migratory birds, the dates of trees coming into and losing their leaves, the blooming of plants, the ripening of fruit and seeds, the building of birds'-nests, the first appearance of various insects, diseases amongst animals and plants, the appearance of abundance or otherwise of crops of fruit, corn, &c. If such registers were extensively kept and carefully recorded, the effect of the weather upon the animal and vegetable kingdoms would be well seen. It is extremely desirable that every precaution should be taken, in order that, year after year, the same object should be the special one on which the remarks are based, and that one species is not mistaken for another. The following examples will show that it is essential to use the utmost care:—

First, "the elm is said to lose its leaves on a certain date." Such an observation is useless. It is requisite to mention the particular kind of elm; thus, the broad-leaf elm is the first tree to become leafless, which it frequently does in September; the Siberian elm, on the contrary, will remain green after all other trees have become leafless,—sometimes it is in leaf as late as December. It will also be found that the same species, in a group whose boughs touch each other, will come into leaf at different dates. In no tree is this more strikingly exhibited than in the beech; two beeches growing close together may be seen to vary a couple of weeks in their period of coming into leaf. The age of the tree also causes a difference to occur. In the different kinds of lilacs and laburnums, there will be a range of some days in their time of coming into bloom. Amongst herbaceous plants, none that have been transplanted should be the objects of record.

In migratory birds, the swallow, sand-martin, and white-martin will appear, at different dates, in places so near together, that a flight of one minute would enable them to reach it. The swallows are seen near the Trent some days previous to coming here, although only a mile distant. The cuckoo and the landrail are invariably heard earlier in the season three or four miles west of this place.

Amongst the observations on the ripening of fruits, the strawberry will serve as an example: the variety called black prince will be ripe before Kean's seedling, Kean's seedling before British Queen, and British Queen before the Elton pine.

In entering the flowering of plants, it is advisable to give the dates when they first come into bloom, as well as the dates when in full glory of flower, and when the blooming is over.

The most conspicuous objects should be entered in a book, space being left which may occupy the remarks for several years. Every observer who, for a course of years, pursues such a course of observation, will find that he has done some good to his kind; and if any plan could be devised for recording and bringing together such a body of observations, they would form a valuable collection of facts for the naturalist and meteorologist to generalize upon.

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